OPTIMAL POWER MINIMIZATION IN TWO-WAY RELAY NETWORK WITH IMPERFECT CHANNEL STATE INFORMATION

by

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Abstract

We study a two-way amplify and forward relay network with two transceivers which communicate through a network of n_r relays while there is no direct link between the two transceivers. Each relay is equipped with a single antenna for transmitting and receiving. We study the minimization of the total transmit power that is used in all of the network nodes given the condition that the transceiver which calculates the optimal transmitting power has a full knowledge about the channels between itself and the relays and the variance with zero mean of the channels between the relays and the other transceiver. The total average power is minimized subject to a soft constraint which guarantees that the outage probability is below a certain level. The optimal solution is derived in closed form and leads to a single relay selection criterion.

Dedication

To my family: with gratitude for their love and support

To my little lion

Acknowledgements

I want to express my gratitude to my supervisor Dr. Shahram Shahbazpanahi for all the support, help, and motivation. Dr. Shahbazpanahi has been always ready to support and direct me. He helped me in establishing the skills of researching and I hope I will carry his inspiration in my future studies and throughout my careers.
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Glossary

$u(\cdot)$	Step function, 33
AF	Amplify and Forward, 15
ALOHANET	The first wireless network used packet radio system, 2
BS	Base Station, 11
CDF	Cumulative Density Function, 26
CDMA	Code Division Multiple Access, 2
CSI	Channel State Information, 9
DF	Decode and Forward, 15
IMT	International Mobile Telecommunications, 2
ITU	The International Telecommunications Union, 2
kbps	kilo bit per second, 2

Mbps	Miga bit per second, 2
MIMO	Multiple Input Multiple Output, 11
MS	Mobile Station, 12
Perceived SNR	the signal power to the average noise power ratio, 31
SNR	Signal to Noise Ratio, 9
TDD	Time Division Duplex, 12
WLAN	wireless local area networks, 2

Nomenclature

$(\cdot)^*$	denotes the complex conjugate transpose of a matrix
$(\cdot)^H$	denotes the hermitian conjugate transpose of a matrix
α	represents the non outage time
$\mathbf{A}$	boldface uppercase letters represent matrices
$\mathbf{a}$	boldface lowercase letters are used to represent vectors
$\mathbf{A}^{-1}$	inverse of matrix $\mathbf{A}$
$\mathbf{A}^T$	transpose of matrix $\mathbf{A}$
$\mathbf{I}$	represents an identity matrix
$\mathbf{f}$	represents the channels' coefficient vector
$\mathbf{w}$	represents weight vector
λ	represents an eigenvalue of a matrix
$\mathrm{tr}\{\cdot\}$	represents the trace of a matrix
$\mathrm{diag}(\mathbf{A})$	convert a diagonal matrix to a vector
$\mathrm{diag}(\mathbf{a})$	convert vector to a diagonal matrix
$E\{\cdot\}$	denotes the statistical expectation

Chapter 1

Introduction

Communications, which is the process of conveying messages from one place to another, is an essential need in the life of not only human being but also different existing species. The closest distance between two communicators is almost zero [1]. For example, a mother touches her infant to convey a love message. The distance increases and varies from one type of communications to another such as direct speech that can be performed over a very short distance (i.e. within the range of hearing without the aid of any mean) and the signaling using different part of the body that is done within a line of sight distance. The development of communications between human beings has taken place since the existence of our species and has passed slow and fast periods of development inspired by various needs in peace or wars. The development in communications took different shapes such as in hard means and soft means, wired communications and wireless communications, and each type of communications has its own branches and development.

One of the most fast growing sectors in communications is wireless communications. Its growth is inspired by the need of minimizing the use of wires and increasing the degree of free movement. Nowadays, wireless systems are replacing many wire based systems at residential and industrial applications. In fact, wireless systems can perform tasks that cannot be done by wired systems such as in some medical applications [2]. As the wireless communications grows, the need of research to further improve the service and addressing the challenges grows. This opens a wide area of research in different aspects and fields such as digital signal processing.

1.1 Historical Review

Wireless communications was employed even in the pre-industrial age. At that time, people use smoke signals, light signals, torch signals, and flag signals in order to exchange some complex messages within a sight distance [3]. When the revolution of technology took place, communications became one of the most progressive fields. It started when Samuel Mores invented the telegraph in 1838 and followed by the invention of the telephone in the 1870s .When the telegraph was first invented, it used digital signals to transmit messages while the telephone used analog signals. The use of analog signal in telephone was inspired by the fact that the main purpose of communicating by telephone is to transmit voice messages [3]. The first radio communications system was demonstrated by Guglielmo Marconi who managed to transmit over a distance of 18 miles [4]. Since then, radio communications has been developing and improving in different aspects. It improved in the distance that it can deliver signals to, in the quality of the received signal, in the power consumption, in the size of transmitters and receivers, and the cost of the service. These improvements have made it possible for different services to exist e.g. private and public radio, television, and wireless networks.

Delivering signals in analog communications has been replaced by digital communications in most types of communications. The main reasons to shift from analog to digital is the improvement in the capacity, efficiency, and power consumption. Furthermore, digital communications can be sent directly using continuous digital bit stream or the bits can be grouped and transmitted in a packet of data [3]. Transmission in packet radio was first used in a network called ALOHANET that was developed in the University of

Hawaii. ALOHANET established the first set of protocols used in channel access and routing in packet radio system. The network in ALOHANET maintains a star network configuration and does not need a fixed infrastructure. This fact attracted the attention of the United States military in the 1970s and 1980s. Commercially, packet radio network did not have the same attraction; however, it is used in applications to support wireless data access such as emails and internet. The data rate of the wide area wireless data service was very low (20 kbps while its cost was high. Besides, the existence of the wired Ethernet that can carry data with a rate of 10 Mbps took over the market from the packet radio network technology. This technology replaced in the early 1990s by the Cellular network and the wireless local area network (WLAN).

1.2 Challenges to Wireless Systems

The cable-free advantage of the wireless communication systems over the wired telecommunications does not give the wireless systems attraction unless the wireless systems satisfy an accepted level of quality and affordability. In fact, for the wireless communications systems to compete with wired systems or at least to find a promising market they should enjoy some attracting features such as reliable service, light weight, a handheld size, small and long lasting battery, and affordable price [3]. Wireless communications has faced a lot of challenges that has inspired and is still inspiring the researchers in order to address them. One challenge is to make the handheld device small enough that can perform multiple tasks and operate in different modes supporting different applications at the same time. It is also desired to perform all these multiple tasks in a very conservative power consumptions [3]. Moreover, the wireless environment is not fixed and maintains a degree of randomness due to the moving objects around the communication nodes. Furthermore, the frequency bandwidth is limited and the wireless service demand is huge.

These challenges require the researchers in the signal processing field to find methods to process the signals in a power and spectrum efficient regime while maintaining a highly secure and reliable connectivity. In order to understand the difficulties and challenges facing wireless and signal processing, it is necessary to understand the environment and other factors affecting the transmission of a wireless signal.

The first phenomenon that faces any wireless signal transmitted from one point to another is the path loss. Path loss is the loss in the signal power as it is traveling over a distance. In the simplest case, the signal loses its power non-linearly with distance [5]. However, the existence of the noise in the receivers affects the quality of the signals randomly. Moreover, the signal itself faces blockers, reflectors, and scatterers which randomize it and make the detection of a signal difficult [6]. Another factor that affects the signal in its way from the transmitter to the receiver is the multipath. Due to the existence of different objects in between the transmitter and the receiver, the signal arrives at the receiver in multiple copies and different time delays. Also, variations in the signals due to movement at the mobile wireless devices; in addition, the wireless communications service is affected by the inter-symbol-interference that happens when two different signals interfere with each other causing the detection of the signal difficult [3].

A remarkable advancement in providing reliable wireless communication links was achieved in the late 1990s when a practical transmit diversity scheme was introduced by Siavash Alamouti [7]. This scheme, which is well-known as Alamouti scheme, has improved the reliability in wireless communication systems and has opened a wide area of research in signal processing and diversity [7]. Diversity is a technique to overcome the effects of multipath fading and shadowing. One type of diversity is the cooperation diversity which depends on cooperating devices (nodes) to relay a transmitted signal from a transmitter to a receiver. Our work in this thesis is continuing the research in this type of diversity techniques. More precisely, we aim to minimize the total perceived transmit power in a network that consists of two transceivers and a group of cooperating relay nodes in between.

1.3 Motivations

The use of wireless systems can be found in variety of applications such as in medical area [2] and industrial area [8]. In many cases, wireless nodes are distributed over a wide area in order to collect information [8]. Each node then needs to share its observations and data with another node, and it may happen that two nodes want to share some information with each other but they are in positions that do not allow them to communicate directly. Therefore, the cooperation diversity technique provide a method that allows these two nodes to communicate with each other when a group of wireless nodes in between cooperate to relay the signals of the two communicating nodes.

There are two relaying schemes in sense of the direction of the communications between two nodes. One-way relay network, in which a group of relays cooperate to relay the signal from one node to another in one direction, has gain a lot of attention and research [30], [14], and [13]. Two-way relaynetwork, where the signals of the two nodes are relayed at the same processing cycle, is of a current research interest. For example, the power minimization problem over two-way relay network was studied in [17]. Havary-Nassab and et al assumed that the perfect channel information over all the system are known. This work by Havary-Nassab et al has motivated the work in this thesis to tackle the problem again by relaxing the assumption and assuming that the channels between the second transceiver and the relay nodes are not available and the transceiver that perform the optimization calculation model them as Gaussian random vector with zero-mean and covariance of $\sigma_{f_2}^2$.

1.4 objective and Methodology

The objective of this thesis is to minimize the total transmit power over a two-way relay network adopting Amplify and forward technique. It is assumed that the channels between the transceiver that calculates the optimization and the relay nodes are available to the transceiver while the channels between the second transceiver and the relay nodes are model as Gaussian random vector with zero-mean and covariance of $\sigma_{f_2}^2$.

First, the system design is set up and the problem is formulated. Then the optimal weight vector that minimizes the total transmit power is found by finding the first derivative and the global minima. Finally, the mathematical result is examined through simulations.

The organization of the remaining of this thesis is as follow; the next chapter will discuss the literature review and elaborate on different approaches prior to our work that established the path to our work. We also discuss some similar works and explain the similarities and differences in details. In chapter three, we present how we modeled our data and our network description. In chapter four, we derive the optimal solution that minimizes the total perceived transmitted power. In chapter five, we present our data simulations and discuss our results. Some further studies are given in the Conclusions.

Chapter 2

Literature Review

In this chapter, we review the advancement and development in wireless communications and signal processing that led to the use of relay networks and discuss some previous approaches that tackle the problem of power minimization in relay networks. We track back the roots of relay networks, review some existing work which is similar to our studies, and show some advantages and limitations in each approach discussed. Then we review an important work that has a direct relation with the work presented in the thesis.

Diversity in its wide meaning is to have different people or things. A multi-cultural society has a diversity of people which in turn might lead to different ideas, opinions, and ways of thinking. In communications, diversity techniques are methods which help to deliver the message from one point to another point as clear as possible. In technical communications for example, a person can use his or her gestures as well as the speech to improve the communications [1]. In this simple example, speech is a channel to convey a message and the gesture is another independent channel to make sure that the message is delivered clearly. This is a diversity in everyday communications.

In wireless communications technology, the main purpose of using diversity is not far from the use in everyday communications. In fact, using diversity improves the quality of communications and increase the probability of conveying the message correctly.

Diversity in wireless technology means to use two or more independent channel to communicate a message. It is used to mitigate the effects of two phenomena that exist in wireless communications channel, which are multipath fading and shadowing. Multipath fading happens when a signal is transmitted as one signal and received as train of signals arriving at different times and different amplitudes at the receiver. The signals are transmitted in omnidirectional manner, and they travel in an environment with many objects and passes through different reflections. This makes the arrival of different copies of the signal vary in time causing interference to other arriving signals. The diversity that addresses the multipath fading is called microdiversity [3]. Shadowing, which is caused by people and objects blocking the path of the signals, is resolved by macrodiversity. Macrodiversity is performed by combining signals received by different access points such as two base stations in cellular systems [3] and thus requires some sort of coordination between these access points. Shadowing is beyond the scope of our work and therefore it is not addressed in this thesis.

2.1 Microdiversity

Microdiversity aims to provide multiple paths that can be utilized to communicate a particular message. The main characteristic that multipaths should have to provide diversity is the independent fading [7]. It is desired that every path should have an independent fading from the other paths to guarantee that if one path faces a deep fading problem the other paths can convey the message. Independent fading paths can be accomplished in different ways. One of the well-known methods is called space diversity, and it deploys multiple antennas that are well-spaced so that they maintain independent fading paths [3]. Another technique is by sending the message over different time slots. This technique that is known as time diversity requires the process of transmission to be repeated twice with the hope that the two transmissions would have independent fading paths [3]. Frequency diversity is also a method that can be used to have independent fading paths. Two different frequency bands face independent fading paths, and if one frequency failed to carry the signal, the other frequency band can convey the message [4]. Diversity is not exclusive to these methods and other methods such as coding [3]and space-time diversity presents in the literature [7].

Space diversity has gained special interest for its flexibility in the locations it can be employed. It can be accomplished in the transmitting side, the receiving side, both sides or in middle points as it will be shown later. When the diversity is accomplished in the receiver(e.g. having two receiving antennas) it is a receiver diversity. If the diversity technique is performed in the transmitter, it is called transmitter diversity. For example, When the transmitter uses two antennas to send a message over two independent paths, it materializes transmitter diversity. Diversity can be done in both sides at the same time if both sides are equipped with diversity mediums such as in the multiple input multiple output scheme. It can also be achieved through cooperation of some devices (e.g. a network of relays) in between two communicating nodes.

2.1.1 Receiver Diversity

Receiver diversity is a power efficient way to achieve diversity since it does not require additional transmission power. The omnidirectional antenna in the transmitter sends the signal in different directions and therefore it can be detected at different places using multiple antennas. When the receiver receives the multiple copy of the signal then different methods can be used to utilize these copies. One way of making use of this diversity is to pass only one signal to the decoding section. The receiver based on certain design parameters passes only one signal to the decoder while ignoring the rest of the images. For example, the receiver can pass the image with highest signal to noise ratio (SNR) [3]. SNR is the power of the signal compared to the power of the noise at the receiving side. Mathematically, it is the ratio of the signal power over the noise power, and it is one of the most important measurement parameters that judge the quality of the service in communications.

In order for the receiver to pass signal with the highest SNR to the decoding section, it requires the receiver to have a monitoring part that continuously monitor all receiving branches and choose the signal with the highest SNR. This also makes the switching from one branch to another is more often. When a level of quality is desired, another selection scheme called threshold combining can accomplish the selection process while the quality of service is maintained. The active branch in threshold combining remains active until the quality of service drops below a certain threshold then a branch with higher SNR is selected to pass the signal [3].

Moreover, the receiver can make use of all copies by coherently adding them together [4]. Combining the signals coherently requires the receiver to have some knowledge about the environment between the transmitter and the receiver. The impact of the environment between the transmitter and the receiver on the transmitted signal is known as the channel state information (CSI), which indicates the quality of the link between the transmitter and the receiver. Mathematically, CSI is modeled as a complex random variable and the signal is multiplied by it. The CSI can be known at the receivers through training signals. Moreover, the environment in one path from one node to another is similar to the environment backward from the second node to the first node, many studies assume that the channel state information enjoy a reciprocal behavior [23] and [17]. The assumption of having slow fading channels guarantees that the receiver estimation of the CSI is accurate and therefore it helps the receiver to process and decode the received signals.

Adding the signals coherently is done by adjusting the phase of each copy, and it is important since combining multiple signals with the same amplitude but with different phases can affect the quality of the signal severely, and might destroy the whole message. For further improvement in the over all SNR, the receiver can perform a maximal ratio combining. Maximal ratio combining not only adjusts the phase of the received copies of the signal but also weighs each signal differently so that the image with higher SNR plays a more significant role in the output, and the copy with lowest SNR plays a less significant role in the outcome. This method is usually used when the channel between the two communicating nodes does not vary quickly with time since the channel phase is required at the receiver [4].

2.1.2 Transmitter Diversity and MIMO Systems

Space diversity can also be applied in the transmitter side. This scheme requires that the transmitter to have at least the phase knowledge about the channels between the transmitter and the receiver [3]. The transmitter knowledge about the channels is essential since the transmitter has to beamform the signal at each antenna before the sending process. In signal processing, beamforming means to multiply the transmitted images by weights before the process of transmission [3]. The beamformer weights have mainly two tasks: They strengthen the signals to ensure the delivery at the desired quality, and adjust the phases of the signals to allow for coherently combining of the multiple copies at the receiver. This method is valid when the channels information is available at the transmitter since the transmitter uses this knowledge to obtain the phases of the beamforming weights.

When the transmitter lacks information of the channels, the best method to achieve diversity is to use Alamouti code. Alamouti code is named after Sivash Alamouti who introduced in 1997 a novel method to mitigate the effects of multipath fading specially for small devices such as cell phones [7]. The classical method before 1997 used to have a single antenna in the transmitting side and two or more antennas in the receiving side [7]. This method however, might not be practical for small devices since it requires two closely spaced antennas. Alamouti code suggests that it is possible to reverse the arrangement and instead of having a single antenna in the transmitter and two antennas in the receiver, we can have two antennas in the transmitter and one antenna (or more) in the receiver. This is assuming that the transmitter does not suffer from lack of space such as in the base station (BS)case, and overcome the problem of size in the receiver specially with handheld devices.

2.2 Cooperative Diversity

The trade off between spatial diversity and the size in the small devices has also been addressed using this approach. This approach was introduced and studied in [9] and [10]. They established for a new spatial diversity technique to overcome the limitations that are caused by the size in small devices such as mobile station (MS) in maintaining a space diversity. The proposed scheme in [9] suggests that each MS should be assigned with another MS to be its partner. Each partner should relay its partner data and send its own data to the receiver (assume it is a base station (BS)) simultaneously (see Fig 2.1).

This process is done in two steps; first, each MS sends its data to the BS as well as to the partner MS. Then both MSs send their next data along with the data received from the partner to the BS. The assumption in this proposal is that the data received at the MSs and at the BS are contaminated with noise. Moreover, the CSI between the partners are assumed to be known to each of them and the CSI between the MS and BS is known to the BS. The proposal was carried on for a slow relay fading so that the CSI between the two MS and BS does not change rapidly. The CSIs between the MSs and BS have three scenario from the MSs point of view; the amplitude and phase are known to the MS, the phase only is known to the MS, and no information is known to the MS. When both amplitude and phase are known to the MS, the MS can adjust its power allocation accordingly . The authors in [9] claim that the limitation of the power peak in the MS forces the MS to keep their peak power within a limited range. Moreover, keeping MS

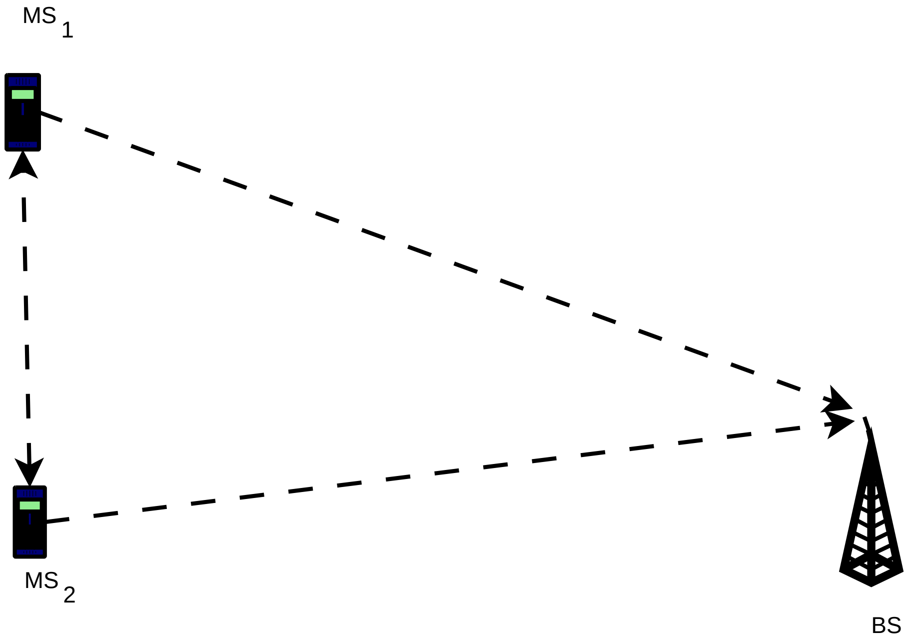

Fig. 2.1: Two MS cooperate to relay each other signal to the BS

as simple as possible is a desired design factor. Thus, this scenario was not considered. The second scenario is that when the MS knows the phase of the channel between the MS and BS. This allows the MS to send its signals so that it offsets the channel phase and thus the receiver can just add the received signals coherently. The phase information can be obtained using a feedback loop or if the system operates in time division duplex (TDD) mood. Considering that the phase knowledge is available to the MSs, Sendonaris and et al in [9] found that in the case when both MSs have the same channel quality between them and the BS, this cooperation scheme increases the data transmission rate of each MS equally as the channel status between the two MSs becomes stronger. In the case when the channel between one MS and the BS is better than the channel between the other MS and the BS, the MS with the stronger channel acts as a relay to the other MS. This scenario benefits the MS with a weaker channel the most, and if the other MS does not benefit, it is not affected from the data rate point of view. It is also found that this cooperative scheme decreases the total outage probability and increases the cellular coverage (in case of cellular system) comparing with no cooperation scheme.

The scheme in [9] focuses mainly on cellular systems and limited the cooperation by the assignment of partners. However, the idea of spatial diversity has been expanded to help improving other wireless systems such as ad hoc networks. There are many situations that when two nodes (in an ad hoc network, for example) want to communicate but the channel between them has a poor quality while there are some other nodes willing to cooperate and relay the signals between these two nodes (Fig. 2.2 shows a general one-way relay network). The cooperating relays volunteer to relay the signals, yet their participation should not consume a lot of their power specially when the cooperating relays are mobiles and have a limited battery life. Thus, methods to minimize the power either in the whole network or at the relays for different configurations have been studied in the literature, and we will try to review some of them in the coming two sections.

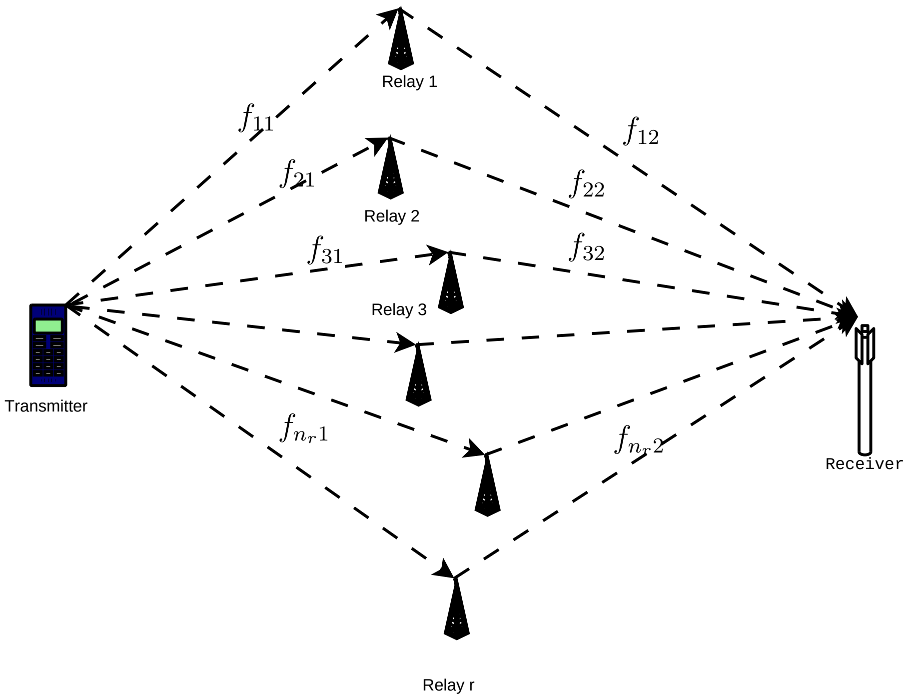

Fig. 2.2: One-way relay network

2.3 Relay Networks and Distributed Beamforming

The advancement in relaying techniques took multiple paths. In the simplest case a single transmitter, a single relay, and a single receiver can be considered as a relay network. For example, the network in [9] that was discussed in section (2.2) adopts this network configuration. One MS is a transmitter, the other MS is a relay and the BS is the receiver. The expansion in the network configurations evolves as the requirements of the network changes. A single-transmitter single-receiver pair equipped with multiple antennas with single relay that is also equipped with multiple antenna is considered in [23]. This configuration might have application in communications between large devices; however, in networks where each node is size-limited (e.g. MS) this configuration faces challenges with space limitation. Another network in [23] is a single relay node that is equipped with multiple antenna assists multiple transmitter-receiver pairs in which each transmitter and receiver is equipped with a single antenna. This scenario can be used in a network that has a large central node that can communicate with all other nodes. Berger et al in [23] also considered the case when multiple transmitters utilize multiple relay nodes to communicate with multiple receivers and all nodes in this scenario is equipped with a single antenna. This configuration provide more freedom for small devices to assist each other such as in large ad hoc network. Another network configuration is when a single antenna transmitter communicates with a single antenna receiver with the help of a group of single antenna relays in one way relaying scheme [14] or a pair of single antenna transceivers communicate with the help of a group of relays in the two way relaying scheme [17].

The relay nodes in [14] and [17] are distributed over a geographical area in a random pattern, and they relay the signal using a beamformer similar to the beamformer adopted in the transmitter diversity (see section 2.1.2). Therefore, the relaying scheme that deploys this method is called a distributed beamforming [18]. This scheme has potential benefits specially for a non-central network such as ad hoc networks. It provides a high spatial diversity since the number of distributed relays is considerably high. Moreover, the failure of a relay node does not affect the relaying process since the contribution of each node is small as compared with the rest of the relays. It might happen in some cases that the optimal solution to the distributed beamforming scenario leads to a relay selection criterion such as in the research approach in this thesis.

2.3.1 One-Way Relaying Scheme

The most time efficient one-way relaying scheme is accomplished in two steps. First, the transmitter sends the signal to the relay network. Then the relay network processes the transmitter signal images and sends them to the receiver. This process can be done in different approaches such as decode and forward (DF) and amplify and forward (AF). (DF) is desired when the noise at the relays is very high and amplifying the signals will amplify the noise also which degrades the performance [11]. Therefore, decoding the signals and forwarding them to the receiver avoids sending the relay noise along with the signal. However, This process consumes power in the processing and requires a more complex design in the relays [12]. Using AF, the relays just amplify the received signals and then send them to the receiver. The simplicity of AF implementation makes it more desirable when the noise at the relays is very low compared to the signal power [11]. Due to its implementation simplicity, AF scheme is adopted in this thesis.

The idea that for the relays to operate in their maximum transmission power was first raised by Yindi Jing and Hamid Jafarkhani [13] where a one-way amplify and forward relaying scheme (see Fig 2.2) was studied, and it was shown that the power of each relay can be at any level between zero to the maximum. Moreover, the power of each relay depends on the (CSI) of itself as well as the (CSI) of the other relays. In the optimal case, the relay with high noise power uses the minimum power to amplify its signal while the relay with lower noise power uses higher level of power to transmit the signal. This is because the amplifying in the relays amplifies both the desired signal power and relay noise power and therefore the desired SNR can be achieved by combining all the relayed signals in the receiver.

Jing and Jafarkhani in [13] assumed that the CSI of all channels are known at the receiver, and the CSIs of each relay are known to the relay itself. A more relaxed assumption was studied in [14] where it is assumed that the second order statistics of the CSI are known instead of the instantaneous information about the channel. It is found that the optimal relay beamforming weights leads to a closed-form solution if the total transmit power of the network is considered with SNR constraint. A closed form solution is also derived for the case when maximizing the SNR is desired and the total power is required to be under a certain level. For maximizing the SNR while the transmit powers of the individual relays are concerned, a quadratic programming solution is suggested in [14].

2.3.2 Two-way Relaying Schemes

The new focus of the relaying networks is directed toward two-way relaying schemes. In two-way relaying schemes, the relays cooperate to relay the signals between two nodes. In fact, the transmitter-receiver pair in the one-way relaying scheme is replaced by two transceivers that have the ability to send and receive signals. Moreover, the relaying schemes in two-way relaying networks can be performed in four steps, three steps, or two steps.

In four-step method (see Fig. 2.3), the first transceiver is active and sends its signal to the relays while the relays are listening and the second transceiver is also silent. In the second step, the first transceiver becomes silent, the second transceiver is listening, and the relays are sending the signal, which originally was received from the first transceiver, to the second transceiver. In the third step, the first transceiver remains silent, the second transceiver is sending its signal to the relays that are listening. In the last step, the first transceiver is listening, the second transceiver is silent and the relays are sending the signals that were originally received from the second transceiver. This scheme is desired

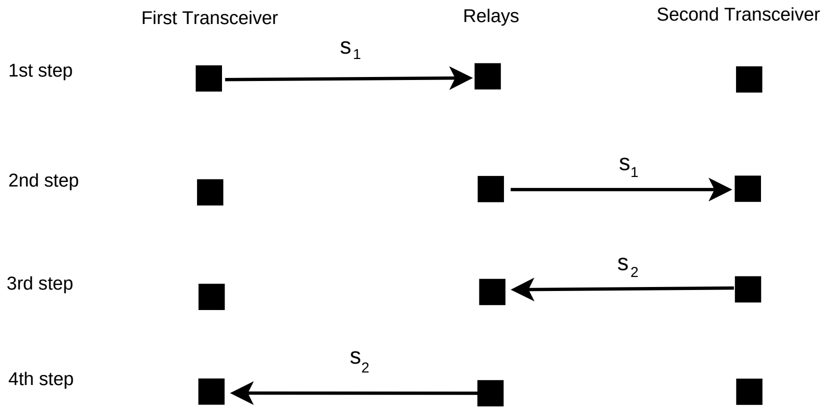

Fig. 2.3: Four-step two-way relaying scheme [17]

in the cases where reducing the interference is of primary concern; however, it is not time efficient.

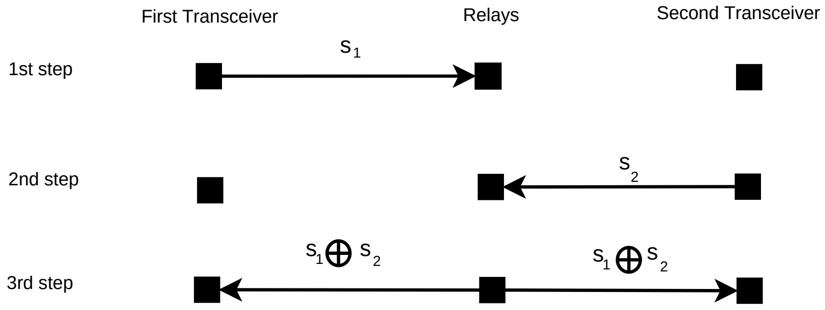

Fig. 2.4: Three-step two-way relaying scheme using time division broadcast technique [17]

In order to save one step, a three-step scheme such as the time division broadcast technique can be used (see Fig. 2.4). In this technique, the relays receive the signals from the first transceiver in the first step while the second transceiver is silent, then the second transceiver send its signal to the relays while the first transceiver is silent. In the third step, the relays send a logically XORed signal of both of the received signals. The transceivers then perform an XOR logic to detect the desired signal.

Four-step and three-step methods are not spectrally efficient compared to a two-step method such as multiple access broadcast channel(see Fig. 2.5). In this technique, the

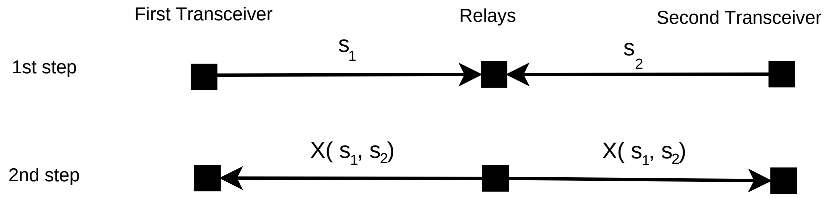

Fig. 2.5: Two-step two-way relaying scheme [17]

relays receive a superposition of the signals that are sent by both transceivers simultaneously in the first step. Then the relays send the amplified-and-phase-steered version of the signals to both receivers. Each transceiver then processes the signal and detects the desired signal. In this thesis, we consider a two-step technique.

A multi-antenna relay assisting a pair of transceivers was considered in [16], the communication process is accomplished using a two-step AF based scheme. First, both transceivers send their signals to the relay. Distorted copies of the transmitted signals from both transceivers are received in the relays simultaneously. The relays then multiply these signals by beamforming weights that strengthen the signals and adjust their phases. The processed signals are then re-transmitted to both transceivers in order for them to detect their corresponding desired signals. Zhang and et al in [16] discussed the optimal relay beamforming structure, and they found the beamformer that maximizes the weighted sum transmission rate subject to the relay processing power. The relay processing power is expressed as an $M \times M$ matrix where M is the number of antennas in the relay node. Let $\mathbf{A}$ be the relay beamforming matrix, $\mathbf{H}_{UL} = [\mathbf{h}_1, \mathbf{h}_2]$ is the uplink channel matrix and $\mathbf{h}_1$ is the channels vector between the relay node and first transceiver and $\mathbf{h}_2$ is the channels vector between the relay node and the second transceiver. Furthermore, it is assumed that the channels between the transceivers and the relays are reciprocal. Moreover, $\mathbf{H}_U L$ can be decomposed using singular-value-decomposition to be;

$$\mathbf{H}_{UL} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^* \tag{2.1}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary square matrices of size 2×2 and $n \times n$, respectively and $\mathbf{\Sigma}$ is a $2 \times n$ real-nonnegative-diagonal matrix with n equal to the number of channels between one of the transceivers and the relay. Then the optimal relay beamforming matrix is expressed mathematically as;

$$\mathbf{A} = \mathbf{U}^{*}\mathbf{B}\mathbf{U}^{H} \tag{2.2}$$

where $\mathbf{B}$ is a 2×2 unknown matrix. Therefore, the authors in [16] found that the number of unknown variables remains 4, which are the elements of the matrix $\mathbf{B}$, as the number of antennas in the relay node increases. Furthermore, for the case when the channels between one transceiver and the relay are perpendicular to the channels between the relay and the other transceiver, the matrix $\mathbf{B}$ has only two unknown elements as it is illustrated in the following matrix

$$\mathbf{B} = \begin{pmatrix} 0 & c \\ d & 0 \end{pmatrix}.$$

When $\mathbf{h}_1 = x\mathbf{h}_2$ for some constant x, the unknown elements in the matrix $\mathbf{B}$ is reduced to one, which is the element in the first-row-first-column while the rest of the elements are zeros.

Moreover, the optimal beamforming matrix that maximizes the transmission rate subject to keeping the relay processing power below a certain level was studied in [16]. It is found that the constraint in the formulated problem is convex, but the objective function itself is not a concave function. The solution to the problem is an algorithm that defines a maximum rate and minimum rate then adjust the controlling parameter in every iteration to reach the optimal beamformer. The authors in [16] studied also the optimal beamforming matrix that minimizes the relay processing power subject to keeping the SNRs in both transceivers above certain levels. The solution in this case leads to a semi definite programming algorithm.

Finally, the authors in [16] presented two sub-optimal but simpler solutions for the beamforming matrix **A** schemes, which are the maximum ratio reception-maximum ratio transmission (MRR-MRT) scheme and the zero-forcing-reception-zero-forcing transmission (ZFR-ZFT) scheme. The (MRR-MRT) maximizes the signal power transmitted from the relay to the transceivers, yet it does not mitigate the interference of the undesired signal. The (ZFR-ZFT) remove the interference between the two transmitted signals.

A network of pair transceivers and a group of relays was introduced in [17] (see Fig. 2.6). The quality of the direct channel linking the transceivers is assumed to be poor, and thus, direct communication cannot be established. The assistance by the group of relays enable each transceiver to communicate with the other transceiver. Havary-Nassab and et al in [17] adopts a two-step two-way relaying procedure based on AF relaying scheme.

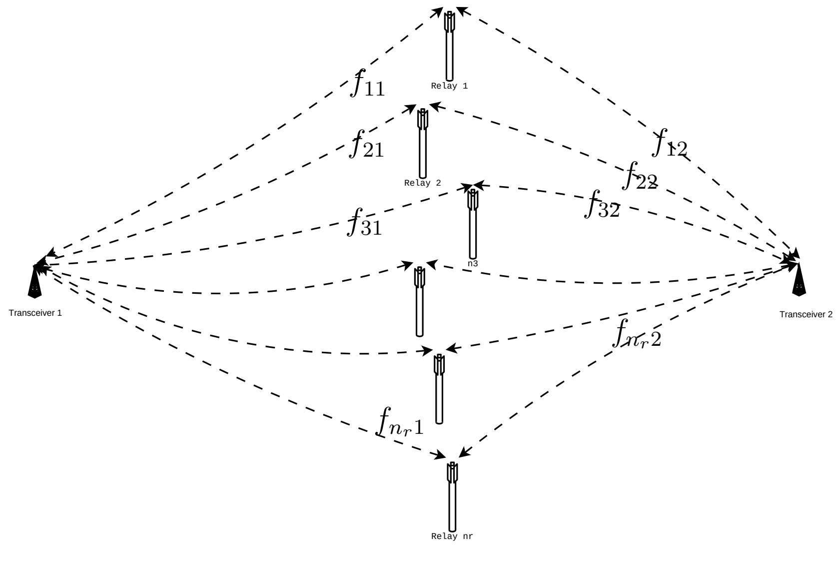

Fig. 2.6: Two-way relay network

The fact that adding coherent signals can improve the quality of the received signal helps to find the optimal beamformer that minimizes the transmit power over the network while the quality of service is kept above a certain level or maximizing the quality of service while keeping the total transmit power below a certain level [17].

Assuming perfect knowledge about the (CSI) in a flat fading scenario, Havary-Nassab and et al studied three aspects. First, the optimal beamforming weight vector was found to minimize the total transmit power in the network constrained to the quality of service to be kept above certain levels. Mathematically the objective function can be written as

$$\begin{aligned} \min_{\mathrm{P_1,P_2,\mathbf{w}}} \quad & \mathrm{P_T} \\ \text{subject to} \quad & \mathrm{SNR_1} \geq \gamma_1 \text{ and } \mathrm{SNR_2} \geq \gamma_2 \end{aligned} \tag{2.3}$$

where $\mathrm{P_1}$, $\mathrm{P_2}$, and $\mathrm{P_T}$ are Transceiver 1, Transceiver 2, and total transmit power, respectively, $\mathbf{w}$ is the beamforming vector weight, and γ_1 and γ_2 are the thresholds that the SNR of Transceiver 1 and Transceiver 2 should be above. The SNR constraints are expressed in [17] as follows

$$\mathrm{SNR_1} = \frac{\mathrm{P_2}\mathbf{f}_2^{\mathrm{H}}\mathbf{F}_1\mathbf{w}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{f}_2}{\sigma^2 + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w}} \tag{2.4}$$

$$\mathrm{SNR_2} = \frac{\mathrm{P_1}\mathbf{f}_2^{\mathrm{H}}\mathbf{F}_1\mathbf{w}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{f}_2}{\sigma^2 + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{F}_2\mathbf{F}_2^{\mathrm{H}}\mathbf{w}} \tag{2.5}$$

where $\mathbf{f}_1$, $\mathbf{F}_1$, $\mathbf{f}_2$, and $\mathbf{F}_2$ are the vector and matrices of the channels between Transceiver 1 and the relays and Transceiver 2 and the relays, respectively. When the total transmit power is minimum, $\mathrm{P_1}$, $\mathrm{P_2}$ can be expressed as functions of $\mathbf{w}$ since the inequalities turn out to be equality [17]. Therefore, the objective problem can be reduced to one equation with only one objective parameter which is $\mathbf{w}$. It is found in [17] that to reach the minimum total transmit power, the phase of the beamforming weight in each relay should be equal to the addition of the phases of the channel coefficients between the relay and transceiver one and two respectively. This will cancel the effects of phases of the two channels. The vector of the beamforming weight amplitudes can be calculated using the steepest decent algorithm.

It is also found that when the desired signal to noise ratios are equal in both transceivers (i.e., $\gamma_1 = \gamma_2$), the transmit power is divided into two equal shares at the optimal case. The first share is divided between the transceivers, and the second share is distributed over the relays.

In [17], the case when only the relays are power constraint and their transmission power need to be minimized is also investigated. That is, minimizing the relay power subject to keeping the quality of service above certain levels. It is found that this minimization can be solved using a second order convex cone programming approach, yet this method may not lead to global solution due to the existence of multiple minima.

Finally, Havary-Nassab and et al studied the method of improving the quality of service subject to keeping the transmit power below a certain level. It is found that for balanced SNRs (i.e $\text{SNR}_1 = \text{SNR}_2$) that both transceivers consume one half of the power while the rest is shared among the relays. This matches the results in the first approach (i.e. minimizing the power subject to SNR constraint). Similar to the results in the power minimization problem, the optimal beamformer can be obtained using a linear computational algorithm.

Two further studies about the network configuration in [17] were presented in [19] and [20]. In [19], the problem of minimizing the transmission power subject to keeping the SNRs in both transceivers above certain thresholds was reconsidered. It was noted in [17] that minimizing the total transmit power subject to keeping the SNRs above threshold does not depend on the change of the threshold SNR as far the sum of the two SNR thresholds is kept constant. In other words, if one of the SNR threshold is increased and the other is decreased while the sum of them are kept constant, the beamforming weight vector does not change. In [19], one of the SNR thresholds was set to be zero and the other was set to be the sum of both thresholds. This is equivalent to modeling the network to be one-way relaying scheme. If one intermediate parameter (e.g. the source power) is given, the solution to the problem can be obtained in a closed form. Similarly, in [20] the problem of balancing the SNRs subject to keeping the total transmit power below a certain level was reconsidered, and a semi-closed-form solution was obtained. The presented solution reduces the computational complexity. The method used in [20] adopts a bisection method to obtain the power of one of the transmitter and then the beamforming weight vector can be obtained in a closed-form solution.

In this thesis, we were inspired by the work in [17] as well as by the results of [33] where the CDF of the indefinite quadratic form in Gaussian random variables was obtained. However, it is interesting before we discuss the inspiring results in [33] to review some work about a relay selection scheme since our results lead to a relay selection scheme.

2.3.3 Relay Selection Scheme

Relay selection scheme is a an independent scheme itself that has been studied in the literature for different network configurations and under various constraints. Similar to the distributed beamforming method, the relay selection scheme is considered in both one-way relaying method [22] and two-way relaying method [15]. In this section we review some work in relay selection criterion for both one-way and two-way methods.

A one-way relay selection criterion was studied in [22], where a source-destination pair is assisted by group of distributed relays. The message from the source is delivered to the destination in two paths: the direct path from the source to the destination (i.e., there is no help from the relays), and the second path is by delivering the message from the source to the destination through the help of the best relay. The receiver then monitors the SNR from both links and passes the signal with the highest SNR. Kenan and Lok in [22] considered the optimal relay is the relay that is positioned so that it can deliver the message with the minimum outage given a total power constraint. The outage is defined as the fall in SNR below a certain level. In other words, the outage occurs when both SNR of the direct path and the SNR at any hop of the relaying path fall below the desired quality of service. The authors in [22] adopted a DF scheme assuming a Rayleigh fading and considered that the total transmit power as the sum of the source transmit power and the relay transmit power. It is also assumed that the relay has a fixed transmit power, and therefore, the only transmit power that needs to be minimized is the source transmit power. The source transmit power is a function of the source-destination, source-relay, and relay destination CSI. Moreover, the CSI is a function of the distance between each two nodes. Therefore, the optimal source transmit power is a function of the location of the three nodes. Kenan and Lok, found the optimal location of the relay in a one- dimensional case. However, the locations of the relays are not in one dimension with the source-destination pair in practice.

In [15], a two-way relay selection scheme was proposed. The network arrangement in [15] is the same as the network in [17] (see Fig. 2.4) as well as that in our work. Then the relay that maximizes the worst SNR is selected to relay the signals to both terminals. In order to amplify the signals, the selected relay multiplies the signals by a pre-defined weight. Jing in [15] studied the diversity order in this scenario and derived an upper bound to the average symbol error rate. Furthermore, a closed-form approximation to the block error rate was derived. The block transmission was defined in [15] as the two signals transmitted in both direction, and the block error rate calculates the error rate that occurs in both transceivers. It is found in [15] that this relay selection scheme which maximizes the worst SNR of both transceivers achieves full diversity.

2.3.4 The CDF of the Indefinite Quadratic Random Variables

As it was mentioned earlier, the work in this thesis was inspired by a result introduced in [33]. In fact, wireless communications and signal processing are mutually connected to randomness and stochastic processes. The wireless channels and the noise behavior follow random patterns, and thus, are modeled as random variables. This makes the advancement in the field of random variables and stochastic processes contribute to advancing the fields of wireless communications and signal processing. In [33], the distribution of indefinite quadratic form was studied. It is considered that the random variable of interest Y is a norm of Gaussian random vector $\mathbf{h}$ in the space of a square Hermitian matrix $\mathbf{A}$. This can be expressed mathematically as

$$Y = \|\mathbf{h}\|_{\mathbf{A}}^2 = \mathbf{h}^H \mathbf{A} \mathbf{h}. \tag{2.6}$$

It is found that in the case when $\mathbf{h}$ has zero mean and unit variance, the CDF of the Y can be expressed as

$$F_Y(y) = u(y) - \sum_{i=1}^{M} \frac{\lambda_i^M}{\prod_{l \neq i}(\lambda_i - \lambda_l)} \frac{1}{|\lambda_i|} \exp(\frac{-y}{\lambda_i}) u(\frac{y}{\lambda_i}) \tag{2.7}$$

where $u(\cdot)$ is the step function, which is equal to zero if the argument inside the brackets is negative and one otherwise λ_i is the ith non-zero eigenvalues of the Hermitian matrix $\mathbf{A}$ and M is the number of non-zero eigenvalues.

In the case of non-zero mean $\mathbf{h}$, the derivation becomes complicated and may not lead to a closed-form unless the eigenvalues of $\mathbf{A}$ are all equal. In our thesis, the Gaussian random vector $\mathbf{h}$ is modeled to have zero mean and therefore the closed form CDF of Y can be used in our problem.

In this chapter we discussed the purpose of using diversity techniques, and reviewed some existing diversity methods. We mentioned some diversity schemes that are used in the receiving side, transmitting side, and in both sides concurrently such as the MIMO systems. We then reviewed the cooperative spatial diversity and tracked its development. We discussed the one-way and two-way relay networks, and paid a special attention to the work in literature that deal with power allocation problems. Then we reviewed some relay selection schemes and discussed an important result about the indefinite quadratic form in Gaussian random variables. In the next chapter, we present the optimal solution to a two-way relay network with imperfect channel information.

Chapter 3

Distributed Beamforming in Two-Way Relay Network with Imperfect CSI

In this chapter, we tackle the problem of a two-way amplify and forward relaying network from another aspect. We consider the case when the optimum weights and power are calculated at one of the transceivers, and this transceiver has the perfect knowledge about the (CSI) between the transceiver itself and the relays while it has only the knowledge of the mean and variance about the channels between the relays and the other transceiver. This assumption relaxes the case in [17] since the instantaneous CSI about the channels between the relays and the other transceiver is not required. The transceiver itself can perform a training process to obtain the CSI between the transceiver itself and the relays, and it can estimate and adjust the variance of the channels between the relays and the other transceiver as it receives the desired signal. Note that we assumed a slow fading which implies that the channels is not changing too fast and therefore the estimation can be accomplished.

In the first section, we describe our network and formulate the objective problem. In section two, we derive the solutions that optimize the problem, discuss these solutions, and provide the practical and optimal solution to the problem.

3.1 Data Modeling

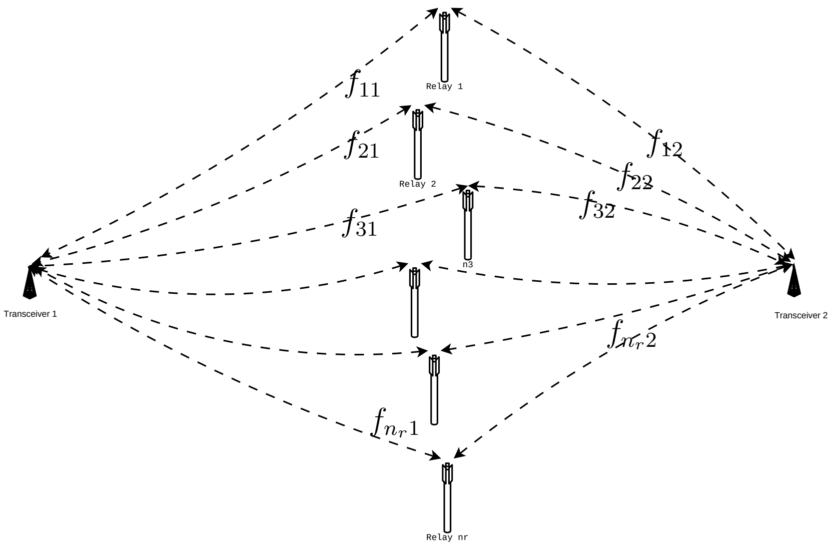

Fig. 3.1: Two-way relay network

In [17], it was considered that there are two wireless transceivers (Transceiver 1 and Transceiver 2) that cannot communicate with each other over a direct path due to the poor quality of the channel between them. Instead, they depend on n_r single-antenna relay nodes in order to communicate (see Fig. 3.1). The communication process is accomplished in two-step AF scheme. Assuming flat fading scenario, both transceivers send their signals to the relay nodes simultaneously. Let $\mathbf{x}$ represents the $n_r \times 1$ complex vector of the signal received at the relays, then it can be expressed as

$$\mathbf{x} = \sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \mathbf{v} \tag{3.1}$$

where P_1 and P_2 are the transmit power of Transceivers 1 and 2, respectively, s_1 and s_2 are the data of Transceiver 1 and 2, respectively $\mathbf{v}$ is the $n_r \times 1$ complex vector of the relay noise, and

$$\mathbf{f}_1 \triangleq [f_{11}\ f_{21}\ \dots\ f_{n_r 1}]^T$$

$$\mathbf{f}_2 \triangleq [f_{12}\ f_{22}\ \dots\ f_{n_r 2}]^T$$

are the vectors of the channel coefficients from/to the relays to/from the Transceivers 1 and 2, respectively, and f_{ij} is the reciprocal channel coefficient between the ith relay and the jth transceiver. It is assumed in [17] that $\mathbf{f}_1$ and $\mathbf{f}_2$ are known to both transceivers. In the second step, the ith relay transmits to both transceivers a scaled version of the received signal. The $n_r \times 1$ complex vector $\mathbf{t}$ retransmitted by the relay can be expressed as

$$\mathbf{t} = \mathbf{W}\mathbf{x} \tag{3.2}$$

where $\mathbf{W}$ is an $n_r \times n_r$ diagonal matrix with its ith diagonal entry equal to the complex weight w_i^* of the ith relay. The received signals y_1 and y_2 at Transceivers 1 and 2, can be respectively written as

$$y_1 = \mathbf{f}_1^T\mathbf{W}\mathbf{x} + n_1 = \mathbf{f}_1^T\mathbf{W}(\sqrt{P}_1\mathbf{f}_1 s1 + \sqrt{P}_2\mathbf{f}_2 s2 + \mathbf{v}) + n_1 \tag{3.3}$$

$$y_2 = \mathbf{f}_2^T\mathbf{W}\mathbf{x} + n_2 = \mathbf{f}_2^T\mathbf{W}(\sqrt{P}_1\mathbf{f}_1 s1 + \sqrt{P}_2\mathbf{f}_2 s2 + \mathbf{v}) + n_2 \tag{3.4}$$

where n_1 and n_2 are the received noise at Transceivers 1 and 2, respectively, and since $\mathbf{a}^T\mathrm{diag}(\mathbf{b}) = \mathbf{b}^T\mathrm{diag}(\mathbf{a})$, then (3.3) and (3.4) can be re-arranged as

$$y_1 = \sqrt{P}_1\mathbf{w}^H\mathbf{F}_1\mathbf{f}_1 s_1 + \sqrt{P}_2\mathbf{w}^H\mathbf{F}_1\mathbf{f}_2 s_2 + \mathbf{w}^H\mathbf{F}_1\mathbf{v} + n_1 \tag{3.5}$$

$$y_2 = \sqrt{P}_1\mathbf{w}^H\mathbf{F}_2\mathbf{f}_1 s_1 + \sqrt{P}_2\mathbf{w}^H\mathbf{F}_2\mathbf{f}_2 s_2 + \mathbf{w}^H\mathbf{F}_2\mathbf{v} + n_2 \tag{3.6}$$

where $\mathbf{F}_k \triangleq \mathrm{diag}(\mathbf{f}_k)$ for $k = 1, 2$ and $\mathbf{W} \triangleq \mathrm{diag}(\mathbf{w})$. In (3.5), the first term is well-known to Transceiver 1 since it depends on s_1, which has been transmitted by Transceiver 1 itself in the first step, while $\sqrt{P}_1\mathbf{F}_1\mathbf{f}_1$ are also known to Transceiver 1 and $\mathbf{w}$ is calculated at Transceiver 1 side and shared with the relays and Transceiver 2. Therefore, this term can be subtracted from y_1 and the residual signal can be processed at Transceiver 1.

Similarly, the second term in (3.6) is well-known to Transceiver 2 and can be subtracted from y_2 and the residual signal can be processed by Transceiver 2 and thus the residual signals are defined as

$$\tilde{y}_1 \triangleq y_1 - \sqrt{P}_1 \mathbf{w}^H \mathbf{F}_1 \mathbf{f}_1 s_1 = \sqrt{P}_2 \mathbf{w}^H \mathbf{F}_1 \mathbf{f}_2 s_2 + \mathbf{w}^H \mathbf{F}_1 \mathbf{v} + n_1 \tag{3.7}$$

$$\tilde{y}_2 \triangleq y_2 - \sqrt{P}_2 \mathbf{w}^H \mathbf{F}_2 \mathbf{f}_2 s_2 = \sqrt{P}_1 \mathbf{w}^H \mathbf{F}_1 \mathbf{f}_2 s_1 + \mathbf{w}^H \mathbf{F}_2 \mathbf{v} + n_2. \tag{3.8}$$

The first terms in the right hand sides of both (3.7) and (3.8) are the desired signal while the other terms are considered as noise. We assume $E(|s_1|^2) = E(|s_2|^2) = 1$ without loss of generality. We also assume that the noise process is zero-mean spatially white with variance σ^2 which means that $E(|n_1|^2) = E(|n_2|^2) = \sigma^2$ and $E(\mathbf{v}^H \mathbf{v}) = \sigma^2 \mathbf{I}$.

In [17], the optimization problem was

$$\begin{aligned} &\min_{P_1, P_2, \mathbf{w}} \ \mathrm{P_T} \\ &\text{subject to} \qquad \mathrm{SNR}_1 \geq \gamma_1 \text{ and } \mathrm{SNR}_2 \geq \gamma_2 \end{aligned} \tag{3.9}$$

$\mathrm{P_T}$ is the total transmit power in the network. SNR_1 and SNR_2 are the signal to noise ratio at Transceiver 1 and Transceiver 2 respectively, and γ_1, γ_2 are the threshold that SNR_1 and SNR_2 should be above. The SNR is defined as the ratio of the desired signal power to the noise power at the receiving transceiver. Therefore, using (3.7) and (3.8), the SNRs can be written as;

$$\mathrm{SNR}_1 = \frac{\mathrm{P}_2 \mathbf{f}_2^{\mathrm{H}} \mathbf{F}_1^{\mathrm{H}} \mathbf{w} \mathbf{w}^{\mathrm{H}} \mathbf{F}_1 \mathbf{f}_2}{\sigma^2 + \sigma^2 \mathbf{w}^{\mathrm{H}} \mathbf{F}_1 \mathbf{F}_1^{\mathrm{H}} \mathbf{w}} \tag{3.10}$$

$$\mathrm{SNR}_2 = \frac{\mathrm{P}_1 \mathbf{f}_2^{\mathrm{H}} \mathbf{F}_1^{\mathrm{H}} \mathbf{w} \mathbf{w}^{\mathrm{H}} \mathbf{F}_1 \mathbf{f}_2}{\sigma^2 + \sigma^2 \mathbf{w}^{\mathrm{H}} \mathbf{F}_2 \mathbf{F}_2^{\mathrm{H}} \mathbf{w}} \tag{3.11}$$

where has been assumed that $E(\mathbf{v}\mathbf{v}^H) = \sigma^2 \mathbf{I}$ and $E(|s_k|^2) = 1$, for $k = 1, 2$. It is also assumed that $\mathbf{v}$, s_1 and s_2 are all zero-mean mutually independent random variables. The total power P_T can be expressed as;

$$\mathrm{P_T} = \mathrm{P_1} + \mathrm{P_2} + \mathrm{P_r} \tag{3.12}$$

where P_r is the relay transmit power;

$$\mathrm{P_r} = \mathrm{E}(\mathbf{t}^{\mathrm{H}}\mathbf{t}) = \mathbf{w}^{\mathrm{H}}\mathrm{E}(\mathbf{X}^{\mathrm{H}}\mathbf{X})\mathbf{w} \tag{3.13}$$

and $\mathbf{X} \triangleq \mathrm{diag}(\mathbf{x})$. Using (3.1), the relay transmit power becomes;

$$\mathrm{P_r} = \mathrm{P_1}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w} + \mathrm{P_2}\mathbf{w}^{\mathrm{H}}\mathbf{F}_2\mathbf{F}_2^{\mathrm{H}}\mathbf{w} + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w}. \tag{3.14}$$

The total power then can be written as

$$\mathrm{P_T} = \mathrm{P_1} + \mathrm{P_2} + \mathrm{P_1}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w} + \mathrm{P_2}\mathbf{w}^{\mathrm{H}}\mathbf{F}_2\mathbf{F}_2^{\mathrm{H}}\mathbf{w} + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w}. \tag{3.15}$$

Using (3.10), (3.11), and (3.12) the optimization problem in (3.9) becomes;

$$\begin{aligned} \min_{\mathrm{P_1},\mathrm{P_2},\mathbf{w}} \quad & \mathrm{P_1} + \mathrm{P_2} + \mathrm{P_1}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w} + \mathrm{P_2}\mathbf{w}^{\mathrm{H}}\mathbf{F}_2\mathbf{F}_2^{\mathrm{H}}\mathbf{w} + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w} \\ \text{subject to} \quad & \frac{\mathrm{P_2}\mathbf{f}_2^{\mathrm{H}}\mathbf{F}_1^{\mathrm{H}}\mathbf{w}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{f}_2}{\sigma^2 + \sigma^2\mathbf{w}^{H}\mathbf{F}_1\mathbf{F}_1^{H}\mathbf{w}} \geq \gamma_1 \\ & \frac{\mathrm{P_1}\mathbf{f}_2^{\mathrm{H}}\mathbf{F}_1^{\mathrm{H}}\mathbf{w}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{f}_2}{\sigma^2 + \sigma^2\mathbf{w}^{H}\mathbf{F}_2\mathbf{F}_2^{H}\mathbf{w}} \geq \gamma_2 \end{aligned} \tag{3.16}$$

In our work, we relaxed the assumption and instead of assuming that both transceivers know $\mathbf{f}_1$ and $\mathbf{f}_2$, we assumed that Transceiver 1, which is the transceiver that calculates the optimization, knows only $\mathbf{f}_1$ and the variance of $\mathbf{f}_2$. In other words, we assume that Transceiver 1 knows the channels between itself and the relay nodes as well as the probability density function of the channels between Transceiver 2 and the relays as $\mathbf{f}_2 \sim \mathcal{N}(\underline{0}, \sigma_{f_2}^2)$. In this case, finding the optimal solution that minimizes the total transmit power in the network is hard if not impossible since the total power requires that $\mathbf{f}_2$ to be known at Transceiver 1. Thus, we find the optimum solution that minimizes the total perceived power as it is seen by Transceiver 1 by applying the expected value with respect to $\mathbf{f}_2$ to the total power. Moreover, instead of finding the minimum perceived transmit power subject to the SNRs to be above a certain level, we make use of the result in [33], and aim to find the optimal solution that minimizes the total perceived power subject to a soft constraint which guarantees that the outage probability is below a certain level.

An outage occurs when the SNR falls below a certain level. Mathematically we aim to solve the following optimization problem;

$$\begin{aligned} \min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & E_{f_2}(\mathrm{P_T}) \\ \text{subject to} \quad & Pr(\mathrm{SNR}_1 \geq \gamma_1) \geq \alpha_1 \text{ and } \mathrm{Pr}(\mathrm{SNR}_2 \geq \gamma_2) \geq \alpha_2 \end{aligned} \tag{3.17}$$

In the next section we derive the optimal solution to our objective function.

3.2 POWER MINIMIZATION

In this section, we first substitute the parameters of the objective function. Then we solve the optimization problem using a derivative based optimization approach. Finally, we find the optimal weight vector that minimizes the total transmit power over the system.

Using (3.10), (3.11), and (3.12) the optimization problem in (3.17) becomes;

$$\begin{aligned} \min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{P}_1 \mathbf{w}^{\mathrm{H}} \mathbf{F}_1 \mathbf{F}_1^{\mathrm{H}} \mathbf{w} + \mathrm{P}_2 \sigma_{\mathrm{f}_2}^2 \mathbf{w}^{\mathrm{H}} \mathbf{w} + \sigma^2 \mathbf{w}^{\mathrm{H}} \mathbf{w} \qquad (3.18) \\ \text{subject to} \quad & Pr\Big(\frac{\mathrm{P}_2 \mathbf{f}_2^{\mathrm{H}} \mathbf{F}_1 \mathbf{w} \mathbf{w}^{\mathrm{H}} \mathbf{F}_1 \mathbf{f}_2}{\sigma^2 + \sigma^2 \mathbf{w}^H \mathbf{F}_1 \mathbf{F}_1^H \mathbf{w}} \geq \gamma_1\Big) \geq \alpha_1 \\ & Pr\Big(\frac{\mathrm{P}_1 \mathbf{f}_2^{\mathrm{H}} \mathbf{F}_1 \mathbf{w} \mathbf{w}^{\mathrm{H}} \mathbf{F}_1 \mathbf{f}_2}{\sigma^2 + \sigma^2 \mathbf{w}^H \mathbf{F}_2 \mathbf{F}_2^H \mathbf{w}} \geq \gamma_2\Big) \geq \alpha_2 \end{aligned}$$

The instantaneous noise power at Transceiver 2 cannot be calculated at Transceiver 1 since the only available information to Transceiver 1 about $\mathbf{f}_2$ is the probability density function. Thus, we considered the perceived SNR in the second constraint. The perceived SNR is the signal power to the average noise power ratio. We will show in the simulation results section that this assumption guarantees the satisfaction of both the perceived SNR as well as the true SNR. When we change the second constraint in (3.18) to consider the average instead of instantaneous noise power, it becomes

$$\begin{aligned}
\min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{P}_1\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w} + \mathrm{P}_2\sigma_{\mathrm{f}_2}^2\mathbf{w}^{\mathrm{H}}\mathbf{w} + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w} \\
\text{subject to} \quad & Pr(\frac{\mathrm{P}_2\mathbf{f}_2^{\mathrm{H}}\mathbf{F}_1\mathbf{w}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{f}_2}{\sigma^2 + \sigma^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}} \geq \gamma_1) \geq \alpha_1 \\
& Pr(\frac{\mathrm{P}_1\mathbf{f}_2^{\mathrm{H}}\mathbf{F}_1\mathbf{w}\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{f}_2}{\sigma^2 + \sigma^2 E_{f_2}(\mathbf{w}^H\mathbf{F}_2\mathbf{F}_2^H\mathbf{w})} \geq \gamma_2) \geq \alpha_2.
\end{aligned} \tag{3.19}$$

Re-arranging (3.19);

$$\begin{aligned}
\min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{P}_1\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w} + \mathrm{P}_2\sigma_{\mathrm{f}_2}^2\mathbf{w}^{\mathrm{H}}\mathbf{w} + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w} \\
\text{subject to} \quad & Pr(\mathbf{f}_2^H\mathbf{F}_1^H\mathbf{w}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_2 \geq \frac{\sigma^2\gamma_1}{\mathrm{P}_2}(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})) \geq \alpha_1 \\
& Pr(\mathbf{f}_2^H\mathbf{F}_1^H\mathbf{w}\mathbf{w}^H\mathbf{F}_1\mathbf{f}_2 \geq \frac{\sigma^2\gamma_2}{\mathrm{P}_1}(1 + \sigma_{f_2}^2\mathbf{w}^H\mathbf{w})) \geq \alpha_2.
\end{aligned} \tag{3.20}$$

Let $\check{\mathbf{f}}_2 = \frac{\mathbf{f}_2}{\sigma_{f_2}}$, then (3.20) becomes

$$\begin{aligned}
\min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{P}_1\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w} + \mathrm{P}_2\sigma_{\mathrm{f}_2}^2\mathbf{w}^{\mathrm{H}}\mathbf{w} + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w} \\
\text{subject to} \quad & Pr(\sigma_{f_2}^2\check{\mathbf{f}}_2^H\mathbf{F}_1^H\mathbf{w}\mathbf{w}^H\mathbf{F}_1\check{\mathbf{f}}_2 \geq \frac{\sigma^2\gamma_1}{\mathrm{P}_2}(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})) \geq \alpha_1 \\
& Pr(\sigma_{f_2}^2\check{\mathbf{f}}_2^H\mathbf{F}_1^H\mathbf{w}\mathbf{w}^H\mathbf{F}_1\check{\mathbf{f}}_2 \geq \frac{\sigma^2\gamma_2}{\mathrm{P}_1}(1 + \sigma_{f_2}^2\mathbf{w}^H\mathbf{w})) \geq \alpha_2
\end{aligned} \tag{3.21}$$

In (3.21), $\sigma_{f_2}^2\mathbf{F}_1^H\mathbf{w}\mathbf{w}^H\mathbf{F}_1$ is a Hermitian matrix and $\check{\mathbf{f}}_2\,\tilde{}\,\mathcal{N}(0,\mathbf{I})$. Evaluating the CDF of the two constraints using the method presented in [33](see section 2.5);

$$\begin{aligned}
\min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{P}_1\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w} + \mathrm{P}_2\sigma_{\mathrm{f}_2}^2\mathbf{w}^{\mathrm{H}}\mathbf{w} + \sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w} \\
\text{subject to} \quad & 1 - u(\frac{\sigma^2\gamma_1}{\mathrm{P}_2}(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})) + \\
& \exp(\frac{-\frac{\sigma^2\gamma_1}{\mathrm{P}_2}(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}})u(\frac{\frac{\sigma^2\gamma_1}{\mathrm{P}_2}(1 + \mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}) \geq \alpha_1 \\
& 1 - u(\frac{\sigma^2\gamma_2}{\mathrm{P}_1}(1 + \sigma_{f_2}^2\mathbf{w}^H\mathbf{w})) + \\
& \exp(-\frac{\frac{\sigma^2\gamma_2}{\mathrm{P}_1}(1 + \sigma_{f_2}^2\mathbf{w}^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}})u(\frac{\frac{\sigma^2\gamma_2}{\mathrm{P}_1}(1 + \sigma_{f_2}^2\mathbf{w}^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}) \geq \alpha_2
\end{aligned} \tag{3.22}$$

$\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}$ is in quadratic form and hence it is always non-negative. P_1, P_2, σ^2, γ_1, γ_2 and $\sigma_{f_2}^2$ are also non-negative terms and thus, the arguments inside the step functions

$u(\cdot)$ in 3.22 are always nonnegative values and thus they are always equal to one. 3.22 then can be rewritten as;

$$\begin{aligned} \min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & \mathrm{P}_1+\mathrm{P}_2+\mathrm{P}_1\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w}+\mathrm{P}_2\sigma_{\mathrm{f}_2}^2\mathbf{w}^{\mathrm{H}}\mathbf{w}+\sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w} \\ \text{subject to} \quad & \exp\left(-\frac{\frac{\sigma^2\gamma_1}{\mathrm{P}_2}(1+\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}\right)\geq\alpha_1 \\ & \exp\left(-\frac{\frac{\sigma^2\gamma_2}{\mathrm{P}_1}(1+\sigma_{f_2}^2\mathbf{w}^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}\right)\geq\alpha_2 \end{aligned} \tag{3.23}$$

equivalently;

$$\begin{aligned} \min_{\mathrm{P}_1,\mathrm{P}_2,\mathbf{w}} \quad & \mathrm{P}_1+\mathrm{P}_2+\mathrm{P}_1\mathbf{w}^{\mathrm{H}}\mathbf{F}_1\mathbf{F}_1^{\mathrm{H}}\mathbf{w}+\mathrm{P}_2\sigma_{\mathrm{f}_2}^2\mathbf{w}^{\mathrm{H}}\mathbf{w}+\sigma^2\mathbf{w}^{\mathrm{H}}\mathbf{w} \\ \text{subject to} \quad & B_1\frac{(1+\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}\leq\mathrm{P}_2 \\ & B_2\frac{(1+\sigma_{f_2}^2\mathbf{w}^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}\leq\mathrm{P}_1 \end{aligned} \tag{3.24}$$

where $B_1=-\frac{\sigma^2\gamma_1}{\ln\alpha_1}$ and $B_2=-\frac{\sigma^2\gamma_2}{\ln\alpha_2}$. The two constraints in 3.24 should satisfy with equality at the optimum scenario. If we assume that one of the constraint (e.g. the first constraint) satisfy with the inequality at the optimum scenario, then we can scale down P_2 until it reach the equality;hence, a further minimization to P_2 and in turn to the total power which makes the inequality contradict with the optimality. Thus, the optimization problem turns to be

$$\begin{aligned} \min_{\mathbf{w}} \quad & B_2\frac{(1+\sigma_{f_2}^2\mathbf{w}^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}+B_1\frac{(1+\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}+B_2\frac{(1+\sigma_{f_2}^2\mathbf{w}^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w} \\ & +B_1\frac{(1+\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}\sigma_{f_2}^2\mathbf{w}^H\mathbf{w}+\sigma^2\mathbf{w}^H\mathbf{w}. \end{aligned} \tag{3.25}$$

re-arranging 3.25;

$$\min_{\mathbf{w}}(B_1+B_2)(1+\sigma_{f_2}^2\mathbf{w}^H\mathbf{w})\left(\frac{1}{\sigma_{f_2}^2\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}+\frac{1}{\sigma_{f_2}^2}\right)+\sigma^2\mathbf{w}^H\mathbf{w} \tag{3.26}$$

Taking the derivative of 3.26 with respect to $\mathbf{w}^H$ and equating to zero yields

$$\begin{aligned} & (B_1+B_2)\left[(\mathbf{w})\left(\frac{1}{\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}+1\right)-(1+\sigma_{f_2}^2\mathbf{w}^H\mathbf{w})\frac{\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}}{\sigma_{f_2}^2(\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})^2}\right] \\ & +\sigma^2\mathbf{w}=0. \end{aligned} \tag{3.27}$$

or equivalently

$$[(1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1})(\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})^2+(\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})\mathbf{I}-\\(\frac{1}{\sigma_{f_2}^2}+\mathbf{w}^H\mathbf{w})\mathbf{F}_1\mathbf{F}_1^H]\mathbf{w}=0. \tag{3.28}$$

Let $\mathbf{D}\triangleq(1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1})(\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})^2+(\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w})\mathbf{I}-(\frac{1}{\sigma_{f_2}^2}+\mathbf{w}^H\mathbf{w})\mathbf{F}_1\mathbf{F}_1^H$ then for 3.28 to have a non trivial solution, the rank of $\mathbf{D}$ has to drop at least by one. For $\mathbf{D}$ to drop rank by one, one of its diagonal entries has to be zero, and the corresponding entry of $\mathbf{w}$ is not zero. Assume that the ith diagonal element of $\mathbf{D}$ is zero then,

$$(1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1})|w_i|^4|f_{1i}|^4+|w_i|^2|f_{1i}|^2-|w_i|^2|f_{1i}|^2-\frac{|f_{1i}|^2}{\sigma_{f_2}^2}=0. \tag{3.29}$$

Solving 3.29 for $|w_i|^2$, we obtain that

$$|w_i|^2=\frac{1}{\sqrt{f_{1i}f_{1i}^H\sigma_{f_2}^2(1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1})}}. \tag{3.30}$$

Substituting 3.30 into the minimization problem 3.25, $\mathrm{P_T}$ can be written as

$$\mathrm{P_T}=(\mathrm{B_1}+\mathrm{B_2})\left(1+\sigma_{\mathrm{f_2}}^2\frac{1}{\sqrt{\mathrm{f_{1i}f_{1i}^H}\sigma_{\mathrm{f_2}}^2(1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1})}}\right)\left(\sqrt{\frac{(1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1})}{\sigma_{\mathrm{f_2}}^2\mathrm{f_{1i}f_{1i}^H}}}+\frac{1}{\sigma_{\mathrm{f_2}}^2}\right)\\+\sigma^2\frac{1}{\sqrt{f_{1i}f_{1i}^H\sigma_{f_2}^2(1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1})}} \tag{3.31}$$

Therefore, this solution leads to a relay selective criterion and the relay with maximum channel coefficientf_1 should be active.

It may happen that the rank of $\mathbf{D}$ drops by more than one. Then two or more of the diagonal entries in $\mathbf{D}$ are equal to zero and their correspondent elements in $\mathbf{w}$ can be non-zero. Let $c=1-\frac{\ln\alpha_1\ln\alpha_2}{\gamma_1\ln\alpha_2+\gamma_2\ln\alpha_1}$, $z=\mathbf{w}^H\mathbf{F}_1\mathbf{F}_1^H\mathbf{w}$, and $y=\frac{1}{\sigma_{f_2}^2}+\mathbf{w}^H\mathbf{w}$ and assume that $w_i\neq 0$ and $w_j\neq 0$ for $i\neq j$, then (3.32) the non-zero elements of $\mathbf{w}$ should satisfy the following two equations

$$(cz^2+z-y|f_{1i}|^2)w_i=0 \tag{3.32}$$

$$(cz^2 + z - y|f_{1j}|^2)w_j = 0. \tag{3.33}$$

Solving for $|f_{1i}|$ and $|f_{1j}|$ in both equations, we obtain

$$|f_{1i}|^2 = \frac{cz^2 + z}{y} \tag{3.34}$$

$$|f_{1j}|^2 = \frac{cz^2 + z}{y}. \tag{3.35}$$

Therefore $|f_{1i}|^2 = |f_{1j}|^2$ for all $i \neq j$ correspond to non zero $\mathbf{w}$ elements. This simplifies 3.28 as

$$\begin{aligned} &|f_1|^2 \sum_{n=1}^{N} |w_n|^2 - (\frac{|f_1|^2}{\sigma_{f_2}^2} + |f_1|^2 \sum_{n=1}^{N} |w_n|^2) + (|f_1|^4 (\sum_{n=1}^{N} |w_n|^2)^2 \\ &\times (1 - \frac{\ln \alpha_1 \ln \alpha_2}{\gamma_1 \ln \alpha_2 + \gamma_2 \ln \alpha_1}) = 0 \end{aligned} \tag{3.36}$$

where $|f_1|$ is the magnitude of one of the equal norm channel coefficients, N is the total number of channels that have equal magnitude, and $\sum_{n=1}^{N} |w_n|^2$ is the summation of the squared norm of the weighting factors corresponding to the channel coefficients that are equal. Rearranging 3.36;

$$(1 - \frac{\ln \alpha_1 \ln \alpha_2}{\gamma_1 \ln \alpha_2 + \gamma_2 \ln \alpha_1})(|f_1|^2 (\sum_{n=1}^{N} |w_n|^2)^2 - \frac{1}{\sigma_{f_2}^2} = 0 \tag{3.37}$$

or

$$\sum_{n=1}^{N} |w_n|^2 = \frac{1}{\sqrt{|f_1|^2 \sigma_{f_2}^2 (1 - \frac{\ln \alpha_1 \ln \alpha_2}{\gamma_1 \ln \alpha_2 + \gamma_2 \ln \alpha_1})}}. \tag{3.38}$$

Substituting 3.38 into 3.25

$$\begin{aligned} \mathrm{P_T} = \quad & (B_1 + B_2)(1 + \sigma_{f_2}^2 \frac{1}{\sqrt{|f_1|^2 \sigma_{f_2}^2 (1 - \frac{\ln \alpha_1 \ln \alpha_2}{\gamma_1 \ln \alpha_2 + \gamma_2 \ln \alpha_1})}})(\sqrt{\frac{(1 - \frac{\ln \alpha_1 \ln \alpha_2}{\gamma_1 \ln \alpha_2 + \gamma_2 \ln \alpha_1})}{\sigma_{f_2}^2 |f_1|^2}} + \frac{1}{\sigma_{f_2}^2}) \\ & + \sigma^2 \frac{1}{\sqrt{|f_1|^2 \sigma_{f_2}^2 (1 - \frac{\ln \alpha_1 \ln \alpha_2}{\gamma_1 \ln \alpha_2 + \gamma_2 \ln \alpha_1})}}. \end{aligned} \tag{3.39}$$

This solution implies that for each $i \neq j$ the channel coefficient between the ith active relay and Transceiver 1 has to be equal to the channel coefficient between the jth active relay and Transceiver 1. This is not the case in general and this group of relays selection is not practical and therefore the only practical solution leads to the single relay selection scheme.

Therefore, the optimal solution that minimizes the total transmit power subject to SNR constraint with certain outage probability in two-way relay beamforming network turns to be a relay selection criterion. The relay that enjoys the highest channel amplitude between Transceiver 1 and the relay itself is the best to relay the signals between the two transceivers.

In this chapter, we have formulated our optimization problem, and we have showed the difference between our work and a previous work was presented in [17]. Then we derived the optimization solution and showed how this solution led to a relay selection criterion. In the next chapter, we will discuss our simulation results.

Chapter 4

Simulation Results

In this chapter, we discuss our simulation results and test our theoretical results whether or not they meet the conditions given during the formulation of the problem. We utilized Matlab programming package to test our theoretical results, and used Monte carlo procedure to average our results over sufficient number of runs to reach an interval of confidence above 95 percent. We assumed a network of two transceivers and 10 relays, and generated the channels between Transceiver 1 and the relays as Gaussian random variables with zero-mean and variance of 0 dB. The noise at both transceivers and the relays were also generated as independent zero mean Gaussian random variables with variance of 0 dB over noise level. The variance of the channels between Transceiver 2 and the relays was set to the variable $\sigma_{f_2}^2$ since in some tests we checked the changes in the total power as the channels between Transceiver 2 and the relays changes. The Matlab code calculates the optimal weight and finds the optimal relay and then calculates the transmit power accordingly. In Section 4.1, we show that our theoretical results meet the given parameters. More specifically, we show that relaying the signals with the optimal total power meets the desired SNR for the given outage probability. In Section 4.2, We then discuss the change in the transmission power as the variance of the channels between Transceiver 2 and the relays change while the other parameters are kept fixed.

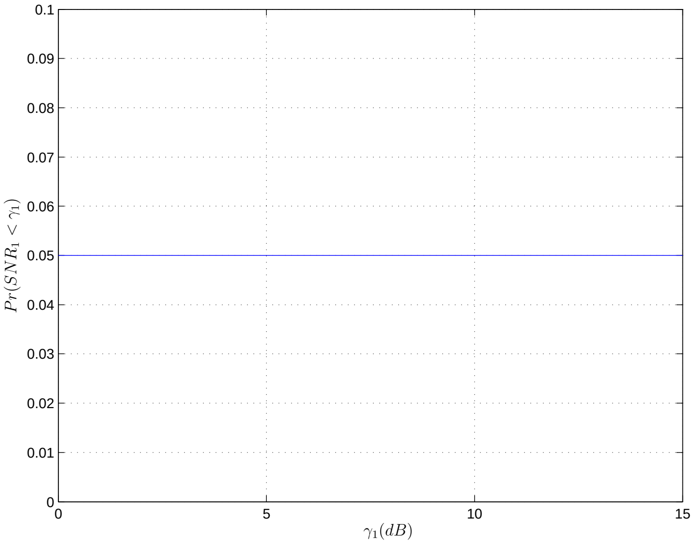

Fig. 4.1: The outage probability at Transceiver 1 for desired outage probability less than 5 percent

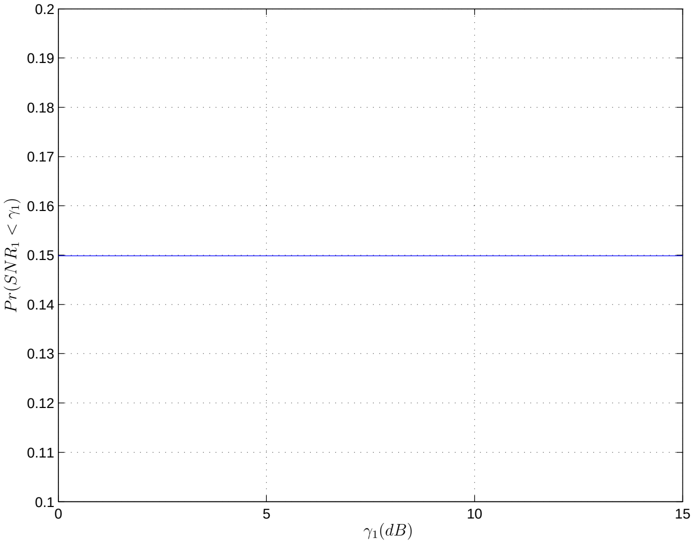

Fig. 4.2: The outage probability at Transceiver 1 for desired outage probability less than 15 percent

In Section 4.3, we discuss the changes at the total transmission power while the desired SNRs are changing and the rest of the parameters are kept fixed.

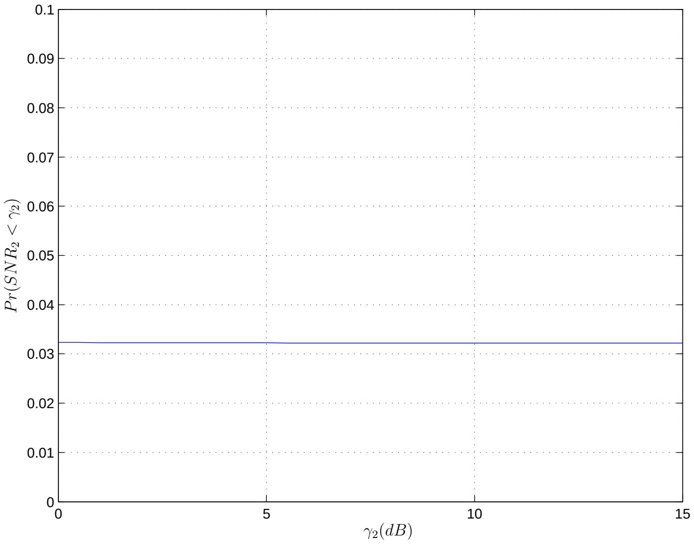

Fig. 4.3: The true outage probability at Transceiver 2 for desired outage less than 5 percent

4.1 Outage Satisfaction Tests

In this section we ran the code 150000 times to ensure that the outage probability is averaged properly. The average of the outage probability may not be precise with few runs specially when the SNR is high. We first checked the outage probability at Transceiver 1 as the desired SNR increases. The variance of the channels between Transceiver 2 and the relays was kept at 0 dB while we set the outage probability to be below 5 percent. As it is shown in Fig. 4.1, the outage probability is constant below 5 percent as the SNR increases. In order to make sure that the system works as the system parameters changes, we ran the program for $\sigma_{f_2}^2 = 0$ dB and the outage probability to be 15 percent and the results satisfied the conditions (see Fig. 4.2) which makes sure that our optimization is valid.

We then checked the true outage probability at Transceiver 2 and compared it to the perceived outage probability at Transceiver 2. For the true outage probability we generated the channels between Transceiver 2 and the relays as Gaussian random numbers with

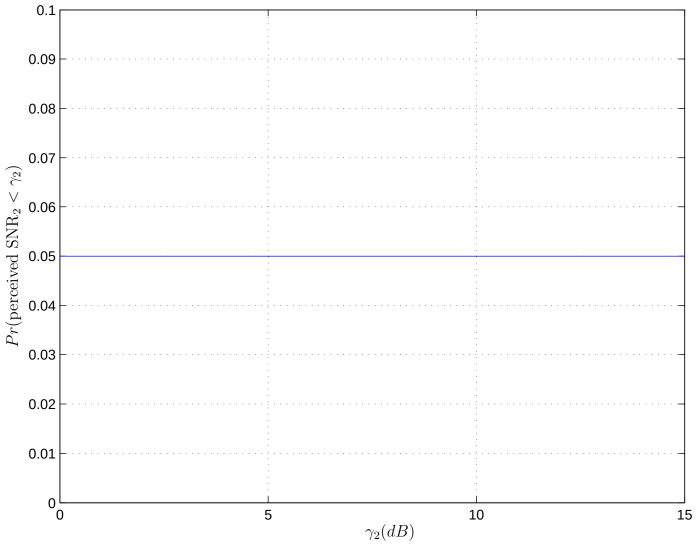

Fig. 4.4: The perceived outage at Transceiver 2 for desired outage less than 5 percent

variance of 0 dB while for the perceived SNR, $\sigma_{f_2}^2 = 0$dB was used in the SNR formula. Fig. 4.3 shows that the true outage probability gives a better result than the perceived outage probability. For a desired outage probability to be below 5 percent the real outage probability becomes below 4 percent which means that our perceived model (see Fig. 4.4 for perceived outage at Transceiver 2) is conservatively accurate and therefore the true outage probability is below the desired outage probability.

4.2 Transmission Powers versus $\sigma_{f_2}^2$

In this section, we averaged our results over 1000 runs and we calculated the average of the total transmit power, the relay transmit power, Transceiver 1 and Transceiver 2 transmit power as the variance of the channels between Transceiver 2 and the relays changes (Fig. 4.5). We set the SNRs in both transceivers to be $\gamma_1 = \gamma_2 = 10$ dB, the channels between Transceiver 1 and the relays are generated as Gaussian random quantities with variance of 0 dB, and the outage probability are set to be below 5 percent at both

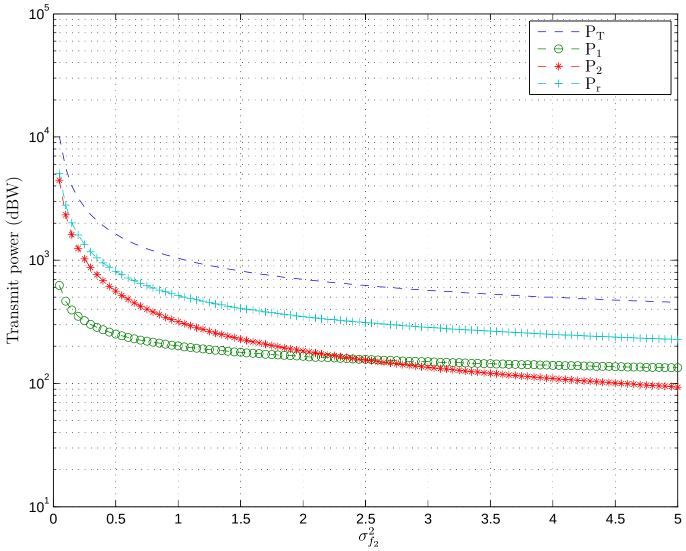

Fig. 4.5: The average transmission power of the over all system, the relay network, Transceiver 1 and Transceiver 2 versus $\sigma^2_{f_2}$

transceivers. It is found that the average total transmit power, relay transmit power and both transceivers transmit power in logarithmic scale are decaying as $\sigma^2_{f_2}$ increases in regular scale. Moreover, Transceiver 1 is the least effected quantity by the changes in $\sigma^2_{f_2}$ since Transceiver 1 is more concerned about successfully transmitting its signals to the relays and thus the dominant factor in Transceiver 1 is the channels between itself and the relays. On the other hand, Transceiver 2 is affected most by the channels between itself and the relays and therefore as the $\sigma^2_{f_2}$ increases the required transmission power from Transceiver 2 decreases. The relay transmit power and the total transmit power are affected by the changes in $\sigma^2_{f_2}$ since they are in direct relationship with Transceiver 2 transmit power.

In Fig. 4.6, the total transmit power is examined for different channels quality. In fact, we plot the total transmit power versus σ_{f_2}, which gives to Transceiver 1 the information about the quality of the channels between Transceiver 2 and the relays, while the variance of the channels between Transceiver 1 and the relays is changing. We kept the SNRs at

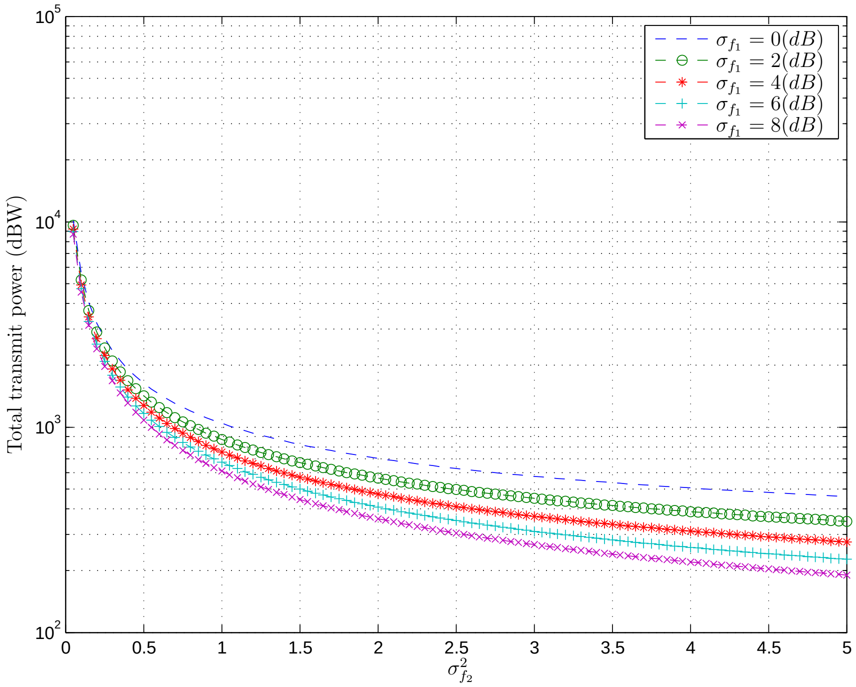

Fig. 4.6: The average of the total transmit power for different channel quality

both transceivers to be $\gamma_1 = \gamma_2 = 10$ dB and the outage probability to be 5 percent. It is found that the total transmit power improves as the quality of the channels becomes better. This result matches the expectations since P_1 and P_2 are in inverse relationship with the variance of the channels between Transceiver 2 and the relay.

4.3 Transmission Powers versus the γ

In this section, we averaged our results over 1000 runs, and calculated the average of the total transmit power, the relay transmit power, Transceiver 1 and Transceiver 2 transmit powers as $\gamma = \gamma_1 = \gamma_2$ changes (Fig. 4.7). We set the variance of the channels between Transceiver 2 and the relays to be 0 dB, we generated the channels between Transceiver 1 and the relays as Gaussian random quantities with variance of 0 dB, and we set the outage probability to be 5 percent at both transceivers. γ varies from 0 dB to 15 dB.

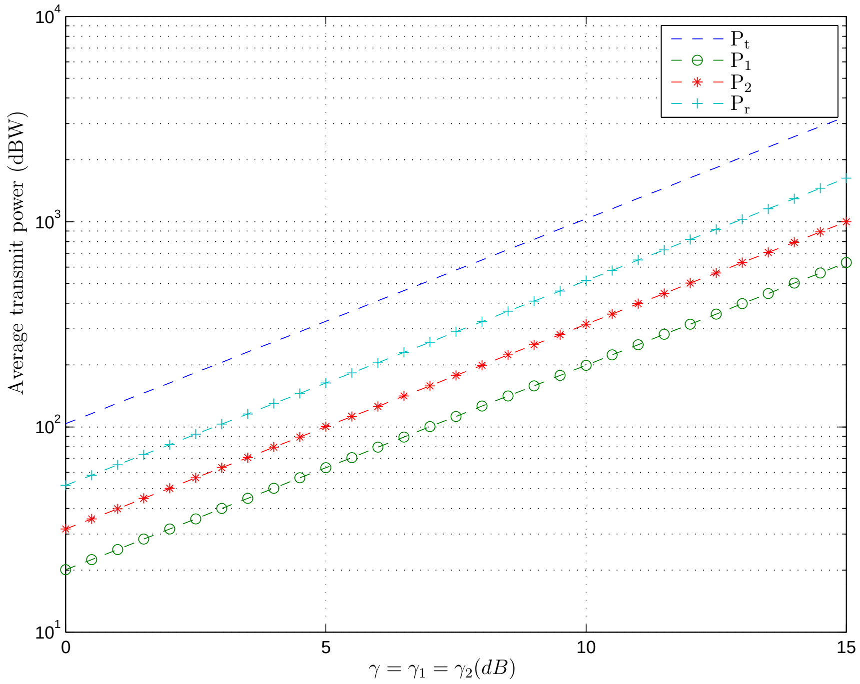

Fig. 4.7: The average transmission power of the over all system, the relay network, Transceiver 1 and Transceiver 2 versus γ

Furthermore, the power quantities are plotted in logarithmic scale. All power quantities increases as the γ increases, and this is logical since as the desired quality of service increases the required power increases. It is noticed that Transceiver 1's transmit power is less than Transceiver 2 transmit power and this supports the results shown in Fig. 4.5 since the channels between the relays and Transceiver 1 is much better than the channels between Transceiver 2 and the relays.

In Fig. 4.8, we plotted the average of the total transmit power versus γ for different channel quality between Transceiver 1 and the relays. The variance of the channels between Transceiver 2 and the relays is kept constant at 0 dB. In Fig. 4.9, we kept the channel quality between Transceiver 1 and the relays constant with variance of $\sigma_{f_1}^2 = 0$ dB, we varied the channels between transceiver 2 and the relays from - 4 dB to 4 dB. We plotted the average of the total transmit power versus $\gamma = \gamma_1 = \gamma_2$. In Figs. 4.8 and 4.9,

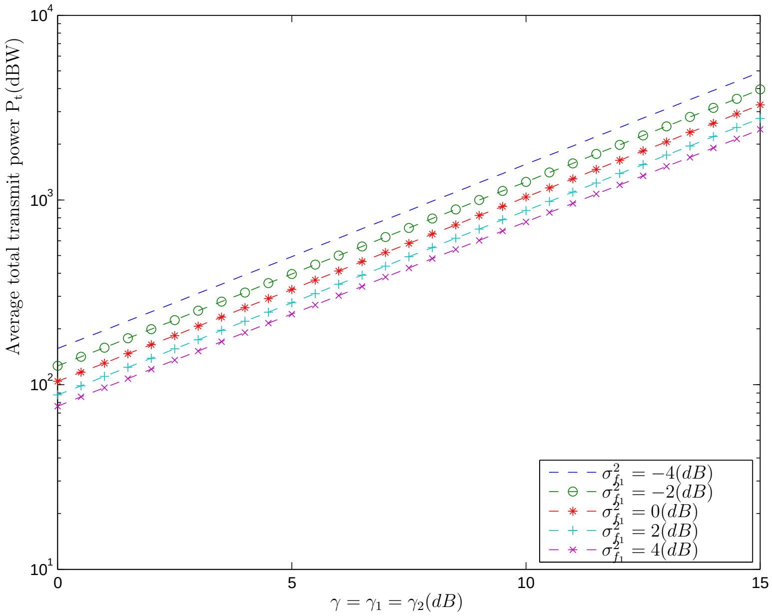

Fig. 4.8: The change in the average total transmit power $\mathrm{P_t}$ versus γ for different quality of the channels between Transceiver 1 and the relays

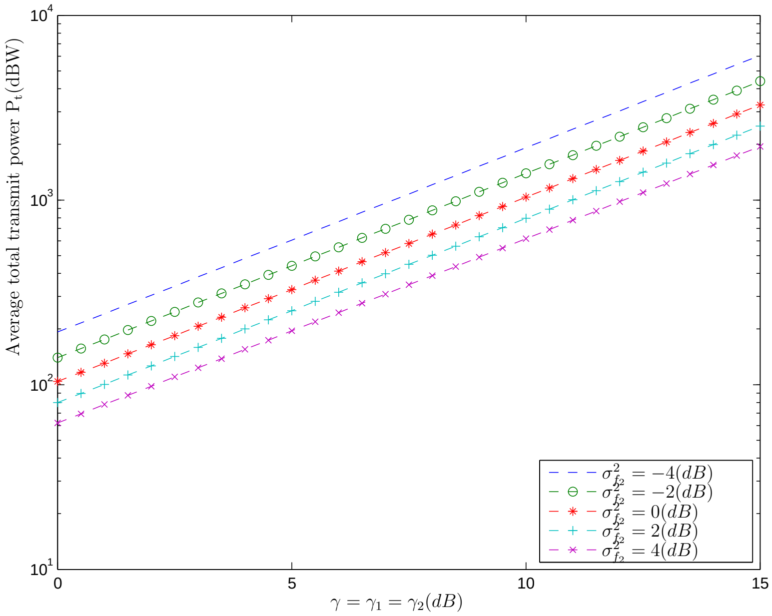

Fig. 4.9: The change in the average total transmit power $\mathrm{P_t}$ versus γ for different $\sigma^2_{f_2}$

the improvement of the channel quality in any side improves the total transmit power. This result was expected since the improvement in the channels minimizes the required transmit power to deliver the desired signals

4.4 Perceived versus true total transmit power

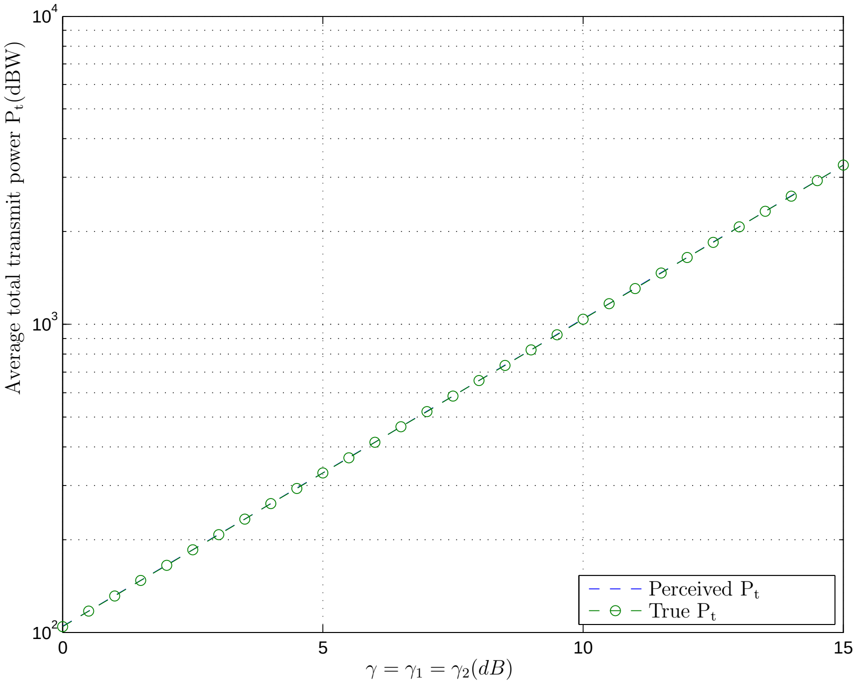

Fig. 4.10: The average perceived total transmit power and average true total transmit power versus γ for different $\sigma_{f_2}^2$

In this section we compare the average of the total perceived transmit power with the average of the true transmit power (Fig 4.10). We averaged both quantities over 1000 runs. The channel quality at both sides of the relays are kept at 0 dB while we plot the average total transmit power for both perceived and true value versus $\gamma = \gamma_1 = \gamma_2$. Note that in the objective function we generated the channels between Transceiver 2

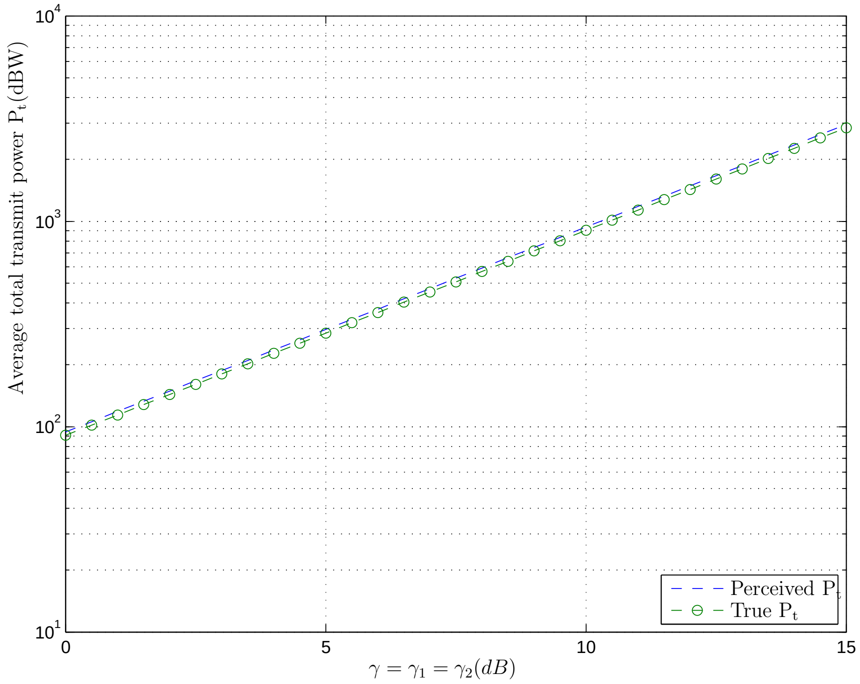

Fig. 4.11: The instantaneous perceived total transmit power is below the instantaneous true total transmit power

and the relays as a Gaussian random quantities with zero-mean and variance equal to $\sigma_{f_2}^2$. It is noticed that the average of the true transmit power is equal to the average of the perceived transmit power. This can be explained since the average of the true value turns off the effects of the true value of $\mathbf{f}_2$ and over the averaging sense, the magnitude of $\mathbf{f}_2$ approaches its variance. The true instantaneous total transmit power differs from one run to another above and below the perceived total transmit power depending on the true status of the channel between Transceiver 2 and the relay that relays the signal (Figs 4.11 and 4.12).

In this chapter, we have presented our simulations and simulation results. We have shown that the optimization problem meets the constraints and the outage probability for different parameter values. Then we have shown how the total transmit power changes with respect to the channel quality. After that, we showed the changes in the transmit

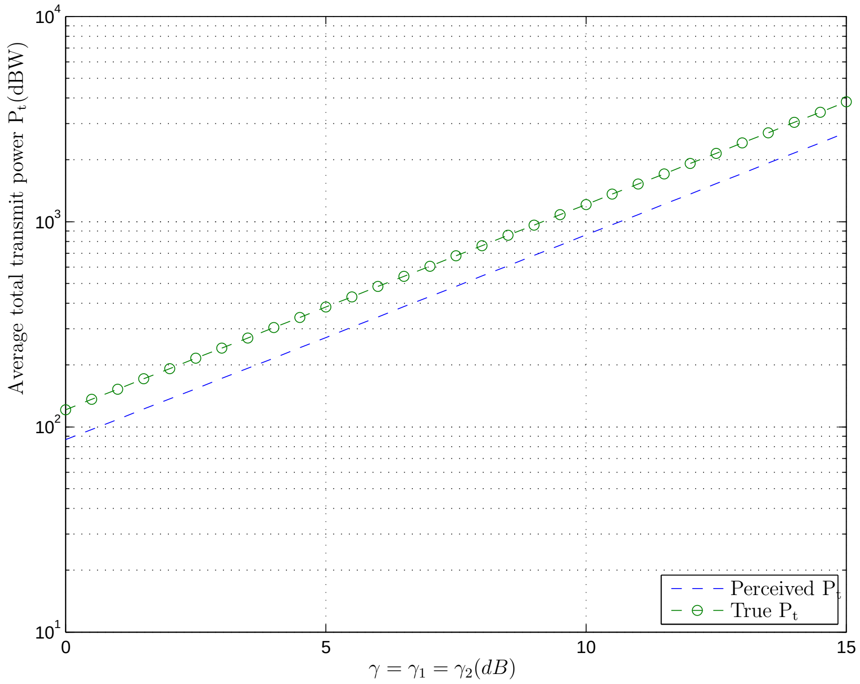

Fig. 4.12: The instantaneous perceived total transmit power is above the instantaneous true total transmit power

power as the desired SNRs changes. Finally, we compared the perceived total power and the true total power in both average scenario and instantaneous scenario.

Chapter 5

Conclusions

In this thesis, we studied the case when two transceivers cannot communicate directly due to a poor channel quality. They instead utilize a network of n_r relays to communicate. Each relay node is equipped with a single antenna for sending and receiving. We deployed two-way two-step relay communication scheme. First, both transceivers send their signals to the relays. Then the relays process the signals, and send them bi-directionally to both transceivers. The transceivers then process and decode the desired signals. The technique that is used in the relay is amplify and forward since the relays multiply the received signals by weights and retransmit them. We assumed that the transceiver that solves the underlying optimization problem knows the perfect channel state information about the channels between itself and the relays and knows only the mean and variance of the channels between the relays and the other transceiver. We then find the solution that minimizes the total transmit power subject to keeping the SNRs in both transceivers above a certain level with outage probability below certain percent.

The optimization problem was solved and a closed form solution was obtained. The solution to this problem leads to a single relay selection criterion, and therefore, there is no need to adjust the phase of the signal relayed since our scheme does not require coherent addition. The solution was tested experimentally and was shown that the given parameters and conditions were satisfied. The relation between the transmission power and the channel state information between the relays and the far transceiver, and the relation between the transmission power and the SNRs were discussed in the simulation section.

The relay network technology can be applied in different applications such as in ad hoc networks. Ad hoc networks nodes have usually very limited power due to the battery life, and therefore, relaying while maintaining the minimum power consumption is desired. Another application can be found in the wireless mesh networks specially for the routers the maintain mobility.

5.1 Future Works

The discussed system deploys a single antenna in both transceivers and in each relays. In many applications today, the nodes are equipped with multiple antennas and therefore this study can be further expanded to test the effect of having multiple antennas in the transceivers. Moreover, the study in this thesis uses two steps in relaying the signals. It would be interesting to study the effects of adding one more step and study the trade off between the power and the time efficiency. Furthermore a further expanding direction would be to study the optimization at both transceivers and then find the best weight and relay that should be used. Finally, it would be interesting to study the trade-offs between the power and the number of active relays.

In our scheme, the optimal values of the weight vector $\mathbf{w}$ and those of the transceivers' transmit powers are calculated in one side of the transceivers.

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