Analytical modelling of annular flow magnetorheological valves for advanced fluid control

by

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Examining Committee:

Chair of Examining Committee	Dr. Amirkianoosh Kiani
Research Supervisor	Dr. Brendan MacDonald
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The above committee determined that the thesis is acceptable in form and content and that a satisfactory knowledge of the field covered by the thesis was demonstrated by the candidate during an oral examination. A signed copy of the Certificate of Approval is available from the School of Graduate and Postdoctoral Studies.

Abstract

Magnetorheological valves have the potential to disrupt the fluid control industry with their numerous advantages over mechanical valves. The current design and development process of these valves largely leverages computer simulation and experimental analysis. This dependency on physical and computer based models is a process that is both costly and time consuming. In this work, an analytical model was developed to reduce the timeline of development for annular flow magnetorheological valves. The model was validated to simulation and experimental results and an application of the model was demonstrated in the design of an magnetorheological valve. This proposed model will facilitate thorough magnetorheological valve design, reduce the design process dependency on computer modelling software, and decrease development time.

Keywords: Magnetorheological fluid; magnetorheological valve; fluid valve; annular flow; analytical model

Author's Declaration

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Finally, a earnest goodbye to my best friend Suzy. Without so much as a word you were able too support and comfort throughout times of ultimate struggle. Your unwavering love, loyalty, and kindness were unparalleled. You defined a new meaning to the position of mans best friend. All those who knew you were thankful to have had the opportunity.

Statement of Contributions

The following piece was the sole responsibility of the author. As of the submission date, none of the work contained within this thesis has been submitted for publication.

Contents

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Nomenclature

Symbols

ΔP Pressure drop (Pa)

ΔP_{MR} Magnetorheological effect pressure drop (Pa)

$\Delta P_{viscous\,active}$ Active region viscous pressure drop(Pa)

$\Delta P_{viscous\,coil}$ Coil region viscous pressure drop (Pa)

$\Delta P_{viscous}$ Viscous pressure drop (Pa)

A_x Magnetic circuit cross sectional area (m^2)

a_x MR yield stress approximation coefficients (-)

B_{gap} Active region magnetic flux density (kg/ s^2 A)

c coil region fluid gap (m)

D_{coil} Actuation coil outer diameter (m)

D_{core} Bobbin core diameter (m)

D_{in} Bobbin outer diameter (m)

D_{out} Valve case diameter (m)

F Coil fill factor(-)

G Active region fluid gap width(m)

L_x Magnetic circuit length (m)

L_{act} Active region length (m)

L_{coil} Coil length (m)

L_{core} Valve length (m)

L_{return} Valve case width (m)

N Number of active turns in coil(-)

Q Valve flow rate (m/s)

W Coil wire diameter (m)

Greek Letters

β Valve on/off pressure drop ratio (-)

η MR fluid dynamic viscosity (kg/m s)

μ Magnetic permeability (N/A^2)

μ_g MR fluid magnetic permeability (N/A^2)

μ_s Valve core material magnetic permeability (N/A^2)

Φ Magnetic flux (kg m^2/ s^2 A)

τ_y MR fluid yield stress (Pa)

Chapter 1

Introduction

1.1 Motivation

Fluid systems are found in nearly every aspect of human life. Whether it be the relatively simple water piping networks in our houses, or the large and complex hydraulic power systems of heavy industrial machinery, the manipulation of fluid flow is a crucial aspect of modern engineering. These fluid systems require control mechanisms in order to facilitate proper operation. One such mechanism is the fluid valve. Quick acting and accurate control of fluid flow is a crucial aspect of fluid system design, and valves provide the ability to either restrict or direct flow within a circuit. Typically, this valving is done by mechanical means, with a multitude of valve types available all centered around a mechanical restriction within the fluid flow path. This restriction can be used to reduce flow, or direct it to another pathway. Mechanical valves can be actuated manually or with electromechanical assistance. These electromechanically actuated valves are typically heavier and larger than their manual counterparts, with actuation times being considerable in those with proportional, modulated valve control.

In recent years, a new technology has emerged which provides a method of fluid control without the use of mechanical mechanisms. This new type of valve harnesses the rheological power of a smart fluid, known as the magnetorheological fluid. The magnetorheological (MR) fluid has the unique ability of changing its rheological properties when exposed to a magnetic field [7]. This change in rheological properties is observed as a “thickening”, or increase in apparent viscosity when a magnetic field is applied to the fluid. This phenomenon is utilized in MR valves by modulating the apparent viscosity of the fluid as is passes through the active region by means of controlling the applied magnetic field [8]. Control of this magnetic field is done by increasing or decreasing the applied current to the valve actuation coil. These new valves prove to be high performing, offering quick acting and powerful control of fluid flow [5]. With their unique versatility MR valves have been extensively used in hydraulic dampers, and can be found in several high end automotive suspension dampers by manufacturers such as Ferrari, Lamborghini, and Audi [9]. The widespread use of MR valves in dampers is largely due to their wide operating range of pressures and quick response time. In these scenarios, the MR valve is almost untouchable in terms of performance.

The development of these MR valves has been largely simulation and experimentally based, with little analytical models available [5]. This reliance on simulation results in lengthy development periods using financially and computationally expensive software. To further restrict the development process, these valves must then be experimentally validated to ensure the original design requirements were met [10, 4, 11, 12]. Should an analytical model be available that can accurately and completely describe MR valve function and performance, design times and costs can be drastically reduced. It is therefore the aim of this thesis to develop and validate an analytical model which mathematically represents the function of an MR valve given all the design constraints and geometric parameters taken into consideration when developing an MR valve.

1.2 Background

1.2.1 MR fluids and valves

Magnetorheological (MR) fluids are a set of smart materials which have a unique ability to have their rheological properties manipulated given an applied magnetic field [13]. This smart fluid consists of iron particles suspended within a carrier oil. These iron particles, being ferromagnetic, have been observed to align themselves along magnetic field lines as depicted in the scanning electron microscope images of a MR fluid in figure 1.1. This alignment of iron particles changes the rheological properties of the fluid and is observed as an increase in apparent viscosity.

In general, an MR fluid will see an increase in apparent viscosity with an increase in applied magnetic flux density [14]. An MR valve is one implementation in which the unique ability of MR fluids is utilized to perform a mechanical function such as regulating the flow of a fluid. The rheological effect of an MR fluid is proportional to an applied magnetic field and quick acting [7]. This makes the MR valve a very attractive option for fluid valving. These MR valves are quick actuating, in the order of milliseconds [2], with proportional control from the off to the on state [15]. MR valves are used in many devices,

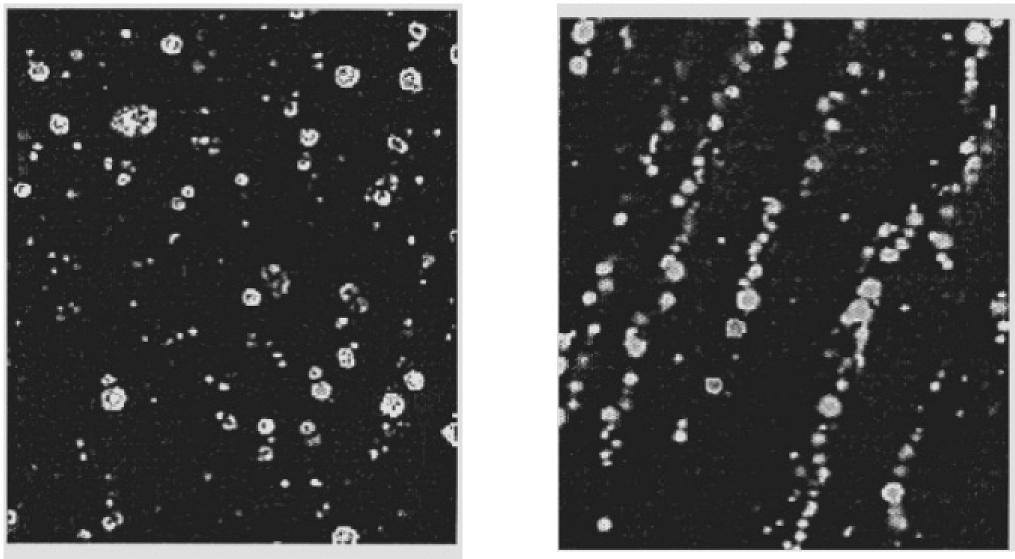

Figure 1.1: Scanning electron microscope images of an MR Fluid
Left - No applied magnetic field **Right -** Applied magnetic field [1]

namely dampers [16] and brakes [17] in place of standard, mechanical valve devices. One particular system of interest in which MR valves have recently been applied to is fluid flow control in a multi loop fluid circuit [15]. An improvement to the field of MR valving would therefore impact a wide range of applications and systems.

Generally, MR valves can be segregated into different flow configurations. Current MR valves are categorized in either annular [6] or radial [8] flow. A comparison of these two flow types is shown in the MR valve schematics presented in figure 1.2.

Advantages of the annular flow include: simpler geometry, narrow valve diameter, high efficiency, and quick actuation time [5]. The major disadvantage to the annular flow valve being that a necessity to increase valve active length results in an increase in overall valve length. This results in the annular valve being best suited to compact applications where smaller valve diameter is more advantageous than short valve length.

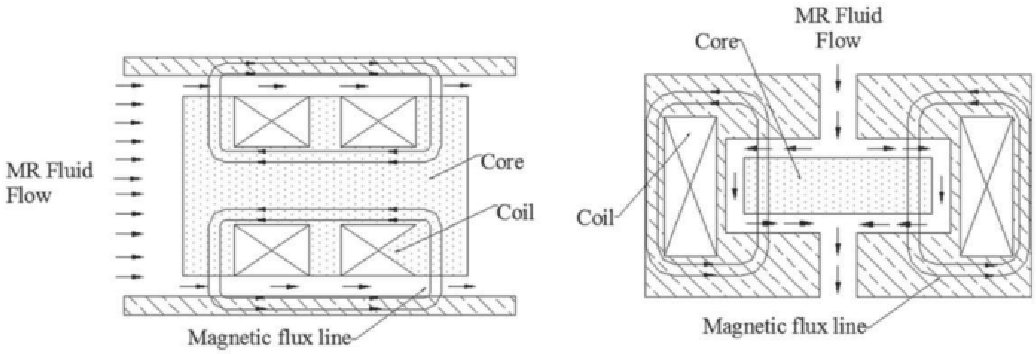

Figure 1.2: MR valve flow configurations
Left - Annular flow **Right -** Radial flow [2]

In contrast, radial MR valves offer the following advantages: larger peak pressure drop and shorter valve length [5]. The radial valve suffers from generally greater viscous losses and a larger valve diameter. Therefore the radial valve is best suited to short, thick valves where maximum pressure drop is considered more beneficial than off state pressure drop (when no magnetic field is applied). Hybrid designs which use more than one method of MR flow valving are possible [18]. The annular configuration offers benefits in terms of efficiency, manufacturing, and sizing and will be considered for this thesis [19].

1.2.2 MR valve design

Annular flow MR valves consist of four main components: a valve body, valve bobbin, actuation coil, and fluid flow pathway as depicted in figure 1.3. Every aspect of these four components impacts the valve performance and must be taken into consideration when designing an MR valve. The components form a magnetic circuit which is excited by an input current fed through the actuation coil. MR fluid flow is restricted to the flow path way and impacted by the magnetic field applied by the coil. Fluid flow is controlled by varying the applied current to affect the rheological properties of the MR fluid accordingly. With proper valve design, MR fluid pressure drop through the valve can be regulated leading to reduced flow or complete blockage [15].

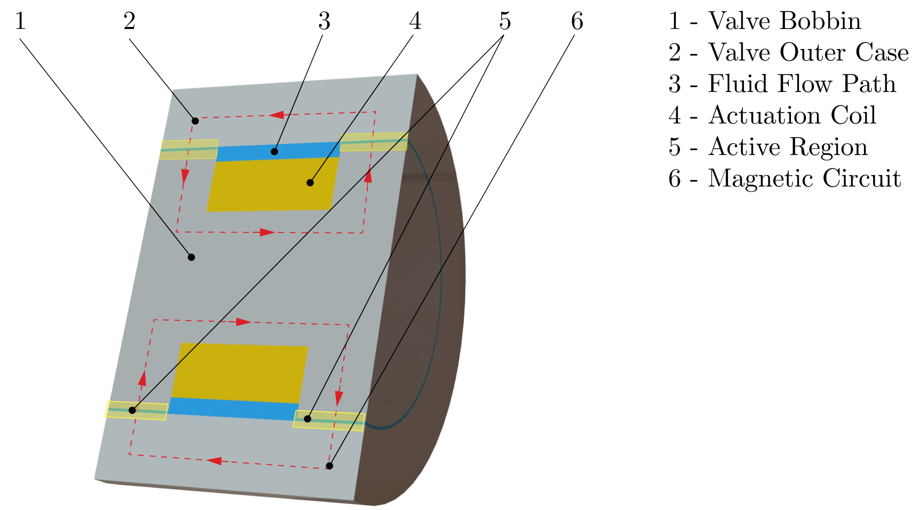

Figure 1.3: Section view of an annular flow MR valve

MR valve design has largely been aimed at providing large pressure drops across the valve in order to provide blocking power in high pressure applications [18]. Expressions for pressure drop across an MR valve exist but are limited in their accuracy and application due to a multitude of geometric dependencies. A unified model which considers all valve geometry as well as the impact valve geometry has on both the fluid flow and magnetic circuit of the valve would provide a more accurate and thorough mathematical understanding of MR valves. This understanding would aid in better and more efficient design.

Complete MR valve design is separated into two subsystems. The first system is the fluid flow path and the valve geometry associated with it. The second system is the magnetic circuit which provides the magnetic field strength necessary to operate the valve. Both the fluid flow path and the magnetic circuit are highly dependent on valve geometry. These two subsystems are therefore not independent and modifying geometric variables of the valve can have a drastic effect on both the flow path and the magnetic circuit. Therefore, it is crucial to have a complete model which considers the impact of valve geometry on both of these subsystems. Thorough and optimized design of MR valves must consider the effect of valve geometry on both the MR fluid flow path and the magnetic circuit which drives valve actuation.

The layout of the annular flow MR valve is depicted in figure 1.3. This figure is a cut section view of a 3 dimensional representation of the valve. In the annular valve, MR fluid flows around the bobbin and through the active regions of the flow path . The active region is where the magnetic flux density within the fluid is concentrated, with magnetic flux density outside of the region considered negligible [6]. This concentration of magnetic flux is due to the relative permeability of the copper coil and MR fluid being negligible in comparison to the high permeability of the valve bobbin and core as seen in table 1.1. Similar to how an electric circuit will follow the path of least resistance, the magnetic flux follows the path of high permeability. The two active regions are separated by an inactive, coil region of the flow path. The inactive coil region experiences little to no magnetic flux [20] and as such the MR effects within that region are negligible. Therefore, rheological changes within the valve are considered to only occur within the active regions. Flow path optimization requires sufficient active region length to provide the pressure drop needed for the valve application, whilst also considering the viscous pressure drop due to flow within the valve when the coil is not actuated.

Table 1.1: Relative permeability of MR valve materials

Component	**Material**	**Relative Permeability**
Valve bobbin and case	Magnetic steel alloy or cast iron	1000+
Actuation coil	Copper wire	1
MR Fluid	Various MR fluids	5

The magnetic circuit of an annular flow MR valve flows from the valve bobbin, through the active region fluid gap, through the outer case of the valve, and back to the valve bobbin. This circuit is depicted in figure 1.3. The magnetic circuit powers the active fluid region by means of an induced magnetic field. This is done by supplying current to the valve actuation coil. Coil designs vary, with single coil systems offering simplicity and higher maximum magnetic flux densities, while multi-coil systems offer great improvements in efficiency [5]. The magnetic circuit is generally designed to supply a magnetic flux density which reaches the saturation point of the MR fluid in the active region [6]. This ensures no further MR effects are feasibly attainable since an increase in coil current past the saturation point will result in negligible increase in magnetic flux density within the active region. Optimized design of the magnetic circuit ensures maximum valve performance for a given valve geometry and MR fluid characteristics.

In figure 1.4, the geometric variables of an annular flow MR valve are shown. These variables describe the complete schematic of an annular flow valve with all design aspects considered. With these parameters, an annular flow MR valve can be designed and manufactured as there is no valve geometry which has not been accounted for. It is important to note that the fluid gap at the coil c does not have to be equal to that of the active region fluid gap g, though in many applications this is advantageous [4]. The

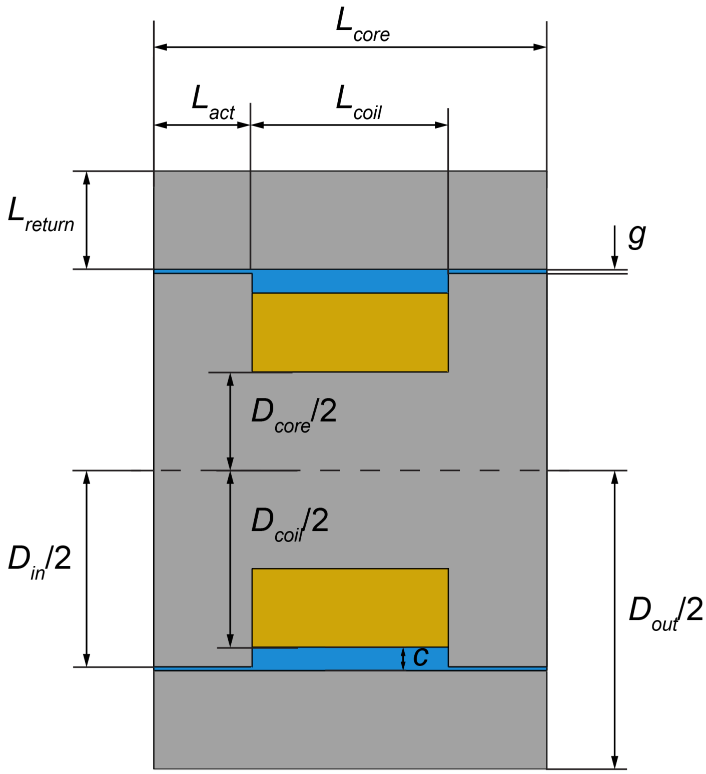

Figure 1.4: Cross section of annular MR valve

dimensions shown can impact both the fluid flow path as well as the magnetic circuit and as such a thorough understanding of the impact of these variables is required for optimized valve design.

1.3 Literature review

The following section is a review of literature relevant to the field of MR valve design. The aim of this section is to uncover the past efforts and current state of the art in terms of the development process taken when designing MR valves for a multitude of applications. Common applications found in research are brakes, dampers, and flow valves. In general, all of these applications follow a similar design approach in that simulation is conducted before an experimental apparatus is created and evaluated. Therefore, the review of literature will follow in this order: an overview of MR fluids and MR valve design, common simulation and experimental analysis approaches, and available MR valve models. Finally, gaps in the literature will be exposed with areas of interest highlighted.

1.3.1 MR fluids and MR valve design

MR fluids were first described by J. Rabinow in 1948, with their first application patented in 1951 [13]. This smart material introduced a new method of fluid control by changing the rheological parameters of the fluid with an applied magnetic field, however interest and research in MR fluids remained minimal until the early 1990's . Although not applied directly to MR fluids, R. Phillips proposed the idea of using the Bingham plastic viscosity model to electric field actuated electrorheological (ER) fluids [21]. Given that ER and MR fluids share a similar rhelogical response with their respective actuating fields, the bingham plastic model began to see use in the field of MR fluid research. Research in the MR fluid field has since been focused around modelling approaches as well as yield stress maximization, with numerous viscosity models being proposed by Norman Wereley [22, 23], J. Yoo [4] and others [6, 24, 25].

MR valve design, as with much of MR fluid device development, saw a surge in interest in the 1990's. In 1993, William Kordonsky published work on a simplistic MR valve based suspension system which was one of the pioneering papers in the MR damper field [26]. Many other valve designs were proposed during this time, with numerous experimental results published discussing the many advantages of MR valves. S. Dyke and colleagues demonstrated that the MR valve viscous effect was able to be actuated within 10 miliseconds [27], greatly increasing interest in the technology. Further increasing valve feasibility, S. Kelso proposed an effective and low cost MR valve utilizing the annular configuration with multiple coils as depicted in figure 1.5[3].

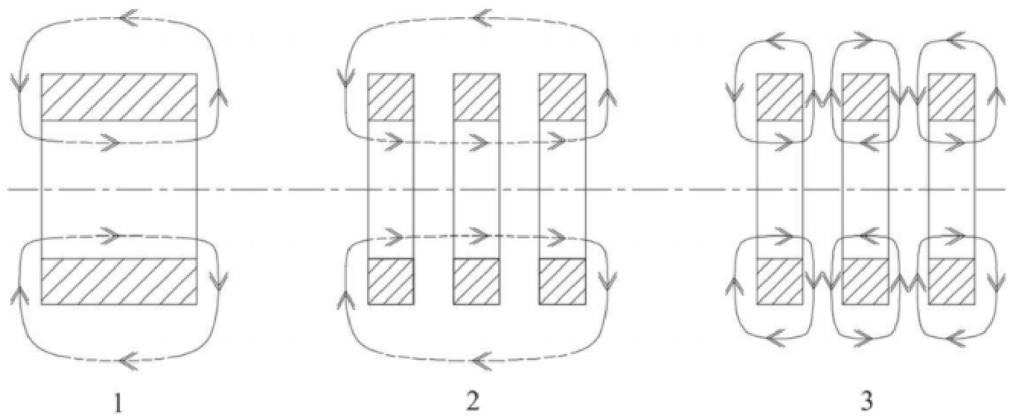

Figure 1.5: Typical MR valve coil arrangements with
1) Single coil **2)** Multiple coils with similar coil polarity **3)** Multiple coils with alternating coil polarity [3]

In 2006, Ai. et al. contructed a hybrid MR valve design, one of the first of it's kind proposed in research [28]. This hybrid valve design utilized both annular and radial flow configurations and was found to combine the benefits of both the flow types at the cost of complexity. With this introduction to hybrid MR valve designs, further annular-radial flow valves were created. Imaduddin et. al. proposed a very complex annular-radial hybrid design utilizing a total of 5 annular active regions and 6 radial active regions as depicted in figure 1.6. This design was high performing at the expense of extreme complexity in design and manufacturing.

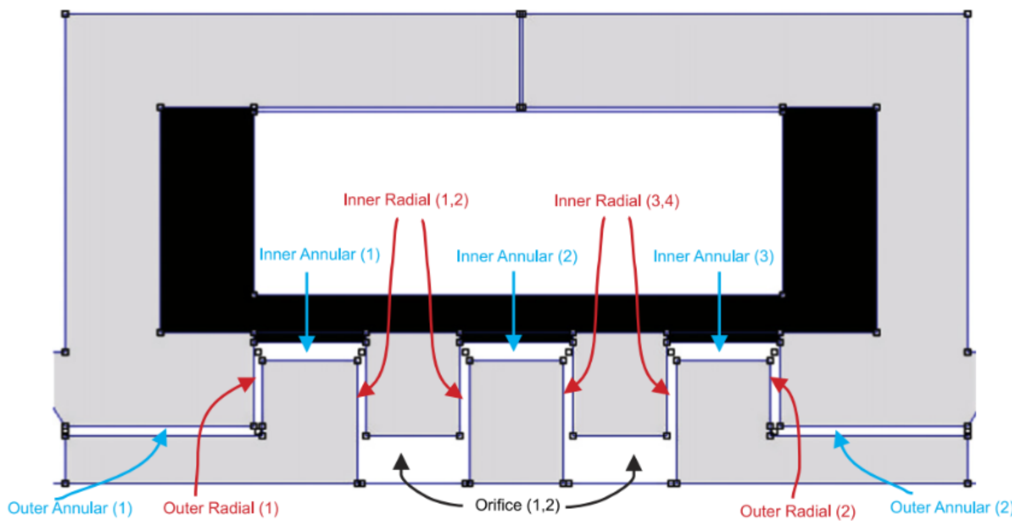

Figure 1.6: Imaduddin hybrid MR valve schematic

1.3.2 Empirical studies: experiments and computer FEM simulations

Due to the complexity of the MR valve, development of these valves often leverages simulation in order to generate valve designs. Li. et al proposed the use of ANSYS simulation in order to optimize the valve geometry such that magnetic flux density within the active region would reach the saturation flux density of the MR fluid in use [6]. Shortly after, Q. Nguyen et al. demonstrated the ability to use ANSYS simulation along with iterative optimization algorithms to design an MR valve based on specific application requirements [29]. The process taken to perform this optimization is outlined in the flow chart of figure 1.7. It is important to note that for every iteration of this process, a simulation step must be performed in order to obtain the performance characteristics of the valve. This equates to numerous simulation steps which require time and computing power to perform.

Generally speaking, most research publications of MR valve designs do not consist of

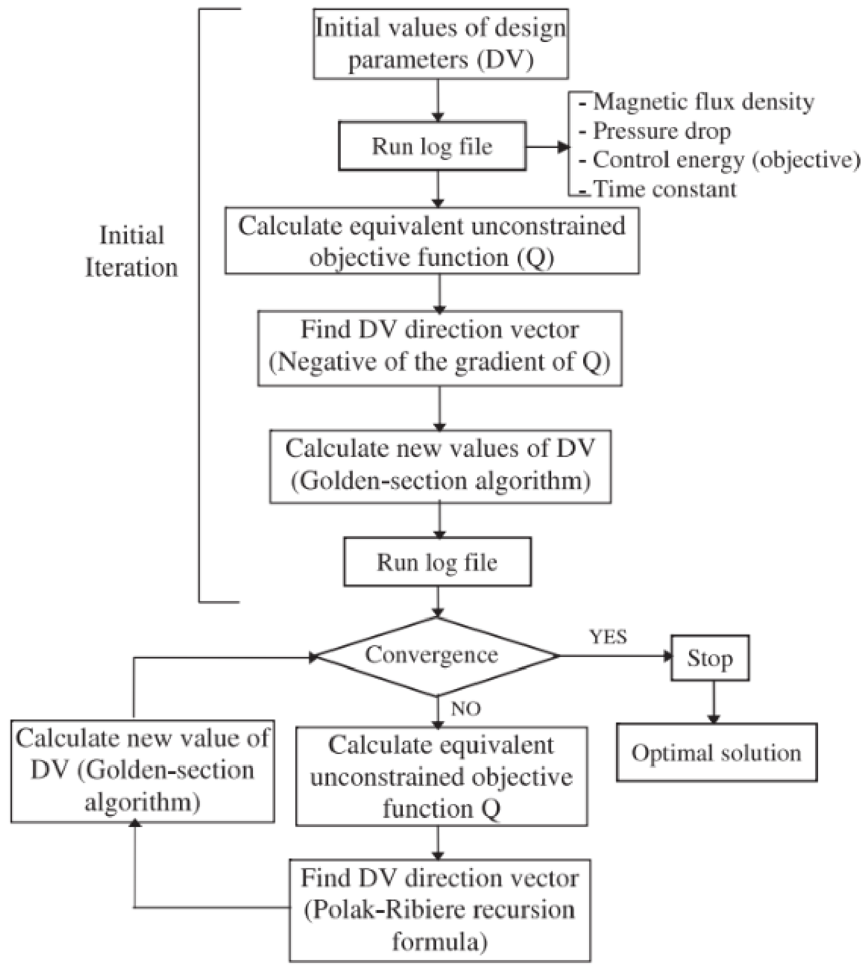

Figure 1.7: Flow chart depicting ANSYS iterative optimization algorithm

only simulated performance analysis. Due to the discrepancies between simulation and real world results, most researchers validate their simulated designs by performing experimental testing of a physical prototype [8]. J. Yoo et al. [4]simulated and experimentally validated a high-efficiency MR valve design and discovered a vast difference between their simulation based predictions and experimental observations as shown in figure 1.8. It was found that the simulation had predicted greater than double the pressure drop that was observed in the experimental prototype. As evident in this study, the requirement for experimental validation after FEM simulation remains in in the current state of the art.

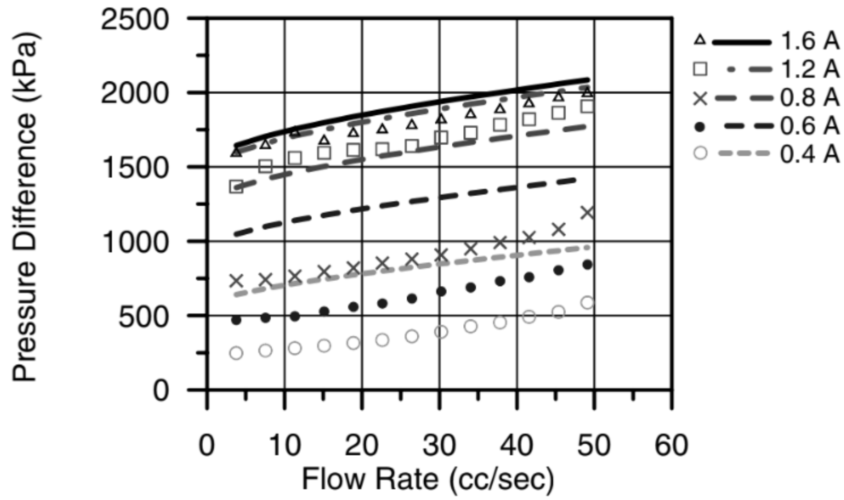

Figure 1.8: Simulation (symbols) vs Experimental results for an annular flow MR valve [4]

1.3.3 MR valve analytical modelling

Bingham plastic model

Around the new millenium, an emphasis on MR valve modelling was observed within the research community. In 1996, Gavin et. al [30] published the first application of the Bingham plastic model previously proposed by Phillips. The Bingham plastic model suggests that a fluid responds to an applied pressure similar to a solid up until a yield stress point where the fluid begins to flow like a Newtonian fluid. This pseudo dual-phase response of the Bingham plastic model lends itself perfectly to the MR fluid as it has two distinct responses to applied pressure with respect to an applied magnetic field and the absence of one. Gavin's approach was to utilize the Bingham plastic model in a rectangular duct flow path for an ER damper, while deviating from the Bingham plastic model slightly such that the fluid's response to an applied magnetic field was accounted for with a yield stress term τ_y. This ER model was subsequently applied to MR valves by Carlson and Jolly in 2000. Li et al. [6] further developed the Bingham plastic model by applying the pressure drop expressions using the geometric parameters of the annular flow MR valve. They proposed that the pressure drop within the valve could be modelled by:

$$\Delta P = \frac{24\eta Q L_{act}}{\pi(D_{in}+g)g^3} + \frac{4L_{act}}{g}\tau_y \tag{1.1}$$

This expression assumed a constant MR term coefficient of 4. This was later found to be an inaccurate assumption, and that the MR term coefficient varied from 2.07 to 3 depending on flow conditions within the valve [19]. Nguyen et. al [29] proposed that the MR term coefficient C was obtained by:

$$C = 2.07 + \frac{12Q\eta}{12Q\eta + 0.8\pi R_1 t_g^2 \tau_Y} \tag{1.2}$$

This expression is a function of flow rate Q, fluid viscosity η, valve radius R_1, valve fluid gap t_g, and fluid yield stress τ_Y. The expression for C would later be applied by many researchers in modelling MR valves using the Bingham plastic model [18].

Herschel-Bulkley model

The Herschel-Bulkley model, itself an extension of the Bingham plastic model, was proposed by X. Wang and F. Gordaninejad [25] in 1999. This model accounted for shear thinning and thickening effects observed in the MR fluid. This phenomenon is a deviation

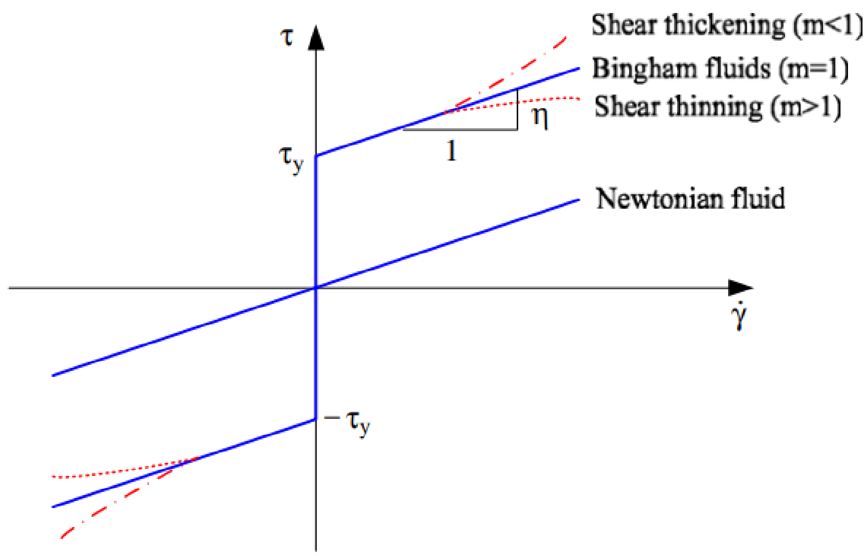

Figure 1.9: Post-yield shear thinning and thickening of MR fluid

in post-yield viscosity which is not considered in the Bingham plastic model. Figure 1.9 illustrates these effects. The Herschel-Bulkley model replaced the τ shear stress term of the Bingham plastic model with a much more complicated yield stress representation based on fluid parameters K and m.

$$\tau = (\tau_y(H) + K|\dot{\gamma}|^{\frac{1}{n}})sgn(\dot{\gamma}) \tag{1.3}$$

Bi-viscous model

Wereley et. al [22] published numerous MR valve designs, with unique models supporting their design process. Specifically, Wereley had proposed a new model, the bi-viscous model, to better approximate the rheological effects of MR fluids when considering hysteresis effects within the valves. This model was experimentally validated and is shown in figure 1.10.

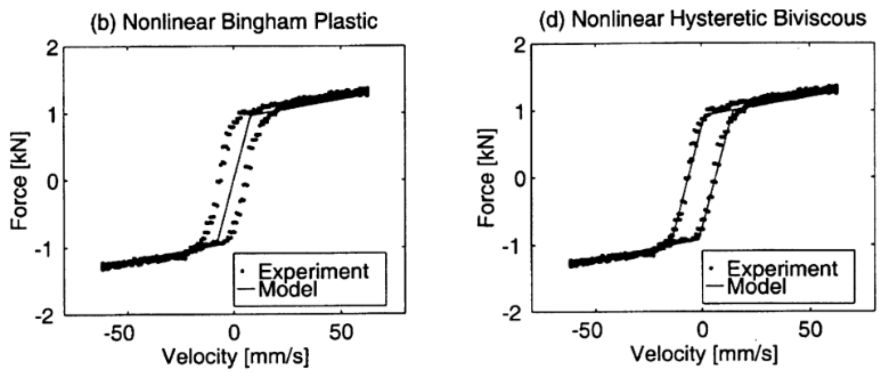

Figure 1.10: Comparison between Bingham plastic and bi-viscous model for nonlinear hysteresis estimation

The Bi-viscous model proposes that the shear stress of the MR fluid be modelled as the piecewise function:

$$\tau = \tau_y(H) + \eta\dot{\gamma} \quad \text{for} \quad |\tau| > |\tau_1 \quad , \quad \eta_r\dot{\gamma} \quad \text{for} \quad |\tau| \leq \tau_1 \tag{1.4}$$

where τ_1 is the transition yield stress and η and η_r are related to the viscous and elastic properties of the fluid. The yield parameters are

$$\tau_y(H) = \tau_1(1 - \frac{\eta}{\eta_r}) \tag{1.5}$$

Herschel-Bulkley model with pre-yield viscosity

The Herschel-Bulkley model was further developed by including aspects of the Bi-viscous model by S.R.Hong in 2007. The shear stress function for the Herschel-Bulkley pre-yield viscosity model is

$$\tau = [\tau_y + k|\dot{\gamma}|^n \mathrm{sgn}(\dot{\gamma}) \quad \text{for} \quad |\tau| > |\tau_t \quad \text{or} \quad |\dot{\gamma}| > \dot{\gamma}_t, \tag{1.6}$$

$$\mu_{pr}\dot{\gamma} \quad \text{for} \quad |\tau| \leq \tau_t \quad \text{or} \quad |\dot{\gamma}| \leq \dot{\gamma}_t \tag{1.7}$$

where k and m are the fluid parameters from the Herschel-Bulkley model, $\mathrm{sgn}(\dot{\gamma})$ is the signum function of the shear strain-rate $\dot{\gamma}$, μ_{pr} is the pre-yield plastic viscosity, and τ_t is the transition yield stress. The numerous dependencies of the Herschel-Bulkley pre-yield viscosity model have limited its use in MR valve research.

In summary, many mathematical models for the rheological effects of the MR fluid have been described in the literature. These models offer their own unique advantages and disadvanatages, with the Bingham plastic model being widely used today due to its simplicity and lack of fluid dependent parameters. A comparison of the available MR valve models has been included below to illustrate the relative accuracy and complexity of the models discussed [5].

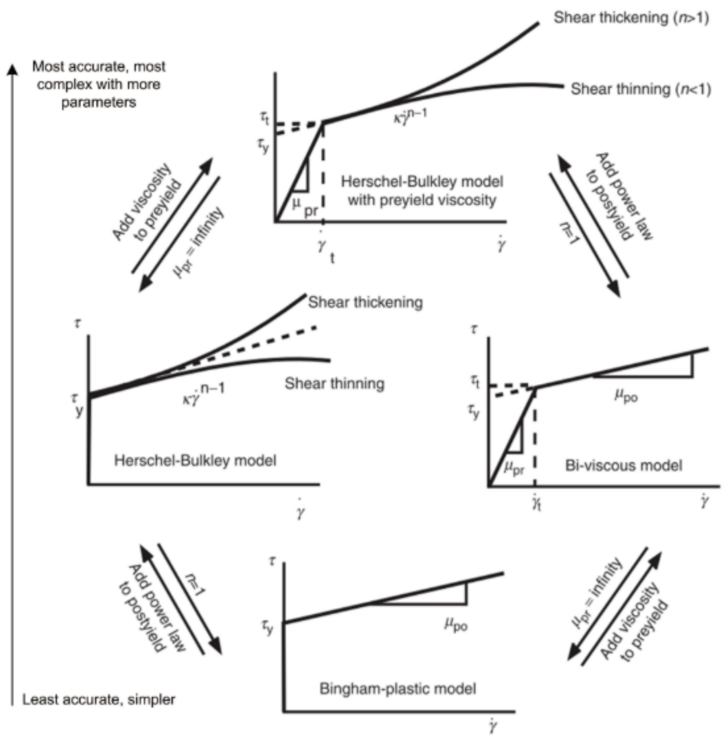

Figure 1.11: Comparison of MR fluid models [5]

1.4 Thesis objectives

The objective of this thesis is to provide a complete mathematical model which can be used to estimate the pressure drop across an annular flow MR valve. The model must take into consideration all aspects of valve geometry, material properties of the valve and MR fluid, and the magnetic and electric circuits which drive the valve actuation. Therefore, the aim of this thesis is to accomplish the following:

- Determine the relevant analytical fluid model for the flow path of an annular flow MR valve along with all related dependencies.
- Develop a model for the magnetic circuit within an annular flow MR valve which takes into consideration all relevant valve geometry.

- Derive an expression for the maximum number of active coil turns within the valve coil given geometric dependencies within the valve and applied current constraints.
- Combine the aforementioned expressions to compile a complete annular flow MR valve model.
- Validate this model to simulation and experimental results
- Demonstrate the application of this model with a proposed valve design

Chapter 2

Mathematical modelling

A useful model for annular flow MR valves must consider all aspects of valve design and performance. A pressure drop equation provides great insight as to the fluid flow performance of an MR valve as well as the valve's impact on the flow of a fluid circuit. An expression for pressure drop within the valve describes the impact the valve has on the fluid circuit flow. In cases where complete blockage is required, the pressure drop within the valve needs to be greater than that of the supplied pressure at the inlet of the valve [18]. For MR valves in a directional application where the valve is being used to divert flow between multiple channels, the pressure drop experienced in each channel affects the flow rate distribution in those channels [15]. In damper applications, pressure drop across the valve is directly relating to the damping coefficient of the damper, which must be tuned given the application. In all these cases, the pressure drop experienced in the fluid caused by the valve is a crucial aspect of design which must be fully understood.

Valve performance can also be described by its on/off pressure drop ratio β [6] which describes the ratio of pressure drop when the valve has full current applied to the pressure drop when the valve has no current applied (on to off state).

$$\beta = \frac{\Delta P(\text{field on})}{\Delta P(\text{field off})} \tag{2.1}$$

This ratio describes the multiple of off pressure that the valve can supply when fully actuated. This metric is important for valve designers as it describes valve efficiency in that it takes into account the maximum pressure drop experienced as well as the minimum, or viscous pressure drop [6]. By considering this viscous pressure drop, the β ratio describes how well a valve is able to increase the pressure drop utilizing the MR effect relative to a minimum pressure drop that will always be present within the valve.

A magnetic circuit expression which isolates the magnetic flux density within the active region fluid gap must also be determined to understand the MR effects of the fluid within the valve. This magnetic circuit expression is dependent on the electrical circuit of the MR valve, and largely depends on the number of active turns within the valve coil. An expression for the maximum number of active turns within the coil will also provide a more detailed understanding of valve performance and design. Ultimately, a complete model which utilizes a pressure drop expression for the fluid path of the valve, an active region magnetic flux density expression for the magnetic circuit of the valve, and an estimation of active turns in the valve actuation coil would provide a thorough and detailed estimation of valve performance and dependencies on valve geometry.

2.1 Mathematical modelling of pressure drop in an annular MR valve

It is understood that the pressure drop across an MR valve is a combination of the viscous pressure drop due to the geometric variables of the flow pathway as well as the MR effect within the fluid as is passes through the active region of the valve.

$$\Delta P = \Delta P_{viscous} + \Delta P_{MR} \tag{2.2}$$

where

$$\Delta P_{viscous} = \frac{24\eta Q L_{act}}{\pi(D_{in} + g)g^3} \tag{2.3}$$

$$\Delta P_{MR} = \frac{4L_{act}}{g}\tau_y \tag{2.4}$$

In the particular case of annular flow MR valves, previously proposed models [6] have only included viscous pressure drop within the active region.

In the proposed model this expression has been modified to include the viscous pressure loss within the valve coil region (delta P proposed). While this region experiences a negligible MR pressure loss due to the near zero magnetic flux density within that region, viscous pressure drop in the coil region can be equal to or greater than that of the active region depending on valve geometry. The equation put for by Li has also been modified to include the MR term coefficient C as proposed by Nguyen [29] to increase the accuracy of the model across various flow conditions. It is therefore suggested that the model consider the pressure drop within the coil region by including a viscous pressure drop term for the geometry of that region, as well as the C coefficient such that:

$$\Delta P = \Delta P_{viscous\,active} + \Delta P_{viscous\,coil} + \Delta P_{MR} \tag{2.5}$$

where:

$$\Delta P_{viscous\,active} = \frac{24\eta Q L_{act}}{\pi(D_{in} + g)g^3} \tag{2.6}$$

$$\Delta P_{viscous\,coil} = \frac{12\eta Q(L_{core} - 2L_{act})}{\pi(D_{coil} + c)c^3} \tag{2.7}$$

$$\Delta P_{MR} = \frac{C L_{act}}{g}\tau_y \tag{2.8}$$

and

$$C = 2.07 + \frac{12Q\eta}{12Q\eta + 0.8\pi\frac{D_{in}}{2}g^2\tau_y} \tag{2.9}$$

Note that the C expression has been modified from its original representation given by Nguyen et. al to reflect the geometric variables used in the analytical model.

In these pressure drop expressions, the MR pressure drop term is dependent on the yield stress of the MR fluid. A function for the yield stress is needed to model the MR fluid response to changing valve actuation states. Yield stress curves for MR fluids can be obtained from the manufacturers [31]. In the proposed model a polynomial fit approximation function was used to model the MR fluid (tauy equation).

$$\tau_y = a_3 B_{gap}^3 + a_2 B_{gap}^2 + a_1 B_{gap} + a_0 \tag{2.10}$$

This expression is dependent on the magnetic flux density in the active region fluid gap (B_{gap}) as well as fit coefficients determined from the manufacturer data.

It is evident that the pressure drop of the MR fluid flow through the valve is dependent on valve geometric variables and the B_{gap} flux density.

2.2 Mathematical modelling of annular MR valve magnetic circuit

A critical aspect of magnetic modelling is the saturation point of a given material. In general, an increase in applied magnetic field H results in an increase in magnetic flux density B. This relationship remains fairly linear below the saturation point, and can be simplified by making an approximation that the relationship between the two variables is a function of a material parameter μ, the magnetic permeability of a material. Saturation occurs within a material when the relationship between the magnetic flux and applied magnetic field is no longer linear. This is generally observed as a decrease in the rate of change of B with respect to an applied H field. In the context of MR valves, the saturation of the valve core is equally as important as the saturation of the MR fluid. An example of the B-H relationship is depicted in figure 2.1 for a valve core material (steel) and an MR fluid (MRF-132LD). Beyond this saturation point the relationship quickly becomes non-linear and thus the approximation utilizing the magnetic permeability becomes invalid.

Due to the more demanding and complex modelling of the magnetic relationship between B and H past the saturation point, the magnetic permeability assumption was used in the proposed analytical model which limits its application to valve operation before the saturation point of any of the valve materials. To account for this limitation, the coding behind the proposed model included warnings for the end user when the saturation point of any of the valve materials was reached.

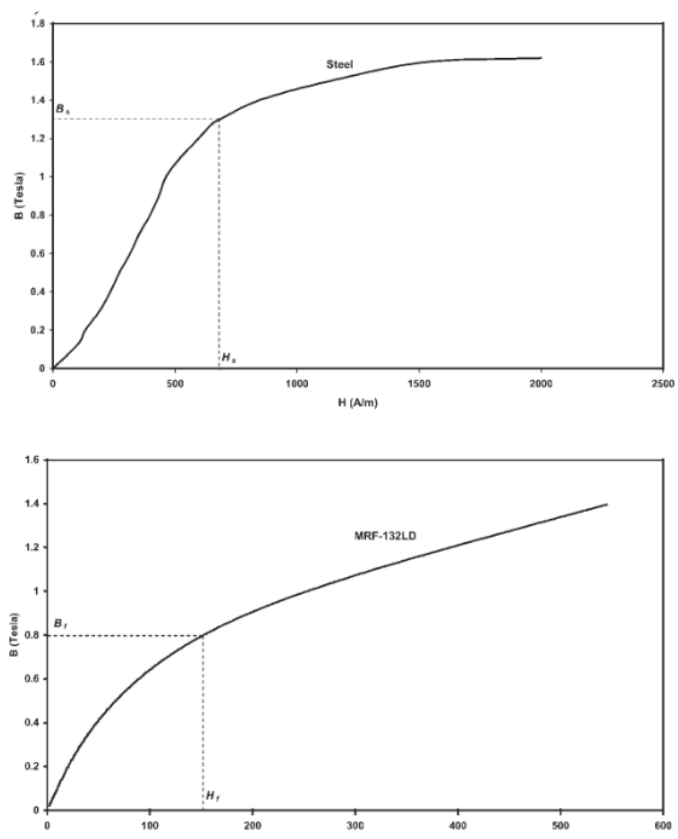

Figure 2.1: Magnetic saturation of steel and an MR fluid [6]

With an understanding of the limitations of the valve materials in terms of their magnetic abilities, a representation of the magnetic circuit of the valve was then analyzed to ensure proper estimation of the magnetic flux density within the fluid. Figure 2.2 shows the magnetic circuit of an annular flow MR valve with the length and area of each link labelled.

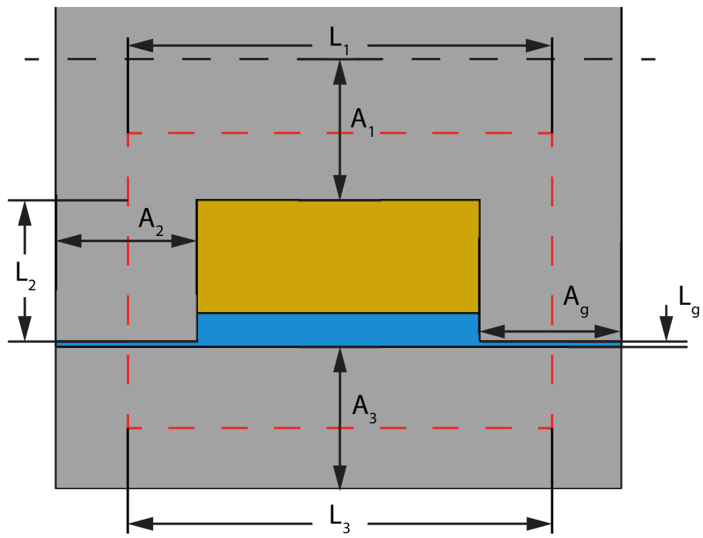

Figure 2.2: Magnetic circuit of an annular flow MR valve

Assuming no magnetic flux leakage, the magnetic flux density within the circuit remains constant such that:

$$\Phi_{Bobbin} = \Phi_{Fluid\,gap} = \Phi_{Valve\,case} \tag{2.11}$$

and

$$\Phi = BA \tag{2.12}$$

where the magnetic flux in each link of the magnetic circuit is defined as the product of the magnetic flux density and cross sectional area of that link.

Table (link lengths and areas) shows the length and cross sectional area of each individual link within the magnetic circuit. Substituting the magnetic flux densities and

Table 2.1: Magnetic circuit link lengths and cross sectional areas

Link	**Length**	**Cross Sectional Area**
1	$L_{core} - L_{act}$	$\pi(\frac{D_{core}}{4})^2$
2	$\frac{D_{in}}{2} - \frac{D_{core}}{4}$	$2\pi(\frac{D_{in}}{2})L_{act}$
3	$L_{core} - L_{act}$	$\pi(\frac{D_{out}}{2})^2 - \pi(\frac{D_{in}+g}{2})^2$
g	g	$2\pi(\frac{D_{in}}{2} + g)L_{act}$

link cross sectional areas into equation (mag flux conservation) becomes:

$$B_1(\pi(\frac{D_{core}}{4})^2) = B_2(2\pi\frac{D_{in}}{2}L_{act}) = B_3(\pi(\frac{D_{out}}{2})^2 - \pi(\frac{D_{in}+g}{2})^2) = B_g(2\pi(\frac{D_{in}}{2} + g)L_{act}) \tag{2.13}$$

In a steady-state scenario, Ampere's law is given as:

$$\oint Hdl = I_{enc} \tag{2.14}$$

Where the line integral of the H magnetic field over the closed path length of the magnetic circuit is equal to the enclosed current within the path. From Ampere's law it is understood that magnetomotive force, or enclosed current, in a circuit is equal to the sum of the length elements of each link in the circuit times the magnetic field at each of those length elements.In the case of the annular flow MR valve, the magnetomotive force is driven by the actuation coil and is the product of the number of active turns within the coil and the applied current.

$$NI = H_1 l_1 + 2(H_2 l_2) + 2(H_g l_g) + H_3 l_3 \tag{2.15}$$

Combining the magnetomotive force due to the coil and the link lengths given by the valve geometric variables (table of mag circuit) results in:

$$NI = H(L_{core} - L_{act}) + 2(H_2(\frac{D_{in}}{2} - \frac{D_{core}}{4}) + 2(H_g g) + H_3(L_{core} - L_{act}) \tag{2.16}$$

Given that the magnetic field and magnetic flux density relationship can be approximated by the relative permeability assumption

$$B = \mu H \tag{2.17}$$

equations (mag flux detailed) and (magnetomotive force detailed) can be combined to form an expression for the magnetic flux density in the active region fluid gap.

$$B_{gap} = \frac{NI\mu_s}{(4L_{act})(D_{in} + 2g)(L_{core} - L_{act})(\frac{1}{(D_{core}^2)} + (\frac{1}{(D_{out}^2 - (D_{in}+g)^2}) + g(\frac{1}{2} + 2\frac{\mu_s}{\mu_g} - \frac{D_{core}}{4D_{in}})} \tag{2.18}$$

Where μ_s and μ_g are the permeabilities of the valve core and MR fluid respectively.

2.3 Mathematical modelling of number of active turns in valve actuation coil

The number of active turns in the valve actuation coil has a great impact on the magnetomotive force within the valve as shown in equation 2.16. In order to further develop the proposed analytical model, an approximation of the number of coil turns within the valve has been proposed. Inclusion of this approximation allows for a complete model which considers the impact of valve geometry on the coil strength. The following expression relates the valve core geometry and wire diameter to a predicted number of active coil turns.

$$N = \frac{F(L_{core} - 2L_{act})(\frac{D_{coil} - D_{core}}{2})}{\frac{\pi}{4}w^2} \tag{2.19}$$

where F is the coil fill factor (determined by manufacturing process capabilities) and w is the copper wire diameter (including insulation) of the coil. This equation is based on similar expressions for solenoid and transformer coil winding , but has been modified to relate to the geometric variables of the annular MR valve. If a given current rating for the valve is specified, the wire diameter can be determined by the continuous current capacity of the wire gauge. This information is generally provided by the manufacturer of the wire. The proposed model uses this expression to estimate the maximum number of active coil turns within the actuation coil. When provided with an applied current and the valve core material, the model can estimate the magnetomotive force within the valve. With this magnetomotive force and given the geometric parameters of the valve design, a complete pressure loss expression is determined.

Chapter 3

Validation

3.1 Validation to ANSYS FEM Simulation

W.H.Li et al [6] have published an analysis of an annular flow MR valve which was conducted using an ANSYS based simulation. This simulation consisted of analysis of the magnetic circuit as well as the pressure drop across their proposed valve. The physical properties of their valve design, including the geometric parameters and material characteristics were included in their publication and have been summarized in table 3.1.

Table 3.1: W.H.Li simulated valve parameters

Item	Value
D_{core}	14mm
L_{core}	16mm
D_{in}	22mm
D_{out}	28mm
L_{act}	3mm
L_{return}	2.5mm
g	0.5
μ_s	1000
μ_g	5
N	94
I_{max}	1.6A
Q	10-50 ml/s
MR fluid	MRF-132LD

With this information given, the model proposed in this thesis was used to predict a pressure drop value given the Li valve design parameters. Figure 3.1 shows the similar outcomes of both the Li ANSYS simulation [6] and the proposed model. The model is shown to be nearly identical to the ANSYS simulation across the entire range of valve operation. This is likely due to the ANSYS simulation consisting of FEM based magnetic and fluid model solving, which in itself follows Ampere's law for magnetic simulation and Navier-Stokes equations for fluid solving. However, it should be noted that the linear magnetic permeability assumption utilized in the proposed analytical model did not impact the accuracy of the model relative to the non-linear magnetic permeability model utilized in the ANSYS based simulation. This is likely due to the valve operation being designed to remain below the saturation point of all valve materials. Since the equations within the model are based on Ampere's law and Navier-Stokes as well, the solutions to both of these models should be very similar if not identical. This illustrates the power of the proposed analytical model, as it is able to produce results very similar to ANSYS simulation at a fraction of the computational power and without the cost of an ANSYS license. The proposed model is built using the MATLAB language, but could be ported to an open source language such as Python, further decreasing the expense of utilizing the model and allowing for a greater range of users.

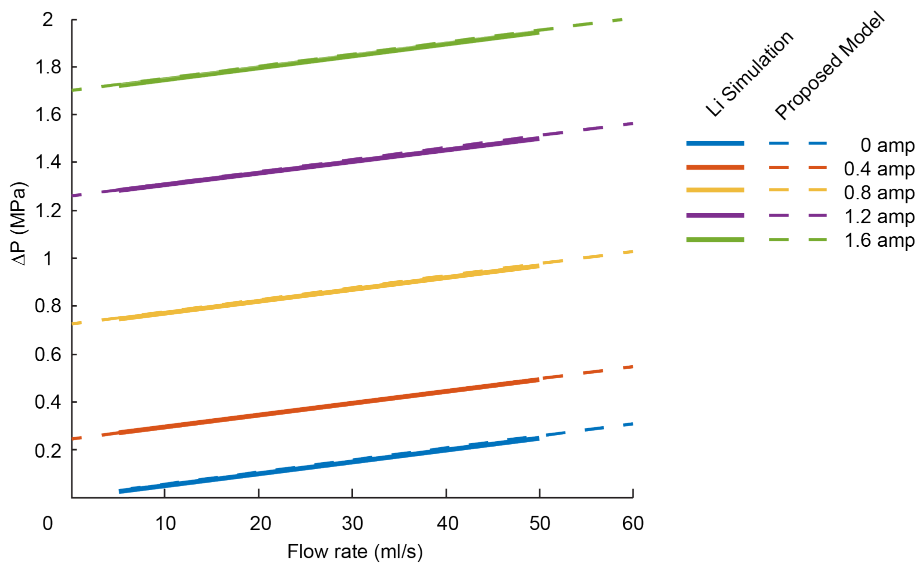

Figure 3.1: Comparison of proposed analytical model and ANSYS simulation conducted by W.H.Li et al.

3.2 Validation to experimental results

Figure 1.8 shows the comparison of results from the proposed model and the experimental results of J. Yoo and N. Wereley [4]. The published results also include a simulation conducted by Yoo and Wereley which showed a vast difference in predicted values relative to the experimental results. The proposed model shows a significant improvement over the simulation performed across the whole range of valve operation as evident in the figure. It is shown that the simulation performed by Yoo and Wereley would drastically over estimate the pressure drop of the valve in all cases, with only the full on state (1.6A) and 75% on state (1.2A) being within 50% accurate of the experimental results. It should be noted that the experimental results suggest that the valve approached a B_{gap} saturation around the 75% on state since the full on state showed a roughly 10% increase in pressure drop for an additional 25% increase in applied magnetic field. This is speculated to be a result of varying testing conditions or valve parameters after construction. The design of the valve requires tight tolerances and specific operating conditions which are difficult to obtain in testing. It is possible that these sources of error contributed to a valve that did not completely reflect the simulated valve. In addition, the proposed analytical model also suggest that the saturation point was reached before the 100% on state, as the B_{gap} magnetic flux density exceeded the 0.8T saturation point of the MR fluid. Some discrepancies between the proposed analytical model and the experimental results do exist, and it is speculated that these discrepancies may be due to a number of factors. Most notably of these factors, the shear thickening and thinning effect of the MR fluid is not taken into consideration with the Bingham plastic model and remains unaccounted for in the analytical model. As well, the mentioned sources of error in the physical prototype testing likely attribute some discrepancy between the experimental results and the model results. Figure 3.2 shows that the analytical model is considerably more accurate than the simulation used. The model shown to be more accurate at the lower flow rate range for lower applied current levels, and higher flow rate range for high current levels. This suggests that while not perfectly depicting the MR valve response to increasing current and fluid flow, the proposed analytical model is suitable as a design tool as its accuracy over the entire range of valve operation is greater than that of simulation. Overall, the proposed model proves to be more accurate that simulation and can thus be used as a method of valve design, with optional experimental validation conducted if further accuracy is required by the valve application.

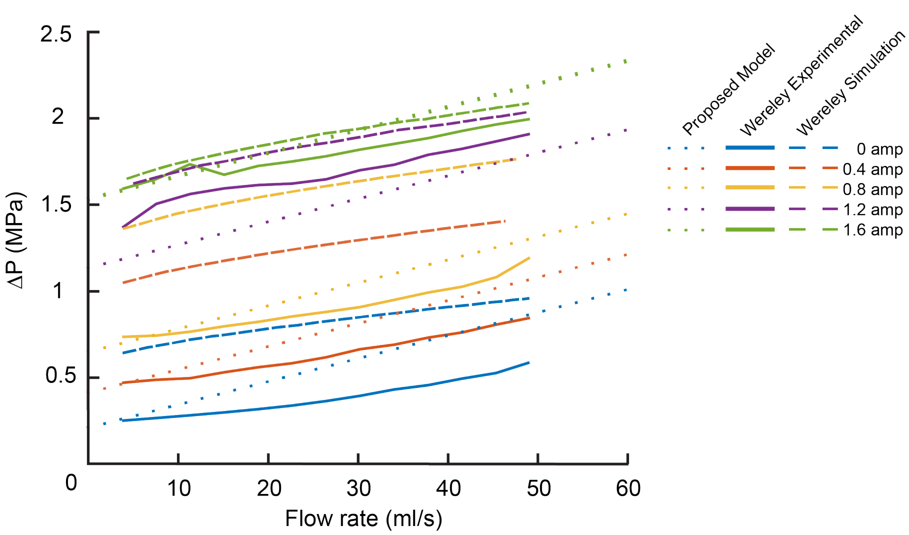

Figure 3.2: Comparison of proposed analytical model and experimentation and simulation conducted by J. Yoo and N. Wereley

Chapter 4

Results and discussion

4.1 Key design terms

One advantage of the complete mathematical model proposed is the ability to investigate the effect of varying parameters on the entire MR valve system. In particular, certain terms have been identified by parametric analysis to provide key performance impacts:

4.1.1 MR term - L_{act}/g

The MR effect pressure drop as outlined in equation 2.8 is product of the yield stress of the fluid, the MR coefficient C, and the ratio of active length to fluid gap width. Since the coefficient C is largely determined by the instantaneous flow rate within the valve, it is likely to be constantly changing in the valve application. The yield stress of the fluid is also frequently changing due to the changing magnetic field while in operation. This leaves the ratio, $\frac{L_{act}}{g}$, as the only parameter within the MR pressure drop that will not be changing during valve operation and is thus a design point for the valve. The MR term

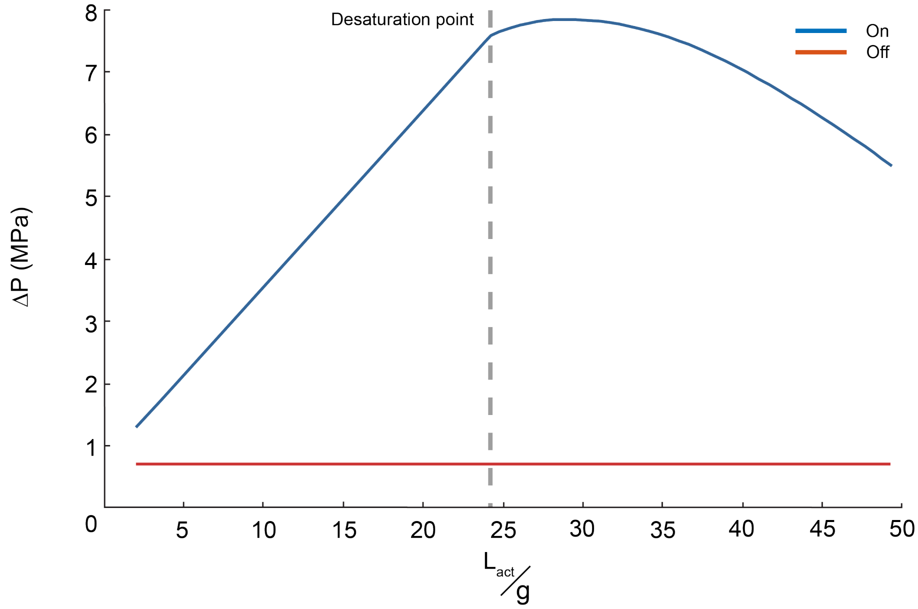

Figure 4.1: Pressure drop vs MR term ($\frac{L_{act}}{g}$)

has been analyzed to understand the impact it has on the performance of the valve. This analysis was done by isolating the $\frac{L_{act}}{g}$ term in the mathematical model, and plotting the pressure drop across the valve versus the varying $\frac{L_{act}}{g}$ term. Figure (MR term pressure drop) shows that increasing the ratio of L_{act} to g will result in a larger pressure drop through the valve due to MR effects. In contrast, increasing the ratio has little effect on the off state pressure drop of the valve. This finding suggests that design of an annular flow MR valve can be optimized by varying this ratio. Should a certain application require a large on/off ratio β the MR term can be increased to meet these needs. It should be noted that with very small fluid gap widths (g), very large viscous pressure drop is experienced. Generally, a fluid gap of 0.5mm or larger is sufficient. Figure 4.1 also shows a plateau and decrease in pressure drop as the MR term is increased. This is due to the magnetomotive force NI within the valve not providing sufficient power to ensure the B_{gap} of the valve approaches saturation. This decreasing trend in pressure drop can be counteracted with stronger magnetomotive force by either increasing the applied current or number of active turns within the coil. This MR term offers a great design parameter that can be adjusted to optimize valve design given application requirements.

4.1.2 Bobbin diameter - D_{in}

Figure 4.2 shows the impact of bobbin diameter D_{in} on the pressure drop of the valve. In many valve designs, the bobbin coil diameter D_{coil} and bobbin outer diameter D_{in} are equal, resulting in a uniform cross section throughout the fluid flow path. In these designs and others which have a D_{coil} less than or equal to D_{in}, the D_{in} term restricts the total number of active coil turns by reducing the cross sectional area of the coil. This is evident in figure 4.2 as the pressure drop is significantly increased with an increase in D_{in}. This is due to the larger volume available for the actuation coil, and thus a larger magnetomotive force and B_{gap} within the valve during full applied current operation. Once saturation occurs, further increasing D_{in} will result in lower viscous pressure drop within the valve as evident in equation 2.7 as the D_{in} parameter is in the denominator of the viscous pressure drop term. This decrease in viscous pressure drop reduces the off state pressure drop, while reducing the on state pressure drop by the same amount since the on state pressure drop is the sum of the viscous and MR effect pressure drops. Since the reduction in pressure drop is due to only viscous pressure drop changes, the on/off ratio, β, will see an increase with an increase in D_{in}. This is a useful design parameter as an MR valve application may require high β ratio which can be greatly increased by increasing the valve bobbin diameter.

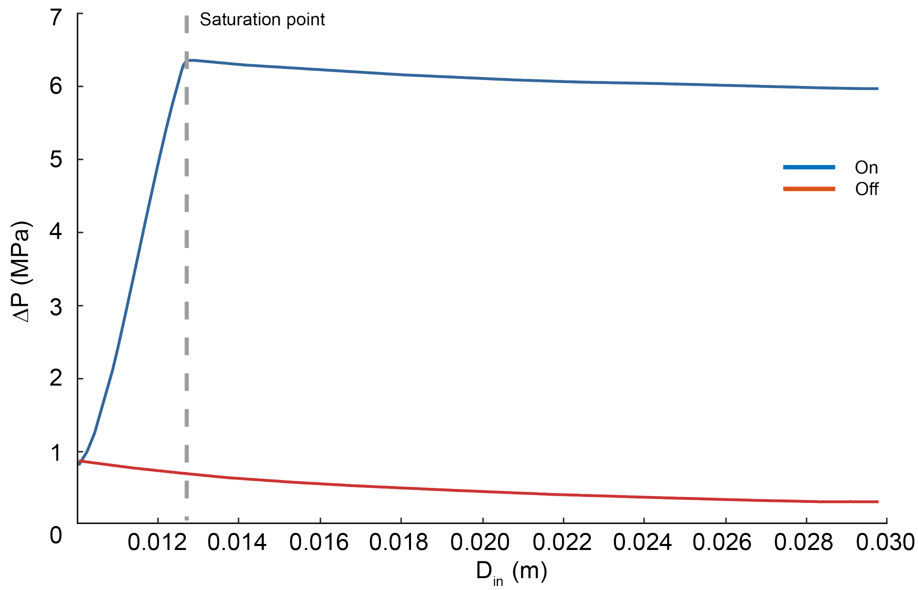

Figure 4.2: Pressure drop vs bobbin diameter (D_{in})

4.2 Application of model

As a demonstration of the capabilities of the proposed model, a valve was designed. This valve was designed such that it could compete with another high performance MR valve. The valve used for comparison was that of Fitrian Imaduddin et. al [18]. Their valve design was a complex, hybrid annular-radial-orifice type with complex meandering fluid flow construction. Given their general design constraints (valve size, current rating, MR fluid type, total pressure drop) a comparable annular valve was designed using the proposed model. The geometric constraints of the variable are shown in table (table of application MR valve geometry).

The annular valve designed as a substitute for the Imaduddin valve was found to outperform the complex meandering flow valve in terms of total pressure drop and β ratio (graph comparison). The proposed valve is shown to have a lower off pressure

Table 4.1: Designed valve parameters

Variable	**Value**
Active length L_{act}	0.0115m
Valve length L_{core}	0.084m
Bobbin diameter D_{in}	0.02m
Active region fluid gap g	0.0005m
Coil diameter D_{coil}	0.02m
Coil region fluid gap c	0.0005m
Bobbin core diameter D_{core}	0.005m
Valve case diameter D_{out}	0.025m
Active coil turns N	1790
Wire gauge and diameter w	24 AWG (0.00051m)

drop, largely due to the easier flowing geometry of the annular flow valve, whille still providing a high pressure drop when the valve is fully actuated. The analytical model was used to ensure the B_{gap} flux density reached saturation, thus ensuring no further MR effect pressure drop could be obtained from the fluid. In this particular case, the maximum operating current was 1A, however, the valve design proposed used 24 AWG wire which has a continuous current carrying capacity of 2A. If further pressure drop was required in this valve, the active region could be enlarged and the maximum operating current increased to 2A. These modifications are outside of the operating requirements set forth by Imaduddin, but are well within the limits of the proposed annular valve. This demonstrates the potential of the proposed model as a tool of complete MR valve design and how it can displace current methods of simulation. Since all of the parameters of the valve are easily observed within the analytical model, it is possible to see where the valve can gain further performance, as in the case of increasing the operating current in this proposed valve. This application of the analytical model shows how the model can be easily integrated into current MR valve applications and how the use of a complete model allows for a greater understanding of the development process for the designer of the valve.

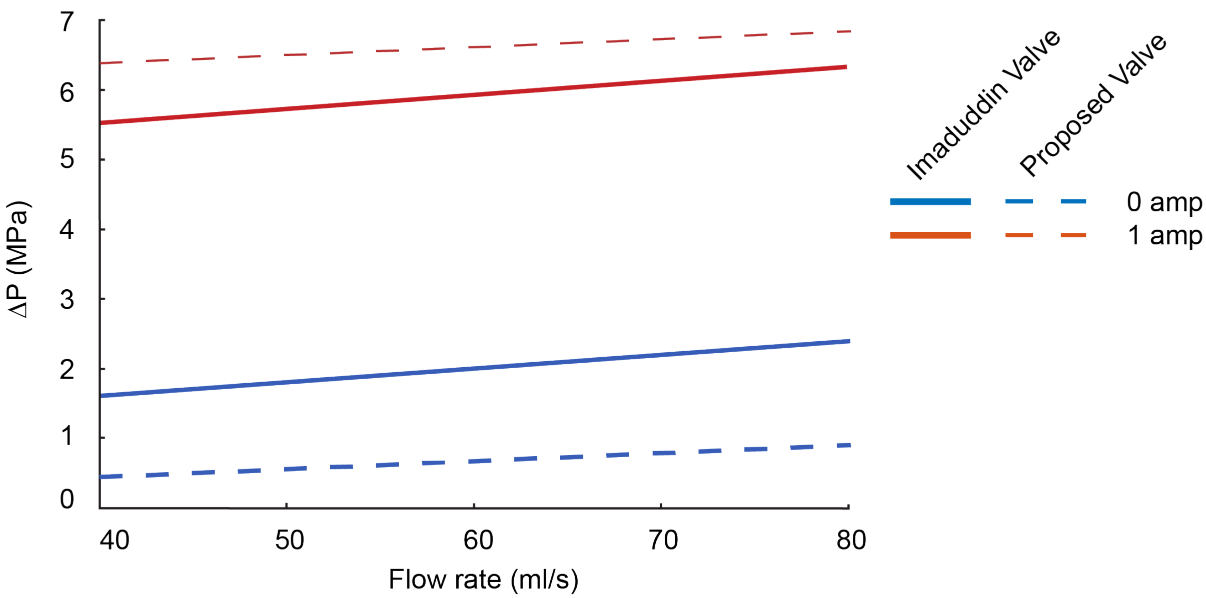

Figure 4.3: Comparison of valve designed with proposed model and Imaduddin hybrid flow Valve

Chapter 5

Conclusions

5.1 Conclusions

In this thesis, a novel complete analytical model of annular flow MR valves was proposed. This model incorporated an understanding of flow characteristics and thorough magnetic circuit analysis with a detailed outline of the geometric dependencies of the fluid and magnetic subsystems within an annular flow MR valve. The model was validated by comparison to simulation and experimental test results. Key terms of the model were analyzed and their effect on valve design and performance was highlighted. Finally, an annular flow MR valve was designed using the model. This valve was designed to application requirements and proved to be an effective design in terms of performance. Although minor discrepancies are present between the model predictions and experimental results, the model proposed in this paper has proven to be an accurate and effective method of annular flow MR valve design. Overall, the thesis completed the initial thesis objectives with the following key tasks completed:

- An analytical fluid model for the flow path of annular flow MR valves was determined which considered all geometric and MR fluid dependencies
- A model for the magnetic circuit within the annular flow MR valve was developed which included all relevant valve geometry
- An expression estimating the maximum number of active coil turns in the valve actuation coil was described
- The aforementioned expressions were compiled into a complete annular flow MR valve model
- The proposed analytical model was validated to simulation and experimental results
- An MR valve was proposed given application requirements utilizing the analytical model to develop and optimize the valve design.

5.2 Contributions

As mentioned in the literature review, the field of MR valve research lacks a complete and thorough model for MR valve performance. It was the ultimate objective of this thesis to provide exactly such am model, which could be used by researches to design and develop better MR valves without the use of expensive and complex simulation and experimental apparatus. My contribution to the field of MR valve research was a Bingham plastic based analytical model which considered all aspects of valve design. This thesis outlined the benefits and application methods of the analytical model, and is the first to demonstrate such a model. MR valve designers and researchers will be able to use this thesis and proposed analytical model to enhance MR valve development and push forward the cutting edge of MR valve research.

5.3 Recommendations

Currently, a major limitation to the accuracy of the proposed annular MR valve model, and any other Bingham plastic based model for MR fluids, is the lack of consideration for pre and post yield changes in MR fluid rheology. As well, the assumption that the B and H permeability relationship is linear limits the model's application to only valve states below saturation. As uncovered in the literature review, other more complex models exist to better account for the varying viscosity representations of MR fluids which could be applied to MR valve modelling. While completely outside the scope of this thesis, a substantial step forward would be investigating and empirically deriving the underlying dependencies of these more complex models, such as the Herschel-Bulkley pre-yield viscosity model. Additionally, identifying the non-linear B-H relationships of valve materials would allow for the use of a non-linear model, increasing accuracy and allowing the model to be applied above the pre-saturation operation limit. If a study could be made to identify the missing dependent parameters of these models, this would greatly increase the feasibility of their use in MR fluid modelling and thus MR valve modelling.

Additionally, while the model contained within this thesis accurately describes fluid flow in an annular MR valve, there remains additional valve configurations that were not accounted for. The radial and orifice flow MR valve configurations are widely used in research and should be accounted for in a future model. The process taken to develop the annular flow model in this thesis could likewise be applied to both radial and orifice flow to establish separate models for each of these valve types. If possible, a model which considers all valve types would be the pinnacle of MR valve modelling and could provide important insight into the benefits of each type of flow for MR valve applications.

A few recommendations for future research efforts have been outlined below in hopes of further focusing the study of MR valve modelling:

- Apply proposed analytical model to radial flow configuration MR valves and experimentally validate analytical model for radial flow configuration MR valves
- Transpose proposed model to open source programming language such as Python or Java for greater access to model
- Empirically determine fluid parameters for Herschel-Bulkley pre-yield viscosity model
- Substitute Herschel-Bulkley pre-yield viscosity model into existing MR fluid flow models

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