OPTIMAL THREE-TIME SLOT DISTRIBUTED BEAMFORMING FOR TWO-WAY RELAYING

by

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Abstract

In this study, we consider a relay network, with two transceivers and r relay nodes. We assume that each of relays and the two transceivers have a single antenna. For establishing the connection between these two transceivers, we propose a two-way relaying scheme which takes three phases (time slots) to accomplish the exchange of two information symbols between the two transceivers. In the first and second phases, the transceivers, transmit their signals, toward the relays, one after other. The signals that are received by relays are noisy versions of the original signals. Each relay, multiplies its received signal by a complex beamforming coefficient to adjust the phase and amplitude of the signal. Then in the third phase, each relay transmits the summation of so-obtained signals to both transceivers. Our goal is to find the optimal values of transceivers' transmit powers and the optimal values of the beamforming coefficients by minimizing the total transmit power subject to quality of service constraints.

In our approach, we minimize the total transmit power under two constraints. These two constraints are used to guarantee that the transceivers' receive Signal-to-Noise Ratios (SNRs) are above given thresholds.

To solve the underlying optimization problem, we develop two techniques. The first technique is a combination of a two-dimensional search and Second-Order Convex Cone Programming (SOCP). More specifically, the set of feasible values of transceivers' transmit powers is quantized into a sufficient fine grid. Then, at each vertice of this grid, an SOCP problem is solved to obtain the beamforming coefficients such that for the given pair of transceivers' transmit powers, the total transmit power is minimized. The pair of the transceivers' transmit powers, which result in the smallest possible value of the total transmit power, leads us to the solution of the problem. This approach requires a two-dimensional search and solving an SOCP problem at each point of the corresponding two-dimensional grid. Thus, it can be prohibitively expensive in terms of computational complexity. As a second method, we resort to a gradient based steepest descent technique. Our simulation results show that this second technique performs very close to the optimal two-dimensional search based algorithm.

Finally we compare our technique with multi-relay distributed beamforming schemes,

previously developed in literature and show that our three-phase two-way relaying scheme requires less total power as compared to the two-phase two-way relaying method. On the other hand, the two-phase two-way relaying achieves higher data rates when compared with three-phase two-way relaying for the same total transmit power. Also, we observe that the three-phase scheme has more degrees of freedom while multi-relay distributed beamforming schemes, previously developed in literature appears to be more bandwidth efficient.

Dedication

To my parents: with respect and gratitude for being the bridge between lonely and love, between glance and gaze.

To my sister: thank you for the love you are sharing with me.

Acknowledgements

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Last but not least, I owe my deepest sense of gratitude and love to my beloved family, to my parents for their unending love, support and giving me the strength to climb up to the place I am standing now and to the kindest and most lovely sister in the world for her love and support.

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Chapter 1

Introduction

Communications simply is defined as transmitting signals over distances in order to establish connection between two points. Ancients used smoke, drums or flags, as early forms of signals to communicate over long distances. Nowadays, communications is implemented using either the wired connections, such as radio, telephone, television or internet, or wireless links, such as cell phones. Today, communications has become one of the inevitable members of the modern society.

While information is transmitted on the air with no aid of conductors or wires, a wireless communications is established. The distance of the points connected through wireless communication can be short, such as television remote controllers or long such as satellite communications that can travel thousands of miles.

Although, the wireless technologies experienced a very fast growing development pace during the past two decades, many technical challenges in designing of wireless systems are still remaining. People are interested in including mobility, portability, and accessibility in their communication systems. Such demands are the motivation behind the on going research in the field of wireless communications [1].

1.1 A History of Wireless Technologies

The very first wireless communication signal, invented by Greek people, was smoke. They used it for asking help, from nearby islands, while their island was under siege by an enemy. After human society grew bigger, the communication needed to be established over longer distances. Inventions such as flashing mirrors, signal flares and telescopes were the answers to this requirement. But the history of using the electronic devices started in 1838 when a single-wire telegraph system was introduced by Samuel F. B. Morse. "Mr. Thomson can you hear me?" was the first conversation through Alexander Graham Bell's "electrical speech machine", in 1876 that was later known as telephone. Wireless communication was born after detecting radio waves by Heinrich Hertz, in 1887. When Hertz was asked about the application of this discovery, his answer was "no important application". But only 17 years later, in 1893, Nikola Tesla transmitted first radio electromagnetic waves. Later on, the radio waves were only used by military, because of the large size, cost, and power utilization of the devices. Therefore, all efforts were focused on reducing the dimension, price and energy consumption of such devices to make them affordable and easy to use. Over the years, from the discovery of radio waves till 1931, there were some improvements by introducing AM, and FM modulation. The next big change was converting such analog systems to digital systems, after the invention of computers. In the period from 1960 to 1980, all attentions of scientists in this field were focused on changing analog signals to digital, and consequently this technology faced big improvements. Afterwards, transmitting data at 2.9 Mbps, allowed by Ethernet technology, was a significant improvement in data communication. By improving the data rate, more applications became available [2].

Nowadays, most attention in wireless communication is focused on cellular phones which were invented in early 1980.

The first generation of the cellular systems was designed based on analog systems but in less than one decade, the digital cellular systems were developed. Cellular mobile has a very successful appeal in public usage, making it one of the important topics to focus on for research in academia and marketing areas [1].

1.2 Fading

In wireless communications, the signal transmitted by a transmitter often goes through multiple propagation paths to arrive at the receiver. Therefore, the receiver receives a superposition of multiple copies of the transmitted signal. Different copies undergo different attenuations and phase shifts. If these attenuations and phase shifts are such that, the signals add up destructively at the receiver, signal power, received at the receiver can be very low. In this case, essentially channel is said to be in deep fading. Deep fading happens quite often when the transmitter and the receiver are moving with respect to each other and/or with respect to the environment where they reside. At a certain position, the receiver may measure a strong signal but as soon as it starts to move around, as the attenuations and the phase of different paths start to change, the signal strength start to change with time. It may happen that at certain times, multiple signal copies add up destructively, and therefore the received signal may not carry any useful information, but it only contains noise. Therefore, the Signal-to-Noise Ratio (SNR) will decrease significantly. This is called multipath induced fading.

Another type of fading is shadow fading. This second type of fading happens when the signal is blocked by an object. For example, the signal may be blocked by a building or a hill. In this case, there will be no Line of Sight (LOS) between the transmitter and the receiver. And, the communication link has to be established through non-line of sight (non-LOS) components of the signal.

The multipath fading leads to receive more than one copy of the transmitted signal by receiver. Therefore, a multipath channel can be modeled as a linear time-varying channel as shown in Fig. 1.1.

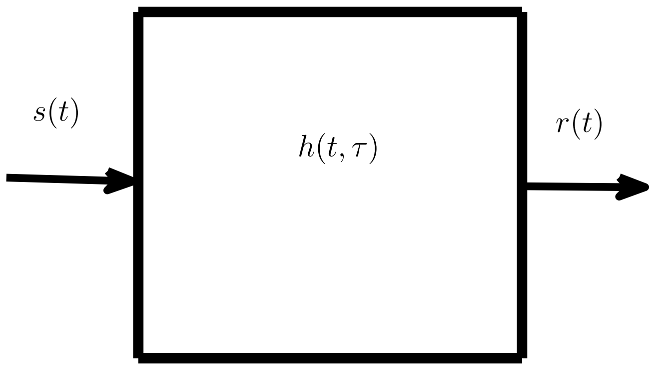

Fig. 1.1: Modeling multipath channel as linear time-varying impulse response

In Fig. 1.1, $h(t, \tau)$ represents the response of the channel to an inpulse locate at $t = \tau$, and depends on different parameters, such as the speed of the mobile. $s(t)$ is the transmitted signal and $r(t)$ is the received signal after going through the multipath fading channel. In order to combat fading, one has to resort to the so-called diversity techniques. There are different methods for diversity such as, time diversity, frequency diversity, polarization diversity, site diversity and the most important one, space diversity [3].

1.3 Diversity

When a signal goes through a fading channel, the signal strength may not be detectable, due to the time-varying nature of the fading. Therefore, the connection between transmitter and receiver could not be established. Providing diversity is one very important way to combat the disturbing effects of fading.

In diversity techniques, several replicas of the signal are transmitted over independently faded channels. As the probability that all channels go into deep fade at the same time is smaller than the probability of each individual channel goes into deep faded, the chance of establishing successful connection between the transmitter and the receiver will be increased. There are different methods of providing diversity, such as:

- Time diversity
- Frequency diversity
- Polarization diversity
- Site diversity
- Space diversity

In time diversity, the same signal is transmitted in different time slots. The time interval that the transmitted signal could successfully be received by the receiver is equal to or larger than the coherence time of the channel. Sending several copies of the signal in different time slots is similar to using repetition code [4].

In frequency diversity, the signal is transmitted on different carriers which are located sufficiently apart from each other. These different carriers serve to provide independently faded version of the transmitted signal. The frequency spacing is chosen equal to or larger than the coherence bandwidth of the channel.

Polarization diversity is another diversity technique which uses either two transmit-antennas or two receive-antennas with different polarizations (e.g., horizontally or vertically polarized waves). This type of diversity has two disadvantages. First, the maximum diversity branch that a system could have is two, because there are only two types of polarization. The second one is losing half of power (3dB) because the transmitter or receiver has to divide the power into two different polarized antennas [4].

Site diversity is a diversity technique used in satellite communication. The downlink transmission of satellites will go through different weather condition. This technique is efficient to limit the effect of rain fade. In site diversity two or more ground stations are linked. All of the stations receive the same signal. Therefore, if one of the ground station cannot receive the signal because of heavy attenuation, the signal will be received by another ground station. The distance between two ground stations cannot be large because the horizontal length of intense rain is few kilometers. By choosing a sufficient distance between two ground stations, the possibility of rain fade will be reduced [5].

In space diversity, there is more than one transmit and/or receive antennas, at the transmitter and/or receiver. The distance between antennas is chosen such that the fading channels are independent, and it should be larger than half of the wavelength of the carrier frequency. By having three possibilities of deploying multiple antennas at the transmitter, receiver, or both, there are three different forms of space diversity; i) receive diversity, which uses one transmit antenna and multiple receive antennas, ii) transmit diversity, which employs multiple transmit antennas and one receive antenna. iii) transmit-receive diversity, which uses multiple antennas at both transmitter and receiver [5].

1.4 Spatial Diversity

In order to combat fading, signals may be transmitted from antennas that are placed spatially apart from each other. This way, receiver will receive different versions of the same signal with different levels of attenuation, noise, and interference, can be used to create more robust connection [6]. This technique is called spatial diversity. Spatial diversity is a form of diversity that utilizes multiple antennas to achieve reliable wireless communication [7].

The importance of diversity in the wireless communications is highlighted by fading effect, which causes users different experience time-varying signal-to-noise-ratios (SNRs). Spatial diversity is grounded in the idea that signals transmitted between spatially separated transmitters or receivers experience independent fading. This phenomenon occurs when multiple copies of the signal emitted from different antennas of the transmitter, travel on different paths and if channel from any of the transmitting antennas to the receiver experiences deep fading, the receiver can use the signal from the other antennas to complete the reception [5].

Multiple-Input, Multiple-Output (MIMO) systems is one of the techniques to introduce spatial diversity. When for example, the base station is equipped with multiple antennas, the quality of uplink and downlink channels will increase, without any extra cost, or power increase of the consumption the mobile devices. There are some simple examples for receive and transmit diversity.

In this case, the distances between receivers' antennas are chosen so that their channels are independent of each other. By using multiple antennas at the receiver, a corresponding bunch of independent fading channels are created.

Such a configuration is usually called Single-Input, Multiple-Output (SIMO) scheme. The multiple-antennas receiver can hence combine the signals received on its antennas to improve the reception performance. One method of combining the received signals of the multiple antennas is Maximum Ratio Combining (MRC). In MRC, the receiver multiplies the signal of each antenna by a gain which is proportional to that antenna's channel gain and adds all the so-obtained signals such that the receiver SNR is maximized.

Another configuration is to equip the transmitter with multiple antennas while the receiver has only one and is known as Multiple-Input, Single-Output (MISO) scheme.

In this case, because there is only one receiver, the received signal is the summation of different copies of the transmitted signal that arrive through different channels [3].

Comparing the two configurations above, higher diversity gain is achievable by using multiple antennas at both sides. A MIMO system with equal antennas on both sides, in a rich scattering environment, has linear increase in capacity with the number of antennas. This increased capacity is achieved without any increase in the transmission power. For example, if number of antennas at transmitter and receiver are N and M, respectively, then the capacity is proportional to $\min\{N, M\}$. On the other hand, MIMO can increase either, number of transmitted symbols per time slot (multiplexing gain) and/or diversity of the system. Note that there is a trade-off between these two parameters [1].

In a MIMO system, two situations are possible. In the first situation, there is no available information for the transmitter about the channels. This MIMO system is called "open-loop" system. The receiver estimates the channel and uses the Channel State Information (CSI) for decoding. The second case happens when the transmitter receives some information about the channels from the receiver via a feedback channel. Such systems are known as "closed-loop" systems. The feedback is used by the transmitter to improve the transmission performance.

Comparing with "open-loop" system, the gain in a "closed-loop" system that comes from the process in the transmitter and receiver, is called "array gain" [1].

1.5 User Cooperation Diversity

While the most desirable means of achieving diversity is through multiple transmit antennas, this may not be practical for some systems such as cellular phones due to the size of the mobile unit.

In cooperative diversity scheme, spatial diversity is achieved not through equipping the transmitter with multiple antennas, but by using the partner(s) antenna and hence forming a virtual antenna array. The partner is merely relaying the original signal to the desired receiver. That means in order to have spatial diversity in each cell, each user has a partner. Each of the two partners responsible to transmit its own information and the received and detected information of the other part, as shown in Fig. 1.2.

Spatial diversity may also play a role in wireless ad-hoc networks that which may not use a fixed infrastructure or central unit such as a base station. In such networks, nodes communicate with each other by forming a connection based on the current signal condition and mobile locations. Such communication occurs directly between mobile users or, can be established, with the aid of a third mobile node [8].

The fact that mobile antennas are unidirectional eliminates any additional cost regards to the transmit power required to transmit signal to the receiver and the partner [8]. This

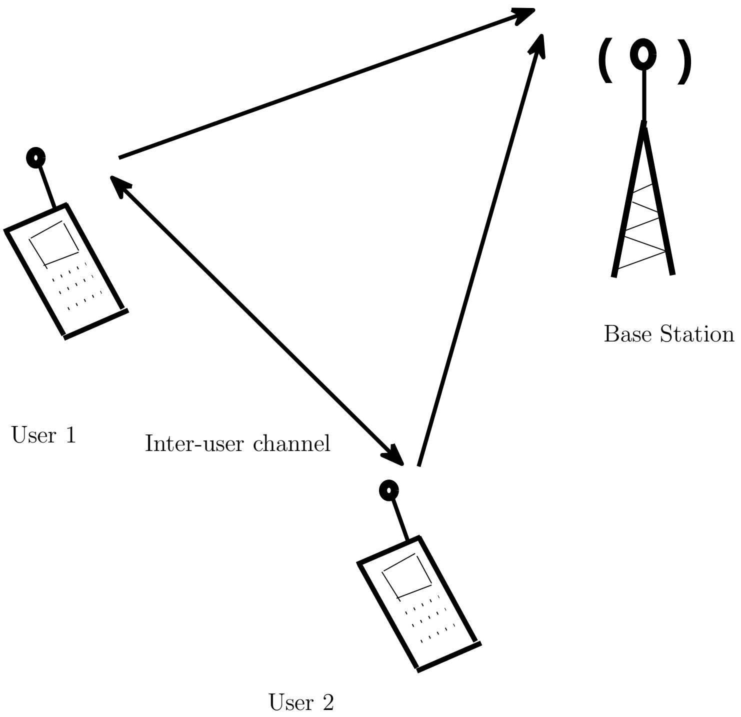

Fig. 1.2: A two-user cooperative scheme

strategy has its own benefits and drawbacks comparing to a non-cooperative strategy.

Cellular users benefit by attaining a higher data rate, decreased sensitivity to channel variations, and potential increase in cellular coverage. The increase in data rate leads to reduced power needs for users, as cooperation requires less total power to achieve a certain rate. This may translate into longer battery life for each user. Cellular coverage area will increase linearly with data throughput [9].

One drawback of user cooperative diversity is the need for increased complexity in the mobile receiver. Users' data must be encrypted prior to transmission so that the partner's receiver can receive the information without understanding it.

However, this drawback may be ignored considering the gains achieved in battery life, data rate, and coverage [8].

Cooperative diversity may be applied to a number of different relaying strategies. In 1968, Van der Meulen introduced the concept of a source-relay-destination link, which has been studied in isolation and from only a limited physical layer approach for many years. There are four different physical layer techniques that the cooperative strategy can be based on:

1. Amplify-and-forward

2. Decode-and-forward

3. Estimate-and-forward

4. Compress-and-forward

In Amplitude-and-Forward (AF) strategy is characterized by a relay that sends a scaled version of its signal to the destination.

On the other hand, in Decode-and-Forward (DF) strategy, the relay receives the signal, decodes the message, and then maps it to the nearest valid codeword before it finally sends the re-encoded information to the destination.

Finally, in the Estimate-and-Forward (EF) strategy, the relay decodes the entire message, rather than individual pieces of it, and then re-encodes it before forwarding it on to the receiver. [10].

A fourth option is compress-and-forward (CF) , in which the relay sends a compressed version of the message to the destination [11]. These four strategies range in complexity, from the memoryless and least complex amplify-and-forward method to the highly complex decode-and-forward or compress-and-forward technique, that require high levels of signal processing.

The literature is somewhat inconclusive as to which strategy is the best. While the AF mode may provide a better diversity order than DF, both are able to outperform the other depending on the underlying channel condition [12]. For example, AF offers greater capacity when the inter-user channel is worse than the user-destination channels; when the reverse is true, DF is superior. However, when these two modes are considered in light of practical situations in which an inter-user outage occurs, both perform equally poor. This suggests that location is more important than the particular cooperative strategy used in achieving effective cooperative diversity [6].

Regardless of which type of relaying scheme is used, creating a virtual array of antennas requires a great deal of planning and coordination. Involving distributed algorithms and synchronization at the packet level, [13] simplifies these requirements by developing a strategy to choose the best relay between the source and destination. Such a relay selection can be done through instantaneous measurements of channel strength rather than distance between the users. The opportunistic relaying scheme chooses the best relay from a set of available relays and then uses this relay as the means of communication between the source and the destination. The advantages of this strategy are the elimination of the need for space-time code algorithms, improved coordination of the cooperating terminals, and a simplified physical layer.

1.6 Distributed Beamforming

A signal received after propagating in space is usually affected by interferences, as both the original signal and the interference occupy the same frequency bandwidth. Therefore, a technique needs to be applied to separate the desired signal from the interference. Beamforming is a technique used to transmit and receive a signal, in the presence of interference. This technique is used in sensor arrays and gives direction to the transmitted or received signal, with the aid of fixed receive or transmit beampatterns. This pattern comes from the idea that the phase and amplitude of each transmitted signal can be controlled by beamforming techniques. At the receiver, beamforming acts as a controller in combining the signals that are received from different nodes such that the receiver could observe the expected pattern of radiation. In other words, a receive beamformer, has to amplify all received signal, by multiplying different weights by the signal. These multiplied weights have different amplitudes and phases. Therefore, the beamformer would not only amplify the received signal, but also adjust the shifted-phase. Typically, beamforming is forming the beam in a desired direction by using the information of antennas' location and direction of the wave. There are two types of beamforming techniques namely conventional beamformers and adaptive beamformers. The conventional beamformer uses a fixed weighting scheme while the adaptive beamformer has the ability to adjust its performance to appropriate the differences exist in its environment.

When the network environment and the number of the nodes increase, a smart solution is using two or more wireless hops to convey the signal of information from a source to a destination. This is known as multiple-hop communication. Therefore, the information signal has to be distributed over the networks using on different hops [14]. Most of the relay networking studies are focused on a strategy that is known as distributed beamforming. In this strategy a group of antennas emulate an array by transmitting a common signal which is focused on the direction of a base station receiver. This goal is not reachable except in the case that the carrier phase of each transmitted signal is adjusted. These phase-adjusted signals are combined in phase at the receiver.

1.7 Motivation

In this thesis, we consider the problem of optimal three-time slot distributed beamforming for two-way relay networks. In a two-way relay network one way to avoid any interference occurring at the relay is to have the source nodes transmit on orthogonal channel. Therefore, to achieve exchange of information, at least four time slots would be needed. This strategy is not however bandwidth efficient. Second strategy reduces the total communication period from four time slots to three. In this strategy, source node one transmits a signal to the relay. During the second time slot, the second source node follows the same procedure. In the third and final time slot, the relay broadcasts one signal to both source nodes that contains separate information for each. The third alternative is two-phase relaying. In this strategy the number of time slots needed is further reduced to two. Through scheduling, both source nodes transmitted information simultaneously while the relays act as the destination node. In the subsequent time slot, roles are reversed, and the relays transmit information while the two source nodes receive information. Between three-time slot and two-time slot strategies, the three-time slot offers more degree of freedom while two-time slot offers a better data rate. We aim to study whether the three-phase scheme performs better in terms of data and/or total transmit power, as compared to the two-phase scheme.

1.8 Objective

Havary-Nassab et. al. considered the problem of optimal two-time slot distributed beamformin for two-way relaying [15]. In our thesis, we have considered the tradeoff between our three-phase distributed beamforming scheme and two-phase distributed beamforming scheme proposed in [15]. Our scheme has more degree of freedom while the two-phase scheme discussed in [15] is more bandwidth efficient. We consider the problem of three-time slot distributed beamforming for a two-way relay network and our goal is to compare the performance of our scheme with the two-phase scheme in terms of data rate and total transmit power.

1.9 Methodology

To find the optimal values of transceivers' transmit powers and the optimal values of beamforming coefficients, the total transmit power is minimized subject to two constraints on quality of service at both transceivers. These two constraints are used to guarantee that the transceivers' receives SNRs are above given thresholds.

The rest of the thesis is organized as follows. In Chapter 2 we review different approaches for one-way and two-way relaying schemes already presented in the literature. Chapter 3 introduces three-phase two-way relay scheme and the corresponding data model develops the problem formulation. Finally, we obtain an optimally distributed beamforming scheme by minimizing the total transmit power subject to two constraints on transceivers' SNRs. In Chapter 4, we present the numerical evaluation of the proposed scheme and compare that with the scheme discussed in [15]. Chapter 5 concludes the thesis.

Chapter 2

Literature Review

This chapter describes the strategies studied in the literature in distributed beamforming and focuses on some results that closely relate to this thesis. Furthermore, different strategies of one-way relay networks specifically the ones that investigate the minimization of the total transmit power is discussed. Next, the capacity scaling and beamforming in two-way relay networks are reviewed. Finally, the results of studies in distributed beamforming are summarized and the closest study to this thesis is presented in more details.

2.1 Distributed Beamforming in Sensor Networks

One strategy that illustrates the use of cooperative user diversity is that of distributed beamforming. In this case, a group of antennas emulates an array by transmitting a common signal that is "focused" in the direction of a receiver. For example, reference [16] considered distributed beamforming in a cluster of wireless sensors. This cluster of sensors, are energy-constrained. These nodes transmit messages to Base Station receiver (BS) with a considerable distance, as shown in Fig. 2.1. In this figure a_1 to a_6 show the sensors. This is accomplished when the carrier phase of each transmitter is adjusted so that all of the signals combine in phase at the receiver.

Distributed beamforming creates significant energy gains in the strength of the receive signal. For instance, if one antenna with total transmit power P_t can achieve a SNR equal to ρ_1, then an array of N antennas will achieve a SNR of ρ_N that is equal to $N \times \rho_1$.

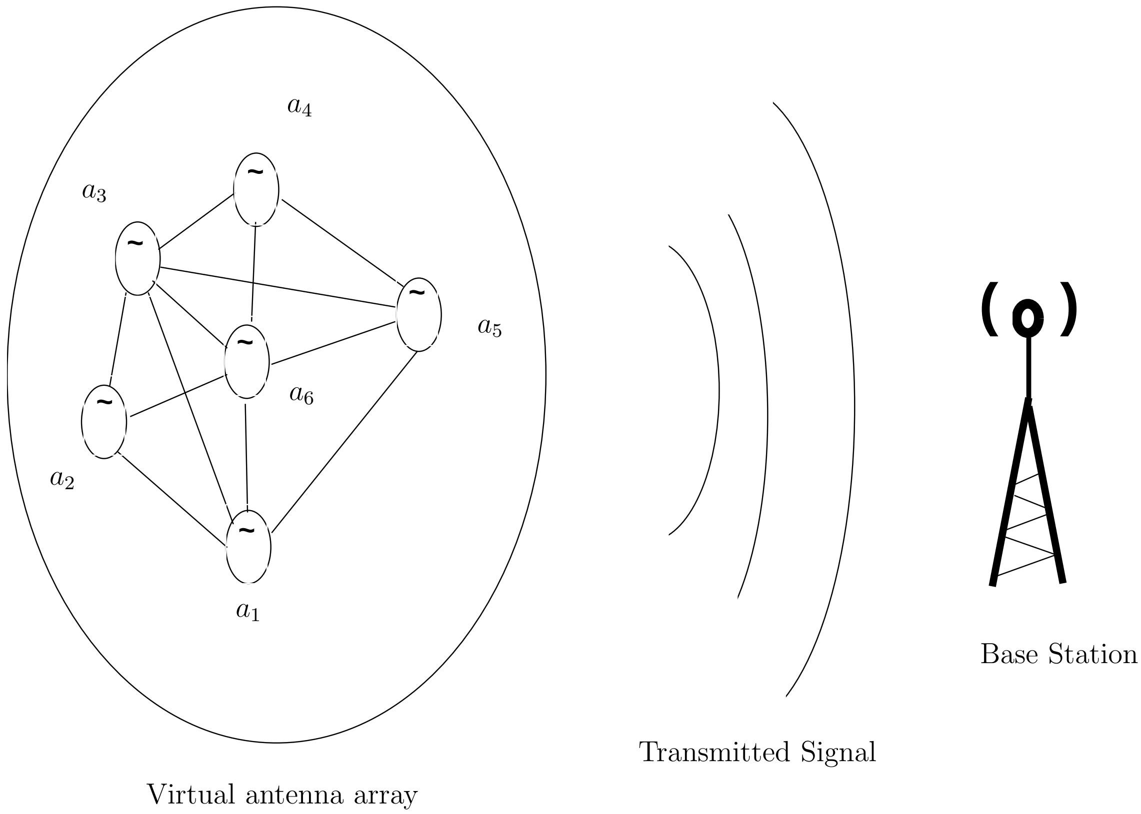

Fig. 2.1: Communication model for a sensor network

While the total transmit power of P_t remains the same. From a physical standpoint, the increase in the SNR is due to the increase in the transmitted electromagnetic energy from multiple antennas to the receiver. In order to achieve these gains, synchronization of signal phase and frequency is necessary [16].

An important distinction between centralized (traditional) beamforming and distributed beamforming is that in the former, beamforming occurs by estimating the channel gains to each antenna using a Radio-Frequency (RF) carrier signal supplied by one local source. In distributed beamforming, each sensor creates its own RF carrier signal, and the group of carrier signals from multiple antennas remains unsynchronized. Therefore, it is not possible to estimate the channel phase responses in order to achieve phase synchronization. However, the reduction of SNR is not more than 30 percent even the error created by lack of synchronization exist [16].

2.2 MIMO Relaying Channels

A MIMO relay network consists of a source, relay, and destination in which each has multiple antennas. The channel capacity of this type of MIMO network with a fixed number of relays is unknown, but changes in capacity as the number of relays increases have been characterized [17]. The advantage to a MIMO system is that it boosts system capacity by transmitting multiple streams of data, using space-time coding to improve the reliability of the transmissions. However, the problem lies in the physical limitations of placing more than one antenna onto a mobile terminal [18].

One example of a MIMO relay channel is the MIMO-OFDM, or MIMO-Orthogonal Frequency-Division Multiplexing. In this type of system, the data signal is split into multiple channels, each transmitted over a different carrier. When the signals are out of phase with each other, fade occurs, causing the signal strength to fluctuate. The OFDM receiver is able to correct distortions present on any of the sub-channels, each of which may distort the signal in different ways, thus correcting distortions that occur with regards to specific frequencies. Hammerstrom and Wittneben considered a system comprised of two terminals, each of which could either be a source or destination, separated by a relay [19].

All nodes in the system were comprised multiple antennas and operated in half-duplex mode, in which none were able to receive and transmit at the same time while using the same frequency. Furthermore, in the system, the authors assumed that although the destination could not receive the signal directly from the source, it did have, as did all the nodes, perfect knowledge of the channels of both hops. Based upon this assumption, the authors examined power allocation within the space and frequency domains in a MIMO-OFDM system [19].

Two approaches are relevant to power allocation in this system. In the first approach, the source and relay were jointly optimized over the sub-channels with respect to the joint power constraint. In doing so, the system is able to more efficiently respond to path losses between the first and the second hops. Additionally, this strategy provides insight into the behavior of the required transmit power over the entire communication link, rather than just among each hop. In the second strategy, the source and relay power allocation are optimized separately within the confines of power constraints existing in each node. It has been shown iterative optimization of the power allocation in the source and relay results in the most significant performance gains.

2.3 Decoding Strategies in Relaying

As mentioned in Chapter 1, number of relay srtategies exist that are relevant to relay networks, such as AF, DF, EF and CF. Of these, the most desirable ones are those strategies which reach towards the limits of cooperative diversity without reaching the limits of processing complexity [20]. Memoryless functions are one example of a relay strategy that attempts to meet these criteria. Not only are they desired for their simplicity, but also their power and increased capacity. Gommadman and Jaffar investigated one such strategy, amplify-and-forward, and compared it with the more complex decode-and-forward in both parallel and serial networks. The authors also proposed a novel strategy, estimate-and-forward [20].

A Parallel relay network and a serial relay network are shown in Fig. 2.2 and Fig. 2.3, respectively.

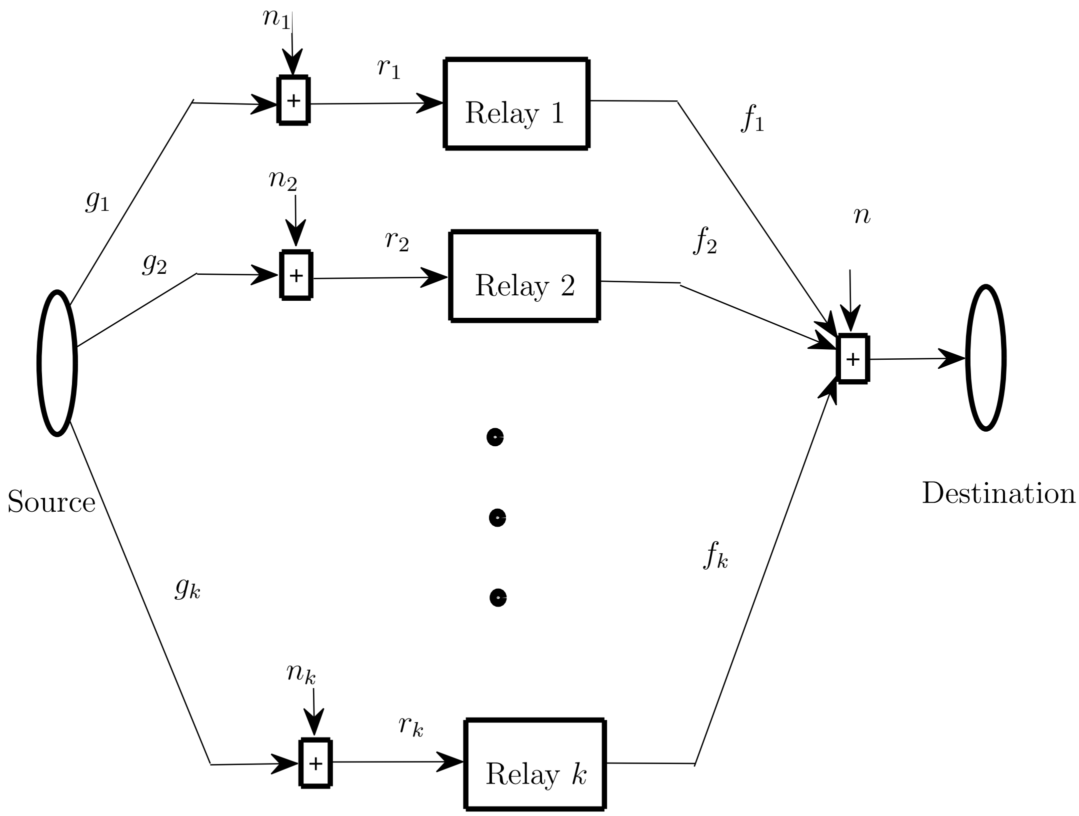

Fig. 2.2: A Parallel Relay Network

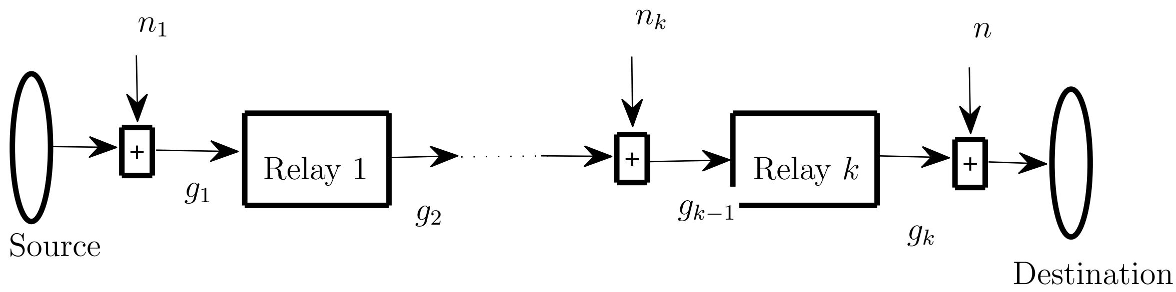

Fig. 2.3: A Serial Relay Network

Both of these networks consist of k relays, a source and a destination. First consider the parallel network of Fig. 2.2. In this configuration g_i and f_i denote the channel gain between the source and the ith relay and the ith relay and the destination, respectively. The signal received at the ith relay shown by r_i and $r_i = g_i x + n_i$, where x and n_i are the source signal and the additive white Gaussian noise in ith relay, respectively. Moreover, w_i is denoted the complex weight that relay i multiplies its received signal before re-transmitting it. Therefore, the signal transmitted by relay i will be $y_i = w_i r_i = w_i(g_i x + n_i)$. Each relay transmitted signal, y_i, goes through the relay destination channel, f_i. Therefore, the signal received by the destination, z will be sum of signals received from all relays.

$$z = (\sum_{i=1}^{k} y_i f_i) + n = (\sum_{i=1}^{k} w_i r_i f_i) + n \qquad (2.1)$$

where n is the noise at the destination. Hence, the symbol transmission in this configuration completes in two phases. A serial relay network is shown in Fig. 2.3. In this figure g_1 and g_k denote the channel gains from source to relay 1 and from relay k to destination, respectively. For $i = 1, 2, \ldots k-1$, g_i denotes the channel gain between $(i-1)$th relay and ith relay. Here x, n_i, w_i and n denote the same parameters as defined for the parallel network. In this network the source transmits the symbol to relay 1. Afterwards, each relay multiplies its received signal by complex gain w_1 and transmits it to the next relay. The total transmission completes in k phases comparing to the two phases communication in a parallel network [21].

Among memoryless strategies, the forwarding techniques of amplification and demodulation are the simplest. Relays that utilize an AF strategy receive a signal, scale it down to meet the specific power constraints of that relay, and then forward it to the destination.

Relays that utilize DF schemes demodulate received symbols and then modulate them within the constraints of their own power. Demodulation results in a transmitted signal that carries no information concerning uncertainty in the relay's choice of demodulated symbols, a factor that can lead to performance degradation. These two strategies may also be considered in terms of their two basic signal processing operations, detection and estimation. In DF, the relay uses the Maximum A posteriori Probability (MAP) detection rule prior to demodulating. In contrast, AF technique uses a Minimum Mean Square Error (MMSE) estimation and normalization to meet the power constraint.

A novel memoryless forwarding strategy designed to overcome these limitations while maintaining the simplicity is the estimate and forward strategy. In this case, signal-to-noise ratio is optimized at the destination by utilizing a scaled version of MMSE, called Minimum Mean Squared Uncorrelated Error (MMSUE) estimation. The EF strategy performs better than the AF and DF strategies in both serial and parallel relay networks [20].

A more in-depth understanding of amplify-and-forward Rely strategies may be obtained by examining statistical properties such as envelope probability density function, autocorrelation, level crossing rate, and performance characteristics such as frequency and duration of outages. This AF strategy is used in relay channels where a signal is sent from a base station to a mobile relay, which in turn forwards it to a destination mobile station, as shown in Fig. 2.4.

In this figure, h_1 and h_2 are the channels between the base station and relay and relay and destination, respectively. The overall channel, from BS to mobile destination, is characterized as double Gaussian, since white Gaussian noise is amplified and transmitted to the destination. Such a channel may be categorized as fixed gain or variable gain. Observations made of simulated relay fading channels, including an analysis of their

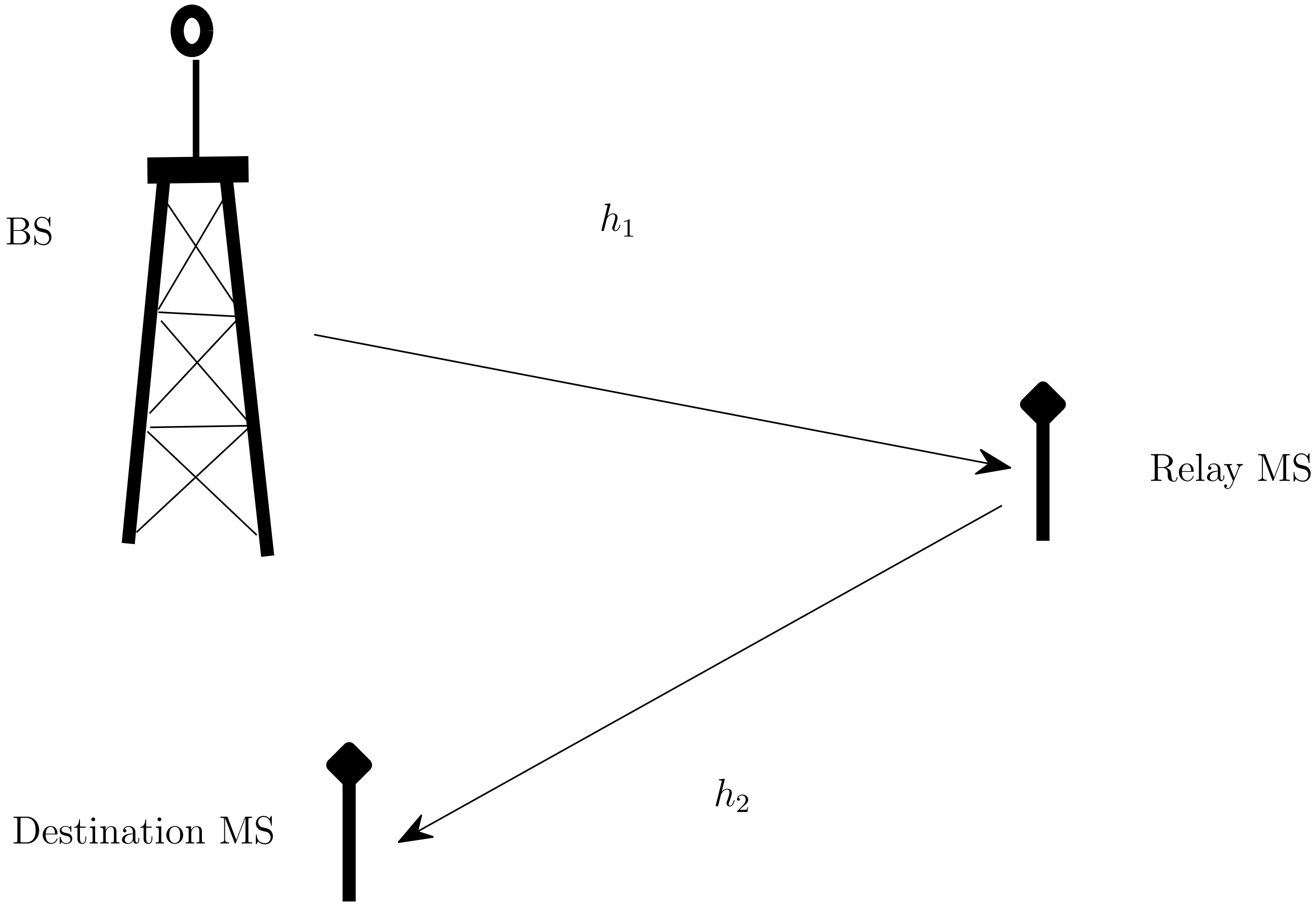

Fig. 2.4: A relay network with fading channels

statistical properties, revealed that for moderate to high SNRs, variable gain systems have less frequent and shorter outages than fixed gain systems. However, when the SNR is low, variable gain systems still experience more infrequent outages but at longer durations than fixed gain systems [22].

2.4 One-Way Relaying

Relay networks recently has been the topic of numerous efforts in wireless communication. One-way relay networks utilizing AF strategies are successful at fostering cooperative communications, in which users share resources. However, in a wireless situation, this requires significant coordination and may be difficult or impractical.

Chen et. al. proposed an alternative architecture using one-way relays called the Multiple-Access Relay Channel (MARC) and an alternative strategy called Multiple Access Amplify-and-Forward (MAF) that enables users to operate as if in a noncooperative multiple access channel [23]. In the literature, the authors of [23] used the notion of high/low multiplexing. As mentioned in Chapter 1, a MIMO system can increase either, number of transmitted symbols per time slot (multiplexing gain) and/or diversity of the system. Diversity-multiplexing tradeoff shows the tradeoff between error probability and the data rate of the system. In the literature [23], the authors used the notion of high/low multiplexing gain instead of high/low data rate. An MARC with two users, multiplicative fading, and additive noise shown in Fig. 2.5.

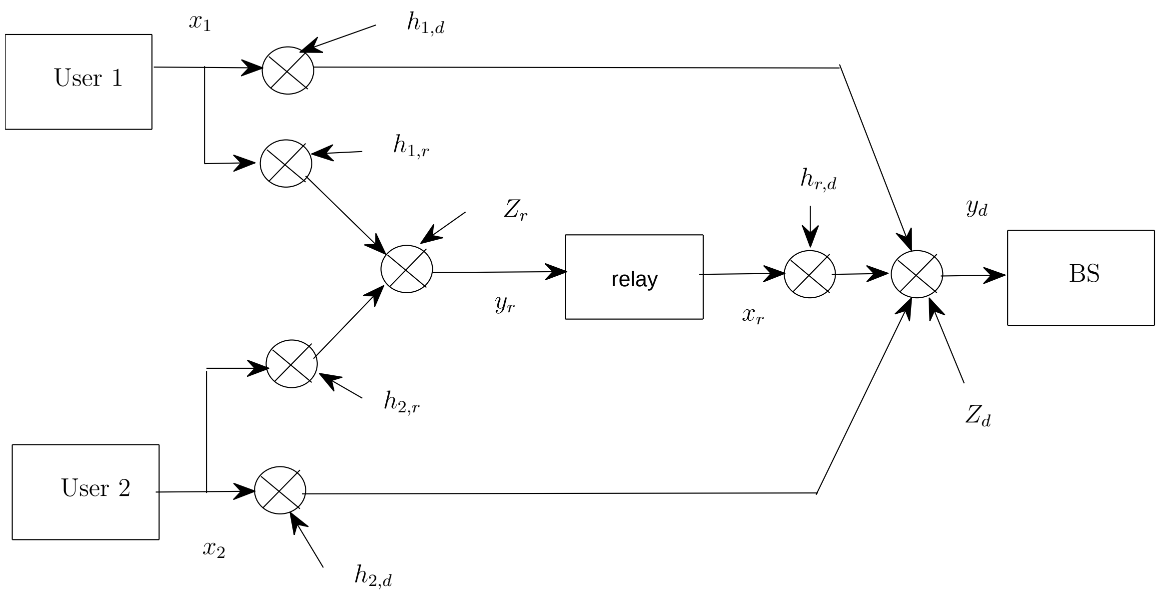

Fig. 2.5: Multiple-Access Relay Channel (MARC) with two users, multiplicative fading, and additive noise

Here, $h_{i,d}$ and $h_{i,r}$ denote the fading coefficients of the ith user destination and ith user-relay links, respectively. $h_{r,d}$ denotes the fading coefficient of the relay-destination link and

x_r shows the signal transmitted by the relay. Z_r and Z_d are additive noise at the relay and the destination, respectively. y_r and y_d are the received signal at the relay and destination, respectively. These strategies are cost-effective in the sense that the cost of adding a shared relay is spread out over many users [23]. Thus, assuming a one-way relay network, the MAF protocol requires simple encoding and decoding and is able to achieve optimal diversity-multiplexing tradeoff, which offsets the performance loss typically observed in AF relay protocols used with high multiplexing systems. Additionally, in low multiplexing systems, users receive benefits from the relay as if each individual was the only user and not sharing resources with others [24]. Thus, the MAF protocol offers low complexity at the relay and performance gains in slow fading environments [23].

Jing et. al. considered a distributed beamforming strategy where each amplify-and-forward relay knows its exact channel state information [25]. In this situation, the transmit directions of the transmitter and relays are adjusted to form a beam at the receiver, and the power used to accomplish this is adjusted to optimize the network performance. A two-step amplify-and-forward protocol is used in order to allow the transmitter and relays to generate a beam at the receiver and adjust power based on channel strength information. This beamforming is accomplished through the application of match filters at the transmitter and relays, which cancels the channel phase effect and allows for a coherent beam. For networks without a direct link between the transmitter and receiver, the authors concluded that the transmitter should use maximum power while the relay should adopt its optimal power. Typically, this optimum value would depend upon the quality of all other channels. However, in a distributed beamforming scenario, the relay needs only its own channel state information and some additional information from the receiver in order to effectively allocate power.

As shown in Fig. 2.6, the relay networks in reference [25] has R relays, f_i is the channel gain from the transmitter to the ith relay and g_i is the channel gain from the ith relay to the receiver. Finally, f_0 is denoting the case where, the direct link between the transmitter and the receiver exists.

This strategy, which is based upon the availability of instantaneous channel information, does not allow for any uncertainty. Havary-Nassab et. al., took this strategy one step further and assumed that the second order statistics channel coefficients were avail-

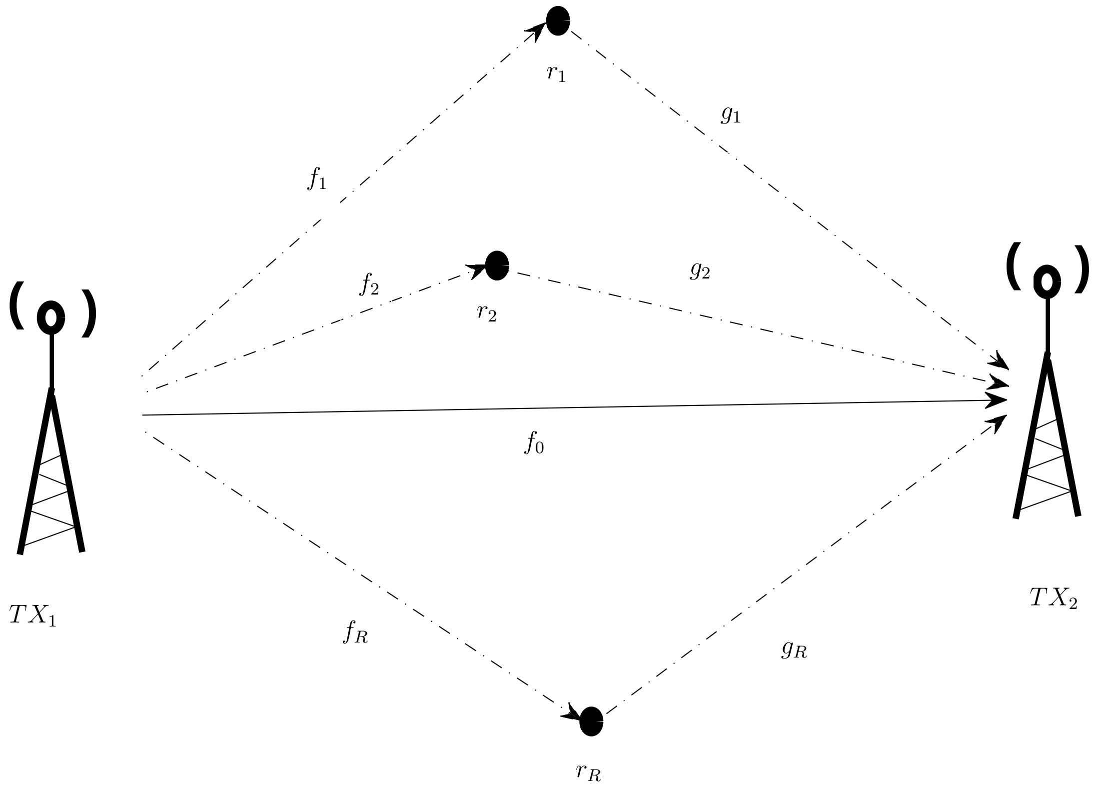

Fig. 2.6: Wireless Relay Network with R Relay Nodes

able, and considered how this information could be used to maximize of receiver SNR subject to power constraints [26]. This might occur when relay nodes have limitations with regards to battery time. The authors considered two different approaches. In the first, beamforming was achieved by minimizing the total transmit power that could be subject to constraint. In the second approach, beamforming was achieved by maximizing the SNR subject to either total transmit power constraint or power constraints for each relay. Results from the algorithms and solutions associated with each of these options suggested that as the level of uncertainty in channel state information increases, it takes a greater amount of power to maintain quality of service [26], [27].

2.5 Two-Way Relaying

In contrast to the one-way relay network, the two-way relay network allows for terminals and relays to both transmit and receive information. The two-way relaying scheme aims to establish connection between two transceivers using a group of relays. This scheme arises from two-way channels which were studied by Shannon [28]. In his studies, he assumed direct link between two transceivers and derived an achievable rate region for full duplex scenarios. Here we focus on studies which have discussed MIMO relay channels, capacity scaling and beamforming in two-way relay networks. Two-way relaying offers the promise of greater efficiency over one-way relay networks and thus provides a promising opportunity for novel network designs.

The two-way relay channel consists of, on the most basic level, two source nodes and a helping relay node. Typically, in order to avoid any interference occurring at the relay, the source nodes do not transmit simultaneously. Thus, four time slots would be needed to achieve exchange of information. In the first time slot the source one transmits its signal of information to the relay. In the second time slot, the relay transmits the received information to the second source node. In the third and fourth time slots the source node two transmits the information signal to the relay and the relay transmits the so-obtained signal to the first node, simultaneously, as shown in Fig. 2.7.

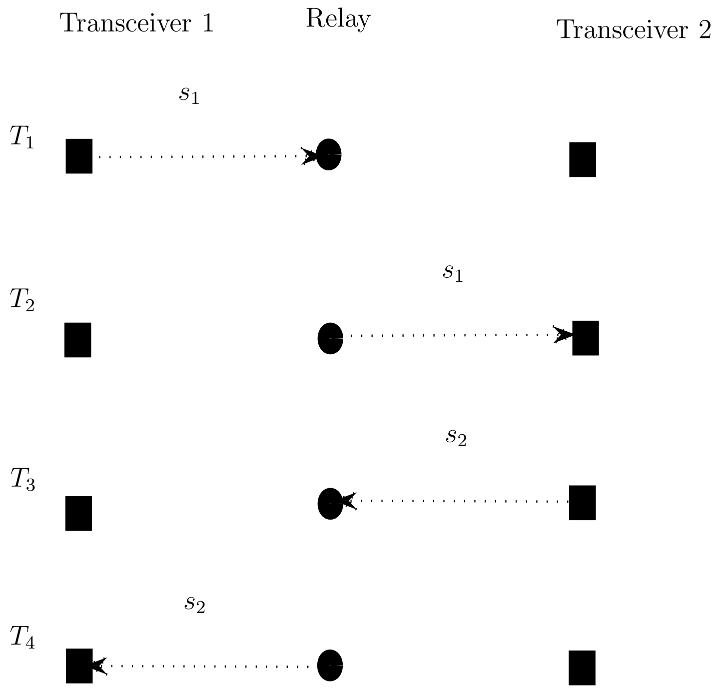

Fig. 2.7: Traditional four-time-slot technique for two-way relaying scheme

However, strategies exist to reduce the number of time-slots needed for information transmission. For example, one method exists that reduces the total communication period from four time slots to three. In this strategy, source node one transmits a signal to the relay, and the relay immediately decodes it. During the second time slot, the second source node follows the same procedure, and the relay, is immediately able to decode it. In the third and final time slot, the relay broadcasts one signal to both source nodes that contains separate information for each. This scheme is shown in Fig. 2.8. The individual source nodes are able to recognize which portion of the signal is relevant to them and only recovers that piece of the information [29].

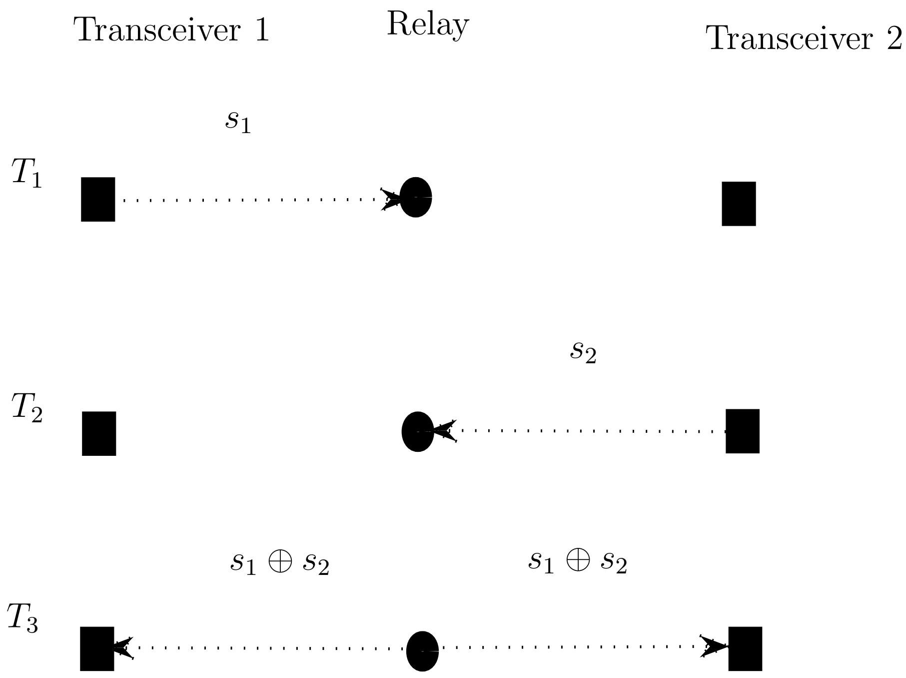

Fig. 2.8: Three-time-slot scheme for two-way relaying

In another scheme, provided by [17], the number of time slots needed was further reduced to two. In this work a two-way relay system was comprised of relays, and two terminals, T_1 and T_2, which wish exchange information with each other. Through scheduling, the terminals both transmitted information simultaneously while the relays act as the destination node. In the subsequent time slot, roles are reversed, and the relays transmit information while the two terminals receive information. Knowledge acquired by the terminals about the signals during the transmission period is used during the receiving period to eliminate interference, as shown in Fig. 2.9.

2.5.1 Capacity Scaling

In two-way relaying one of these three strategies would apply to the system. Vaze et. al considered a two-way relay network as shown in Fig. 2.10.The authors assumed that both

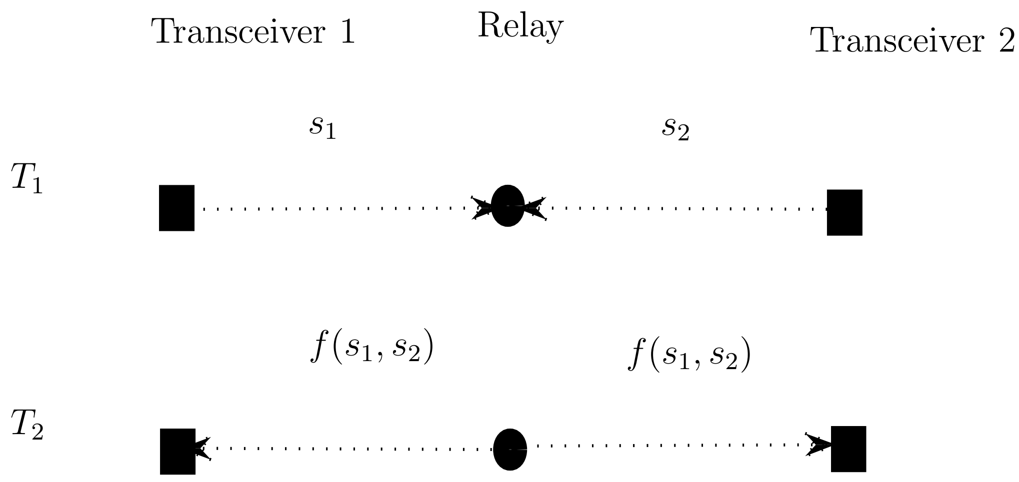

Fig. 2.9: Two-time-slot scheme for two-way relaying

the terminals and relays were equipped with multiple antennas and that perfect channel state information is available at the terminals and relays.

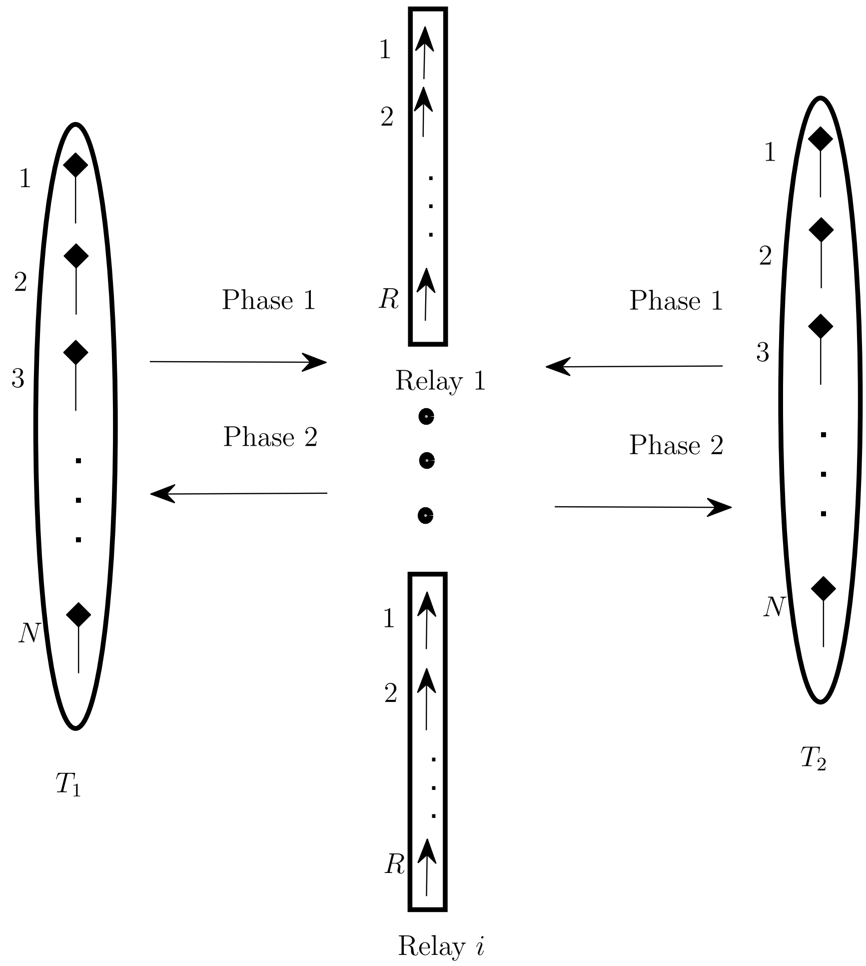

Fig. 2.10: A two-way relay networks

Here, T_1 and T_2 are two terminals, with N antennas. They are communicating with each other via i relays, and each relay has R antennas [17]. These terminals use the knowledge of the signals, obtained during the first time slot, to cancel any interference received from the relay during the subsequent time slot. In this system, the communication is established during two phases. With a single relay and full-duplex nodes, different protocols will result in different achievable rates. When compared to half-duplex nodes, two-way relaying reduces rate loss [17].

In their two-way relaying system model, the relays hold no data of their own, but merely assist the terminals in transmitting information. Additionally, the relays are located uniformly and randomly in a fixed-size geographical area. Terminals transmit in spatial multiplexing mode, where each antenna generates an individual stream of data. The protocol used in this system consists of two components. First, T_1 and T_2 transmit signals, which are then received by all relays. Second, in the next time slot, the relays transmit simultaneously and the two terminals are set to receive the information. A final component of this model path loss and shadowing occurred independently, randomly, and constantly over the entire time period of interest.

The authors calculated upper and lower bounds of two-way relaying capacity and demonstrated the gains achieved by two-way MIMO relaying. This strategy removes one-half of the rate loss factor associate with capacity, translating into increased capacity. This gain is due to the perfect channel state information available at each terminal, thus allowing the terminals to cancel interference present in its own signals [17].

The authors of [30] determine the capacity of a full-duplex two-way relay channel, which has not yet been fully characterized. Using a deterministic approach, the authors focus on the interaction of the signals arriving from different nodes rather than the background noise. The system model utilized in this study was one in which communication occurred between relay and nodes, although channel reciprocity was not assumed.

The deterministic channel model used in this study may be defined in terms of a wireless network. At each time point in the system, a node transmits and receives a vector. These received signals are deterministic functions of the transmitted signals arising from the other node. Using this model to interpret a proposed scheme for two-way relay networks, the first event that occurs is that the relay receives and decodes a message from one of the nodes. This message arrives above the signal level of the other node and can be subtracted out from the overall signal. The part of the signal remaining represents a simple summation of the messages from both nodes. The relay now creates a transmit signal by using a code to quantize it [30].

2.5.2 Different Strategies of Decoding

In a similar study, Zhang et. al investigated the two-way exchange of information between two source nodes and a relay by applying the principle of analog network coding [29]. Both Analog Network Coding (ANC) and another type of coding, Physical-Layer Network Coding (PNC) allow for the reduction in the number of time slots needed to send information between two source nodes using a relay. However, these two types of network coding differ in the exact relay operation used, as ANC utilizes amplify-and-forward while PNC utilizes estimate-and-forward. Thus, in ANC, the relay amplifies the composite signal received from both source nodes and broadcasts it back to them. As described previously, interference is not a problem, as it is negated by the source nodes upon receiving the signal from the relay. In contrast to ANC, PNC uses estimate-and-forward to allow the relay to estimate the signal composition, differentiating between information specific to each source node. The decoded signal is then re-encoded into a new signal transmitted to both sources, which are then able to extract information specific to them.

In this study, the authors focus on an AF/ANC, multiple-antenna-based two-way relay network. As such, it was assumed that both source nodes had one antenna, while the relay contained multiple antennas. Using this model, they investigated an optimal beamforming design that would achieve an optimum capacity under transmit power constraints associated with source nodes and the relay. Results indicated that an optimal beamforming structure exists that achieves the desired capacity, leading the authors to propose two suboptimal beamforming strategies, one based on "matched-filter" and the other based on "zero-forcing" principles, which require less complexity to implement. This beamforming structure differs from that employed with one-way relay networks due to bidirectional transmission and the cancelation of self-interference at the source node.

The Maximal-Ratio Reception and Maximal-Ratio Transmission (MRR-MRT) utilize "matched-filter"-based beamforming at the relay to maximize the signal power transmitted to both source nodes. The relay itself makes no attempt to diminish the interference between the two nodes. In contrast, the Zero-Forcing Reception and Zero-Forcing Transmission (ZFR-ZFT) utilizes "zero forcing"-based beamforming to remove all interference at the relay and at the receiving end on the source nodes. The primary advantage of the latter over the former is that it removes the need for the source nodes to cancel their self-interference, creating simpler receivers. However, MRR-MRT has demonstrated improved performance over ZFR-ZFT in ANC-based two-way relay networks. This is due to the ability of MF-based beamforming to maximize the signal power at the source nodes, achieving close-to-optimum capacity under varying SNR and channel conditions [29].

While in most two-way relay networks, the source nodes communicate with each other, this is not always the case. In a separated *two-way relay channel*, the two terminals only receive signals from the relay, not from each other. Such a case might occur when the two terminals are physically separated and can only communicate through the relay. In this model, the relay assists both users by broadcasting the decoded parts of the message and the compressed version. As such, the achievable rate region is based upon partial Decode-and-Forward (pDF) and compress-and-forward schemes. However, this differs from the typical network in that both users already know their own messages.

This particular model has achieved gains in popularity due to its practicality in communication and its ability to demonstrate the benefits of network coding directly within the physical domain. The relay component only decodes and forwards the binary sum of the two messages, which is adequate, as each user already knows its own message. In a study of achievable rate regions associate with this type of network, reference [31] considered coding functions that are independent of previously received messages. In the first scheme, each user simply splits the message into part. While the relay decodes the first part of the message, the other user decodes the second part. The relay then broadcasts decoded parts of the message with the understanding that each user already knows its own message. Additionally, the relay compresses its own received signal and broadcasts that as well, taking advantage of the fact that each user knows the decoded parts in addition to the message. The rate region may be further enhanced by adopting two levels of quantization in which a better description of the signal is transmitted to only one of the users.

Lee et. al. also investigated zero-forcing filters in relation to two-way relay networks. Based on multiple antenna relay, the authors investigated whether an AF relay beamforming strategy would increase the amount of bidirectional communication between two nodes when the relay and destination node have perfect channel state information, as opposed to both source nodes having access to this knowledge. The beamforming scheme created, termed a sum-rate maximizing two-way beamforming scheme, arose out of an algorithm, the general power iterative algorithm used to solve an optimization problem. The system used in this investigation contains source and destination nodes exchanging information with each other and a relay node with multiple antennas. In order to increase system capacity through beamforming, the relay is situated between the two nodes. No direct link exists between the nodes due to path loss, and each source node has only one antenna. The relay node has perfect knowledge of forward and backward channel state information, as does the destination node but not the source node [11].

In this model, during the first time slot and as with other models described in this paper, the two source nodes transmit their information to the relay simultaneously. A transmitted signal vector is generated at the relay through a beamforming matrix. During the second time slot, the signal that was combined by the beamforming matrix at the relay is now transmitted to the destination nodes. The most important component of this protocol is that it has the ability to effectively utilize having the knowledge. The channel state information provides the capability for the nodes to cancel self-interference. Two different beamforming schemes were compared in this model, two-way ZF and AF. Researchers found that while both provided poor sum-rate performance, the two-way ZF demonstrated improved performance over its counterpart and they both reduced the complexity of the beamforming matrix.

2.5.3 Distributed Beamforming in Two-Way Relaying

Much of the research surrounding two-way relay networks utilizes source and destination nodes with only one relay. Song et. al. studied a situation where two relays were present between source nodes. A two-way two-relay network consists of a total of four nodes, including two terminals and two parallel relays. Direct links are absent between the terminals and between the two relays. As with one-relay networks, communication time is divided into time slots, in each of which a specific action occurs. The first time slot is occupied with terminals sending signals and both relays receiving, while the second time slot involves just the opposite [32].

Several transmission strategies are relevant to this type of network confirmation, including physical layer network coding and multiplexed coding. As described earlier, in physical layer network coding, no direct link exists between the two source nodes and data exchange occurs through a rely located between them. Signal transmission occurs in two phases. In the first phase, the relay receives the signal plus noise, while in the second phase, amplifies the signal and transmits a scaled version of it to the destination nodes. Since the source recognizes its own message, and this appears as interference, the source is able to subtract out the interference. Decoding at the source node occurs after this process. In multiplexed coding, some users know the message of other users. Two nodes that wish to communicate send a message to each other through the relay, which is able to decode both messages, creating an array of codewords. In the next phase, the relay sends the codeword back with the message for the source nodes to decode.

Hu et. al., developed a number of transmission schemes for the two-way two-relay network. The amplify-forward scheme, prevalent in the literature, involves the relay receiving a signal, buffering it, and transmitting it onto the next stage. The receiving terminal, with knowledge of interference, receives the signal and subtracts out its own message to obtain the desired message. The hybrid decode-amplify forward with linear combination strategy begins as relay one decodes a message from both terminals while relay two utilizes the amplify-forward strategy. After decoding the messages, the first relay creates codewords and passes them along to the second stage of the scheme. It is during this first stage that the first relay and both terminals form a multiple-access channel, with this relay serving as the destination. During the second stage, this relay decodes and reconstructs messages for transmission. Meanwhile, the second relay amplifies and transmits the message [33].

In the third type of scheme, namely hybrid decode-amplify-and-forward with multiplexed coding, refer relay once again decodes and forwards the messages from both sources while the second relay amplifies and transmits the signal. In contrast to the previous scheme, however, the first relay now re-encodes the message and creates codewords to be used in the second stage. Both terminals used multiplexed coding to decode the signals [33].

There are two other schemes called decode-forward and partial decode-forward. In the former, each terminal splits the signal into two parts, common and private. Both relays are used to decode the common parts and the messages are then transmitted. However, the private parts are only decoded by one relay and can only be interpreted when following a path that initiates with the terminal that initiated the signal and continues through the relay and onto the second terminal. In the final scheme, partial decode-forward, the message from one terminal is sent to both relays for decoding. After decoding, the reconstructed signal from the terminal is subtracted out from the received signal. The message becomes a new codeword which is then combined with the portion that was subtracted out for transmission to the other terminal.

The authors of [33] devise several transmission strategies for a two-way two-relay networks with only two relays. Through all the transmission strategies there is no transmission strategy that can dominate all other strategies under all channel realizations. Their goal is that the ideas of their schemes can be applied to a network with more than two relays. They conclude that AF and pDF can be directly implemented without any changes. The other strategies need complicated design, since it is necessary to determine which relay to decode which source's message.

2.5.4 Two-Phase Two-Way Distributed Beamforming

Distributed beamforming offers a promising way to overcome the physical limitations of cellular hardware and provide enhanced capacity and coverage to users.

Havary-Nasab et. al. considered a relay network which consists of two transceivers and n_r relay nodes, as shown in Fig. 2.11.

In order to send and receive signal of information, s_1 and s_2, as mentioned before three schemes are available. As shown in Fig. 2.7, the data transmission can be done in four time-slots. In the first time-slot, s_1 is sent to the relays and in second time-slot, the

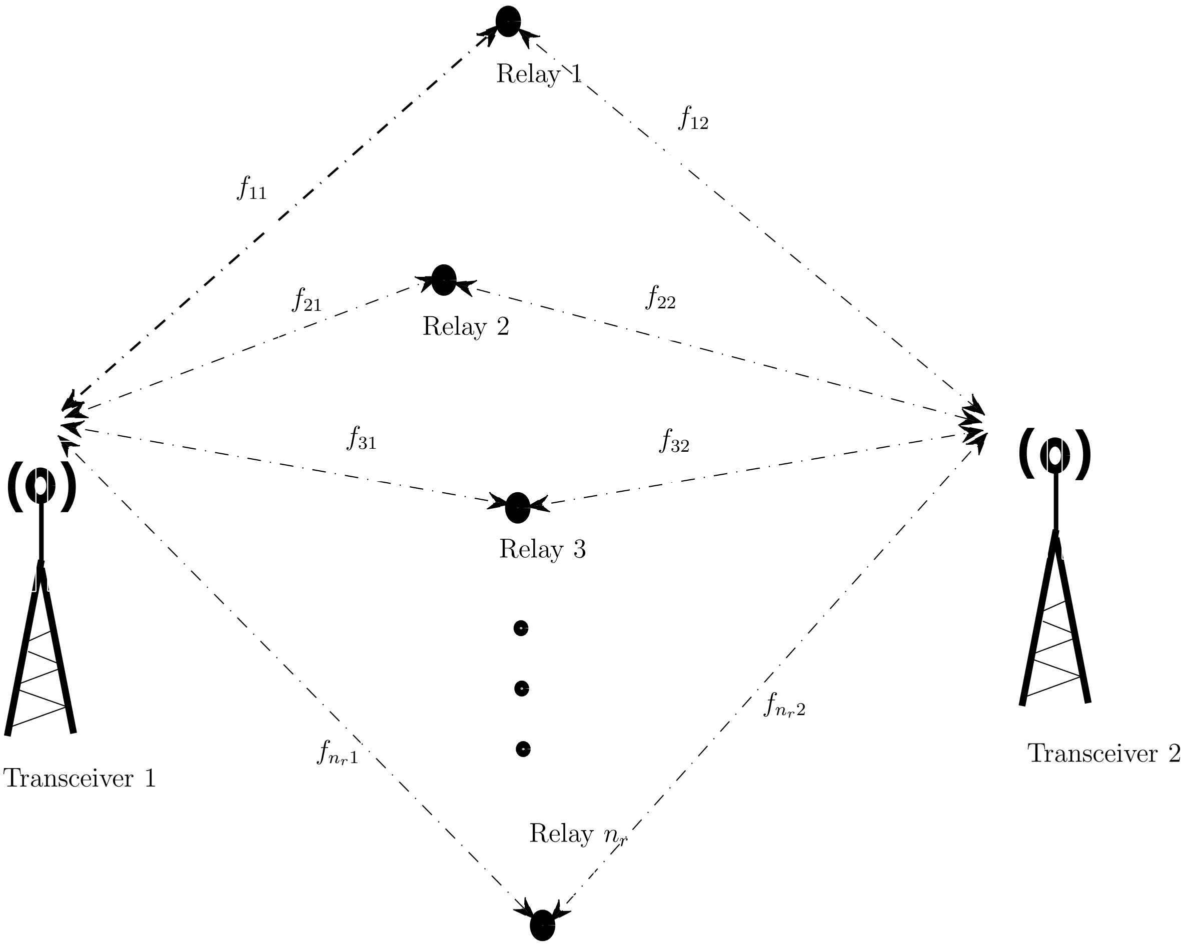

Fig. 2.11: A two-way relay networks with n_r relay nodes

relay re-transmits the signal to the second transceiver. In third and fourth time-slot, s_2 is sent to the relays and then the relays re-transmit it back to the first transceiver [15] .

Further research into the field includes reducing the time requires for such a transmission. A new scheme was proposed that potentially reduced the time taken for the transmission, thus is the major focus of my thesis. Here the communication takes in three time-slots. The authors of [34] proposed this scheme for a single relay and referred to it as single-relay Time Division Broadcast Channel (TDBC) method . This strategy takes three time-slots to establish the connection as shown in Fig. 2.8. This thesis is using this new sequence and then compares the results with that of [15].

A more bandwidth-efficient scheme is called Multiple Access Broadcast Channel (MABC) where the two transceivers transmit their information concurrently and each relay node re-transmits a processed version of its received signal. Study [15] focused on designing optimal MABC-based relaying schemes for a network with multiple relays and the problem of network beamforming for two-way relaying schemes.

Havary-Nasab et. al aimed to use two different approaches to calculate the beamforming coefficients as well as the transceivers' transmit powers. The authors used both the beamforming weight vector and the transceivers' transmit powers as design parameters to minimize the total transmit power subject to two constraints on the Quality of Service (QoS) at both transceivers. In this approach, they proved that power minimization technique have a unique solution and that optimal weight vector can be obtained through a simple iterative algorithm [15].

The authors of [15] also proved that for a symmetric relaying scheme, half of the minimum total transmit power will be allocated to the two transceivers and the remaining half will be shared among the relaying nodes. In the second approach, the authors studied an SNR balancing technique. In this technique, the smallest of the two transceivers' received SNRs is maximized while the total transmit power is kept below a certain power threshold. This approach also leads to a power allocation scheme where half of the power budget is allocated to the two transceivers and the remaining half will be shared among all the relay nodes.

As mentioned above, this model considered wireless network that consists of two transceivers and n_r relay nodes. Each relay has a single antenna for both transmission and reception. The authors also assumed a flat fading scenario. As mentioned previously, in the first time-slot, both transceivers transmit signals s_1 and s_2 to the relays. In this phase, the received signal at the ith relay can be express as a vector:

$$\mathbf{e} = \sqrt{P_1}\mathbf{f}_1 s_1 + \sqrt{P_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu} \tag{2.2}$$

where $\mathbf{e}$ is the $n_r \times 1$ complex vector of the relay received signals, P_1 and P_2 are the transmit powers of Transceivers 1 and 2, respectively, $\boldsymbol{\nu}$ is the $n_r \times 1$ complex vector of the relay noise, and $\mathbf{f}_1$ and $\mathbf{f}_2$ are the $n_r \times 1$ vectors of the channel coefficients from/to the relays to/from the transceivers and defined as $\mathbf{f}_1 \triangleq [f_{11}\ f_{21}\ \ldots\ f_{n_r 1}]^T$ and $\mathbf{f}_2 \triangleq [f_{12}\ f_{22}\ \ldots\ f_{n_r 2}]^T$.

In the second time-slot, the ith relay multiplies the received signal by a complex weight and transmits the obtained signal. The signal transmitted by the relay to the destination is represented by $\mathbf{u}$ which is a $n_r \times 1$ complex vector and expressed as:

$$\mathbf{u} = \mathbf{V}\mathbf{e} \tag{2.3}$$

where, $\mathbf{V} \triangleq \text{diag}([v_1^* \;\; v_2^* \quad \ldots \quad v_{nr}^*])$.

Then both transceivers obtain the optimal values of the weight vector and through has transceivers' transmit powers. In this network, the goal is to minimize the total transmit power P_T, while satisfying the two constraints on the SNRs of the two transceivers, guaranteeing then to be larger than certain thresholds. The problem can be defined mathematically:

$$\begin{aligned} \min_{P_1, P_2, \mathbf{v}} \quad & P_T \\ \text{subject to} \quad & \text{SNR}_1 \geq \gamma_1 \quad \text{and} \quad \text{SNR}_2 \geq \gamma_2 \end{aligned} \tag{2.4}$$

where, SNR_k is the signal power to noise power ratio of P_1 and P_2, respectively for $k = 1, 2$ and γ_1 and γ_2 are the thresholds for SNR_1 and SNR_2, respectively. The total transmit power is defined as

$$P_T = P_1 + P_2 + P_r \tag{2.5}$$

where, P_r is the relay transmit power, with following mathematically definition:

$$P_r \triangleq E\{\mathbf{u}^H \mathbf{e}\} = \mathbf{v}^H \mathbf{L} \mathbf{v} \cdot \tag{2.6}$$

Here $\mathbf{L} \triangleq E\{\mathbf{E}^H \mathbf{E}\}$ and $\mathbf{E} \triangleq \text{diag}(\mathbf{e})$. Also, the SNR_k for $k = 1, 2$ are defined as:

$$\text{SNR}_1 = \frac{P_2 \mathbf{v}^H \mathbf{h} \mathbf{h}^H \mathbf{v}}{\sigma^2 + \sigma^2 \mathbf{v}^H \mathbf{F}_1 \mathbf{F}_1^H \mathbf{v}} \tag{2.7}$$

$$\text{SNR}_2 = \frac{P_1 \mathbf{v}^H \mathbf{h} \mathbf{h}^H \mathbf{v}}{\sigma^2 + \sigma^2 \mathbf{v}^H \mathbf{F}_2 \mathbf{F}_2^H \mathbf{v}} \tag{2.8}$$

where, $\mathbf{h} \triangleq \mathbf{F}_1 \mathbf{f}_2 = \mathbf{F}_2 \mathbf{f}_1 = \mathbf{f}_1 \odot \mathbf{f}_2$ and σ^2 is defined as $E\{\boldsymbol{\nu}\boldsymbol{\nu}^H\} = \sigma^2 \mathbf{I}$. Furthermore, the authors of [15] have shown that in terms of its weight vector $\mathbf{v}$ the optimization problem (2.4) is equivalent to the following unconstrained optimization problem.

$$\min_{\mathbf{v}} \quad \sigma^2(\gamma_1 + \gamma_2) \frac{(1 + \mathbf{v}^H \mathbf{f}_1 \mathbf{f}_1^H \mathbf{v})(1 + \mathbf{v}^H \mathbf{f}_2 \mathbf{f}_2^H \mathbf{v})}{\mathbf{v}^H \mathbf{h} \mathbf{h}^H \mathbf{v}} + \sigma^2 \mathbf{v}^H \mathbf{v} \tag{2.9}$$

It is then proved that the optimization problem (2.9) has only one global solution. Therefore, the optimal weight vector can be obtained through a simple iterative algorithm.

Also, the authors of [15] proved that if the two constraints on the transceivers' SNRs are similar (e.g., $\gamma_1 = \gamma_2$) then the minimum total transmit power will be in two equal half. A half of it, goes to the both transceivers and second to the n_r relay nodes.

In the second approach, SNR balancing with the constraint that required the total transmit power to be below a certain threshold is studied. Mathematically, the optimization problem is expressed below

$$\begin{aligned} \max_{P_1, P_2, \mathbf{v}} \quad & \min(\mathrm{SNR}_1, \mathrm{SNR}_2) \\ \text{subject to} \quad & P_T \leqslant P_T^{max} \end{aligned} \tag{2.10}$$

This solution only re-enforced the fact that this solution has an unique global solution obtained by an iterative process. Again, is proved that half of the maximum power budget is allocated to the two transceivers and the remaining half is shared among the relay nodes.

Finally, a distributed scheme in reference [15] is developed such that minimal cooperation between two transceivers and relays is needed. The authors concluded that even if the size of the network increases, only a constant bandwidth for having beamforming weights in a distributed manners is required.

In this chapter one-way and two-way relaying networks were introduced and some studies in optimal distributed beamforming were reviewed. Furthermore, different decoding and strategies to exchange information in two-way relaying was presented. In this thesis a parallel network as discussed in this chapter is considered to establish a two-way communication between two transceivers. A TDBC distributed beamforming scheme is adopted. The goal is to optimally obtain the relays weight vector as well as the transceivers' transmit powers so that the total power dissipated in the relay networks and in the transceivers is minimized. Subject to QoS constraints in the transceivers. The total transmit power is minimized. In the next chapter, first problem setup is discussed. Second, the optimization problem is formulated and solved and the uniqueness of the solution is further investigated. This problem has been analyzed in [15] for MABC scheme. It is shown that using TDBC, results less total transmit power, although one phase is added in TDBC.

Chapter 3

Optimal Three-Phase Distributed Beamformer

In this chapter, we discuss our three-phase distributed beamforming which is based on the minimization of total transmit power subject to two constraints on transceivers' SNRs. We consider a relay network with two transceivers and r relays and we assume no-LOS between them. We have used TDBC strategy to design our bidirectional relaying scheme. Transceiver 1 and 2 send their symbols to the relays in the first two phases. Each of these signals, is multiplied by two different complex weights (beamforming weight coefficient). In the third phase, relays send a combination of their received signals to both nodes at the same time. The optimal weight coefficients and optimal transceivers' transmit powers are found. Finally, we compare our results with those obtained in [15]. Our design parameters are two beamforming weight vectors and transceivers' transmit powers. Since the signal is being transmitted from two transceivers in two different phases (time-slots), we have two different weight vectors, $\mathbf{w}_1$ and $\mathbf{w}_2$ for Transceiver 1 and Transceiver 2, respectively. Our goal is to minimize the total transmit power, considering two constraints on the QoS at both transceivers. In this thesis we first solve the problem using an iterative steepest descent algorithm. Then to further ensure our obtained solution performs closely to the optimal sloution, we develop second solution based second order cone programming. More specifically, the set of feasible values of transceivers' transmit powers is quantized into a sufficient fine grid. Then, at each vertices of this grid, a SOCP problem is solved to obtain the beamforming coefficients such that for the given pair of transceivers' transmit powers, the total transmit power is minimized. The pair of the transceivers' transmit powers which result in the smallest possible value of the total transmit power, leads us to solution of the problem. This approach requires a two-dimensional search and solving an SOCP problem at each point of the corresponding two-dimensional grid. This second technique performs very close to the steepest descent algorithm.

The rest of this chapter is organized as follows. Section 3.1 describes the data model. The optimization problem has been formulated and expanded in Section 3.2. Section 3.3 is devoted to explain the SOCP based approach.

3.1 Data Model

Consider a wireless relay network consisting of two transceivers and r relay nodes. For this network, the following assumptions are used throughout the paper:

A1 The channel between each transceiver and each relay is frequency flat.

A2 The channel from a certain transceiver to each relay is the same as the channel from that relay to the same transceiver, i.e., the channels are reciprocal.

A3 Let f_{i1} (f_{i2}) denote the reciprocal flat fading channel coefficient between Transceiver 1 (2). Each transceiver has the knowledge of all channel coefficients $\{f_{i1}\}_{i=1}^{r}$ and $\{f_{i2}\}_{i=1}^{r}$.

Our communication scheme consists of three time slots (phases). In the first time slot, Transceiver 1 transmits its data to all relays. Similarly, in the second time slot, Transceiver 2 transmits its data to all the relays. The ith relay linearly combines the signals it received during the the first two time slots and transmit the so-obtained signal in third time slot. Using vector notations, the vectors of the signals received by the relays in time slots 1 and 2 can be written, respectively, as

$$\mathbf{x}_1 = \sqrt{p_1}\mathbf{f}_1 s_1 + \upsilon_1 \tag{3.1}$$

$$\mathbf{x}_2 = \sqrt{p_2}\mathbf{f}_2 s_2 + \upsilon_2 \tag{3.2}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the $r \times 1$ complex vectors of the signals the relays receive during the corresponding time slots, p_1 and p_2 are the transmit powers of Transceivers 1 and 2, respectively, s_1 and s_2 are the information symbols transmitted by Transceivers 1 and 2, respectively, $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$ are the $r \times 1$ complex vectors of the relay noises, and

$$\mathbf{f}_1 \triangleq [f_{11}\ f_{21}\ \ldots\ f_{r1}]^T$$
$$\mathbf{f}_2 \triangleq [f_{12}\ f_{22}\ \ldots\ f_{r2}]^T$$

are the vectors of the channel coefficients between the relays and Transceivers 1 and 2, respectively. Having p_1 and p_2 defined as the transmit powers of the two transceivers implies that $E\{|s_1|^2\} = E\{|s_2|^2\} = 1$. We assume that each transceiver knows both channel vectors $\mathbf{f}_1$ and $\mathbf{f}_2$. In the third time slot, the ith relay multiplies the signals it received in the first and second time slot, by complex weights w_{i1}^* and w_{i2}^*, respectively, and then, transmits this new signal to both transceivers. The $r \times 1$ complex vector $\mathbf{t}$ of the relay transmitted signals can then be expressed as

$$\mathbf{t} = \mathbf{w}_1 \odot \mathbf{x}_1 + \mathbf{w}_2 \odot \mathbf{x}_2 \tag{3.3}$$

where $\mathbf{w}_1 \triangleq [w_{11}^*\ w_{21}^*\ \ldots\ w_{r1}^*]^T$ and $\mathbf{w}_2 \triangleq [w_{12}^*\ w_{22}^*\ \ldots\ w_{r2}^*]^T$. The signals y_1 and y_2 received at the two transceivers can be written as

$$\begin{aligned} y_1 &= \mathbf{f}_1^T[\mathbf{w}_1 \odot \mathbf{x}_1 + \mathbf{w}_2 \odot \mathbf{x}_2] + n_1 \\ &= \mathbf{f}_1^T[\mathbf{w}_1 \odot (\sqrt{p_1}\mathbf{f}_1 s_1 + \boldsymbol{\nu}_1) + \mathbf{w}_2 \odot (\sqrt{p_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}_2)] + n_1 \end{aligned} \tag{3.4}$$

$$\begin{aligned} y_2 &= \mathbf{f}_2^T[\mathbf{w}_1 \odot \mathbf{x}_1 + \mathbf{w}_2 \odot \mathbf{x}_2] + n_2 \\ &= \mathbf{f}_2^T[\mathbf{w}_1 \odot (\sqrt{p_1}\mathbf{f}_1 s_1 + \boldsymbol{\nu}_1) + \mathbf{w}_2 \odot (\sqrt{p_2}\mathbf{f}_2 s_2 + \boldsymbol{\nu}_2)] + n_2 \end{aligned} \tag{3.5}$$

where n_k is the receive noise at the kth transceiver, for $k = 1, 2$. As $\mathbf{a}^T(\mathbf{b}\odot\mathbf{c}) = \mathbf{b}^T(\mathbf{a}\odot\mathbf{c})$, we rewrite (3.4) and (3.5), respectively, as

$$\begin{aligned} y_1 &= \sqrt{p_1}\mathbf{w}_1^H(\mathbf{f}_1 \odot \mathbf{f}_1)s_1 \\ &\quad + \sqrt{p_2}\mathbf{w}_2^H(\mathbf{f}_1 \odot \mathbf{f}_2)s_2 + \mathbf{w}_1^H(\mathbf{f}_1 \odot \boldsymbol{\nu}_1) + \mathbf{w}_2^H(\mathbf{f}_1 \odot \boldsymbol{\nu}_2) + n_1 \end{aligned} \tag{3.6}$$

$$\begin{aligned} y_2 &= \sqrt{p_1}\mathbf{w}_1^H(\mathbf{f}_2 \odot \mathbf{f}_1)s_1 \\ &\quad + \sqrt{p_2}\mathbf{w}_2^H(\mathbf{f}_2 \odot \mathbf{f}_2)s_2 + \mathbf{w}_1^H(\mathbf{f}_2 \odot \boldsymbol{\nu}_1) + \mathbf{w}_2^H(\mathbf{f}_2 \odot \boldsymbol{\nu}_2) + n_2\,. \end{aligned} \tag{3.7}$$

In our scheme, both transceivers calculate the optimal values of the weight vectors as well as optimal transceivers' transmit powers. It is noteworthy that the first term in (3.6) is known as *self-interference* as it depends on the signal s_1 transmitted by Transceiver 1 during the first time slot. Given the fact that s_1, p_1, and $\mathbf{f}_1$ are known to Transceiver 1 and that the weight vectors $\mathbf{w}_1$ is going to be calculated at this transceiver, therefore, the first term in (3.6) is known at Transceiver 1. Hence, this term can be subtracted from y_1 and the residual signal can be used by Transceiver 1 to detect the information symbol s_2. Similarly, the second term in (3.7) can be subtracted from y_2 and the residual signal can be used by Transceiver 2 to extract the information symbol s_1. That is, the residual signals $\tilde{y}_1$ and $\tilde{y}_2$, defined as

$$\tilde{y}_1 \triangleq y_1 - \sqrt{p_1}\mathbf{w}_1^H(\mathbf{f}_1 \odot \mathbf{f}_1)s_1 = \underbrace{\sqrt{p_2}\mathbf{w}_2^H(\mathbf{f}_1 \odot \mathbf{f}_2)s_2}_{\text{desired signal}} + \underbrace{\mathbf{w}_1^H(\mathbf{f}_1 \odot \boldsymbol{\nu}_1) + \mathbf{w}_2^H(\mathbf{f}_1 \odot \boldsymbol{\nu}_2) + n_1}_{\text{noise}} \tag{3.8}$$

$$\tilde{y}_2 \triangleq y_2 - \sqrt{p_2}\mathbf{w}_2^H(\mathbf{f}_2 \odot \mathbf{f}_2)s_2 = \underbrace{\sqrt{p_1}\mathbf{w}_1^H(\mathbf{f}_2 \odot \mathbf{f}_1)s_1}_{\text{desired signal}} + \underbrace{\mathbf{w}_1^H(\mathbf{f}_2 \odot \boldsymbol{\nu}_1) + \mathbf{w}_2^H(\mathbf{f}_2 \odot \boldsymbol{\nu}_2) + n_2}_{\text{noise}}, \tag{3.9}$$

can be processed at their corresponding transceivers to extract the desired information symbols.

We make two more assumptions:

A4 We assume, without loss of generality, that the noise process is zero-mean and spatially white with variance 1. That is, $E\{|n_1|^2\} = E\{|n_2|^2\} = 1$ and $E\{\boldsymbol{\nu}_1\boldsymbol{\nu}_1^H\} = E\{\boldsymbol{\nu}_2\boldsymbol{\nu}_2^H\} = \mathbf{I}$.

A5 The information symbols s_1, s_2 and the relay noise $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$ are all assumed to be zero-mean mutually independent random variables/vectors.

To obtain the transmit powers p_1 and p_2 as well as the relay weight vectors $\mathbf{w}_1$ and $\mathbf{w}_2$, we minimize the total transmit power consumed in the whole network subject to two constraints on the transceivers' received SNRs to ensure a minimum level of quality of service (QoS) is guaranteed for each transceiver.

3.2 Power Minimization

In this section, our goal is to find the beamforming weights $\{w_{i1}\}_{i=1}^{r}$ and $\{w_{i2}\}_{i=1}^{r}$ as well as the transceivers' transmit powers p_1 and p_2 such that the total transmit power P_T, dissipated in the whole network, is minimized while the received SNRs are required to be larger than given pre-defined thresholds $\gamma_1 > 0$ and $\gamma_2 > 0$. Mathematically, we solve the following optimization problem:

$$\min_{p_1,p_2,\mathbf{w}_1,\mathbf{w}_2} \quad P_T \qquad (3.10)$$
$$\text{subject to} \quad \text{SNR}_1 \geq \gamma_1 \quad \text{and} \quad \text{SNR}_2 \geq \gamma_2$$

where SNR_k is defined as the ratio of the desired signal power $P_{\text{s},k}$ to the noise power $P_{\text{n},k}$ at Transceiver k, for $k = 1, 2$. The total transmit power P_T can be expressed as

$$P_T = p_1 + p_2 + p_r \qquad (3.11)$$

where p_r is the relay transmit power during the third time slot, and it is given by

$$p_r \triangleq E\{\mathbf{t}^H\mathbf{t}\} = \mathbf{w}_1^H\mathbf{B}_1\mathbf{w}_1 + \mathbf{w}_2^H\mathbf{B}_2\mathbf{w}_2\,. \qquad (3.12)$$

Here, $\mathbf{B}_k \triangleq E\{\text{diag}(\mathbf{x}_k^*)\text{diag}(\mathbf{x}_k)\}$, for $k = 1, 2$ and we have used the fact that $\mathbf{w}_k \odot \mathbf{x}_k = \text{diag}(\mathbf{x}_k)\mathbf{w}_k$. Using (3.1) and (3.2) along with Assumptions A4 and A5, the matrices $\mathbf{B}_1$ and $\mathbf{B}_2$ can be expressed as

$$\mathbf{B}_1 = p_1\mathbf{D}_1 + \mathbf{I} \qquad (3.13)$$
$$\mathbf{B}_2 = p_2\mathbf{D}_2 + \mathbf{I} \qquad (3.14)$$

where $\mathbf{D}_k = E\{\text{diag}(\mathbf{f}_k^*)\text{diag}(\mathbf{f}_k)\}$, for $k = 1, 2$. Using (3.8) and (3.9), the receive SNRs can be written as

$$\text{SNR}_1 = \frac{p_2\mathbf{w}_2^H\mathbf{h}\mathbf{h}^H\mathbf{w}_2}{1 + \mathbf{w}_1^H\mathbf{D}_1\mathbf{w}_1 + \mathbf{w}_2^H\mathbf{D}_1\mathbf{w}_2} \qquad (3.15)$$
$$\text{SNR}_2 = \frac{p_1\mathbf{w}_1^H\mathbf{h}\mathbf{h}^H\mathbf{w}_1}{1 + \mathbf{w}_1^H\mathbf{D}_2\mathbf{w}_1 + \mathbf{w}_2^H\mathbf{D}_2\mathbf{w}_2} \qquad (3.16)$$

where $\mathbf{h} \triangleq \mathbf{f}_1 \odot \mathbf{f}_2$. Using the following definitions:

$$\mathbf{A}_1 \triangleq [\mathbf{0} \;\; \mathbf{I}]$$
$$\mathbf{A}_2 \triangleq [\mathbf{I} \;\; \mathbf{0}]$$
$$\mathbf{w} \triangleq [\mathbf{w}_1^T \;\; \mathbf{w}_2^T]^T$$
$$\mathbf{Q}_1 \triangleq \text{diag}(\mathbf{D}_1, \mathbf{D}_1)$$
$$\mathbf{Q}_2 \triangleq \text{diag}(\mathbf{D}_2, \mathbf{D}_2),$$

we can rewrite (3.11) as

$$\begin{aligned} P_T &= p_1 + p_2 + p_1 \mathbf{w}_1^H (p_1 \mathbf{D}_1 + \mathbf{I}) \mathbf{w}_1 + p_2 \mathbf{w}_2^H (p_2 \mathbf{D}_2 + \mathbf{I}) \mathbf{w}_2 \\ &= p_1 + p_2 + p_1 \mathbf{w}_1^H \mathbf{D}_1 \mathbf{w}_1 + p_2 \mathbf{w}_2^H \mathbf{D}_2 \mathbf{w}_2 + \mathbf{w}_1^H \mathbf{w}_1 + \mathbf{w}_2^H \mathbf{w}_2 \\ &= p_1 + p_2 + p_1 \mathbf{w}^H \mathbf{A}_2^H \mathbf{D}_1 \mathbf{A}_2 \mathbf{w} + p_2 \mathbf{w}^H \mathbf{A}_1^H \mathbf{D}_2 \mathbf{A}_1 \mathbf{w} + \mathbf{w}^H \mathbf{w} \end{aligned} \tag{3.17}$$

where we have used the fact that $\mathbf{w}_1^H \mathbf{w}_1 + \mathbf{w}_2^H \mathbf{w}_2 = \mathbf{w}^H \mathbf{w}$. Also, the Transceivers' SNRs can be written as

$$\text{SNR}_1 = \frac{p_2 \mathbf{w}^H \mathbf{A}_1^H \mathbf{h} \mathbf{h}^H \mathbf{A}_1 \mathbf{w}}{1 + \mathbf{w}^H \mathbf{Q}_1 \mathbf{w}} \tag{3.18}$$
$$\text{SNR}_2 = \frac{p_1 \mathbf{w}^H \mathbf{A}_2^H \mathbf{h} \mathbf{h}^H \mathbf{A}_2 \mathbf{w}}{1 + \mathbf{w}^H \mathbf{Q}_2 \mathbf{w}} \,. \tag{3.19}$$

Using (3.18)-(3.17), the optimization problem in (3.10) can be written as

$$\begin{aligned} \min_{p_1, p_2, \mathbf{w}} \quad & p_1 + p_2 + \mathbf{w}^H \boldsymbol{\mathcal{D}}(p_1, p_2) \mathbf{w} + \mathbf{w}^H \mathbf{w} \\ \text{subject to} \quad & \frac{p_2 \mathbf{w}^H \mathbf{A}_1^H \mathbf{h} \mathbf{h}^H \mathbf{A}_1 \mathbf{w}}{1 + \mathbf{w}^H \mathbf{Q}_1 \mathbf{w}} \geq \gamma_1 \\ \text{and} \quad & \frac{p_1 \mathbf{w}^H \mathbf{A}_2^H \mathbf{h} \mathbf{h}^H \mathbf{A}_2 \mathbf{w}}{1 + \mathbf{w}^H \mathbf{Q}_2 \mathbf{w}} \geq \gamma_2 \end{aligned} \tag{3.20}$$

where $\boldsymbol{\mathcal{D}}(p_1, p_2) = \text{diag}(p_1 \mathbf{D}_1, p_2 \mathbf{D}_2)$. Note at the optimum, the inequality constraints in (3.20) will be satisfied with equality. Otherwise, if, for example, the first constraint in (3.20) is satisfied with inequality at the optimum, then the optimal p_2 can be scaled down to satisfy this constraint with equality. This, however, further reduces the objective function in (3.20), thereby contradicting the optimality. Using this observation, the transceivers' transmit powers can be obtained as

$$p_1 = \frac{\gamma_2 (1 + \mathbf{w}^H \mathbf{Q}_2 \mathbf{w})}{\mathbf{w}^H \mathbf{A}_2^H \mathbf{h} \mathbf{h}^H \mathbf{A}_2 \mathbf{w}} \tag{3.21}$$
$$p_2 = \frac{\gamma_1 (1 + \mathbf{w}^H \mathbf{Q}_1 \mathbf{w})}{\mathbf{w}^H \mathbf{A}_1^H \mathbf{h} \mathbf{h}^H \mathbf{A}_1 \mathbf{w}} \,. \tag{3.22}$$

It follows from (3.21) and (3.22) that for any SNR threshold pair (γ_1, γ_2), the SNR constraints can always be satisfied by increasing the transceivers' transmit powers. Using (3.21) and (3.22), the optimization problem (3.20) can be turned into the following *unconstrained* optimization problem:

$$\min_{\mathbf{w}} \left(\frac{\gamma_2(1+\mathbf{w}^H\mathbf{Q}_2\mathbf{w})(1+\mathbf{w}^H\mathbf{R}_2\mathbf{w})}{\mathbf{w}^H\mathbf{h}_1\mathbf{h}_1^H\mathbf{w}} \right) + \left(\frac{\gamma_1(1+\mathbf{w}^H\mathbf{Q}_1\mathbf{w})(1+\mathbf{w}^H\mathbf{R}_1\mathbf{w})}{\mathbf{w}^H\mathbf{h}_2\mathbf{h}_2^H\mathbf{w}} \right) + \mathbf{w}^H\mathbf{w} \tag{3.23}$$

where $\mathbf{R}_2 \triangleq \mathbf{A}_2^H\mathbf{D}_1\mathbf{A}_2 = \text{diag}(\mathbf{D}_1, \mathbf{0})$, $\mathbf{R}_1 \triangleq \mathbf{A}_1^H\mathbf{D}_2\mathbf{A}_1 = \text{diag}(\mathbf{0}, \mathbf{D}_2)$, $\mathbf{h}_2 \triangleq \mathbf{A}_1^H\mathbf{h} = [\mathbf{0}^T \ \ \mathbf{h}^T]^T$, and $\mathbf{h}_1 \triangleq \mathbf{A}_2^H\mathbf{h} = [\mathbf{h}^T \ \ \mathbf{0}^T]^T$.

Let $\boldsymbol{\alpha} \triangleq [\boldsymbol{\alpha}_1^T \ \boldsymbol{\alpha}_2^T]^T$ and $\boldsymbol{\theta} \triangleq [\boldsymbol{\theta}_1^T \ \boldsymbol{\theta}_2^T]^T$, where $\boldsymbol{\alpha}_k$ and $\boldsymbol{\theta}_k$ are, respectively, the vectors of the amplitudes and the phases of different entries of $\mathbf{w}_k$, for $k = 1, 2$. As $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{Q}_1$, and $\mathbf{Q}_2$ are real-valued diagonal matrices, the numerators of the first and the second terms of the objective function in (3.23) as well as the third term in this objective function can be written, respectively, as $(1 + \boldsymbol{\alpha}^T\mathbf{Q}_2\boldsymbol{\alpha})(1 + \boldsymbol{\alpha}^T\mathbf{R}_2\boldsymbol{\alpha})$, $(1 + \boldsymbol{\alpha}^T\mathbf{Q}_1\boldsymbol{\alpha})(1 + \boldsymbol{\alpha}^T\mathbf{R}_1\boldsymbol{\alpha})$, and $\boldsymbol{\alpha}^T\boldsymbol{\alpha}$, and therefore, they do not depend on $\boldsymbol{\theta}$, As a result the optimization (3.23) can be equivalently rewritten as

$$\min_{\boldsymbol{\alpha}\succcurlyeq 0} \frac{\gamma_2(1+\boldsymbol{\alpha}^T\mathbf{Q}_2\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_2\boldsymbol{\alpha})}{g_1(\boldsymbol{\alpha})} + \frac{\gamma_1(1+\boldsymbol{\alpha}^T\mathbf{Q}_1\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_1\boldsymbol{\alpha})}{g_2(\boldsymbol{\alpha})} + \boldsymbol{\alpha}^T\boldsymbol{\alpha} \tag{3.24}$$

where

$$\begin{aligned} g_k(\boldsymbol{\alpha}) &\triangleq \max_{\boldsymbol{\theta}} \mathbf{w}^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w} = \max_{\boldsymbol{\theta}} \mathbf{w}_k^H\mathbf{h}\mathbf{h}^H\mathbf{w}_k \\ &= \max_{\boldsymbol{\theta}} \left| \sum_{i=1}^{r} \alpha_{ik} b_i e^{j(\phi_i - \theta_{ik})} \right|^2 = |\boldsymbol{\alpha}_k^T\mathbf{b}|^2 = \boldsymbol{\alpha}_k^T\mathbf{b}\mathbf{b}^T\boldsymbol{\alpha}_k \end{aligned} \tag{3.25}$$

for $k = 1, 2$. Here, b_i and ϕ_i, are the amplitude and the phase of the ith entry of $\mathbf{h}$, respectively, $\mathbf{b} \triangleq [b_1, b_2, \ldots, b_r]$, and, α_{ik} and θ_{ik} are the ith entries of $\boldsymbol{\alpha}_k$ and $\boldsymbol{\theta}_k$, respectively. The maximum in (3.25) is achieved when $\theta_{ik} = \phi_i$ is chosen for any i and for $k = 1, 2$. This means that the phase of the ith entry of $\mathbf{w}_k$ has to match to the phase of ith entry of $\mathbf{h}$ which is equal to the aggregated phase of the channel coefficients from the ith relay to the two transceivers. That is $\theta_{ik} = \phi_i = \angle f_{i1} + \angle f_{i2}$. Using (3.24) and defining $\mathbf{b}_1 \triangleq [\mathbf{b} \ \mathbf{0}]$ and $\mathbf{b}_2 \triangleq [\mathbf{0} \ \mathbf{b}]$, we rewrite (3.24) as

$$\min_{\boldsymbol{\alpha}\succcurlyeq 0} \frac{\gamma_2(1+\boldsymbol{\alpha}^T\mathbf{Q}_2\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_2\boldsymbol{\alpha})}{\boldsymbol{\alpha}^T\mathbf{b}_1\mathbf{b}_1^T\boldsymbol{\alpha}} + \frac{\gamma_1(1+\boldsymbol{\alpha}^T\mathbf{Q}_1\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_1\boldsymbol{\alpha})}{\boldsymbol{\alpha}^T\mathbf{b}_2\mathbf{b}_2^T\boldsymbol{\alpha}} + \boldsymbol{\alpha}^T\boldsymbol{\alpha} \,. \tag{3.26}$$

The optimization problem (3.26) can be solved using a gradient based steepest descent method. This method may however lead to a local minimum. In the next section, we develop a second order cone programming (SOCP) based method to obtain the globally optimal solution to (3.20).

3.3 An SOCP-based Approach

To develop an SOCP-based approach to solve the optimization problem (3.20), we first obtain the set of feasible values of p_1 and p_2. To do so, we rewrite the two constraints in (3.20) as

$$\mathbf{w}^H(p_2\mathbf{A}_1^H\mathbf{h}\mathbf{h}^H\mathbf{A}_1 - \gamma_1\mathbf{Q}_1)\mathbf{w} \geq \gamma_1 \quad (3.27)$$

$$\mathbf{w}^H(p_1\mathbf{A}_2^H\mathbf{h}\mathbf{h}^H\mathbf{A}_2 - \gamma_2\mathbf{Q}_2)\mathbf{w} \geq \gamma_2. \quad (3.28)$$

It follows from (3.27) and (3.28) that if for a given value of p_1, the matrix $(p_1\mathbf{A}_2^H\mathbf{h}\mathbf{h}^H\mathbf{A}_2 - \gamma_2\mathbf{Q}_2)$ is negative definite, then the optimization problem (3.20) becomes infeasible. Similarly, if for a given value of p_2, the matrix $p_2\mathbf{A}_1^H\mathbf{h}\mathbf{h}^H\mathbf{A}_1 - \gamma_1\mathbf{Q}_1$ is negative definite, then the optimization problem (3.20) becomes infeasible. Therefore, for any given pair of p_1 and p_2, the optimization problem (3.20) becomes feasible if and only if the two matrices

$$p_2\mathbf{A}_1^H\mathbf{h}\mathbf{h}^H\mathbf{A}_1 - \gamma_1\mathbf{Q}_1 = \mathbf{Q}_1^{-1/2}(p_2\mathbf{Q}_1^{-1/2}\mathbf{A}_1^H\mathbf{h}\mathbf{h}^H\mathbf{A}_1\mathbf{Q}_1^{-1/2} - \gamma_1\mathbf{I})\mathbf{Q}_1^{-1/2} \quad (3.29)$$

$$p_1\mathbf{A}_2^H\mathbf{h}\mathbf{h}^H\mathbf{A}_2 - \gamma_2\mathbf{Q}_2 = \mathbf{Q}_2^{-1/2}(p_1\mathbf{Q}_2^{-1/2}\mathbf{A}_2^H\mathbf{h}\mathbf{h}^H\mathbf{A}_2\mathbf{Q}_2^{-1/2} - \gamma_2\mathbf{I})\mathbf{Q}_2^{-1/2} \quad (3.30)$$

are non-negative definite. This implies that p_1 and p_2 should satisfy

$$\lambda_{max}(p_2\mathbf{Q}_1^{-1/2}\mathbf{A}_1^H\mathbf{h}\mathbf{h}^H\mathbf{A}_1\mathbf{Q}_1^{-1/2} - \gamma_1\mathbf{I}) \geq 0 \quad (3.31)$$

$$\lambda_{max}(p_1\mathbf{Q}_2^{-1/2}\mathbf{A}_2^H\mathbf{h}\mathbf{h}^H\mathbf{A}_2\mathbf{Q}_2^{-1/2} - \gamma_2\mathbf{I}) \geq 0 \quad (3.32)$$

or, equivalently:

$$p_2 \geq \frac{\gamma_1}{\mathbf{h}^H\mathbf{A}_1\mathbf{Q}_1^{-1}\mathbf{A}_1^H\mathbf{h}} = \frac{\gamma_1}{\mathbf{h}^H\mathbf{D}_1^{-1}\mathbf{h}} \quad (3.33)$$

$$p_1 \geq \frac{\gamma_2}{\mathbf{h}^H\mathbf{A}_2\mathbf{Q}_2^{-1}\mathbf{A}_2^H\mathbf{h}} = \frac{\gamma_2}{\mathbf{h}^H\mathbf{D}_2^{-1}\mathbf{h}}\,. \quad (3.34)$$

Without loss of optimality, we can add the two constraints in (3.33) and (3.34) and rewrite the optimization problem (3.20) as

$$\begin{aligned} \min_{p_1,p_2} \quad & p_1 + p_2 + \min_{\mathbf{w}}(\mathbf{w}^H \boldsymbol{\mathcal{D}}(p_1,p_2)\mathbf{w} + \mathbf{w}^H\mathbf{w}) \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{A}_1^H \mathbf{h}\mathbf{h}^H \mathbf{A}_1 \mathbf{w} \geq \frac{\gamma_1}{p_2}(1 + \mathbf{w}^H \mathbf{Q}_1 \mathbf{w}) \\ \text{and} \quad & \mathbf{w}^H \mathbf{A}_2^H \mathbf{h}\mathbf{h}^H \mathbf{A}_2 \mathbf{w} \geq \frac{\gamma_2}{p_1}(1 + \mathbf{w}^H \mathbf{Q}_2 \mathbf{w}) \\ \text{and} \quad & p_2 \geq \frac{\gamma_1}{\mathbf{h}^H \mathbf{D}_1^{-1} \mathbf{h}} \\ \text{and} \quad & p_1 \geq \frac{\gamma_2}{\mathbf{h}^H \mathbf{D}_2^{-1} \mathbf{h}} \end{aligned} \tag{3.35}$$

For any fixed p_1 and p_2, the inner minimization can be written as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H (\boldsymbol{\mathcal{D}}(p_1,p_2) + \mathbf{I})\mathbf{w} \\ \text{subject to} \quad & |\mathbf{w}^H \mathbf{A}_1^H \mathbf{h}| \geq \sqrt{\frac{\gamma_1}{p_2}(1 + \mathbf{w}^H \mathbf{Q}_1 \mathbf{w})} \\ \text{and} \quad & |\mathbf{w}^H \mathbf{A}_2^H \mathbf{h}| \geq \sqrt{\frac{\gamma_2}{p_1}(1 + \mathbf{w}^H \mathbf{Q}_2 \mathbf{w})} \end{aligned} \tag{3.36}$$

Using the fact that $(\boldsymbol{\mathcal{D}}(p_1,p_2) + \mathbf{I})$, $\mathbf{Q}_1$, and $\mathbf{Q}_2$ are all diagonal matrices, we observe that if the optimal vector $\mathbf{w}_{opt} = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{bmatrix}$ is replaced with $\tilde{\mathbf{w}} = \begin{bmatrix} e^{\beta_1}\mathbf{w}_1 \\ e^{\beta_2}\mathbf{w}_2 \end{bmatrix}$, for any values β_1 and β_2, the value of the objective function and the constraints will not change. This means that the new vector $\tilde{\mathbf{w}}$ is also a solution to (3.36). Hence, without loss of optimality, we can rotate the phase of $\mathbf{w}_2$ such that $\mathbf{w}^H \mathbf{A}_1^H \mathbf{h} = \mathbf{w}_2^H \mathbf{h}$ is real and positive number. Similarly, we can rotate the phase of $\mathbf{w}_1$ such that $\mathbf{w}^H \mathbf{A}_2^H \mathbf{h} = \mathbf{w}_1^H \mathbf{h}$ is real and positive number. Therefore,

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H (\boldsymbol{\mathcal{D}}(p_1,p_2) + \mathbf{I})\mathbf{w} \\ \text{subject to} \quad & \mathfrak{Re}\{\mathbf{w}^H \mathbf{A}_1^H \mathbf{h}\} \geq \sqrt{\frac{\gamma_1}{p_2}(1 + \mathbf{w}^H \mathbf{Q}_1 \mathbf{w})} \\ \text{and} \quad & \mathfrak{Re}\{\mathbf{w}^H \mathbf{A}_2^H \mathbf{h}\} \geq \sqrt{\frac{\gamma_2}{p_1}(1 + \mathbf{w}^H \mathbf{Q}_2 \mathbf{w})} \\ \text{and} \quad & \mathfrak{Im}\{\mathbf{w}^H \mathbf{A}_1^H \mathbf{h}\} = 0 \\ \text{and} \quad & \mathfrak{Im}\{\mathbf{w}^H \mathbf{A}_2^H \mathbf{h}\} = 0 \end{aligned} \tag{3.37}$$

For any feasible pair (p_1, p_2), the optimization problem in (3.37) is a SOCP problem and can be solved efficiently using interior point methods. Therefore, the set of feasible values can be divided into a two-dimensional grid. At each point on the grid, the corresponding

SOCP problem can be solved and the respective total transmit power can be saved. The point on the grid which results in the lowest total transmit power introduces the optimal values of the transceivers' transmit powers.

Chapter 4

Simulations

In this chapter, the results of the numerical evaluation are presented. Throughout our simulations, we assume that the transceivers' noise powers $\sigma_1^2 = \sigma_2^2 = 0$ dBW. Also, we assume that the network consists of $r = 10$ relay nodes. The channel vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ are assumed to be complex zero-mean Gaussian random vectors with variances of $\sigma_{f_1}^2$ and $\sigma_{f_2}^2$, are going to be generated randonly in each simulation run. We also assume that both transceivers know $\mathbf{f}_1$ and $\mathbf{f}_2$. Therefore, $\sigma_{f_1}^2$ and $\sigma_{f_2}^2$ are the measures of the quality of the corresponding channel vectors.

In the simulations, we also use the gradient function $\mathbf{g}(\boldsymbol{\alpha})$ obtained as

$$\begin{aligned}\mathbf{g}(\boldsymbol{\alpha}) \triangleq (2\gamma_2)\frac{(1+\boldsymbol{\alpha}^T\mathbf{Q}_2\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_2\boldsymbol{\alpha})}{\boldsymbol{\alpha}^T\mathbf{b}_2\mathbf{b}_2^T\boldsymbol{\alpha}}(\mathbf{M}_1(\boldsymbol{\alpha})\boldsymbol{\alpha} - \frac{1}{\boldsymbol{\alpha}^T\mathbf{b}_2}\mathbf{b}_2)+\\ (2\gamma_1)\frac{(1+\boldsymbol{\alpha}^T\mathbf{Q}_1\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_1\boldsymbol{\alpha})}{\boldsymbol{\alpha}^T\mathbf{b}_1\mathbf{b}_1^T\boldsymbol{\alpha}}(\mathbf{M}_2(\boldsymbol{\alpha})\boldsymbol{\alpha} - \frac{1}{\boldsymbol{\alpha}^T\mathbf{b}_1}\mathbf{b}_1)\end{aligned} \tag{4.1}$$

where, $\mathbf{M}_1$ and $\mathbf{M}_2$ are $r \times r$ diagonal matrices defined as:

$$\begin{aligned}\mathbf{M}_1(\boldsymbol{\alpha}) \triangleq \frac{1}{1+\boldsymbol{\alpha}^T\mathbf{Q}_1\boldsymbol{\alpha}}\mathbf{Q}_1 + \frac{1}{1+\boldsymbol{\alpha}^T\mathbf{R}_1\boldsymbol{\alpha}}\mathbf{R}_1\\ +\frac{\frac{1}{2\gamma_1}\left|\boldsymbol{\alpha}^T\mathbf{b}_1\right|^2}{(1+\boldsymbol{\alpha}^T\mathbf{Q}_1\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_1\boldsymbol{\alpha})}\mathbf{I}\end{aligned} \tag{4.2}$$

$$\begin{aligned}\mathbf{M}_2(\boldsymbol{\alpha}) \triangleq \frac{1}{1+\boldsymbol{\alpha}^T\mathbf{Q}_2\boldsymbol{\alpha}}\mathbf{Q}_2 + \frac{1}{1+\boldsymbol{\alpha}^T\mathbf{R}_2\boldsymbol{\alpha}}\mathbf{R}_2\\ +\frac{\frac{1}{2\gamma_2}\left|\boldsymbol{\alpha}^T\mathbf{b}_2\right|^2}{(1+\boldsymbol{\alpha}^T\mathbf{Q}_2\boldsymbol{\alpha})(1+\boldsymbol{\alpha}^T\mathbf{R}_2\boldsymbol{\alpha})}\mathbf{I}\,.\end{aligned} \tag{4.3}$$

Finally, the simulations are repeated 100 times and the obtained parameters are averaged over all runs.

We first investigate the validity of the results obtained via steepest descent technique by comparing them with those obtained by solving the SOCP problem in (3.36). To do this, we grid the values of p_1 and p_2 from 0 dBW to 10 dBW with 0.1 dBW increments. Then, for each pair of p_1 and p_2, we solve the minimization in (3.36) and find the optimal values for $\mathbf{w}_1$ and $\mathbf{w}_2$. The pair of the transceivers' transmit powers which result in the smallest possible value of the total transmit power, P_T leads us to solution of the problem.

Fig. 4.1 shows that both methods have resulted in the same values for the minimized total transmit power which means that the steepest descent technique performs very closely to the optimal solution (i.e., that obtained via SOCP).

Fig. 4.2 illustrates the average values of the total transmit power, P_T, relay transmit power, P_r, and the transceivers' powers, p_1 and p_2 versus different values of $\gamma_1 = \gamma_2 = \gamma$. In this simulation the variance of the channel vector is defined as $\sigma_{f_1}^2 = \sigma_{f_2}^2 = 0$ dB.

Fig. 4.3 depicts the same quantities as in Fig. 4.2 for $\sigma_{f_1}^2 = 3$ dB and $\sigma_{f_1}^2 = 7$ dB.

In Fig. 4.2, the power of the two transceivers, p_1 and p_2 are almost equal because of having same channel qualities (i.e., $\sigma_{f_1}^2 = \sigma_{f_2}^2 = 0$ dB). On the other hand, different channel qualities in Fig. 4.3 ($\sigma_{f_1}^2 = 3$ dB and $\sigma_{f_2}^2 = 7$ dB) leads to different values for p_1 and p_2. As seen in this figure, choosing $\sigma_{f_1}^2 > \sigma_{f_2}^2$ results in $p_1 > p_2$. This means that the transceiver with better channel consumes less power.

Figs. 4.4, 4.5 and 4.6 show the average power of Transceiver 1, the average power of Transceiver 2, the average total transmit power, respectively, versus $\sigma_{f_1}^2$ for different values of $\sigma_{f_2}^2$. Figs. 4.7, 4.8 and 4.9 illustrate the average power of Transceiver 1, the average power of Transceiver 2 and the average total transmit power, respectively, versus $\sigma_{f_2}^2$ for different values of $\sigma_{f_1}^2$. In all of these figures, we assumed that $\gamma_1 = \gamma_2 = \gamma = 10$ dB.

It is seen in these two sets of figures that the values of p_1, p_2 and P_T versus $\sigma_{f_1}^2$ for different $\sigma_{f_2}^2$ are similar to the values of those parameters plotted versus $\sigma_{f_2}^2$ for different values of $\sigma_{f_1}^2$. This occurs because the whole problem is symmetric with regard to the two transceivers. Also, it can be seen from 4.4 to 4.9 that the achievable average power of Transceiver 1, Transceiver 2 and total transmit power are increased as the quality of the channel coefficients are improved.

Figs. 4.10 and 4.11 depict the total transmit powers of the TDBC scheme studied in this thesis and the MABC method of [15] versus $\gamma_1 = \gamma_2 = \gamma$. Data rates have been calculated using $\frac{1}{2}\log_2(1+\gamma)$ for two-phase MABC and $\frac{1}{3}\log_2(1+\gamma)$ for our TDBC scheme, where γ is the signal to noise ratio in linear scale. The total transmit powers illustrated in Fig. 4.10 have also been divided by the number of phases for a fair comparison. It is evident from Fig. 4.10 that the minimum total power for our TDBC scheme is lower than that obtained in the two-phase MABC scheme of [15]. However, Fig. 4.11 shows that the two-phase MABC scheme achieves higher data rates with equal total transmit power.

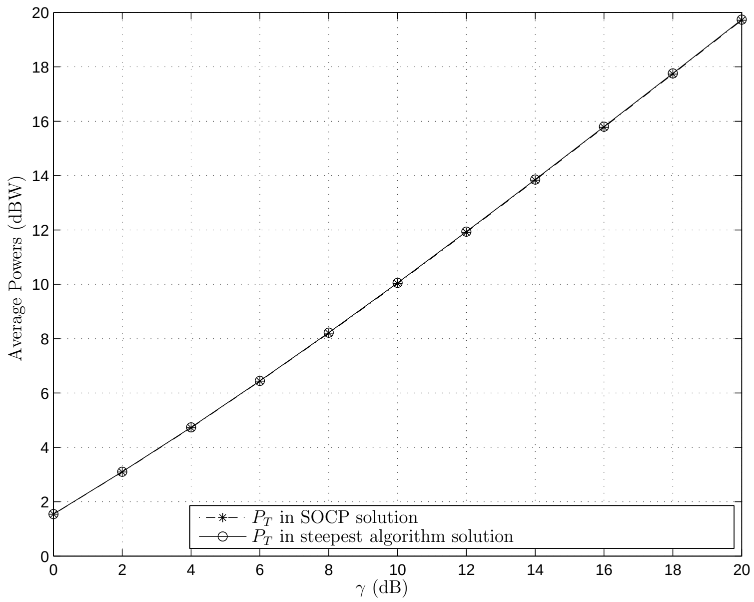

Fig. 4.1: The average total transmit power, P_T, obtained from the SOCP approach and steepest descend algorithm, versus $\gamma_1 = \gamma_2 = \gamma$ for $\sigma^2_{f_1} = \sigma^2_{f_2} = 0$ dB.

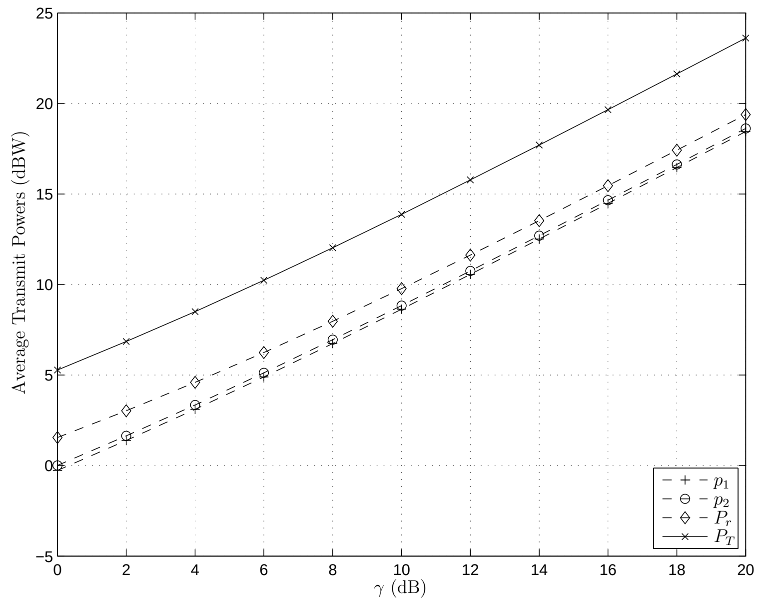

Fig. 4.2: The average total transmit power, P_T, the corresponding average relay transmit power P_r, and the corresponding average transceivers' powers p_1 and p_2, versus $\gamma_1 = \gamma_2 = \gamma$ for $\sigma^2_{f_1} = \sigma^2_{f_2} = 0$ dB.

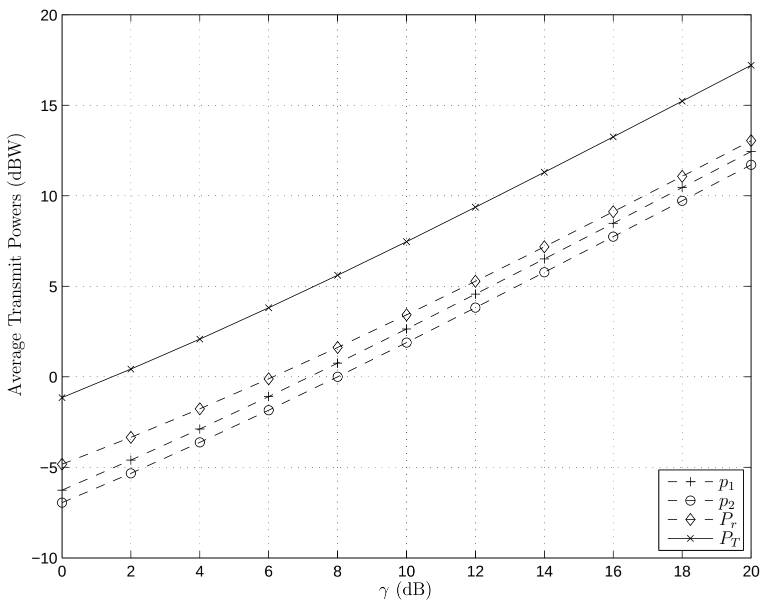

Fig. 4.3: The average total transmit power, P_T, the corresponding average relay transmit power P_r, and the corresponding average transceivers' powers p_1 and p_2, versus $\gamma_1 = \gamma_2 = \gamma$ for $\sigma^2_{f_1} = 7$ dB and $\sigma^2_{f_2} = 3$ dB.

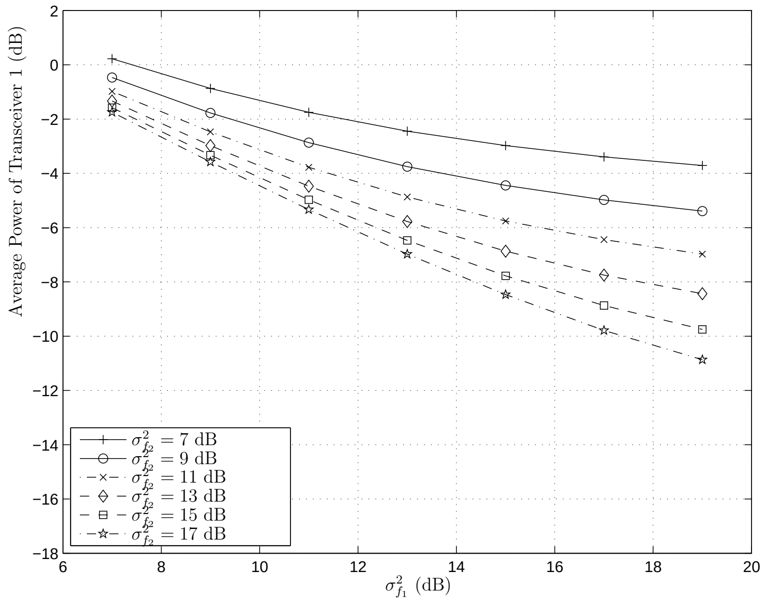

Fig. 4.4: The average power of Transceiver 1, p_1, versus $\sigma_{f_1}^2$ for different values of $\sigma_{f_2}^2$.

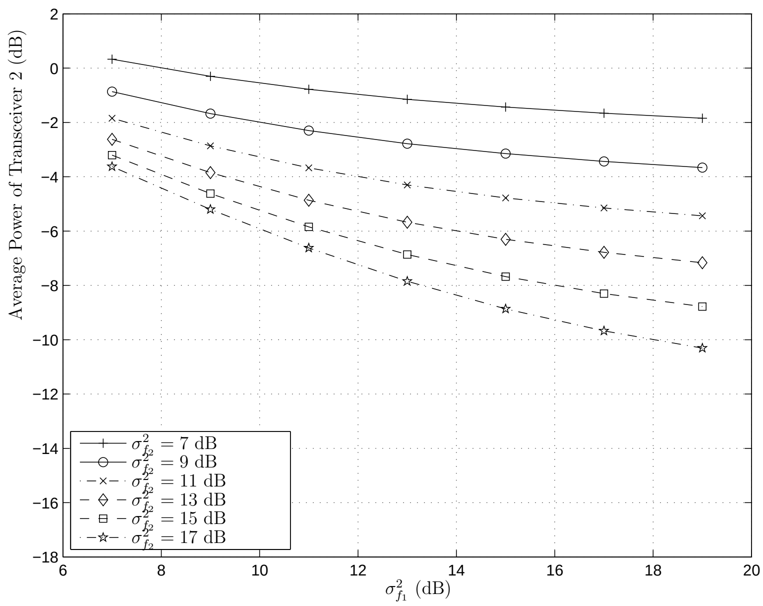

Fig. 4.5: The average power of Transceiver 2, p_2 versus $\sigma_{f_1}^2$ for different values of $\sigma_{f_2}^2$.

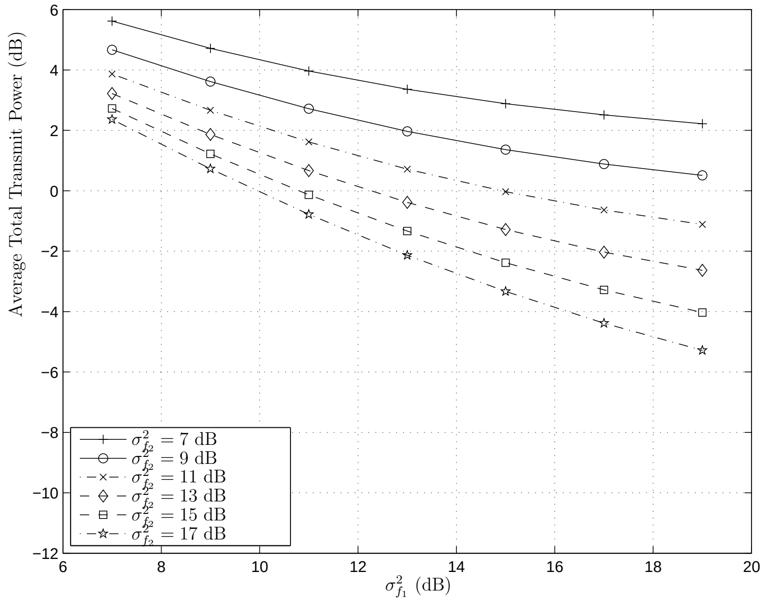

Fig. 4.6: The average total transmit power, P_T versus $\sigma_{f_1}^2$ for different values of $\sigma_{f_2}^2$.

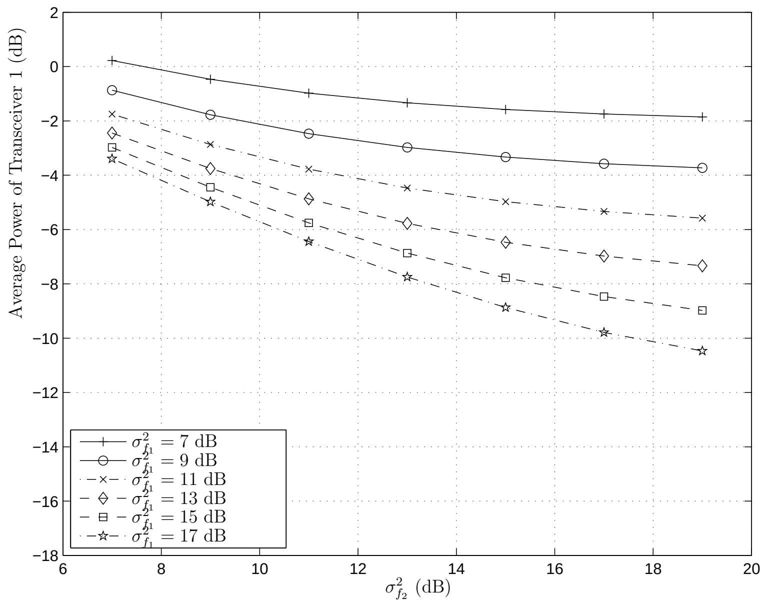

Fig. 4.7: The average power of Transceiver 1, p_1 versus $\sigma_{f_2}^2$ for different values of $\sigma_{f_1}^2$.

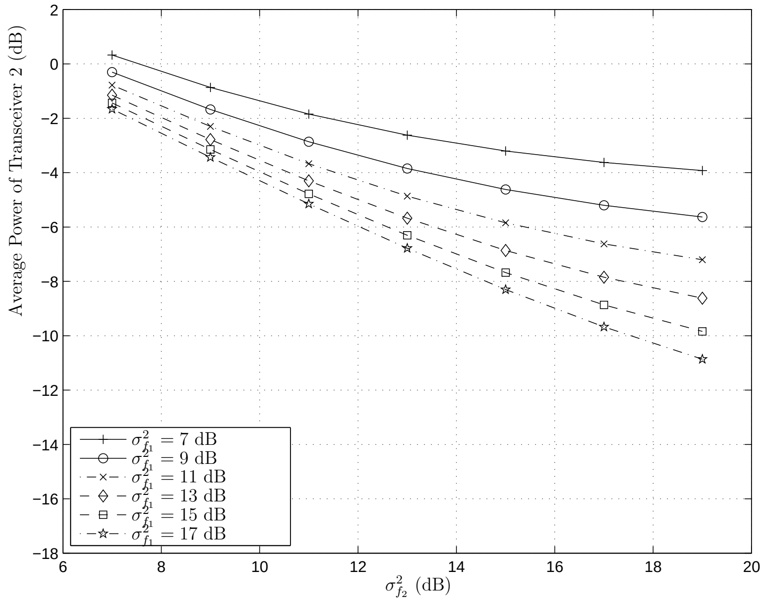

Fig. 4.8: The average power of Transceiver 2, p_2 versus $\sigma^2_{f_2}$ for different values of $\sigma^2_{f_1}$.

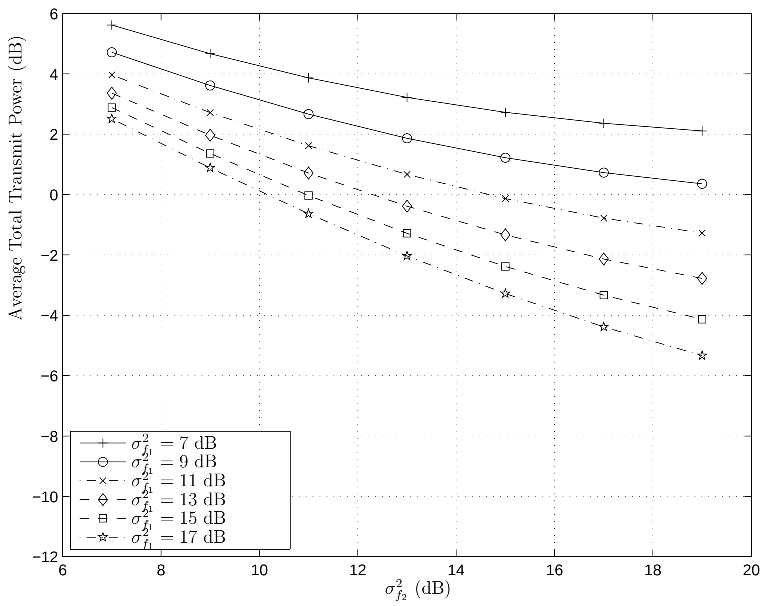

Fig. 4.9: The average total transmit power, P_T versus versus $\sigma^2_{f_2}$ for different values of $\sigma^2_{f_1}$.

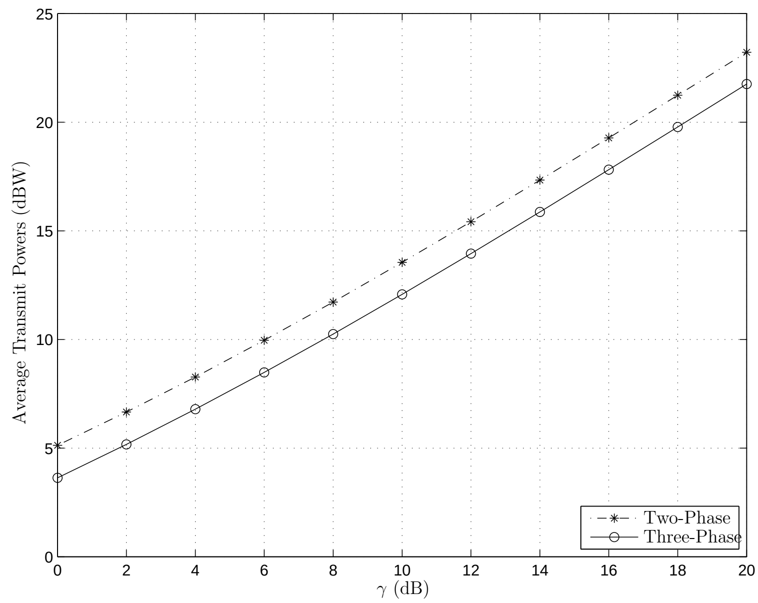

Fig. 4.10: The average total transmit power, P_T, versus $\gamma_1 = \gamma_2 = \gamma$ for the two methods of three-phase TDBC and two-phase MABC for $\sigma_{f_1}^2 = \sigma_{f_2}^2 = 0$ dB.

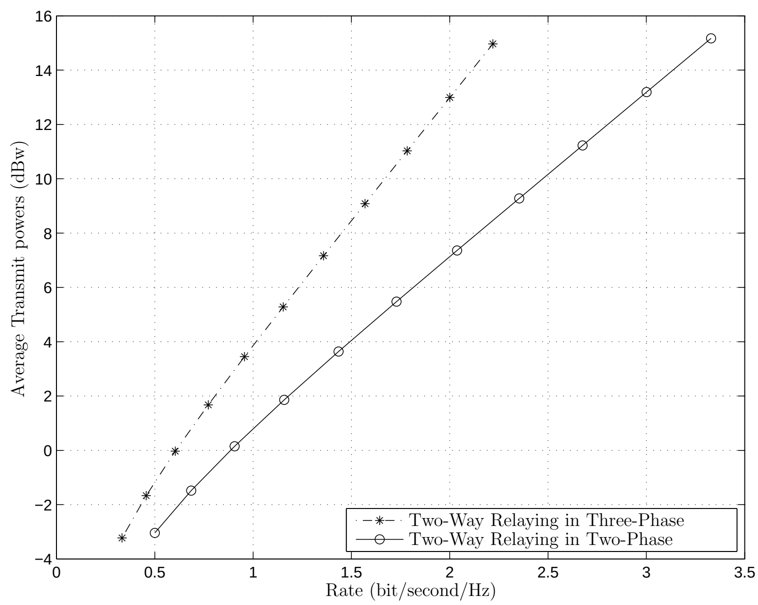

Fig. 4.11: The average total transmit power, P_T, versus data rate in two-phase MABC and three-phase TDBC for $\sigma_{f_1}^2 = \sigma_{f_2}^2 = 0$ dB.

Chapter 5

Conclusions

In this thesis, we studied the optimal distributed beamforming in two-way wireless relay networks. Our system was made up of r relays and two transceivers. Each of the relay node and transceivers were equipped with a single antenna. For establishing the connection between these two transceivers, we proposed a two-way relaying scheme which takes three phases to accomplish the exchange of two information symbols between the two transceivers. Transceiver 1 and 2 send their symbols to the relays in the first two phases. The signals that are received by the relays are noisy versions of the original signals. Each of these signals, is multiplied by two different complex weights (beamforming weight coefficients) to adjust the phase and amplitude of the signal. In the third phase, relays send a combination of their received signals to both nodes at the same time. Our goal was to find the optimal values of transceivers' transmit powers and the optimal values of the beamforming coefficients by minimizing the total transmit power subject to quality of service constraints. In our approach, we minimized the total transmit power under two constraints. These two constraints were used to guarantee that the transceivers' SNRs were above given threshold. To solve the underlying optimization problem, we developed two techniques. The first technique is a combination of a two-dimensional search and Second-Order Convex Cone Programming (SOCP). More specifically, the set of feasible values of transceivers' transmit powers is quantized into a sufficiently fine grid. Then, at each vertices of this grid, an SOCP problem is solved to obtain the beamforming coefficients such that for the given pair of transceivers' transmit powers, the total transmit power is minimized. The pair of the transceivers' transmit powers which results in the smallest possible value of the total transmit power, leads us to the solution of the problem. This approach requires a two-dimensional search and solving an SOCP problem at each point of the corresponding two-dimensional grid. Thus, it was prohibitively expensive in terms of computational complexity. As a second method, we resorted to a gradient based steepest descent technique. Our simulation results showed that this second technique performs very close to the optimal two-dimensional search based algorithm. Moreover the numerical results illustrated that as the level of quality of services increases, the total required power to satisfy given SNRs increases. Also, different channel qualities leads to different values for the powers of transceivers while we have the same values for such powers if the qualities of channels are equal. Finally, we compared the total transmit powers and the data rates of two-phase two-way relaying scheme proposed in [15] with our three-phase two-way relaying scheme. Through our simulation we showed that our three-phase two-way relaying require less total power as compared to the two-phase two-way relaying method. On the other hand, the two-phase two-way relaying achieves better data rates when compared with our three-phase two-way relaying for the same total transmit power. Also, we observed that the three-phase scheme has more degrees of freedom while the scheme of [15] appears to be more bandwidth efficient.

5.1 Future Work

The main goal of this thesis was to optimally find the beamforming complex weights of the network for given SNRs. Therefore, future work in this field for a TDBC scheme would be to develop an SNR balancing approach [15]. In other words, the beamforming weights and transceivers' transmit powers can be obtained by maximizing the smallest of the two SNRs under a given maximum total transmit power budget.

Moreover, most commercial applications today require a MIMO system. In this study we had only one antenna for each node. Therefore, our communication scheme could be expanded into the use of multi-antenna for transceivers and relays instead of having single-antenna nodes. In this network, one could make use of spatial diversity to increase the SNRs of the transceivers, which could be another direction for research in this field.

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