Conceptual Videos in Mathematics: Theory to Practice

by

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PROJECT REVIEW INFORMATION

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The Project was approved on April 18, 2023 by the following review committee:

Review Committee:

Research Supervisor	Dr. Ann LeSage
Second Reader	Dr. Robin Kay

The above review committee determined that the Project is acceptable in form and content and that a satisfactory knowledge of the field was covered by the work submitted. A copy of the Certificate of Approval is available from the School of Graduate and Postdoctoral Studies.

ABSTRACT

This project begins by providing an overview of the research on the purpose and criteria for creating quality educational videos and the development of a conceptual understanding of proportional reasoning. The paper then shifts from theory to practice, illustrating how I used research to create a four-part video series titled *Is it Proportional?* Designed for middle school students, the videos demonstrate identifying and solving proportional and non-proportional situations. The videos are analysed through the lens of technological design, content and pedagogical choices, focusing on the practical application of theory. The issues and successes of putting research into practice are critiqued, finding potential for conceptual videos in the middle school classroom, with a need for content and pedagogical understanding and acknowledgement of barriers such as time and access to technology.

Keywords: videos; education; conceptual; proportional reasoning

AUTHOR'S DECLARATION

I hereby declare that this project consists of original work of which I have authored. This is a true copy of the work, including any required final revisions, as accepted by my committee.

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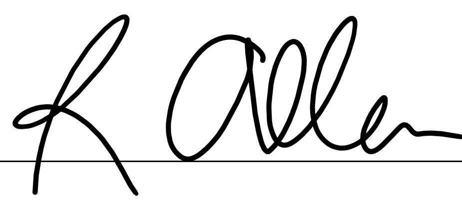

Rachel K. Allen

DEDICATION

To my Dad, my ultimate supporter, who listened to a lifetime of thoughts, of all kinds. I would not be the student, teacher or human I am without you. To my Mom, who instilled my love of learning and research from a young age. I know you would have loved this. I hope I've made you proud and I'm sorry there isn't more data. And to the rest of my friends and family- we did it. Thank you for your continued support, even when you thought I had finished this degree years ago.

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To my second reader, Dr. Robin Kay, thank you for your time and feedback in the world of educational videos and my work. Thank you for all the insight throughout my journey from thesis to this final project.

STATEMENT OF CONTRIBUTIONS

I hereby certify that I am the sole author of this work and that no part of this work has been published or submitted for publication. I have used standard referencing practices to acknowledge ideas, research techniques, or other materials that belong to others. Furthermore, I hereby certify that I am the sole source of the creative works and/or inventive knowledge described in this document.

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Chapter 1. Introduction

1.1 Overview

Educational videos are not a new phenomenon, particularly with the advancement of virtual and online learning in recent years. For over two decades, researchers have described the benefits of educational videos, including transferring new information to long-term memory (Mayer, 2001;2003), improving student learning (Kay & Kletskin, 2012; Khan & Slavvit, 2013; Shoufan, 2019; Weinberg, 2022), improving in-class engagement (Mykhenko, 2016), and increasing accessibility (Dinmore, 2019). However, not all educational videos have the potential to influence students' learning and engagement (Guo et al., 2014). Simply increasing the number of videos used is not the solution. Similar to Li and Ma's (2010) meta-analysis of the effects of computer technology on mathematics learning, found that while technology can positively affect student learning, the mere use of technology does not necessarily transform students' learning experiences. The same can be said of educational videos. Much educational video research has focused on the characteristics of quality educational videos (Kay, 2014; Brame, 2016; Shoufan, 2019; Costa & Pacansky-Brock, 2020; Pandey et al., 2022). From video creation frameworks (Kay, 2014; Shoufan, 2019) to video selection models (Pandey et al., 2022) or video creation guidelines (Brame, 2016; Costa & Pacansky-Brock, 2020), researchers have identified specific characteristics of quality educational videos.

The field of mathematics education has long been using educational videos, mainly worked example videos where the instructor solves a mathematics problem in a step-by-step manner (Kay, 2012). This style of mathematics video is easily accessible on

YouTube through sites such as the Khan Academy (Khan & Slavitt, 2013; Light & Pierson, 2014). There are, however, limited videos available on YouTube or other similar platforms that support students' conceptual understanding of mathematics (Kay, 2012; Light & Pierson, 2014).

The lack of conceptual mathematics videos is particularly problematic concerning proportional reasoning. More than 30 years ago, the National Council for Teachers of Mathematics (NCTM) stated the importance of teaching proportional reasoning and stated that it "merits whatever time and effort must be expended to assure its careful development." (NCTM, 1989, p.82). Sufficient time and effort are necessary as the concepts of proportional reasoning do not develop naturally (Sowder et al., 1998). Therefore, the instruction of proportional reasoning necessitates particular pedagogical strategies to develop student knowledge and understanding (Van Dooren et al., 2005). Extensive research into proportional research and problem solving has been explored and tested with students over the past 30 years. Researchers have identified types of proportional reasoning problems (Lamon, 1993), levels of strategies for solving proportional reasoning problems (Carpenter et al., 1999) and categorizations of students' problem-solving solutions specific to proportional reasoning (Langrall & Swafford, 2000). This body of research provided the foundational knowledge I used for creating educational videos addressing the development of a conceptual understanding of proportional reasoning. Throughout this paper, I explored the research on quality educational videos and the development of a conceptual understanding of proportional reasoning. Then, I put the theory into practice by analyzing the series of videos I created for this project.

1.2 My Background & Journey to Mathematics Video Creation

My journey of researching and creating educational videos began before the global pandemic forced a shift to online learning. I have been an Ontario Certified Teacher since 2009 and am qualified to teach K-12. I have taken some additional qualification courses in mathematics, and am passionate about teaching mathematics and bringing concepts to life for students. In 2017, I began a position with one of the largest school boards in Ontario as a mathematics consultant. I researched and developed professional learning in mathematics for teachers, leveraging the school board and Ontario Ministry of Education priorities. When I began my Master of Education at Ontario Tech University in 2018, Dr. Ann LeSage invited me to be a Research Assistant (RA) because of my background in teaching mathematics. The RA position was for a cross-faculty project where two Education faculty and a group of Education students supported two sections of a first-year business mathematics course.

For this cross-faculty project, the Education faculty taught two sections of Business Math during the winter (January-April) term. Approximately half of the students in this project had failed or withdrawn from the course at least once and were considered "at-risk" by the researchers and Education faculty (LeSage et al., 2019). The Education faculty used the flipped classroom approach to support these at-risk Business students. This meant the students watched short, direct instruction videos at home before class and then were actively problem-solving with their peers during class time.

As the RA for the project, I worked with the Education faculty to create videos for students to view prior to attending weekly lectures. I created two types of videos: business vocabulary and business mathematics worked examples. Throughout this process, I received feedback from Dr. LeSage about aspects of the video content, scaffolding, video length, animations, and highlighting key visuals. Dr. LeSage would also share specific strategies and mathematical models used in class so that I could reflect them in my videos. The feedback helped further my understanding of best practices in video creation and improved my pedagogical explanation of mathematics. This was the first time I had created educational video material for students I was not teaching. As such, I had to ensure my instructions and the content were clear and concise, as students would be viewing the videos independently. The course videos are publicly available on YouTube (Playlist: Business Math I) at http://bit.ly/video-atrisk-math (LeSage, 2019).

Throughout the course, LeSage and Typelo, from the Faculty of Education, collected data about the Business students. Over 80% of the students agreed that the videos were helpful with respect to learning mathematics and business concepts (LeSage et al., 2019). While it is important to note that student success cannot be linked to the videos in isolation of the flipped classroom approach, I want to highlight some responses from the open-ended questions on the survey. Some student responses that referenced the usefulness of the video clips included:

> "The videos summarized everything quickly."
>
> "... the videos were a must in order to pass. Without them, this course would be a lot harder."
>
> "The online video tutorials with examples [were most useful]."
>
> "The videos [were most useful for learning]." (p.5540)

As a teacher and novice researcher, I was excited to hear that students believed the videos had positively influenced their understanding of the mathematics explored in the course. In particular, our research found that students "who were worried about their math performance level rated the helpfulness of videos significantly higher than students who were confident about their math performance level" (LeSage et al., 2019, p. 5540). These findings lead me to wonder how educational videos could support struggling mathematics students beyond this Business course.

1. 3 From Research Assistant to Practitioner

Prior to my involvement with the Business Math research project, I had yet to consider creating videos to support student learning of mathematics. However, upon observing the influence of the videos on at-risk undergraduate students, I was intrigued and wondered how I could use similar videos to support students in middle school. After my time as an RA, I completed an independent study course during the Fall 2019 term on educational videos. I completed a literature review as a part of the course and created a guide for educators titled *How and Why Make Videos* in both written and infographic form (Allen, 2020). The research completed during the course provided the foundational knowledge I applied to create the videos for my thesis research. This research will be referenced throughout the literature review in Chapter 2.

As I began to approach the end of my master's coursework in early 2020, I was yet to decide on a particular curriculum focus for my research on educational videos. However, 2020 saw two significant educational changes that guided my final decisions: a new Ontario mathematics curriculum and the COVID-19 Pandemic. In June 2020, the Ontario Ministry of Education released a new Mathematics curriculum for grades 1-8 (Ontario Ministry of Education, 2020). The new mathematics curriculum was released while student learning was disrupted by the COVID-19 Pandemic. The sudden switch to online learning meant that the need for virtual resources such as videos became more pertinent beyond my research interests. As I reviewed the new curriculum, I noticed a new proportional reasoning expectation in the Grade 7 curriculum. The new specific expectation stated: "identify proportional and non-proportional situations and apply proportional reasoning to solve problems." (Ontario Ministry of Education, 2020, Grade 7, B2.10). Although proportional reasoning was included in the 2005 Ontario Mathematics curriculum, it did not include expectations requiring students to differentiate between proportional and non-proportional situations. This shift in focus was particularly interesting to me, as I discovered that students (Van Dooren et al., 2005) and teachers (Brown et al., 2020) struggle to differentiate between proportional and non-proportional situations.

This new addition to the Grade 7 Ontario mathematics curriculum became the content focus for my research on educational videos. With this curriculum expectation in mind, I developed the following research questions for my thesis research:

1. What are student perceptions of videos as a learning tool in mathematics?
2. How does student understanding of proportional reasoning change after viewing conceptual mathematics videos?
3. How does student confidence change after viewing proportional reasoning videos?

With my research questions established, I began working on my thesis but encountered multiple roadblocks. One significant roadblock was school closures due to the COVID-19 Pandemic. As students were not in the brick-and-mortar classrooms, the school board's research department was not allowing any research to be conducted. Once students returned to the classroom, the next significant roadblock resulted from union disputes in my school board. Teachers were mandated to follow work-to-rule action and were not permitted to have researchers in their classrooms. In the Spring of 2022, when the labour disputes were finally resolved, I was granted permission from the school board to conduct my research. However, there were multiple delays with the Research Ethics Board (REB) at Ontario Tech University. The REB expressed concerns about students participating in this research as a component of their classroom learning. The continual delays and ongoing issues with REB approval lead me to the decision to transition from a thesis to a project. Before making this decision, I completed much of the literature review, designed the videos, and created the pre- and post-surveys focused on student attitudes towards videos and their ability to identify and solve proportional problems. As such, I reference much of the work I completed throughout this project.

With my shift from a thesis to a project, my research questions also changed. For this project, I focused my literature review (Chapter 2) on research specific to videos in education and proportional reasoning understanding, to inform my revised research questions:

1. What is the criteria for a good educational video?
2. How do students develop their conceptual understanding of proportional reasoning?
3. What are the principal elements to consider when creating an educational video focused on developing a conceptual understanding of proportional reasoning?

In Chapter 3, I discuss how I applied the research on videos and proportional reasoning to designing videos for middle school students about identifying proportional and non-proportional situations. I also analyse elements of the videos I created, highlighting key design and pedagogical choices throughout the practical application of theory. Finally, in Chapter 4, I explore further research connections to my analysis, discuss limitations in the research and share issues putting the research into practice. I also identify potential next steps for future research for videos and proportional reasoning.

Chapter 2. Literature Review

This literature review covers two main concepts: educational videos and proportional reasoning. First, I explore research on (a) the use of videos in education, (b) how videos can be used as a learning tool, (c) what makes a good educational video, and (d) video use in mathematics. In the latter part of the chapter, I shift my focus to research on proportional reasoning, including the conceptual understanding of proportions and the types of proportional reasoning problems.

2.1 Videos in Education

Prior to the COVID-19 pandemic in 2020, young adults spent almost 9 hours daily on entertainment media such as phones, tablets, and computers (Rideout, 2015). Of those 9 hours, teenagers spent a daily average of 1 hour and 18 minutes watching online videos (Rideout, 2015). The onset of social lockdowns during the COVID-19 pandemic resulted in a significant shift in the time children and youth spent online. A meta-analysis including 46 studies conducted after the 2020 lockdowns of the COVID-19 pandemic found that overall screen time increased by 52% compared to pre-pandemic time. That is an increase of 84 minutes per day for children and adolescents (Madigan et al., 2022). While overall screen time increased during the lockdowns, little of the increase was connected to screen time for educational purposes. Aside from when students were attending online classes through virtual conferencing platforms like Zoom or Google Meet, few of the students' hours on phones, tablets, and computers were using educational technology (Christopoulos & Sprangers, 2021).

As screen time increased with the COVID-19 lockdowns, the use of educational technology did not. Therefore integration, or lack thereof, of technology resources in educational settings must be addressed. Pandemic aside, research highlights many barriers to integrating technology in educational settings. These barriers are categorized as first- and second order (Rikala et al., 2014). First-order barriers are extrinsic obstacles that are out of the educator's control. Some first-order barriers include a lack of technology available to teachers, a lack of funding allocated to purchasing technology (Nikolopoulou & Gialamas, 2015), or limited funding for teacher training on technology use (Fishman & Davis, 2006). Second-order barriers are intrinsic and personal to the teacher (Rikala et al., 2014). Some second-order barriers include teachers' beliefs in the importance of technology, confidence in technology use, and overall technology skills (Christopoulos & Sprangers, 2021). The second-order barriers can either reduce or magnify the effects of first-order barriers for technology integration. For example, suppose a teacher believes that technology is important. In that case, they are more likely to seek out teacher training or their own technology devices to bring technology to their classroom. However, suppose a teacher does not believe technology is important, or lacks confidence in technology use. In that case, they are less likely to seek out opportunities to learn and integrate technology in their classrooms (Christopoulos & Sprangers, 2021).

Some first-level barriers were quickly rectified when the global COVID-19 pandemic thrust educational institutions into online learning. Teaching and learning in an online environment require educational institutions to provide devices to both teachers and students. The province of Ontario, for example, invested $35 million in the acquisition of tablets and iPads for students, as well as connectivity supports such as remote wifi access student use at home (Ontario Ministry of Education, 2022). This investment in technology may have alleviated some first-order barriers allowing more students one-to-one access to technology. However, research from a pre-pandemic study in Peru found that simply providing students with a laptop does not significantly influence student learning outcomes (Beuermann et al., 2015). In this study, nearly 1,000 public school students in grades 1 to 6 were given laptops as a part of the One Laptop per Child program. However, no significant impact on mathematics and reading scores or cognitive skills.

Even though more Ontario students were provided access to technology due to COVID-19 lockdowns and online learning, not all first-order barriers were addressed. School boards and districts focused little on the first-order teacher training barrier. Teachers were suddenly teaching entirely online without training, which further amplified teachers' second-order barriers of lack of confidence with technology. As such, teachers and students turned to resources such as online videos to support their virtual learning. The viewership of educational YouTube videos, such as Wayne Breslyn's channel "Dr. B's Science Videos", more than doubled since the start of the pandemic in 2020, from just under 500 000 views to just under 1.5 million views (Breslyn & Green, 2022). With the viewership of educational videos increasing, and the elimination of some first-order barriers to technology integration, videos have the potential to be an important learning tool.

2.2 Videos as a Learning Tool

With the pandemic increasing the viewership of educational videos, it is important to define what is known about videos as a learning tool. At their most basic, videos combine words and images. For decades, research has shown that students do not retain key information when it is presented only through written or oral words (Mayer and Anderson, 1992). In his 2008 book, John Medina states, "Vision trumps all other senses" (p.221). He cites over a century's research demonstrating that the more visual the learning artifact, the more likely it is to be recognized and recalled (Medina, 2008). Videos also have the advantage of leveraging multimedia learning, which combines words and visuals such as images or animations (Mayer, 2003). By engaging the audio and visual pathways, learners can better select, organize and integrate the new information and transfer it to long-term memory (Mayer, 2001;2003)

More recently, Kay (2012) conducted a literature review of 53 studies on videos in education that found benefits, including improved study habits and increased learning performance. A decade later, other studies continue to cite the positive effects of videos on student learning and understanding from high school (Khan & Slavitt, 2013) to university (Guy & Marquis, 2016; Mykhnenko, 2016; Shoufan, 2019) and medical school (Weinberg, 2022). Although it is difficult to isolate the influence of video on student performance, multiple studies have shown students who actively engage with educational videos show an increase in learning (Kay & Kletskin, 2012; Khan & Slevitt, 2013; Guy & Marquis, 2016). The increase in learning may be a component of the value of accessing videos on demand (Kay, 2012; Mykhnenko, 2016; Costa & Pacansky-Brock, 2020). Dinmore's (2019) research on Australian University students (n=240) found that over 93% of students revisited videos provided throughout the course when preparing for assignments. With 99% of the students also leveraging the ability to pause or rewind videos while reviewing materials to understand the content better.

When watching videos to learn, students can also turn on the subtitle feature (Dinmore, 2019). Adding subtitles can support learners who are deaf, hard of hearing, or English Language Learners (Dinmore, 2019). A video platform such as YouTube can auto-generate subtitles with adequate accuracy and allow viewers to turn subtitles on and off as needed. Allowing users to control the subtitle feature is also important for some students who may find subtitles distracting. For such students, watching the video sufficiently engages their visual and auditory channels; as such, adding subtitles can cause cognitive overload (Mayer, 2003). Therefore, including subtitle control increases accessibility for students.

Research also highlights the benefits of using videos for educators. The aforementioned advantages for students are also benefits for educators. However, additional teacher benefits include increased in-class engagement and improved communication (Mykhnenko, 2016). However, the potential influence of videos as a learning tool depends on how the teacher uses them. For example, if students believe the videos replace in-class activities / time, student attendance decreases (Kay, 2012). However, if students perceive videos as a tool to supplement their learning, student engagement in the class increases (Guy & Marquis, 2016). Beyond student engagement, Mykhnenko (2016) found that teachers who started using video as a supplementary learning tool were more highly rated by their students than average ratings in previous terms. In particular, Mykhnenko (2016) found that students rated teachers who started using videos as better communicators than they had rated the same teacher in previous years. A possible explanation for the positive rating in communication may have resulted from the increased opportunity for in-class engagement or the opportunity to revisit content presented in the video.

The ability to leverage visual and auditory pathways, increase student learning, and provide subtitle controls make videos a viable learning tool for students. However, simply using more videos is not the solution. Just as Li and Ma's (2010) meta-analysis of the effects of computer technology on mathematics learning found, while technology can have a positive effect on student learning, the mere use of technology does not necessarily transform the learning experience. The same can be said for the use of videos: more does not mean better. Therefore, the following section highlights the components of a good educational video.

2.3 What Makes a Good Educational Video?

As there is potential for videos to positively impact students and teachers, it is important to consider the elements of a good educational video. Research has found that not all videos influence students' learning and engagement (Guo et al., 2014; MacHardy & Parados, 2015). Numerous researchers have explored the characteristics of quality educational videos (Kay, 2014; Brame, 2016; Shoufan, 2019; Costa & Pacansky-Brock, 2020; Pandey et al., 2022). Some researchers have video creation frameworks (Kay, 2014; Shoufan, 2019), video selection models (Pandey et al., 2022), or video creation guidelines (Brame, 2016; Costa & Pacansky-Brock, 2020) detailing the characteristics of good quality educational videos. Within this body of research, some studies focused specifically on video creation (Kay, 2014; Shoufan, 2014; Brame, 2016; Costa & Pacansky-Brock, 2020), while other studies focused on video selection (Pandey et al., 2022). Some video creation research lists as few as three main elements (Brame, 2016) of good videos, while Costa and Pacansky-Brock's (2020) book includes 99 tips to consider. Although the considerations may vary across these sources, four key themes emerge across the frameworks, models, and guidelines for quality educational videos. The four common themes I identified across the research are pre-planning, content covered, verbal explanations, and visuals. These four themes will guide the remainder of the literature review, which focuses on the qualities of a good educational video. I use these same four themes to guide my analysis of the videos I created in Chapter 3. The remaining sections of this literature review chapter will be conducted from the lens of video creation.

2.3.1 Pre-planning

The four major themes I identified in the research on creating good quality educational videos were: (1) pre-planning, (2) content covered, (3) verbal explanations, and (4) visuals. I begin the discussion with pre-planning, as chronologically it is the first action taken to create an educational video. "Pre-production preparedness" is essential in video creation (Dinmore, 2019, p.2). Dinmore (2019) suggests that teachers new to video creation should create a script and practice the script multiple times before recording the video. Costa and Pacansky-Brock (2020) suggest the opposite and advise against using a script. They assert that scripting the video too extensively detracts from the conversational nature of the presenters. As such, teachers need to balance being prepared for the content to be covered and managing to sound natural and conversational. The importance of verbal flow will be discussed more extensively in Section 2.3.3. Regardless of whether a script is used, researchers state a short educational video clip of under 10 minutes in length can take 40-90 minutes to create (Kay, 2014; Costa & Pacansky-Brock, 2020). Kay's (2014) time estimate of 60-90 minutes is based on two secondary school instructors with advanced knowledge in mathematics that were trained to develop video podcasts using his 2014 framework. Costa and Pacansky-Brock's (2020) shorter estimate of 40 minutes is based on personal experience recording video messages to college students about an upcoming assignment or the course content for the week. I could not find research documenting time requirements for K-12 educators looking to create videos to support their class content.

Beyond deciding whether to create a script, during the pre-planning phase of video creation, teachers must also determine the recording tools they will use. Recording tools include video software, cameras, and microphones (Shoufan & Mohamed, 2022). Selecting the appropriate video software is important, because creating good quality videos generally requires editing software such as Moviemaker, Adobe Premiere, iMovie, and Camtasia Studio (Shoufan & Mohamed, 2022).

When creating a video, teachers must also consider where the video will be posted and shared. Dinmore (2019) found that over 90% of the 240 university students surveyed viewed educational videos on a computer, while Madigan et al. (2022) found that 12 to18-year-olds are more likely to use a mobile device such as a tablet or phone to view videos. Consequently, teachers need to consider their end users, and ensure that the videos they create are compatible with multiple platforms. Considering a platform such as YouTube that is accessible across a variety of devices is recommended. The additional advantage of the YouTube platform is automatic subtitle generation, as discussed in Section 2.2.

The components of pre-planning, including script writing/planning, selecting appropriate recording tools, and identifying appropriate video platforms, must be done in conjunction with considering the content covered in the video. The theme of content coverage is explored in the next section. Many of the elements discussed in this section will overlap with teachers' decisions regarding the content to be addressed.

2.3.2 Content Covered

While planning to create a video, teachers must establish their desired learning outcomes (Mayer, 2019). For example, the content covered in the video must be explicit and established early in the planning process (Kay, 2014; Costa & Pacansky-Brock, 2020). Shoufan (2019) found that the most common reason students dislike a particular video is that they perceive the content as unimportant. To avoid perceptions of unimportance, the content must be clearly stated near the beginning of the video. In particular, the video content and rationale should be obvious in the video title and introduction (Kay, 2014; Costa & Pacansky-Brock, 2020).

When establishing the content to be covered, teachers must also consider the duration of the completed video. A team of researchers analyzed 6.9 million video-watching sessions from online courses (Guo et al., 2014) and found that when the video duration is less than 6 minutes, almost all students watched the entire video. However, as the video length increased to 9 -12 minutes, the median engagement time dropped to

50%. When the video length was 12-14 minutes, the median engagement time dropped to only 20%. Guo et al. (2014) concluded that the maximum median engagement time for videos was approximately 6 minutes. Therefore, teachers should strive to create videos with a viewing time of about 6 minutes. Given these time constraints, segmenting more complex content into multiple shorter videos versus videos that extend beyond the 6-minute time frame may be necessary. Brame (2016) found segmenting videos into smaller parts was important to student engagement and how much content students learned from the video. Creating shorter videos may also require weeding (Brame, 2016) to eliminate extraneous information that can cause unnecessary cognitive load. Extraneous noise and visuals will be discussed in Sections 2.3.3 and 2.3.4, respectively.

Although this review focuses on the creation of videos, models used to analyse video selection can also be applied as guidelines during the design process or as feedback once a video has been created. For example, in 2022 (Pandey et al.) developed a theoretical model called VUER to rate videos on their potential to impact student learning. VUER is an acronym for the qualitative model named for the four characteristics it assesses: "Visual Appeal, Understanding of Content, Engagement with Topic, and Recommendation Preference" (Pandey et al., 2022, p.11181). Although all four VUER characteristics are not directly relevant to this section; the 'U' of the VUER model, Understanding of Content, directly connects to the content covered. Other components of this model will be discussed in later sections of this chapter.

In the VUER model, learners are asked to rate a video based on the following five statements specific to Understanding of Content:

1. The video does a good job of explaining the concepts/issues/topics.

2. The video demonstrates the concepts/issues/topics with examples.

3. The information provided in the video is useful.

4. The video discusses concepts/issues/topics in-depth.

5. I need to watch the video again to fully understand what was discussed.

(Pandey et al., 2022, p.11184)

These five questions can serve as prompts for teachers as they design educational videos. Teachers can also use these five questions to ensure their video covers the appropriate content and intended learning goals. This component of the VUER model needs to be used in conjunction with the other considerations discussed previously in the chapter, including length of video, segmenting, and weeding.

2.3.3 Verbal Explanations

Once the content is selected and the appropriate pre-planning is complete, the next phase is recording and creating the video. One component of video creation is verbal explanations. The verbal explanations provided in a video are different from the verbal explanations provided in a live lesson. When breaking down student feedback on videos, issues around the quality of explanations were the source of the most likes and dislikes when rating a video (Shoufan, 2019). In particular, verbal explanations included in a video should involve simple language. Since humans can intake limited amounts of verbal information, using simple language will assist viewers in processing the information presented (Mayer, 1999). The video should show all steps so that students can follow the flow of learning (Kay, 2014). Teachers should not make assumptions about student knowledge and should describe all their steps. They should also explicitly state connections between ideas and previous videos.

Verbal explanations need to consider what is being said and how it is said (Mayer, 2003; Kay, 2014; Brame, 2016; Dinmore, 2019; Costa & Pacansky-Brock, 2020). For example, tone of voice is important, as students tend to respond best to conversational voices (Kay, 2014; Brame, 2016; Costa & Pacansky-Brock, 2020). As mentioned in the "pre-planning" section, maintaining a conversational tone while sticking to pre-planned or scripted material can be challenging (Costa & Pacansky-Brock, 2020). Beyond maintaining a conversational tone, teachers should personalize their videos by using terms such as "you" or "your" throughout the video (Mayer, 2003). By personalizing the video, students are more likely to attend to the information in the video in the same way they would listen to a personal conversation. This attention will lead to higher levels of engagement and will help students better understand the material presented.

Beyond verbal explanation, the quality of the audio recording is also an important consideration. For example, extraneous noise, including music or -quality audio can distract the viewer (Kay, 2014). Quality audio is crucial when covering more difficult or complex topics (Pandey et al., 2022) as viewers must dedicate all of their audio comprehension capacity to following the presented information and cannot handle extraneous or distracting noise. The recording quality also impacts the efficiency of auto-transcriptions on platforms like YouTube (Dinmore, 2019). For example, when a video is created using a high-quality microphone, auto-transcription software (i.e., Google transcription) such as the one on YouTube's platform is 90-95% accurate (Dinmore, 2019). Higher levels of transcription accuracy allow students access to accurate subtitles if they are deaf, hard of hearing, or ELLs. However, to obtain 100% accuracy on YouTube subtitles, teachers will need to manually check and edit the video transcriptions

(Dinmore, 2019). This accessibility feature allows more students to engage with the videos.

Although the VUER model (Pandey et al., 2022) has merit, it does not include a verbal or audio category for assessing videos. Instead, the model consists of generalized statements concerning the audio quality within the category of Visual Appeal. For example, the third statement in the Visual Appeal category is: "The audio is of good quality," and the fourth statement is "I had no problem understanding the speaker" (Pandey et al., 2022, p.11184). Including audio quality within the category of Visual Appeal is misleading and may lead to overlooking an essential element of video creation.

Verbal explanations are an essential consideration in video creation as the quality of audio, and the quality of the explanation significantly influence student opinions of videos. What teachers say in a video and how they say it will also influence student reception, understanding, and access to accessibility tools.

2.3.4 Visuals

The VUER model (Panday et al., 2022) may have combined verbal and audio in the same category as visuals, but video visuals merit an independent section when discussing video creation. Previous research indicates the more visual a learning artifact, the more likely it is to be recognized and recalled (Medina, 2008). Many of the same recommendations listed for verbal explanations, such as keeping things simple and excluding extraneous information, apply to visual explanations. Similar to the quantity of verbal information a human can process, there are limits to how much visual information a human can process (Mayer, 1999). For example, Costa and Pacansky-Brock (2020) recommend limiting the text on the screen to a 10-word maximum at any given time.

Presentation experts share similar advice to drastically limit the text on the screen (Reynolds, 2012).

In addition to the quantity of information presented, the quality of the visual information needs to be considered when creating educational videos. For example, any text on the screen in a video should be clear and easy to read. Just as the quality of the audio recording could be a distraction, text that is difficult to read could cause similar concerns (Kay, 2014; Brame, 2016). Research also indicates that humans do not listen well while reading (Horvath, 2014). Consequently, video creators should not present text visuals that are not based on their verbal explanations. However, some research suggests that video creators can draw viewers' attention to particular text or information by using signalling or cueing to highlight important information (Brame, 2016; Mayer, 2017). Signalling can include such techniques as highlighting or circling text but needs to be timed to correspond with the verbal information being shared (Mayer, 2017). Mayer (2017) refers to this timing connection between written text and verbal information as temporal contiguity. Temporal contiguity allows viewers to better process the information being shared. In addition to temporal contiguity, careful attention needs to be given to spatial contiguity or the placement of words on the screen (Mayer, 2017). For example, words located directly beside the corresponding graphic can ease cognitive load issues (Kay, 2014; Brame, 2016; Mayer, 2017).

As mentioned in the previous section, the category of Visual Appeal in the VUER model (Pandey, et al., 2022) is not exclusively visual. The three VUER model statements that focus solely on the visuals of a video include: "1. The video is of good production quality. 2. The video is distracting… [and] 5. I want to watch it again because I liked the video.” (p. 11184). These three statements are too general as they do not focus on the aforementioned visual elements in a video, including clear text, use of signalling, and the placement and timing of on-screen images and text. Consequently, while the VUER model is simple and user-friendly, it lacks cognitive research in its foundation, particularly in relation to the visual elements of an educational video.

Many elements make educational videos an effective learning tool. Technology advancements are allowing video creation to be more accessible for teachers. However, there are numerous elements for a teacher to consider during the video creation process, including pre-planning, selecting content, verbal explanations, and visual quality. The quality of the video will also be reliant on the rigour of its subject matter. To address appropriate curriculum expectations, teachers will also require a strong understanding of the subject matter. In a subject such as mathematics, for example, there is good potential to leverage educational videos. Still, many pedagogical and instructional factors must be considered beyond the aforementioned basics of video creation.

2.4 Videos in Mathematics

Videos are not a new phenomenon in the mathematics classroom. Research from a decade ago found videos to be a supportive tool in post-secondary mathematics as they can be effective when reviewing material before an examination (Kay & Kletskin, 2012). Similarly, at the secondary school level, mathematics classes that allocated additional time each day for students to use videos and practice problems scored in the 85th percentile for student growth on state exams (Khan & Slavitt, 2013). Although research indicates videos can be effective in mathematics education, it is vital to examine the types and purposes of mathematics videos to assess their potential as a mathematics learning tool.

To examine videos in mathematics, a definition of video types is necessary. Kay's (2012) literature review of 53 articles identified four types of mathematics videos: lecture-based, enhanced, supplementary, and worked examples. Lecture-based videos are recordings of a lecture, in part or in its entirety. Enhanced videos are screen captures of a slide presentation enhanced with an audio explanation. Supplementary videos augment course content with real-world examples, summaries of material covered in class, or additional material to extend student understanding. Worked example videos are recordings of an instructor solving specific problems step-by-step. Worked examples are commonly used in mathematics (Kay, 2012). Worked example videos are commonplace on online platforms such as YouTube or Khan Academy. Khan Academy, an American non-profit educational organization established in 2008 by Sal Khan, is probably the most recognized source of worked example mathematics videos. For example, as of March 2023, Khan Academy's YouTube channel has over 7.76 million subscribers and includes more than 8,300 videos, with thousands of them being worked examples of mathematics problems (Khan Academy, n.d.). Some research indicates that viewing Khan Academy videos can influence student achievement (Khan & Slavitt, 2013; Light & Pierson, 2014). Light and Pierson (2014) found achievement was specific to procedural skills rather than a conceptual or deeper understanding of mathematics. There is limited research on using videos to support students' conceptual understanding of mathematics (Kay, 2012; Light & Pierson, 2014). However, there is some research on using conceptual mathematics videos with pre-service elementary teachers (LeSage, 2011). The pre-service teachers in this study reported watching conceptual mathematics videos influenced their understanding of rational numbers (n=34, 85%). While all (n = 40) pre-service teachers stated, the videos influenced their teaching efficacy/confidence in teaching the concept. However, only 7 of the 40 pre-service teachers felt the videos increased their self-efficacy and confidence to do the mathematics. This finding is particularly note-worthy, as the goal for students would be to increase their understanding of the topic and their confidence to do the mathematics, which did not seem to be the case for the pre-service teachers.

As pre-service teachers reported, conceptual videos can strongly influence their confidence to teach a mathematical concept. Creating videos can test teachers' ability to explain mathematical concepts. For example, in a recent study (Moreno et al., 2020) a group of 50 teachers pursuing their Master's degree in mathematics education were tasked with creating mathematics instructional videos to be used in a flipped classroom environment. The researchers then evaluated the videos using a rubric validated in a previous study on four dimensions they describe as video quality and appropriateness, technical, pedagogical, and instruction. The researchers reviewed the graduate students' videos on a scale of 0-10. The median score for the videos overall was 7.29 points, which researchers considered satisfactory. However, when breaking down the scores on their components of technical elements, pedagogical and instructional aspects, the videos scored a lower median of 5.84 for the instructional components. Some of the instructional components include: presenting the math in order of increasing difficulty, liking the real-life scenarios, linking math concepts to problem solving, linking math with the historical role math has played in human development and stimulating the development of math competency. The study results indicate that instructing teachers/graduate students on how to create quality educational videos is inadequate for creating good quality mathematics videos. Although the teachers in this study (Moreno et al., 2020) were pursuing Master's degrees focused on mathematics education, they struggled to adequately address the conceptual elements of mathematics in their videos. This finding is significant, because most K-8 teachers do not possess the same level of specialized mathematics knowledge (Ball & Hill, 2009; Hill et al., 2005; Ma, 1999) as the teachers in the Moreno et al., study. For teachers to create quality mathematics videos, they require deep mathematics pedagogical content knowledge (Ball & Hill, 2009) and knowledge of video creation. Teachers need deep knowledge of three interconnected understandings: pedagogical knowledge for teaching mathematics (Hill et al., 2005), mathematical content knowledge (Brown et al., 2020) and an understanding of the components required to create a good quality educational video. With this in mind, the next section of the literature review shifts the focus to the mathematics concept of my Master's Project: proportional reasoning. In the next section, I explore the pedagogy and research specific to proportional reasoning, including the pedagogical and conceptual knowledge required to create quality proportional reasoning videos.

2.5. Proportional Reasoning

The 2020 update to the Ontario Mathematics curriculum included differentiating proportional and non-proportional situations in Grade 7 (Ontario Ministry of Education). The focus on proportional reasoning at this grade level is particularly important as research indicates that the mathematics concepts explored in grades 6, 7, and 8 in Ontario are the foundation for success in college-level mathematics courses (Orpwood et al., 2011). For example, over one-third of first-year Ontario college students taking mathematics courses in the first semester are at risk of not completing their college program due to weakness in numeracy skills (Orpwood & Brown, 2015). These numeracy skills/knowledge include fractions, ratios and percentages, which are the focus of the middle school mathematics curriculum and are essential for post-secondary success. As the cornerstone of middle and secondary school, proportional reasoning is essential for success in post-secondary mathematics (Lesh et al., 1988).

Proportional reasoning is particularly important, as an area of mathematics that has long been taught during middle school. Almost 35 years ago, in the *Curriculum and Evaluation Standards for School Mathematics,* the National Council for Teachers of Mathematics (NCTM) stated its importance:

> The ability to reason proportionally develops in students throughout grades 5-8. It is of such great importance that it merits whatever time and effort must be expended to assure its careful development. Students need to see many problem situations that can be modelled and then solved through proportional reasoning. (NCTM, 1989, p. 82).

Although there is no doubt that teaching proportional reasoning "merits whatever time and effort [required] to assure its careful development…" (NCTM, 1989, p.82) stating that the ability "develops in students throughout grades 5-8" (NCTM, 1989, p.82) is misleading. Sowder et al. (1998) showed that understanding proportional reasoning concepts does not develop naturally. Students will need explicit support and examples to help them fully develop their understanding of proportional reasoning (Van Dooren et al., 2005; de la Torre et al., 2013). Researchers have identified the different levels or stages of reasoning students need to progress through their conceptual understanding of proportional reasoning. The following section will define a conceptual understanding of proportional reasoning and explore its development.

2.6 Conceptual Understanding of Proportional Reasoning

As understanding proportional reasoning does not develop naturally (Sowder et al., 1998), students require multiple opportunities to explore proportional situations. Proportional reasoning involves understanding of what Piaget called a "second-order relationship" (de la Torre et al., 2013). As such, students need to understand the relationship between two quantities and two relationships. When students do not identify two relationships in a proportion, they struggle to differentiate between additive and multiplicative relationships (Van Dooren et al., 2005). For example, students looking for additive relationships may see a proportion $\frac{3}{4} = \frac{12}{x}$ and think *x* is 13, one more than 12, just like 4 is one more than 3.

Recognizing two relationships in a proportion can be difficult (Langrall & Swafford, 2000; Van Dooren et al., 2005; Markworth, 2012), and it, takes time to develop. The Piagetian perspective considers proportional reasoning to develop in the formal operational stage at approximately 11 years of age (Inhelder & Piaget, 1958). Vanluydt et al. (2021a; 2021b) believe proportional reasoning may begin earlier, having found general vocabulary levels (2021a) and patterning skills (2021b) at ages 4-5 to be predictors of a students' proportional reasoning skills measured at ages 6-7.

Regardless of when students begin to develop proportional reasoning, Carpenter et al. (1999) identified four levels of strategies for solving proportional reasoning problems. Interestingly, there is no consensus on whether these four levels serve as a classification for student solution strategies or present a developmental trajectory of student understanding (Steinthorsdottir & Sriraman, 2009). In their work with Icelandic students, Steinthorsdottir and Sriraman (2009) interpreted the levels as conceptual development. Students working at Level 1 show limited ratio knowledge, as they often resort to additive strategies in place of multiplicative strategies. For example, if looking at $\frac{8}{5} = \frac{24}{x}$ students see 8 as 3 more than 5 and think *x* to be 21 (i.e., 3 less than 24). Students working at Level 2, primarily use a build-up strategy of either addition or multiplication or a combination of both. For example, a student could solve a ratio like $\frac{8}{5} = \frac{24}{x}$ because they could continue to add +8+8 to get to 24 or multiply by 3, and understand that *x* = (5)(3) However, when the known portion of the proportion cannot be reached through repeated addition of the multiplication a whole number students will struggle. For example, if given the following proportion to solve $\frac{8}{12} = \frac{42}{x}$, students who try to add +8+8 to get to 42 will struggle because 42 is not a multiple of 8. Students at Level 3 continue to see the ratio as a single unit, but they can scale by nonintegers, meaning they can solve $\frac{8}{12} = \frac{42}{x}$, but do not see the relationship between and within the ratios. By Level 4, students understand ratios as more than unit quantities. Level 4 students recognize relationships between and within ratios. This means they could solve $\frac{8}{12} = \frac{42}{x}$ by looking at the relationship between 8 and 42 or the relationship between 8 and 12. This flexibility allows students to use the strategies that best fit the context of the problem.

Like Carpenter et al. (1999), Langrall and Swafford (2000) organized students' proportional reasoning solutions into four categories, numbered 0-3, based on how they solved a word problem. Level 0 is non-proportional reasoning. Much like Carpenter et al.'s Level 1, students do not identify any proportional relationship. Level 1: Informal reasoning about proportional situations, students use manipulatives, pictures, or models to make sense of the situation. This level is the least like any of Carpenter et al.'s. Level 2: Quantitative reasoning is when students can use quantitative reasoning without manipulatives or can link their models with numerical calculations. Finally, when students are operating at Level 3: Formal proportional reasoning, they can "set up a proportion using a variable and solve for the variable using the cross-product rule or equivalent fractions, with a full understanding of the structural relationships that exist" (Langrall & Swafford, 2000, p.258). Langrall and Swafford's (2000) levels focus on the strategies students use to arrive at a solution. Carpenter et al.'s (1999) levels focus on which relationships a student uses to solve a proportion. Both levels provide insight into students' progression as they further their understanding of proportional relationships. Focusing on proportional relationships (Carpenter et al., 1999) and the transition from visuals and manipulatives to numerical representations (Langrall & Swafford, 2000) are necessary for a full conceptual understanding of proportional reasoning.

As students begin to develop proportional reasoning, they rely on both additive and building on strategies (Carpenter et al., 1999) and visual representations or manipulatives (Langrall & Swafford, 2000). Young children explore proportional reasoning through small inquiries and can eventually extend their understanding and apply it to complex problem-solving contexts (de la Torre et al., 2013). When students enter Langrall & Swafford's Level 1, teachers should share qualitative comparisons alongside visual representations, with observations like "if the speed is faster, you cover more distance in less time" (Langrall & Swafford, 2000, p.260). That being said, while well known-measures like speed can be helpful, they might also mask a student's understanding (Langrall & Swafford, 2000) as they struggle to apply their understanding to less familiar contexts. As students progress towards a conceptual understanding of proportional reasoning, it is also important to be cognizant of the types of proportional reasoning problems they are presented with to support their progression.

2.7 Problem Types in Proportional Reasoning

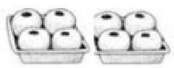
As students progress through the levels of conceptual understanding about proportions, shifting from an additive understanding to multiplicative understanding, they should be exposed to a greater variety of proportional problems. Studies of American curricula and textbooks found missing-value problems to be the most common proportional reasoning task (Kilpatrick et al., 2001). Missing-value problems are when three of four values are given in a proportion , $\frac{a}{b} = \frac{c}{d}$ and students are required to find the missing value. In addition to missing-value problems, de la Torre et al. (2013) included comparison problems and qualitative problems as other forms of proportional reasoning tasks. Comparison problems are when students are given two ratios and need to determine if the ratios are equivalent. A qualitative problem asks students to consider the effect of an increase or decrease to one part of a proportion on the other parts of the proportion.

The categorization of the types of proportional problems is not new. Two decades prior to de la Torre et al. (2013), Lamon (1993) suggested four similar types of proportional reasoning problems: associated sets, well-chunked measures, part-part-whole, and stretcher and shrinker. Lamon (1993) suggested introducing proportional reasoning problems by beginning with associated sets and leaving stretchers and shrinkers until students have a better grasp of proportional reasoning. Associated sets problems focus on the association between sets that are not always associated. For example, the relationship between crayons and students. Students were more likely to use ratio and proportion constructs, like unit ratios, in associated sets, than in other problem types (Lamon, 1993). Well-chunked measures problems involve "the comparison of two extensive measures, resulting in an intensive measure (or rate)" (Lamon, 1993, p.42). Examples of well-chunked measures problems include using familiar measures like kilometres per hour (speed) to find the distance travelled. Part-part-whole problems focus on subsets of a whole. Examples of part-part-whole problems include exploring ratios such as the ratio of starter players (part) to bench players (part) on a basketball team (whole). Part-part-whole problems tend to produce more informal methods of reasoning (Langrall & Swafford, 2000). Finally, stretcher and shrinker problems involve scaling up (stretching) or scaling down (shrinking) various measures. Examples of stretcher and shrinker problems include stretching a smaller image to create a larger version of the same image by using a scale factor so the image does not appear distorted and remains the same shape. Lamon (1993) found that stretcher and shrinker problems to be the most difficult, as the students applied the most visual or additive strategies or avoided the question altogether.

Using some of the research on problem types (Lamon, 1993; Langrall & Swafford, 2000), Belgian researchers conducted a pencil and paper proportional reasoning test with over 1000 students in grades 2-8 (Van Dooren et al., 2005). Students were asked to answer eight questions, including two proportional situations and six non-proportional situations. The non-proportional situations came from three categories: constant, linear or additive problems. An easy and difficult version of each category type was presented. The questions used in the assessment are presented in Figure 1:

Figure 1

Student Survey Questions

Mathematical model	Difficulty level: *Easy*	*Difficult*
Proportional	 Mama buys 2 trays of apples. She then has 8 apples. Grandma buys 10 trays of apples. How many apples does she have?	In the shop, 4 packs of pencils cost 8 euro. The teacher wants to buy a pack for every pupil. He needs 24 packs. How much must he pay?
Constant	Mama put 3 towels on the clothes-line. After 12 hours they were dry. The neighbour woman put 6 towels on the clothesline. How long did it take them to dry? 	A group of 5 musicians plays a piece of music in 10 minutes. Another group of 35 musicians will play the same piece of music. How long will it take this group to play it?
Linear	In the hallway of our school, 2 tables stand in a line. 10 chairs fit around them.  Now the teacher puts 6 tables in a line. How many chairs fit around these tables?	The locomotive of a train is 12 m long. If there are 4 carriages connected to the locomotive, the train is 52 m long. If there would be 8 carriages connected to the locomotive, how long would the train be?
Additive	Today, Bert becomes 2 years old and Lies becomes 6 years old. When Bert is 12 years old, how old will Lies be?  *Bert* *Lies*	Ellen and Kim are running around a track. They run equally fast but Ellen started later. Whe n Ellen has run 5 rounds, Kim has run 15 rounds. When Ellen has run 30 rounds, how many has Kim run?

Adapted from "Not Everything Is Proportional: Effects of Age and Problem Type on Propensities for Overgeneralization" by W. Van Dooren, D. De Bock, A. Hessels, D. Janssens, and L. Verschaffel, 2005, *Cognition and Instruction, 23*(1), p.65.

The two constant problems on the test did not require students to complete any calculations. Instead, they were created to mirror proportional problems. While these constant problems did not require mathematical calculations, they did require students to examine the context of the problem. The linear problems used the underlying function of

$f(x) = ax + b$; however, $b \neq 0$. The easy version of the linear problem involved the real-life task of lining up tables (end to end) and chairs in a hallway. If students inappropriately applied proportional reasoning to this problem, they would not consider the chairs that would be removed as two tables were brought together. The easy problem included a visual representation, while the difficult problem did not. The additive problems on the test involved an additive relationship between the variables. In the difficult question, students may recognize the scenario of running and speed and incorrectly assume the relationship to be proportional.

The results of the Van Dooren et al. (2005) study indicate that the students seemed to struggle to identify non-proportional situations, as only 44% of students correctly solved the non-proportional questions, while 85% of students correctly solved the proportional questions. The study findings indicated that students began to inappropriately apply proportional strategies to non-proportional situations starting in grade 2 and continued to do so with increasing frequency up to Grade 5. These findings support previous research, which asserts that because extensive attention is paid to proportional reasoning situations/problems in early elementary, students tend to over-rely on proportional methods in the other mathematics domains, including algebra, geometry and probability (De Bock, 2002). Consequently, students begin to assume any problem with a proportional element or theme (e.g., speed, price) must be a proportional problem. This overapplication of proportional reasoning, regardless of context, further justifies focusing instruction on conceptual understanding and real-world contexts to support students' development of proportional reasoning.

As students' conceptual understanding of proportional reasoning does not develop naturally, teachers must consider the content and pedagogy they use to support students as they build their understanding. Students also need exposure to a variety of problem types to support their transition through the stages of conceptual understanding. The need for explicit instruction and various problem types lends to using educational mathematics videos to support students' development of proportional reasoning. However, there are other concerns beyond mathematical pedagogy to generate quality videos specific to proportional reasoning. For example, quality videos require appropriate visuals and specific and concise verbal explanations. Each video should explore only one concept and not extend beyond 6 minutes of viewing time. These combined elements allow students to leverage their visual and verbal pathways for a more effective learning experience.

The research from this literature review informed my work, as discussed in Chapter 3. I applied the previous research on quality educational videos and proportional reasoning pedagogy to create mathematics videos focused on proportional reasoning for middle school students. In Chapter 3, I discuss how I applied the theory into practice by planning and scripting videos on proportional reasoning that leveraged multiple problem types across the stages of conceptual understanding. In Chapter 4, I analyse segments of the created videos to bring attention to the creation process and highlight issues and limitations encountered in the practical application of this research.

Chapter 3. Theory into Practice

As discussed in Chapter 2, videos can be an impactful tool to support student learning. As such, I wanted to put this research into practice to teach students about proportional reasoning. In particular, I was interested in implementing the new Ontario

Mathematics curriculum-specific Grade 7 expectation: “identify proportional and non-proportional situations and apply proportional reasoning to solve problems” (Ontario Ministry of Education, Grade 7, B2.10). This specific expectation was a new addition to the Ontario curriculum in 2020; previous iterations of the Ontario mathematics curriculum did not include reference to identifying non-proportional situations. However, the ability to differentiate between proportional and non-proportional relationships or situations is considered by some to be an overall sign of one’s competency in proportional reasoning (Lim, 2009). Therefore, I created a series of videos focused on this theme. I titled the video series *“Is it Proportional?”*. The development of these videos takes into account the stages of conceptual understanding (Carpenter et al., 1999) and the different problem types (Lamon, 1993) in proportional reasoning research. To address this concept, I chunked the videos into smaller components to introduce one concept at a time (Brame, 2016) and strive to create each video with a viewing time of less than 10 minutes (Guo et al., 2014). With these goals in mind, I made four videos. Throughout this chapter, I describe the video creation process. I use the four themes identified in Section 2.3 to frame my video creation process: pre-planning, content covered, verbal explanations and visuals.

3.1 Video Pre-Planning

Once I identified the new Grade 7 curriculum expectation as the starting point for my video series, the theme of *“Is it Proportional?”* was the guiding question for the content of all the videos. To create impactful educational videos, there are numerous logistical aspects of pre-planning to consider. When deciding on recording tools, I wanted to leverage tools that would be accessible to the majority of educators. As such, I used

Canva slide decks to create the visuals and used Zoom to create the video recordings. I decided to use Canva as its slide deck has a free premium subscription available to educators and a variety of images and content available for free use. I chose to use a slide deck format as, based on my experiences at the school board working with K-12 teachers, they are familiar with creating slide decks for educational use. The slide deck format worked well for most of the content I was creating; however, it lacked the personalized touch of handwriting or filling in the information in real-time that students would be accustomed to in the classroom. More details on my visual choices will be discussed in Section 3.4.

When recording the video, I chose to use Zoom. I selected Zoom because it was a recording tool I used to record lessons while teaching in a virtual classroom throughout the COVID-19 pandemic from 2020-2022. Connecting the audio and visuals was simple. My device automatically detected my computer's camera and attached headset when I entered Zoom. When recording, the user can choose whether or not to show the face of the video's narrator. Dinsmore's (2019) research with 240 university students found that 88% of students preferred a speaker presenting to the camera with background slides over a recording of a computer screen. However, since I was creating the videos for teachers to use in their classrooms, I decided that having my face on the screen was neither necessary nor appropriate. I used a headset and microphone with noise-cancelling features for the recordings to ensure sound quality and improve the clarity of my voice.

By using Canva and Zoom to create the educational videos, I selected tools accessible to other educators. However, these programs' limitations and affordances will be discussed in Chapter 4. With my pre-planning logistics in place, I shifted my focus to the proportional reasoning content I planned to present in the video series.

3.2 Video Content Covered

To address the research on proportional reasoning, I needed to ensure that I included a variety of situations and problem types in the videos. To help me plan, I initially created a table of some big ideas that would form the foundation for the content covered in the four videos (see Table 1).

Table 1

Research Guiding Video Creation

Content to cover	Big Ideas and Related Research
What is proportional reasoning	Definition of proportional reasoning based on Izzatin (2020)
Real-Life Examples	Show how proportional reasoning is used beyond the classroom. Students see these problems as puzzles with little relation to the real world (Van Dooreen et al., 2015)
Speed	Well-known-measures like speed can be helpful, but they might also mask a student's understanding (Langrall & Swafford, 2000).
Two relationships	When students cannot understand there are two relationships present in a proportion, they struggle to differentiate between additive and multiplicative relationships (Van Dooren et al., 2005).

After determining the big ideas I wished to include in the four videos, I then turned my focus to the four problem types: well-chunked measures, part-part-whole, associated sets, and stretcher and shrinker (Lamon, 1993). I wanted to include the four problem types throughout the video series. Initially, I thought I would present each problem type in separate videos. However, with further consideration, I decided that the focus of the video series would be on identifying if a problem situation was proportional or non-proportional. This consideration led me to create a theme for each video and present two problems within each video. Some videos would include both a proportional and non-proportional situation on the same topic. Other videos would consist of both proportional problems to (a) be able to address all of the problem types across the video series and (b) stop students from assuming one scenario will always be proportional and one will not. Students often learn to decode word problems superficially to determine which operation to apply (Van Dooren et al., 2005). To encourage students to move beyond superficial applications of knowledge, I wanted the proportional and non-proportional problems to be within the same theme. For example, students see variables like time and distance (speed) and assume the relationship will always be a proportional relationship (Van Dooren et al., 2005). However, it is possible to have a situation that involves time and distance, like Van Dooren et al.'s (2005) difficult additive problem about running, that is non-proportional. This problem, along with the linear non-proportional problem, inspired the non-proportional situations in the videos. I purposefully did not use constant non-proportional questions, as these types of questions did not require calculations to solve. I did not want students to view these questions as redundant or assume they were being deceived or tricked.

Combining the key research elements from Table 1, Lamon's (1993) four problem types and Van Dooreen et al.'s (2005) non-proportional problems, I had sufficient content to create four videos. Tables 2 to 5 illustrate the content covered in each video, the problem type as classified by Lamon (1993) and the non-proportional problems, as classified by Van Dooren et al. (2005). Additional background and justifications follow each table.

Video *#1* is the introduction to the video series. It was created to establish a formal definition of proportional reasoning and show students proportional situations they may be familiar with daily. The definition of proportional reasoning used in the video series was adapted from Izzatin (2020) as the understanding of the multiplicative relationship between quantities. This definition is revisited at the beginning of each video in the series to reinforce the definition and connect the videos (Kay, 2014).

To introduce the definition, two basic proportional situations are presented in *Video #1*. First, an associated sets problem is presented, exploring the relationship between the number of students and the number of skates using simple quantities. In the video, I explain that this problem could be solved using addition, as students in the earlier stages of their proportional reasoning understanding may be inclined to do so.

Table 2

Video #1

Video Title	**Video #1- Introduction to Proportional Reasoning**
Content Covered	What is proportional reasoning? Where do we find proportions in everyday life? Speed, Screen Ratios, Recipes, Grocery Store Deals Sample problems with proportional relationships. • Renting skates • Doubling a recipe "Is it proportional"? This is the question we will be asking ourselves before we do ANY kind of calculation or try to "solve" the problem.
Lamon (1993) Problem Type	**Associated Sets:** A class is having a skating party. Their teacher is looking to rent skates for everyone. If one student attends, they need to rent two skates. If a second student attends, they will need to rent four skates. How many skates will they need if 10 students attend? **Part-Part-Whole:** If my Mom's mustard sauce recipe serves four people, and needs ½ c sugar, 1 egg, ⅓ c of vinegar and 3 tsp of dry mustard, how much will we need to serve eight people?
Van Dooren et al. (2005) Non-proportional situation	None. This introduction video focuses on establishing proportional reasoning.

The second problem in *Video #1* uses a recipe context with more complex quantities, including fractions, for a part-part-whole problem. I chose to include a personal recipe for the second problem as I wanted to speak candidly to better connect with my audience and have a more conversational tone (Kay, 2014; Brame, 2016; Costa & Pacansky-Brock, 2020). The recipe in this proportional situation is examined and solved by looking at the multiplicative relationship of both *within-measure-space,* comparing eggs to eggs when doubling the recipe, and *between-measure-space*, comparing the ratio of servings to dry mustard in each recipe. Riehl & Steinthorsdottir (2014) highlight the importance of solving problems using both within and between-measure space. It shows students thoroughly understand both relationships in proportion and is what Carpenter et al. (1999) would consider Level 4 understanding. The first video concludes by inviting students to engage with future videos in the series, with a caveat that not all of the future relationships examined will be proportional.

Table 3

Video #2

Video Title	**Video #2- Need for Speed**
Content Covered	Review of what is a proportional situation. Two scenarios connected to speed. • One proportional • One non-proportional
Lamon (1993) Problem Type	**Well-Chunked Measures:** In Ontario, the law is the max speed you can travel on an electric bike is 32km/h. If you are travelling at max speed, how far will you travel in 2.5 hours?
Van Dooren et al. (2005) Non-proportional situation	**Additive:** You and your cousin Lee got electric scooters for your birthdays last week and love racing them around the bike paths near your house. Your parents are very cautious, so they have set the controls to max speed of 20 km/h on both scooters. You and Lee are racing at max speed, but Lee took off first. When Lee has travelled 15 km, you have travelled 5 km. When Lee has completed 30 km, how far have you travelled?

As the concept of speed is a proportional relationship, in *Video #2,* I chose to highlight both a proportional and non-proportional situation involving distance, time and speed. Common measures such as speed can be helpful in the exploration of proportional reasoning; however, they can also mask student understanding (Lamon, 1993).

Table 4

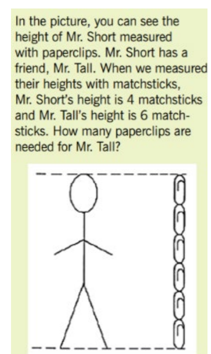
Video #3

Video Title	**Video #3 - Party Planning**
Content Covered	Review of what is a proportional situation. Two problems are connected to planning a party. • One non-proportional (non-proportional purposely presented first so students do not begin to look for patterns in the videos instead of looking at the actual question • One proportional
Lamon (1993) Proportional Problem Type	**Well-Chunked Measures:** Party Planet offers two different brands of pop. They charge $1/50mL for Patsy's Pop & $4/400mL for Safia's Soda. If you want to save money on this party, which is the better deal?
Van Dooren et al. (2005) Non-proportional situation	**Linear:** Party Planet sets up tables for kid parties in one long line. Two tables can fit 10 chairs. If they set up 6 tables in a line, how many chairs can they fit?

Video #2 begins with a well-chunked measure problem, a common missing value problem most often presented in a unit or textbook chapter on proportional reasoning (de la Torre et al., 2013). For the first problem, I chose to use the speed of 32 km/h as it is currently the maximum legal speed limit for an e-bike in Ontario (Ontario Ministry of Transportation, 2022). For the second problem, I continued the theme from the first problem: speed and electric bikes/scooters. The second question is an adaptation of the "difficult" additive question from Van Dooren et al. (2005). I chose this question because Van Dooren et al.'s (2005) research highlighted a few interesting associated patterns. For example, they found that students in Grades 3 and 4 outperformed students in Grades 6-8 on this question, with older students being more likely to answer the question proportionally. Additionally, this question requires students to use real-world logic and stops them from over-applying proportional reasoning (Van Dooren et al., 2005). I intentionally positioned this question second in the video because it is the first non-proportional situation. I did not want students to assume all situations are proportional based on the theme of the video series (De Bock et al., 2002). I model both problems in *Video #2* using number lines as the visual representation to help students see the importance of visualizing a problem.

Table 5

Video #4

Video Title	**Video #4- Mr. Short & Mr. Tall**
Content Covered	Review: what is proportional? Based on Karpus et al.'s 1974 problem as cited in Riehl & Steinthorsdottir (2014). Two different kinds of proportional reasoning problems.
Lamon (1993) Proportional Problem Type	**Associated Sets:** "In this picture, you can see the height of Mr. Short measured with paperclips. Mr. Short has a friend, Mr. Tall. When we measured their heights with matchsticks, Mr. Short's height is 4 matchsticks and Mr. Tall's height is 6 matchsticks. How many paperclips are needed for Mr. Tall?" (Riehl & Steinthorsdottir, 2014, p. 222)  **Stretcher and Shrinker:** Mr. Short and Mr. Tall are going to be featured in a new cartoon show. The graphic artist is making promotional images but wants to make sure she keeps their size ratio right every time. She has a picture that is 6cm wide by 8 cm high. If she stretches the width to 12cm, what will the height of the image be?
Van Dooren et al. (2005) Non-proportional situation	None.

The third video in the series is set in a party scenario, something familiar to many students. I present the non-proportional scenario first, hoping that students do not simply guess about the situation's proportionality based on the previous video's order (proportional and then non-proportional). The first problem presented in *Video #3* is based on Van Dooren et al.'s (2005) "easy" linear problem and focuses on setting up tables for the party. I chose this question because Van Dooren et al.'s (2005) research highlighted a few interesting associated patterns. For example, in an analysis of the 1000+ students' solutions, only about 14% included a visual model to illustrate their thinking or expanded on the graphic in the problem. Van Dooren et al. (2005) also found that almost all of the students who utilized visual strategies to solve the problem arrived at the correct solution. The only exceptions were younger students who were imprecise with their drawings and missed some chairs. Therefore, for the first problem in *Video #3*, I focused on using visual representations. The research indicates that students believe all necessary information is in the written portion of a word problem and that information should not be deduced from any accompanying drawings (De Bock et al., 2002). To encourage students to use visual models as a viable problem-solving strategy, I focussed on the table to show students how this strategy works. I also included a more colloquial name to describe what was happening in the problem as the "squish factor" to reiterate why proportional reasoning would not work for this linear problem situation. The second problem in *Video #3* was another well-chunked measure problem, but it was designed as a comparison problem instead of the missing value problem, as presented in *Video #2*. I decided to purposefully integrate this type of problem so that students were presented with situations other than the typical = structure (de la Torre et al., 2013). As with the first problem in *Video #3*, I continued to use visuals to model the second problem so that students saw visuals as a valuable aid/tool for thinking about proportional and non-proportional situations. I also discussed unit rate as an additional strategy for this problem. Being able to unitize a problem would put a student at the highest level of Langrall & Swafford's (2000) categorization, which is why I presented it after a visual solution. I also wanted to present unitizing later in the video series, as students who learn to unitize often rely on this strategy, even when inappropriate (Singh, 2000).

The final video of the series is an adaptation of the associated sets problem, Mr. Short and Mr. Tall, created by Robert Karplus for his research in the late 1960s (Riehl & Steinthorsdottir, 2014). The simple graphic (seen in Table 5) has been used frequently by researchers and classroom teachers to assess proportional reasoning (Khoury, 2002; Riehl & Steinthorsdottir, 2014). I decided to use this particular problem because it allows students to solve it by applying within-measure (comparing the paperclip to paperclip relationship) or by applying between measures (using the paperclip to matchstick relationship). Students can be successful if they understand proportions, achieving Carpenter et al.'s Level 3. However, when students understand the relationships between and within the proportions, "they will flexibly choose to use whichever relationship permits an efficient solution." (Riehl & Steinthorsdottir, p.222, 2014). In the video, I present the Mr. Short and Mr. Tall problem using both relationships (e.g., within and between measures) so that students can see both relationships. When creating the video, I purposely introduced the less common (Riehl & Steinthorsdottir, 2014) between-measure approach first. In Riehl and Steinthorsdottir's (2014) study, only 4 out of 412 students solved the Mr. Short and Mr. Tall problem using the between-measure relationship.

However, 22 of the 412 students used the within-measure relationship to solve the problem. The other 54 students who answered correctly did so using a build-up strategy, or their strategy was ambiguous and impossible to distinguish. As such, I modelled both between and within-measure so that students could see both relationships.

The final problem in the video series is a stretcher and shrinker problem that continues with the Mr. Short and Mr. Tall theme. In Lamon's (1993) research, she identified stretcher and shrinker problems as the most challenging problem type for sixth graders, adding that students were also more easily frustrated with this problem type. As I considered Lamon's (1993) findings, I wondered if stretcher and shrinker problems are more challenging for students because they are difficult to illustrate or visualize in a traditional textbook or on a static classroom chalkboard. Consequently, this problem type lends itself well to videos' visual, dynamic capabilities. For example, using technology, I can easily stretch and shrink an image multiple times to illustrate the scaling effects.

Moreover, with the ubiquitous nature of technology, students commonly engage with stretching and shrinking digital images, be it on their slide decks, word documents or favourite social media platforms. I recognize stretching images on a screen vastly differs from solving a proportional problem. Therefore, further research to compare student understanding when learning about stretcher and shrinker problems on a static board versus a dynamic screen is necessary to properly assess this dynamic learning platform's potential impact.

I used research about how students progress in their understanding of proportional reasoning as well as research on the different problem types to create the content covered in my videos. Focusing on four themes, I developed a series of videos that established a definition of proportional reasoning and exposed students to Lamon's (1993) four proportional reasoning problem types and two of Van Dooreen et al.'s (2005) non-proportional problems. In the next section, I shift from content coverage to discussing the central role of verbal explanations in creating educational videos.

3.3 Video Verbal Explanation

Pre-planning a video and its content is a lengthy process. As evidenced in Sections 3.1 and 3.2, there are many factors to consider in the pre-planning stage, particularly for novice video creation teachers. For example, audio quality needs to be considered when recording a video. However, the quality of explanation is more important, including simple language, detailed steps, connections to previous videos, and tone of voice. Throughout this section, I use video clips with examples of verbal explanations to illustrate my audio choices while creating my video series.

3.3.1 Audio Quality

The quality of the content of a video will be quickly overshadowed by any issues with the audio quality (Shoufan, 2019). When creating the videos for this project, I used a Poly brand headset with a microphone boom that can be adjusted to fit in front of the mouth for the best sound quality. I also used the feature on Zoom, which increases background noise suppression, to eliminate external audio distractions. The audio quality also impacts the quality of the transcriptions (Dinsmore, 2019). For example, because I took the necessary steps to ensure my videos included good quality audio when using the auto-transcription feature on YouTube for Video 1, there were only three transcription errors in the almost 10-minute-long video recording. Once Video 1 was transcribed, it included the following three errors: "gates" instead of "skates," "slots" instead of “sauce,” and when referring to Instagram reels, it transcribed it as “reals.” Consequently, I could edit the video’s transcription quickly and easily to correct these minor errors. Although the auto-transcription was accurate, it was inconsistent in how numbers/quantities were transcribed. In some circumstances, the transcription included the worded number (e.g., “four,”) while other times it appeared as the numeric digit “4”. I did not edit this transcription aspect, but I found it interesting. No punctuation was included in the auto-transcription, and there were some unusual capitalizations. However, I do not believe these inconsistencies would deter student understanding if left unedited. With a few minor edits, I created videos that were more accessible to a broader audience of students, including English Language learners and students who are deaf or hard of hearing (Dinsmore, 2019).

3.3.2 Verbal Explanation Quality

By creating videos with good-quality audio, students can focus their attention on the content being shared and are not distracted by external stimuli. For example, in the video found in Figure 2, click to hear that I am showing students an additional strategy to identify if a situation is proportional, using the relationship between quantities at zero. This relationship is often only addressed when students begin graphing linear relationships, referring to graphs where the line passes through the origin (De Bock et al., 2002). However, for students to understand this concept situationally, I addressed it without using more complex ideas such as graphing and coordinates.

Figure 2

Content Explanation

Research indicates that verbal explanations are not just what is being said but also how it is being said (Mayer, 2003; Kay, 2014; Brame, 2016; Dinmore, 2019; Costa & Pacansky-Brock, 2020). As such, for each video I created, I purposely used simple language (Mayer, 1999), a conversational tone (Kay, 2014; Brame, 2016; Costa & Pacansky-Brock, 2020), and terms such as “you” and “yours” to engage and connect with the viewer (Mayer, 2003). In the following video clip in Figure 3, I shifted my strategy to better engage the students. In addition to speaking directly to the student, I incorporated the students into the presented problem. I continued to use simple language and a conversational tone as I moved on to the second problem about speed from *Video #2*.

Figure 3

You and Yours Language

As I created this series of videos, I strived to ensure a smooth flow from one video to the next and explicit connections between videos (Kay, 2014). For example, in the video in Figure 4, the original definition of proportional reasoning introduced in *Video #1* is re-introduced at the beginning, as it is in each subsequent video. Students are also encouraged to return to the introduction video if they did not view *Video #1* or want to revisit the definition in greater detail.

Figure 4

Re-introduction

Thus far in this chapter, I have discussed the importance of carefully considering the content covered and the centrality of the audio quality and explanations when creating educational videos. I decided to write and use a script to ensure that I addressed the appropriate content in a short time frame. Although my videos are longer than the recommended length of 6 minutes, they are within the upper time limit of 10 minutes (Guo et al., 2014). It was not until I began practicing my scripts for timing that I realized I had not considered the additional time required to write or highlight words, symbols or objects on the screen or include natural pauses necessary for a conversational tone.

Considering not only quality audio but also the quality of the explanation that shows all of the steps to the solution, simple language, and conversational tone, I used a script to guide my creation of this video series. The script helped keep me on track, and I was able to pre-plan the aforementioned considerations in my videos. While the audio needs to be simple and conversational, the visuals need to be clear and precise. The following Section will analyse and critique my use of visuals.

3.4 Video Visuals

Educational videos' main advantage is leveraging audio and visual pathways (Mayer, 1999). Adding a visual element to an explanation increases students' likelihood of recognizing and recalling information (Medina, 2008). The considerations for the visual components of a video are similar to the verbal components, including limited extraneous distractions, choice of text, use of signalling, and placement or timing of on-screen images. In this section, I leverage the power of video to analyse and illustrate the visual choices I made in creating the videos for this project. This will allow me to more accurately point to the elements I am discussing and show how a short video can change and enhance communication. I include two videos in this section.

Figure 5 highlights the importance of text that is clear and easy to read. In the video, I explain some of the visual choices I made when presenting the definition of proportional reasoning and connect those choices to the supporting research.

Figure 5

Visual Elements- Text

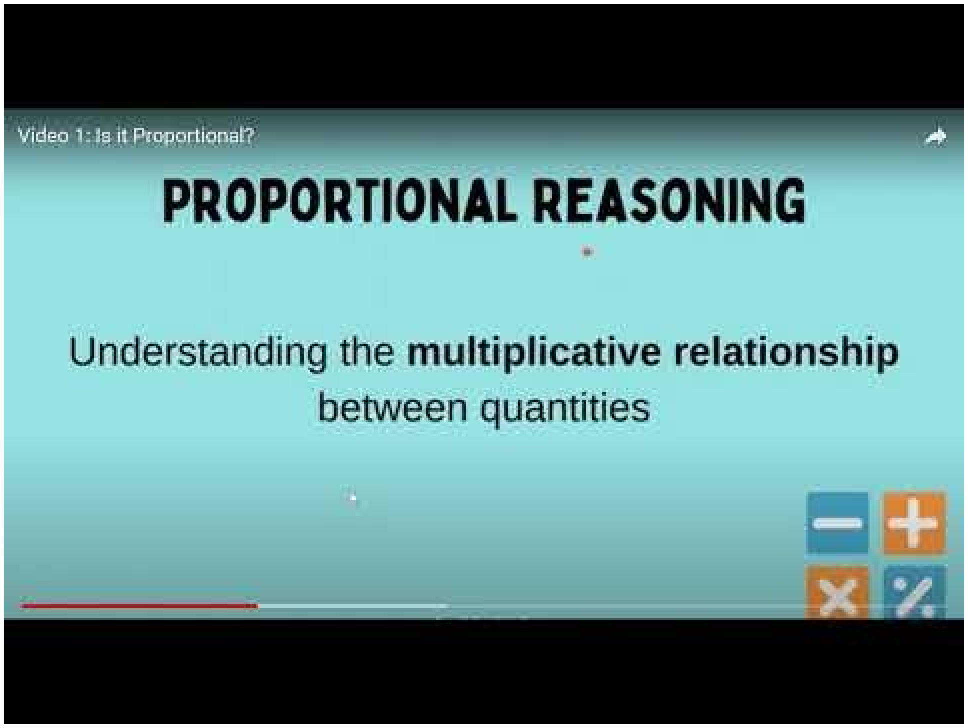
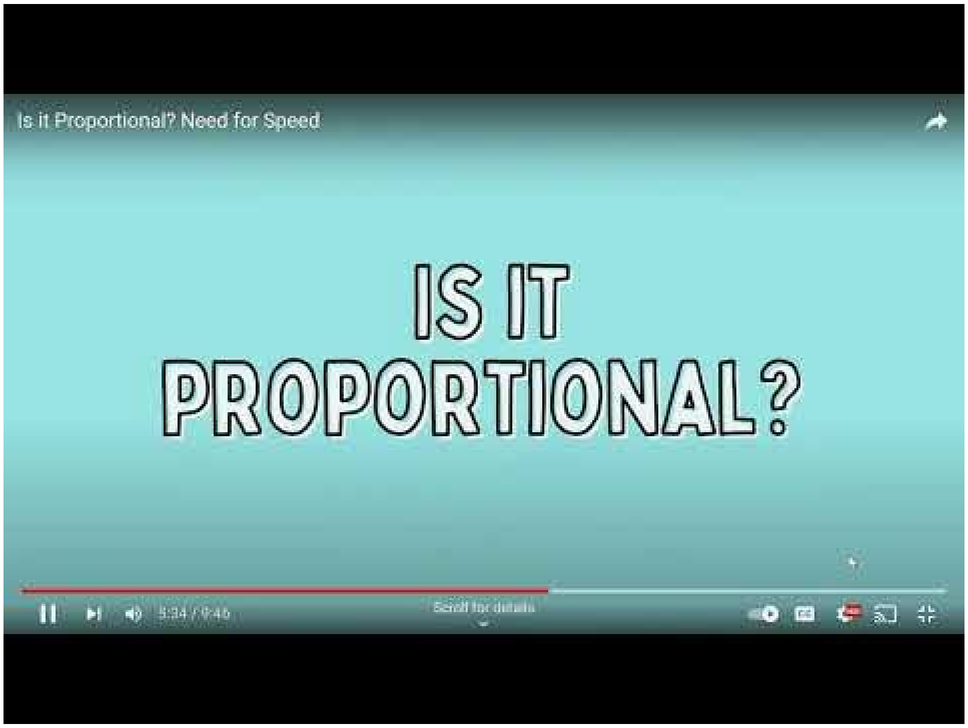


Figure 6 highlights the importance of signalling and cueing to direct students' attention. In the video, I illustrate the importance of cueing and signalling by comparing the presentation of information with and without to show how signalling and cueing can effectively guide the viewer's attention.

Figure 6

Visual Elements- Signaling and Cueing



The potential to leverage visuals and use techniques like cueing and signalling to direct a viewer's attention is quite powerful. Some requirements, such as clear text, can be simple for teachers to check and monitor. However, finer details such as cueing and signalling, timing and appropriate visual use are far more complicated and involved. While cueing, signalling and timing rely on teachers' technical understanding of video creation, the visuals and any mathematical models rely on teachers' pedagogical and conceptual understanding of proportional reasoning (Mishra & Koehler, 2006). This intricate and complicated overlap of technical, conceptual and pedagogical knowledge will be discussed in Chapter 4.

In the literature review in Chapter 2, I discussed the body of research specific to quality educational videos. In this chapter, I illustrated how this research could be applied in practice to create mathematics educational videos. As I travelled this journey of putting theory into practice, I discovered that there are significant implementation considerations for teachers. The application of research is complex, particularly for teachers new to the video creation process. The four main themes of the video curation process: pre-planning, deciding on the content to be covered, and verbal and visuals serve as guides to assist teachers as they begin to implement researched strategies for effective video creation. Chapter 4 discusses some connections to additional research I discovered during my analysis. I also suggest next steps I recommend based on my experiences completing this project. I offer suggestions regarding how teachers can continue to bring theory into practice and some limitations to this process. I also provide considerations for potential future research in educational mathematics video creation.

Chapter 4- Connections, Limitations and Next Steps on putting Theory into Practice

My journey to this project began over four years ago during my first course at Ontario Tech when Dr. Ann LeSage invited me to be a Research Assistant (RA), creating videos for a first-year business mathematics course. From developing videos for first-year business students to researching educational videos in my independent study course, I began to see the potential for educational videos. Two significant educational changes in 2020 solidified the direction of my research: the new Ontario mathematics curriculum and the COVID-19 Pandemic. I intended to use a new curriculum expectation from Grade 7 as the content focus of my thesis research on educational videos. However, as I worked to create videos to test with students, I encountered many roadblocks. These led to the shift from researching student perceptions of videos and the impact of videos on learning performance to researching the elements of a quality educational video and analyzing my experience in the video creation process. Regardless of the shift from a thesis to a project, my literature review, exploring videos in education and proportional reasoning, remained similar. However, as I began to look at putting theory into practice in Chapter 3, I noticed some themes emerging that were not in my literature review. I needed to understand information from various domains to implement the theory of quality videos and proportional reasoning to create my own videos. I needed knowledge of the elements of a quality video, technological abilities to make the videos, an understanding of the development of conceptual understanding of proportional reasoning, how students struggle to identify proportional situations, and the varied types of problems and ways to represent and solve proportions. I realized that all of these elements, in combination, can be framed through Mishra and Koehler's (2006) Technological Pedagogical Content Knowledge (TPACK) model.

The TPACK framework is the interplay of three forms of knowledge: Technology (TK), Pedagogy (PK) and Content (CK) (Mishra & Koehler, 2006). As I ventured through this video creation process, I needed Technology Knowledge of how to use digital tools to make, record and edit the videos. The pedagogical knowledge I needed was twofold: the pedagogy of proportional reasoning and educational videos. The triad's final component is Content Knowledge of proportional reasoning to address the subject matter adequately. Mishra & Koehler (2006) visualized this framework in a Venn diagram, so there are also technological, pedagogical and content knowledge overlaps. In my work, Pedagogical Content Knowledge (PCK) represented proportions, the progression of problem types and common student misconceptions of proportional reasoning. Technological Content Knowledge (TCK) was knowing when and how videos are an appropriate technology to explore this particular mathematical concept. Technological Pedagogical Knowledge (TPK) was also essential to understand how videos can change learning. My video creation process was, according to the TPACK model:

> The basis of effective teaching with technology requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn, and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones. (Koehler & Mishra, 2009, p.66)

As such, the TPACK framework supports my experiences and underscores that creating a conceptual mathematics video with a viewing time of under 6 minutes covering one specific curriculum topic is intricate and involved. While my initial work was not framed using TPACK, this model justifies the many details and angles I have explored throughout this paper.

The remainder of this chapter will explore how the application of both video and proportional reasoning research has led me to consider some practical limitations of the theories and potential opportunities for future research.

As I pose these limitations and next steps, it is imperative to consider my position and background. I have been a certified Primary/Junior Ontario teacher since 2009, with additional qualifications in intermediate grade-level mathematics. I have held school board positions as a lead mathematics teacher for three years and am a master's student focusing my coursework and research on technology and mathematics. As a graduate student, I worked as a Research Assistant (RA) creating mathematics videos for a first-year business mathematics course at Ontario Tech University as a part of an inter-faculty research project. I also co-authored a conference proceeding paper related to the research (LeSage et al., 2019). As such, I believe that my experiences as an educator, school board lead and researcher, combined with my mathematical content knowledge, theoretical foundation, and practical experiences with video creation, provided me with a unique perspective to offer insights on potential limitations and next steps in research in this field.

I will continue this chapter guided by the four themes of pre-planning, content covered, verbal explanations and visuals to discuss limitations and next steps.

4.1 Pre-planning Limitations

There has been limited focus on conceptual videos in mathematics (Kay, 2012; Light & Pierson, 2014). Teachers considering using videos to explore a conceptual topic such as proportional reasoning will likely have to create videos. The research discussed in Section 2.3.1 asserts that video creation requires a time commitment and that an educational video of under 10 minutes in length can take 40-90 minutes to create (Kay, 2014; Costa & Pacansky-Brock, 2020). Given my experience creating videos for this project and the Business mathematics project (2019), I believe the 40-90 minutes estimate is a significant underestimation. This time range does not give adequate consideration to the time required to research the selected content to be covered in the videos, let alone the time to plan, script, create and edit the videos. It might be possible to create a 10-minute video within the 40-90 minute timeframe if teachers were editing lecture-based videos, creating enhanced videos based on pre-existing slides or creating screen capture videos of a worked example to review content from class or for an assignment. However, I believe the time commitment is significantly more substantial when creating videos that support students' conceptual understanding of mathematics concepts. As an example, to make one video in this project after I had completed the pre-planning stage, it took me, on average, around 2 hours. While working as an RA creating videos for Business students, I logged over 60 hours of work, mainly dedicated to video creation. I created 25 videos for the course, including supplementary videos, terminology videos, and worked examples. This workload equates to over 2 hours per video, although there was variance in time commitment from creating short vocabulary videos to more extensive worked examples. I became more efficient in creating business mathematics videos as my confidence in video-making increased, but I had the unique opportunity of creating multiple videos per week over a semester.

This significant time commitment can be incredibly daunting for teachers new to video creation with limited time in their schedules. This commitment is further magnified given that the payout of a video on a specific and narrow topic has applicability for a very brief window of time. For example, in the case of my research, *Is it Proportional?* is a four-video series to address one specific grade 7 curriculum expectation that could take almost 8 hours, more than an entire school day, to create.

In addition to the substantial commitment of time required to create the videos for this project, I was mindful of the technology resources readily available to most K-12 teachers in Ontario. I used Canva, a web-based software with a free premium subscription for educators, to create my slide deck. This platform choice meant I had to pre-program all the text instead of writing it live on the screen during the video recording. This process was incredibly time-consuming and led to issues around timing/time lags, as discussed in Section 4.4. Although numerous quality screen capture software/programs are available to create clear annotations, they require a digital stylus or pencil for ease of use. Given the lack of basic technology that serves as a first-order barrier to technology use and video creation, I knew an additional piece of hardware would not be freely accessible to most public school teachers in Ontario, and therefore I did not consider it for this project.

To record the videos, I used Zoom, as it is a conferencing platform funded by many Ontario school boards and has a free version that could record shorter-length videos. It was straightforward to share my screen and record my videos. I used the basic Microsoft video editing software, which comes free with many devices, to edit my recordings. Most of my edits were simple cuts and trims to remove any long pauses at the beginning of the recordings or remove segments of the recording when I stopped and had to reread a portion of my script. There are other screen-capture technologies available that may have made this process smoother. However, they required a paid subscription, and I wanted to use as much freely available software as possible. With the advancement of software technology, this may become a more straightforward process for teachers looking to record videos. However, teachers will require technology training to create quality educational videos beyond the software's basics, just as the TPACK model (Koehler & Mishra, 2009) highlights the need for pedagogical and content knowledge beyond technical knowledge.

In the next section, I provide suggestions identifying some limitations and the next steps in how research might support the practical application of mathematical content in educational videos.

4.2 Teacher Content Knowledge Limitations

The technical elements of video creation may present barriers to some teachers regarding technology and time commitment. As mentioned in Section 4.1, the time commitment to create a conceptual mathematics video is much longer than most other video types. A conceptual video requires teachers to make choices about the content covered in a video. My literature review in Section 2.3.2 focused on teachers ensuring the videos were important and divided into segments to fit within an ideal time limit. What became apparent as I attempted to select the content for my videos was the ability to choose the most important topics to put in the video relied heavily on research and Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008). Ball et al. (2008) identified multiple domains entailed in teaching mathematics. These include a basic understanding of the content and the ability to solve problems. The ability to unpack mathematics using precise mathematical terms and vocabulary. The knowledge of content and students to anticipate gaps and errors. Finally, teachers need to have mathematical knowledge of the design of instruction, including the order to represent information, mathematical representations and how and when to introduce strategies. For the specific expectation I covered in *Is it Proportional?* I conducted an extensive literature review to gather the appropriate MKT (Ball et al., 2008) about proportional reasoning. This review included how to unpack the topic and terms, the common misconceptions with proportions, the best order to introduce the topics, the problem types, and how to represent the solutions. I have taught Grade 7 mathematics for more than five years, taken an additional qualification course in mathematics and served as a school board mathematics lead. Still, I needed to devote a substantial amount of time and effort to learn the details about the order and progression of proportions.

The discussion around the variety of domains in MKT is based on teaching in a synchronous face-to-face classroom setting (Ball et al., 2008). I would argue that the domains are amplified when creating a video recording because students can replay the information multiple times, and teachers do not get immediate student feedback to gauge student understanding. As such, teachers need to anticipate all gaps in understanding before creating conceptual videos.

Regarding content for conceptual mathematics videos, future research would benefit from focusing on MKT as it significantly impacts video creation. For example, how can the video creation process support teachers' developing stronger MKT? Or how do teachers learn to scaffold / chunk mathematical information into smaller components? How can videos be used at the Faculty of Education or in Teacher professional development to develop stronger MKT? How could a database of conceptual understanding videos support teachers' conceptual understanding and be used for students? This research would provide MKT insight and could also support teachers in developing strategies to create short 6-minute conceptual mathematics videos. Future research might also explore how teachers anticipate student misconceptions and errors. Not only is anticipating student errors a domain of MKT, but it is an essential understanding for teachers creating videos in the absence of a live student audience.

4.3 Verbal Limitations

Much of what research suggests regarding the verbal execution of educational videos, such as using simple language and maintaining a conversational tone, can be easily monitored and implemented by teachers. However, if a teacher needs a script to address the content accurately, they may struggle to maintain a conversational tone. To keep the video in the appropriate time domain, I would recommend teachers create a script that is no longer than 5 minutes long when practising if they wish their completed video to sit within Guo et al.'s (2014) recommended 6-minute mark.

Many of the limitations around the content of quality verbal explanations are directly related to the teachers' depth of Mathematical Knowledge for Teaching (MKT), as explored in Section 4.2. In particular, in the domain of specialized content knowledge (Ball, et al., 2008), teachers possess sufficient knowledge of patterns in student errors and are able to use appropriate mathematics terminology and vocabulary. As such, potential future research questions could include: How can we (i.e., pre-service teacher educators, professional development providers, mathematics school board leads / administration) leverage videos to enhance teachers' content knowledge in mathematics? This research question could be two-fold, focussing on (a) using conceptual videos created by experts to support teachers' content knowledge as LeSage (2012) did for prospective teachers, or (b) having teachers/pre-service teachers create videos as a tool to refine their content knowledge as Moreno et al. (2020) did with Master's of Education students.

Finally, when creating conceptual videos, it is essential that teachers limit their assumptions about students' prior and existing content knowledge to provide appropriate verbal instructions. Teachers require strong MKT to prepare videos without instantaneous student feedback. As such, it would also be interesting to investigate how students perceive the information teachers say in class versus the information teachers provide on a video. Do students perceive information teachers deem important enough to record as more critical than information shared in class? How can a teacher leverage this perception in their selection of video content?

4.4 Visual Limitations

Unlike the similarity of verbal explanations used in the classroom and video, the visual capabilities of a video have the potential to outperform traditional chalkboard/whiteboard visual models used in the classroom. Despite the time and effort required to create quality video visuals, they do hold the potential to bring complex visual concepts, like stretcher and shrinker problems, to life for students. As such, another avenue for future research would be exploring dynamic mathematical models. For example, are some dynamic mathematical models better suited for video presentation than on a static chalkboard or whiteboard? Do students need to interact with a dynamic model on their devices, or can they improve their understanding by watching a video recording of the model? What are the affordances and constraints of using dynamic mathematical models in video recordings? Are specific dynamic mathematical models better suited to video demonstrations?

More research is needed on video visuals as they can be quite labour-intensive to create. For example, I created the visuals using Canva in a slide-deck format in my videos. To add a new number to a chart, I had to duplicate the original chart slide, then add in the new number. I had to click to advance the slide deck when I was discussing the following number in the pattern. There were occasional lags. As such, my audio addressed the new number before it appeared on the screen.

Prepping signalling and cueing can also be difficult, depending on the tool choice. As technology continues to advance, some of these concerns will dissipate. However, as research on video creation develops, researchers must keep the technology classroom teachers have access to in mind. Access to technology will rely not only on the hardware available to teachers but also on the software that is free, accessible and approved for use in schools.

4.5 Conclusions and Reflections

While putting theory into practice, I discovered a few things, some anticipated and some unexpected. First, although I possess a good understanding of the research recommendations specific to video creation and comprehensive content and pedagogical content knowledge in mathematics, video creation was complex. I struggled to balance the proportional reasoning pedagogy, problem types, content choices, script planning, technology requirements, and visual elements. Second, research on both educational videos in middle school mathematics and conceptual mathematics videos, in general, is limited. For example, much of the research I found on mathematics education videos was conducted at the post-secondary level (Kay & Kletskin, 2012; Kay, 2014; Guy & Marquis, 2016; Mykhnenko, 2016; Schoenfeld, 2017; Dinsmore, 2019; Shoufan, 2019) or in laboratory settings (Mayer, 1999; 2001; 2003).

Consequently, applying this research into practice was, at times, challenging. Creating conceptual mathematics videos for middle school students is different from creating mathematics videos for post-secondary students, leading to issues and limitations with the existing research. Younger students have less content knowledge to draw from when learning about mathematical concepts and less experience with independent learning, such as watching educational videos independently. We also know little about how middle school-aged students perceive and use educational videos. As I had proposed in my original thesis research, I believe research needs to examine student perceptions of conceptual videos as a learning tool in mathematics. Until the research community better understands how middle school students perceive educational videos, we will not have an accurate understanding of how to leverage their potential. In the interim, I believe that educational videos can be used to explore the conceptual understanding of mathematical topics like proportional reasoning. However, to create the videos, teachers need quite a vast knowledge of video creation and the pedagogy and content of proportional reasoning.

Despite the challenges and limitations described in this chapter, I believe there is great potential to use and create quality conceptual mathematics videos to support students' learning and understanding. Previous research supports this potential, highlighting the following benefits of video: they can leverage both the auditory and visual channels, help improve study habits, and increase student learning performance. Moreover, unlike teachers, videos can be available to students at a time and place that best suit their learning. The Closed Captioning feature in videos also has the potential to provide better access to students who are English Language Learning or deaf or hard of hearing. Finally, the dynamic visual capabilities of videos have the potential to outperform the static visuals commonplace in traditional chalkboard/whiteboard classrooms. Dynamic mathematical models can provide more visual-based conceptual understanding opportunities for students.

I believe that the creation and use of conceptual mathematics videos will continue to increase as the technology becomes more accessible and manageable for teachers. However, more research and interventions are required to determine how to best support classroom teachers in creating quality conceptual videos. For example, interventions need to focus on developing teachers' conceptual and pedagogical knowledge before exploring technology knowledge specific to video creation. There is also an opportunity to explore how videos can support student and teacher understanding of conceptual mathematical topics.

On a more personal note, this research has allowed me to expand my knowledge and skill set. When I created the initial videos for the Business students, I was confident in my understanding of the mathematical content covered. I thought sharing that knowledge via video would not be much more complicated than teaching the lesson in the classroom. Reflecting on those early videos, I see how much of my pedagogical and technical knowledge (Mishra & Koehler, 2006) was lacking. My first videos had extraneous music and graphics and lacked visual prompts like cueing and signalling. I was striving to make what I believed to be a professional-looking video, with all the extras, without understanding the more critical aspects of quality educational videos. As I began my research on educational videos and discovered the many factors I had not considered, I saw a real learning opportunity. As a classroom teacher or a school board leader, I rarely have the chance to dive deeply into a topic, to practice and improve my teaching. With my newfound knowledge of video creation and a better appreciation for the effort, knowledge and time required to make quality videos, I look forward to leveraging videos as I return to the classroom in 2023. I believe there is a real potential to document sound conceptual instruction and share this knowledge with students in my classroom, as well as fellow teachers. Perhaps my own YouTube channel is on the horizon? Education video creation is not an easy task, but one I believe is worth doing.

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