Multi-Antenna Relay Network Beamforming Design for Multiuser Peer-to-Peer Communications

by

Jiangwei Chen

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Supervisor: Dr. Min Dong

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Abstract

In this work, we consider a multi-user peer-to-peer relay network with multiple multi-antenna relays which employ amplify-and-forward relaying protocol. Assuming distributed relay beamforming strategy, we investigate the design of each relay processing matrix to minimize the per-antenna relay power usage for given users' Signal-to-Noise Ratio (SNR) targets. As the problem is NP-hard, we develop an approximate solution through the Lagrange dual domain. Through a sequence of transformations, we obtain a semi-closed form solution which can be determined by solving an efficient semi-definite programming problem. We also consider the semi-definite relaxation (SDR) approach. Compared with this SDR approach, the proposed solution has significantly lower computational complexity. The benefit of such a solution is apparent when the optimal solution can be obtained by both approaches. When the solution is suboptimal, simulations show that the SDR approach has better performance. Thus, we propose a combined method of the two approaches to trade-off performance and complexity. Simulations showed the effectiveness of such a combined method. In the next step, we change the previous objective and constraints to turn the optimization problem into a total power minimization problem for the relay network. We use an approximation by solving this problem in the Lagrange dual domain, and we finally obtain a semi-closed form solution through the dual approach. The use of the SDR approach to solve this problem is also discussed. After analysis, we find the

two methods have an advantage over different aspects, thus we propose a combined method for this problem. We eventually compare the two combined methods to see the performance difference in the per-antenna power case and the total relay power case, and discuss reasons for this difference.

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Chapter 1

Introduction

With the boost of wireless communication techniques, wireless technology is now applied everywhere: mobile phones, bluetooth, relay and Wi-Fi are examples. There are currently billions of mobile phone users world-wide. However, with more people enjoying the convenience of wireless facilities, more technical challenges are appearing: it is becoming more important and urgent to improve the quality of signals, shorten transmission delay, reduce costs and eventually build a more reliable wireless system. Thus, a increasing number of wireless techniques have been developed to cater to these requirements, which also shows huge demand and a promising future for the wireless industry. Thanks to these new technologies, everyone enjoys more convenience from mobile facilities, e.g., our cell phones now use 4G instead of 3G, we are experiencing higher data speed and shorter delays. The concept of 5G is now also under discussion to be the key technology in the next ten years.

Signal fading is the major issue that can destructively affect the reliability of communication and lower the data rate. Thus, performance of wireless communication is much worse than that of wired communication. For peer-to-peer communication, employing multiple antennas at the transmitter and receiver can provide a better performance as it allows signals to travel through independent fading paths. In a multi-antenna scenario, diversity techniques such as receive/transmit beamforming can be applied to increase the robustness of the system by extending the transmission range and mitigating inter-user interference. Moreover, some emerging techniques such as real-time radio broadcasting also require network beamforming.

Despite the benefits of employing multiple antennas, sometimes it is impractical to equip a mobile terminal with multiple antennas due to its size and power limitation. To overcome this practical difficulty, an alternative is to add a relay network to employ beamforming, especially when the direct link between the source and the destination does not exist. There are different schemes that can be applied at the relay network, such as amplify-and-forward (AF), code-and-forward (CF) or other schemes. With the help of the relay network, the signal can be transmitted to a longer distance and quality of service (QoS) for the user can be improved, and also outage probability can be mitigated. Moreover, with multiple relays, we can adopt diversity technology to either improve the data rate or enhance the signal quality.

1.1 Relay Network

If a user receives the signal directly from the transmitter, the received signal can suffer from severe fading. The fading condition is related to the distance between the source and the destination. With longer transmission distances, the signal at the receiver may be too weak. In this case, the user will have a very low SNR.

To solve this problem, the system can introduce a cooperative relay network to improve the SNR at the user end. A relay network is shown in Fig. 1.1. A single source-destination pair with one relay is studied in [1–5]. Later researchers consider multiple relays [6–9]. Current work has extended to multiple source-destination pairs

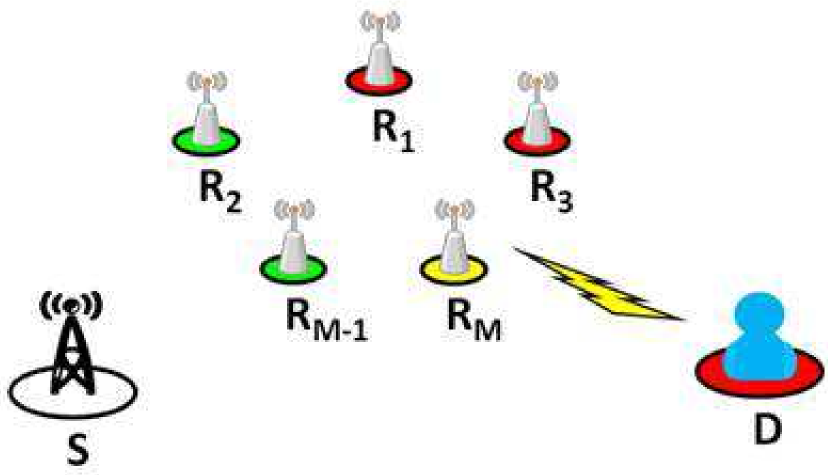

Figure 1.1: Peer to Peer Communication with Relay Network.

[10–12]. With the relay network assisting signal transmission, the quality of the signal at the receiver can be improved. However, each relay not only contributes to the desired signal component at the destination, it also forwards to the user interference from other sources as well as noise at the relay. To mitigate the effect of interference, we can adopt different strategies at the relay. Here, we give a brief introduction to two typical strategies: AF and decode and forward (DF). For AF relaying, the relay will amplify the received signal and forward it to the destination. For DF relaying, the relay will first decode the received signal, then send it to the destination.

The AF and DF strategies are adopted in different scenarios. AF is very easy to apply, but its performance can be worse than that of DF. On the other hand, DF can produce better signals at the cost of more complex hardware, as it needs a decoding block at the relay. In this work, we focus on AF relaying protocol for our problem.

1.2 Beamforming Technique

Beamforming is a very powerful technique to receive, transmit, or relay signals with the existence of interference and noise. Beamforming is a classic but continuously

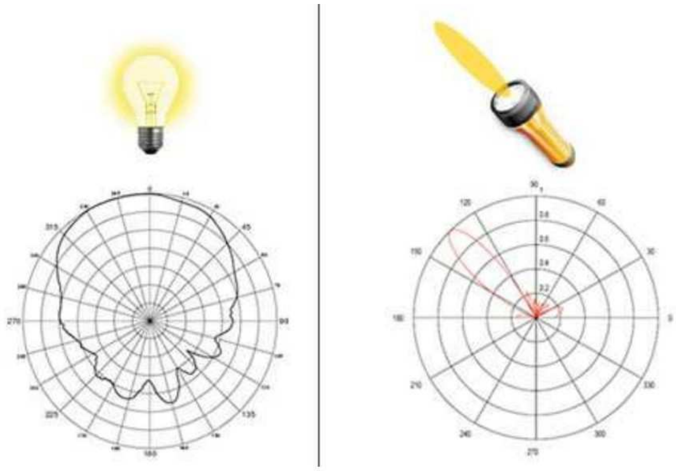

Figure 1.2: Illustration of beamforming

developing field that has significant theoretical research and practical applications, such as radar, communication, radio and other fields. In the last decade, there has also been interest in beamforming applied to wireless communications, where multi-antenna techniques have emerged as one of the key technologies to accommodate the rapid increase of mobile phone users and the urgent demands for high data-rate transmission.

An example of beamforming is given in Fig. 1.2. The left part in this figure describes the condition that the signal is transmitted in all directions, similarly as a bulb does. However, the right part of the figure shows if the beamforming technique is applied, it will focus all the energy into one direction, thus result in stronger signal compared with the case in the left part. The beamformer output SNR is maximized by means of enhancing the desired signal and rejecting the interferers through concentrating all the energy to the channel direction.

Here, we give an example of a transmit beamforming design problem. We consider a single base station equipped with N antennas, transmitting a data stream to a single user with a single antenna. The signal transmitted at base station is given as

$$\mathbf{x}(t) = \mathbf{w}s(t)$$

where $s(t)$ and $\mathbf{w}$ are the transmitted signal for the user and the beamforming vector for this user.

The received signal for this user is given as

$$y(t) = \mathbf{h}^H(t)\mathbf{x}(t) + n(t)$$

where $\mathbf{h}^H$ is the channel vector of the user and $n(t)$ is noise with power σ^2 .

The design of beamformer $\mathbf{w}$ is obtained by solving the following optimization problem:

$$\begin{aligned} &\min_{\{\mathbf{w}\}} \|\mathbf{w}\|^2 \\ \text{subject to } &\frac{\|\mathbf{w}^H\mathbf{h}\|^2}{\sigma^2} \geq \gamma, \end{aligned}$$

where γ denotes the SINR target for the user.

Such a beamforming design can be used at the transmitter and receiver to obtain diversity gain. Under this scheme, all the sources transmit the same symbol, and diversity gain is obtained by coherently combining signal paths. If the channel state information is perfectly known at the transmitter, the maximum diversity gain can be reached. Using beamforming will make the channel more robust to environment disturbance and the SNR at the receiver will be improved. However, using all the channels to transmit the same signal will not improve the capacity. Beamforming and other diversity techniques are discussed in [13].

Relay Centralized Beamforming: Relay centralized beamforming is used in a specific kind of relay network that constitutes a single relay with multiple antennas. When the relay forwards signals to the destination, multiple antennas will send signals through independent paths, and at the receiver all paths are coherently added to maximize the SNR. If the total number of antennas in the relay network is fixed, centralized relay beamforming always has a better or equivalent performance compared to distributed relay beamforming.

Relay Distributed Beamforming: Relay distributed beamforming is adopted when multiple independent antennas transmit signals among multiple relays. There can be some performance loss compared with centralized relay beamforming. This is due to the fact that distributed relay beamforming can only process the signal by each relay. A distributed relay network can also employ centralized beamforming with global CSI known at relays.

1.3 Relay Power Minimization

Cooperative relaying is one of the key techniques to support dynamic ad-hoc networking for next generation wireless systems, thus an efficient physical layer design of cooperative relaying to support such simultaneous transmissions is important. The relay works as a transceiver in this system, therefore it is important to design an energy-efficient scheme for relay network power usage.

At the destination, there is typically a SNR requirement for the received signal to ensure the signal quality is acceptable, so the goal of minimizing the relay power is to reduce the power usage at the relay while satisfying the SNR target. The relay power minimization can be generally classified into three categories: total power minimization studied in [10, 11, 14] , joint power minimization of relay and source researched in [4, 8, 12], and per antenna power minimization considered in [5, 15, 16].

- Total relay power minimization: It targets at reducing the total relay power usage.

- Joint power minimization of relay and source: It considers both source and relay power allocation.

- Per antenna relay power minimization: The goal is to minimize the antenna power usage for all relays.

Among the current research, many efforts have focused on the total power minimization problem and the joint power minimization problem. For the total relay power minimization problem, although the original problem is not convex, after standard relaxation, it can be turned into a convex problem. For the joint source and relay power minimization problem, the optimization problem is convex, so it is rather easy to solve. However, the per antenna relay power minimization problem considers a more practical scenario where each relay antenna has its own power budget. This corresponds to the reality of the individual RF front-end power amplifier at each antenna. Such a per-antenna power control makes the design problem significantly more challenging than the total relay power constraint considered in most of the existing works.

1.4 Channel State Information

To achieve full diversity in relay beamforming, global channel state information (CSI) is required at the relays. CSI represents the knowledge of the channel, and gives information on how a signal propagating from a transmitter to a receiver is affected. The CSI can help relays decide the beamforming matrix for signal forwarding, so it is vital to get a precise CSI at relays.

At the receiver, CSI is obtained through channel estimation technology, and the receiver usually has feedback to the transmitter. Depending on different estimation schemes, the CSI at the the transmitter is generally classified into two cases:

- Instantaneous CSI : Instantaneous CSI means that the channel conditions are perfectly known at the transmitter. Under this condition, optimal relay beamforming can be implemented.

- Statistical CSI: Statistical CSI means statistical characterization of the channel is known (e.g., 2nd order). This estimation can give the distribution of the channel, but it cannot describe the instantaneous channel. Employing beamforming with partial CSI will have some performance loss compared to the case of instantaneous CSI.

1.5 Literature Survey: Relay Power Minimization

1.5.1 Single Source Destination Pair Beamforming

Many existing works focus on the relay processing design in a single source-destination pair setting, either with a multi-antenna relay [1–5] or with multiple single-antenna relays forming distributed beamforming [6–9].

Total Power Minimization

Relay beamforming weights as well as the transceiver transmit powers are designed in [8], and an achievable beamforming rate region is characterized under a constraint on the total network transmit power consumption. Authors of [17] aim at minimizing total relay transmit power with a SNR constraint using imperfect CSI knowledge. It provides a robust design that can guarantee the SNR at the receiver with imperfect CSI. This is a worst-case design and the optimization problem is convex. The total relay power minimization is also considered in [9].

Per Relay Power Constraint

Optimal power usage for distributed beamforming is also studied in [18] and [6]. These two papers describe the distributed beamforming for a single source-destination pair. These two papers also discuss the optimal power usage at the source and at the relay. This shows that the source will always use full power for transmission, and the relay power usage is a fraction value between zero and maximum power budget. The relay power usage depends on the channel condition of its own and all other channels. This conclusion contradicts the natural assumption that relay will just use full power or shut down. Relay power allocation algorithms for non-coherent and coherent AF relay networks are developed in [9]. The goal is to minimize the total relay transmission power under individual relay power constraints, while satisfying a QoS requirement. This study also introduces a robust method to optimize the power allocation in the absence of global CSI. The proposed method outperforms the naive scheme that always uses maximum transmission power at each relay.

Per Antenna Power Minimization

AF multi-antenna relaying between a single pair of source and destination is considered in [5]. Assuming relay per-antenna power constraint, the relay processing matrix is designed to minimize the maximum antenna power under the receiver SNR target. By solving the problem through the dual approach, the author obtains a semi-closed form solution.

1.5.2 Multiuser Peer-to-Peer Beamforming

Total Power Minimization

For Multiuser Peer-to-Peer (MUP2P) relay networks, distributed relay beamforming with multiple single-antenna AF relays is studied in [10–12] for total power minimization among relays or among all network nodes. The beamforming coefficient for each relay is designed in [10] through the minimization of total relay power consumption, while the signal-to-interference-plus-noise ratio (SINR) at the destinations is guaranteed to be above a certain level. As this optimization problem is not convex, the authors use semidefinite relaxation to convert this problem into a semidefinite programming (SDP). Its simulation results show that in their solution, the relays consume less power than other orthogonal multiplexing schemes. A similar problem is also studied in [11]. The main difference is that the relaxation method is different. After approximation, the problem is in a convex second-order cone programming form, and this method has a much lower complexity than the semidefinite relaxation used in [10]. Distributed beamforming is also studied in [14]. In this work, the system model is set to be K user pairs and R relays, with Rx and Tx equipped with a single antenna, and rth relay is equipped with M_r antennas. The power for each symbol s_i is p_i. Its goal is to minimize the total relay power under the Rx SNR constraint. This paper is a general discussion for the total power minimization. The antenna number and transmission power can vary for different Tx and relay, and this paper also uses SDP and randomization.

In addition to the AF relay scheme, distributed beamforming using Filter and Forward (FF) is also studied in [19]. Using finite impulse response (FIR) filters at the relay, and with the CSI being available at the receiver, the relay transmit power is minimized subject to the destination QoS constraint. This problem has a closed form solution. It also compares FF with AF distributed beamforming techniques regarding feasibility and relay power consumption.

Apart from total relay power minimization, the authors in [12] consider the joint optimization of the source power allocation and the relay beamforming weights in distributed MUP2P relay networks. The authors minimize the total power transmitted from all sources and relays while guaranteeing the QoS at the receivers. The paper proposes an iterative feasibility search algorithm (IFSA) to extract a solution for the problem.

A setting with M source-destination pairs and N relay nodes is considered in [20]. With perfect CSI at the relay, the goal is to find optimal beamforming weights subject to the receiver SNR targets. Meanwhile, total relay power is minimized. There are two cases considered in this work: no power control at the relay and the per relay power constraint. The problem can be formulated as a non-convex quadratically constrained quadratic program (QCQP). Through the SDR approach and the Lagrangian duality relaxations, the problem can be solved by convex programming. A later extended work [21] minimizes the total relay power with guaranteed QoS, and the main difference from the previous work is that it assumes the orthogonal channels in the system model. The work provides an iterative algorithm to solve the problem instead of using convex second-order conic programs (SOCPs).

With multiple source-destination pairs assisted by multiple multi-antenna relays, relay beamforming matrices are jointly designed in [22] by minimizing the received power at all the destination nodes, while preserving the desired signal at each destination. This work provides two algorithms to computer beamforming matrices in a processing center and locally. Designing the beamforming matrices of the cooperating relays, by minimizing both the noise received at each destination node and the interference caused by the sources not targeting this node, is studied in [23]. The problem is convex and can be directly solved.

Per Relay Power Constraint

Apart from total power minimization, research is also conducted in per relay power control. The problem of beamforming (BF) design for orthogonal frequency division multiplexing (OFDM) based relay networks over frequency-selective channels is addressed in [24]. The BF vectors are designed by maximizing the minimum SNR over all subcarriers at the destination, both under the total power constraint (TPC) and the per-relay power constraint (PPC). A secrecy scheme to maximize the secrecy sum rate of the two terminals is studied in [25], subject to the per node power constraint for a two-way relay network. The optimal per relay power control, via maximizing the smaller SNR of the two end users in the network, is studied in [26].

Per Antenna Power Minimization

Due to the inherent complexity of the per antenna power problem, numerical methods were proposed to obtain approximate solutions. Most existing designs focus on the total power constraint, either among relay antennas, or across relays, which lead to more analytically tractable problems. However, these results or techniques cannot be applied to the problem where the per-antenna/per-node power constraints are imposed. Under such a per-antenna power budget, the relay processing design was recently studied for a single source-destination pair [5] and for a multicasting scenario [15], both with a single multi-antenna relay. The work [15] is an extension of the previous work [5], and the setting changes from a single source-destination pair to multiple pairs. This paper designs the relay processing matrix to minimize the maximum individual antenna power for a fixed receiver SNR target. By using the dual approach to approximate the original problem, a semi-closed form solution is obtained by solving a semi-definite programming formulation. This approach has much lower complexity than that of the SDR approach, thus the dual approach can serve as an alternative to the SDR method.

The per antenna problem is further studied in [16]. This paper studies the difference in nature between relay distributed beamforming and relay centralized beamforming, and investigates when distributed beamforming will have less performance loss. This paper shows that for noiseless relay, distributed relay beamforming has no loss, but in noisy relay, loss will incur. However, this work assumes single multi-antenna relaying or multiple single-antenna relaying for centralized and distributed beamforming. This can be further extended to multiple relays with multi-antenna.

1.5.3 Optimization Approach

SDR Approach

For the total power minimization problem, there exists the traditional SDR approach to solve this problem, either for single source-destination or multiple pairs. The SDR approach is adopted in [10] to study the total power minimization among multiple users. This approach expresses the problem in the format of a SDP, and this kind of problem can be solved using standard SDP software, such as SeDuMi [27]. However, the SDR approach is not computationally efficient, as it first requires running SDP several times for a feasibility check, then turns to solve the problem itself. After using the SDR approach, the solution is not always rank-one, so for those non rank-one cases, some randomization techniques need to be used to extract a rank-one solution. These randomization techniques are discussed in [28].

The SDR approach is used in [29] and its extended work [10] to solve their respective problems. The authors design a distributed beamformer such that the total relay transmit power among all single-antenna relays is minimized subject to the receivers' SNR targets. The signal transmission has two stages. In the first stage, all sources transmit the signals to the relays. In the second stage, all relays forward the signals to all destinations. Using the SDR approach, this problem can be solved by SDP software and randomization techniques. In work [30], the sources, destinations and relays are equipped with multiple antennas, and semidefinite relaxation is applied to minimize the total source and relay transmit power, such that a minimum SNR at the receiver is guaranteed.

Dual Approach

The dual approach is also adopted in relay power minimization problems. This appraoch solves the original non-convex problem in the Lagrange dual domain with approximation. This approach has been studied in a unicast scenario [5] and a multicast scenario [15]. These two papers studied the individual power budget for a relay network under QoS constraints. The dual approach is also adopted in [16] to compare centralized and distributed beamforming.

The dual approach has a great advantage in computation complexity compared to the SDR approach. Regarding performance, the dual method and the SDR approach can obtain an optimal solution at the same time. For sub-optimal solutions, the SDR approach has better performance than the dual approach.

1.6 Literature Survey: SNR and Rate Maximization

There are also works studying the SNR maximization with power constraints for a single source-destination pair. The author of [31] maximizes the SNR at the destination with the total power constraint under exploited CSI. A near-optimal numerical solution for the collaborative-relay beamforming (CRBF) weights is first found from solving an unconstrained multi-variant minimization problem. After subspace averaging, a semi-closed form solution can be obtained. Relay weights are optimized in [32] to maximize the SNR at the receiver with individual and total power constraints. This work provides the solution that obtains full diversity in the MISO system, and also develops an algorithm that allows each individual relay to independently find its weight. A review of convex optimization approaches is given in [33] to solve beamforming problems. The author analyzes different beamforming types: transmit beamforming and multicast beamforming. The author then discusses the design of beamformers. The authors of [4] design a beamforming vector through maximizing the receiver SNR. The authors provide a rank beamforming approach to obtain a closed-form solution. They also compare their approach with separable receiver and transmitter beamforming.

Optimal distributed beamforming design to jointly maximize the SNR margin in a multiuser multi-relay network is studied in [34]. In this work, the total relay power constraint and per-relay power constraints are considered. Unlike the bisection method, the author proposes a fast converging iterative algorithms to directly solve the two problems. A new approach is proposed in [35] for relay beamforming where a FF relay scheme is adopted at the relays to combat channel distortion, and a close-form solution is obtained to maximize the received SINR. Some attention is also given to throughput in the relay network. A AF relay network is used in [36] to find an optimal gain allocation which results in a coherent combining of all signal contributions at the destination and maximizes the instantaneous throughput of the link. Perfect CSI at the relay is assumed in [37], and the direct link between the source and the destination exists. This paper designs the optimal beamforming weight and the source power allocation that maximize mutual information. Decentralized beamformer design through sum-rate maximization for two-way relay networks is studied in [38]. With the total transmit power constraint, the sum-rate maximization is equivalent to an SNR balancing approach. The problem is converted to maximize the smallest SNR at the receiver. In an extended work [8], sum-rate is maximized to obtain the jointly optimal relay beamforming weights and transceiver transmit powers. Rate maximization is also discussed in [38]. This paper aims to design the beamformer for a relay-aided multiuser multi-antenna cognitive radio (CR) network.

1.7 Literature Survey: Limited CSI Feedback

Optimal beamforming requires perfect CSI at the transmitter. However, the instantaneous CSI is not always available for the transmitter. Thus, researchers also study the beamforming design with partial CSI. The authors of [39] study the performance gap between unlimited and limited feedback for AF relay network beamforming. Research is also conducted to study how the feedback will affect receiver performance. Generalized Lloyd algorithm (GLA) is used in [40] to design the quantizer of the feedback information and the bit error rate (BER) performance of the system is optimized. Two scalable perturbation schemes are considered in [41] for adaptive relay beamforming, and practical implementation aspects are addressed. The result shows the performance is close to optimum performance in time-varying environments. The above papers all consider AF relaying, and efforts are also made in other schemes. Beamforming with limited CSI in regenerative cooperative networks where DF relays are deployed is investigated in [42].

Some researchers focus on limited CSI to be 2nd order statistics. Distributed beamforming with 2nd order statistics available at relays is studied in [7]. This work designs two approaches to minimize the total transmit power with a QoS constraint, and maximizes the receiver SNR, subject to the total power constraint and the per relay power constraint. Analysis for 2nd order statistics is also mentioned in [43]. This paper tries to maximize the SNR at the receiver with source and relay total power constraints.

1.8 Literature Survey: Secure Beamforming Design

Some research is performed in secure transmission. A AF multiple-input multiple-output (MIMO) relay network, composed of a single relay and a single Tx-Rx pair, is considered in [44], where transmit beamforming is adopted both at the source and at the relay. This paper considers two ways to transmit confidential information from the source to the destination: non-cooperative secure beamforming and cooperative secure beamforming. Secure relay beamforming for the two-way relay is studied in [45]. Relay beamforming designs under total and individual relay power constraints are studied in [46] with the goal of maximizing secrecy rates. The authors of [47] propose two sub-optimal null space beamforming schemes to optimize the performance of the cognitive relay beamforming system.

1.9 Motivation

Most of the current research on distributed beamforming relay network design focus on single or multiple source-destination pairs with a single relay, or with multiple single-antenna relays, and they usually try to minimize the total power of the relay network. To study the most generalized distributed relay beamforming, we need to consider MUP2P communication with multiple multi-antenna relaying. The existing research has not thoroughly studied this aspect. Moreover, the current research uses the SDR method as a standard way to extract a solution for the above problem. However, the SDR approach is very inefficient in computation. In addition, most of the papers consider total relay power control, and individual power control is not sufficiently discussed. Considering these factors, we proposed a dual approach to solve the per antenna power minimization for MUP2P communication. The novelty in our research compared to previous works is that firstly we use a different method to solve the problem, and the dual approach has a very close performance to the SDR approach, while the dual approach's complexity has a great advantage over that of the SDR approach. Secondly, unlike total power control other research which focuses on, we study individual power control in our research, and this makes the problem significantly more intractable, as the problem is no longer convex. We eventually use a combined method to trade-off two approaches and the simulation has shown effectiveness. In the last part, we also study total power minimization using the dual and SDR approaches.

1.10 Summary of Results

In this thesis, we study the design of AF multi-antenna relaying in MUP2P relay networks, where multiple source-destination pairs communicate with the assistance of multiple relays. We consider using a distributed relay beamforming technique among relays, each equipped with multiple antennas, to maximize the transmission power gain for data forwarding. We investigate the design of each relay processing matrix to minimize the per-antenna relay power usage for given users' SNR targets. As the problem is NP-hard, we developed an approximate solution through the Lagrange dual domain. After solving the per antenna power minimization, we again use the dual approach to solve the total relay power minimization problem.

In Chapter 2, we first build a system model for the per antenna power minimization, then we solve the problem in the Lagrange dual domain. After a sequence of transformations, we obtain a semi-closed form solution, which can be determined by solving an efficient semi-definite programming problem. After solving this problem with the dual approach, we study per antenna power minimization through the SDR approach. We write all constraints in trace form. After relaxing the rank constraint, the problem can be written in a SDP format. We then compare the two approaches: the dual problem has much lower computational complexity, and when the solution is optimal, the dual approach can obtain the same result with the SDR approach. While the solution is not optimal, the SDR approach has better performance. The simulation results are given to show their performance and complexity. After the comparison between the dual and the SDR approaches, we introduce a combined method to have a trade-off between performance and complexity. We first obtain the solution from the dual approach, and then compare it with a predefined threshold to see if the performance is satisfactory. If so, we will adopt the dual solution as the final solution, otherwise we will run the SDR approach to obtain a more accurate solution. This combined method has a performance between the two approaches, and its performance can be adjusted by modifying the threshold. Some simulation figures are given to show the effectiveness of this combined method.

In Chapter 3, we study the total power minimization with the dual and the SDR approaches. With a different optimization objective and constraints, we first write the problem in Lagrange dual domain, then propose a solution for the dual approach. Next, we study the problem through the SDR approach. After comparing the solutions to the two methods, we introduce the combined method to maintain a balance between complexity and performance. With these three methods, we compare the performance of the three methods to see the actual effect of the trade-off method. At the end of this chapter, we compare the performance of two optimization problems we have studied in the thesis: the per antenna power problem and the total power problem.

1.11 Thesis Organization

In Chapter 2, we build a system model for the per antenna power minimization problem. The dual approach is then derived to solve this problem. The performance of the dual approach is analyzed, then analyze the SDR approach for the per antenna power problem. The two methods are compared, and a combined method is introduced for the trade-off purpose. Chapter 3 uses the three methods in the previous chapter to analyze the total power minimization problem. The performances of three methods are also compared. Finally, this chapter compares the performance of the two power minimization problems. Chapter 4 gives the final conclusion.

1.12 Notation

Notations: Kronecker product is denoted as $\otimes$. Hermitian and transpose are denoted as $(\cdot)^H$ and $(\cdot)^T$, respectively. Conjugate is denoted as $(\cdot)^*$. $(\cdot)^\dagger$ is the pseudo-inverse of a matrix. The semi-definite matrix $\mathbf{A}$ is denoted as $\mathbf{A} \succeq 0$. The vectorization vec($\mathbf{A}$) vectorize matrix $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_N]$ to a vector$[\mathbf{a}_1^T, \cdots, \mathbf{a}_N^T]^T$

Chapter 2

MUP2P under Per Antenna Relay Power Minimization

2.1 System Model

We consider a system with K source-destination pairs communicating through M AF relays. All sources and destinations are equipped with a single antenna, as shown in Fig. 2.1. Each relay m is equipped with N antennas, and processes the incoming signals using the $N \times N$ processing matrix $\mathbf{W}_m$ before forwarding them to the destinations. The $N \times 1$ channel vectors between source k and relay m and between relay m and destination k are denoted as $\mathbf{h}_{1,km}$ and $\mathbf{h}_{2,mk}$, respectively.

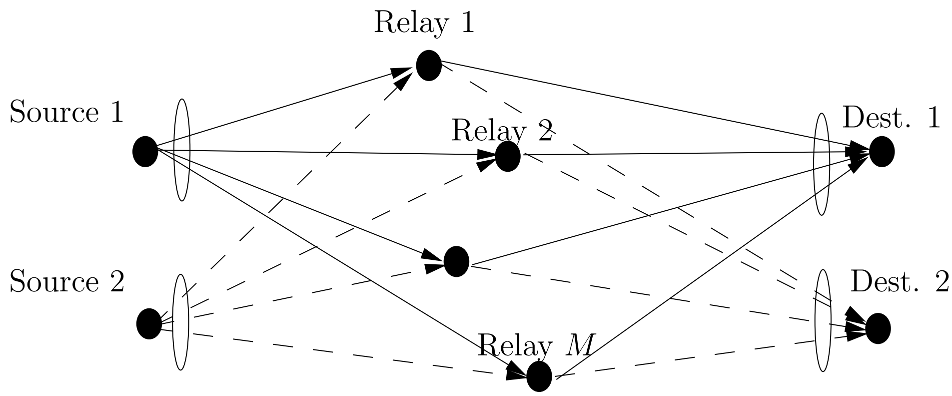

Figure 2.1: The multiuser peer-to-peer relay network model

The received signal at destination k, denoted as $y_{d,k}$, is given by

$$\begin{aligned} y_{d,k} &= \sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_m \left(\sum_{l=1}^{K} \mathbf{h}_{1,lm} \sqrt{P_0} s_l + \mathbf{n}_{r,m} \right) + n_{d,k} \\ &= \sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_m \mathbf{h}_{1,km} \sqrt{P_0} s_k \\ &\quad + \sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_m \left(\sum_{l=1, l \neq k}^{K} \mathbf{h}_{1,lm} \sqrt{P_0} s_l \right) \\ &\quad + \sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_m \mathbf{n}_{r,m} + n_{d,k} \end{aligned} \tag{2.1}$$

where s_k is the signal sent from source k with $\mathrm{E}|s_k|^2 = 1$ and $\mathrm{E}(s_l s_k) = 0$, $\forall l \neq k$, P_0 is the transmit power, $\mathbf{n}_{r,m}$ is the $N \times 1$ complex AWGN vector at relay m with covariance $\sigma_{r,m}^2 \mathbf{I}$ and is independent from that of the other relays, and $n_{d,k}$ is the complex AWGN at destination k with variance $\sigma_{d,k}^2$. The received signal $y_{d,k}$ consists of the desired signal s_k, the interference from other sources s_l, $l \neq k$, and the amplified noises from the relays and the receiver noise at the destination.

From (2.1), we will derive the SNR expression at kth user.

From (2.1), the nominator of the SNR expression (the desired signal) is given as

$$\begin{aligned}
&\mathrm{E}\left[(\sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} \mathbf{h}_{1,km} \sqrt{P_0} s_k)(\sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} \mathbf{h}_{1,km} \sqrt{P_0} s_k)^{H} \right] \\
&= P_0 \sum_{m=1}^{M} \left\| \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} \mathbf{h}_{1,km} \right\|^{2}. \qquad (2.2)
\end{aligned}$$

The denominator of the SNR expression consists of two parts:

The interference from other sources can be written as

$$\begin{aligned}
&\mathrm{E}\left[(\sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} (\sum_{l \neq k} \mathbf{h}_{1,lm} \sqrt{P_0} s_l))(\sum_{m'=1}^{M} \mathbf{h}_{2,m'k}^{T} \mathbf{W}_{m'} (\sum_{l \neq k} \mathbf{h}_{1,lm'} \sqrt{P_0} s_l))^{H} \right] \\
&= \sum_{m=1}^{M} \sum_{m'=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} \left(\sum_{l=1, l \neq k}^{K} \mathbf{h}_{1,lm} \mathbf{h}_{1,lm'}^{H} \right) \mathbf{W}_{m'}^{H} \mathbf{h}_{2,m'k}^{*} \\
&= \mathbf{I}_{\text{int}} \qquad (2.3)
\end{aligned}$$

where

$$\mathbf{I}_{\text{int}} \triangleq \sum_{m=1}^{M} \sum_{m'=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} \left(\sum_{l=1, l \neq k}^{K} \mathbf{h}_{1,lm} \mathbf{h}_{1,lm'}^{H} \right) \mathbf{W}_{m'}^{H} \mathbf{h}_{2,m'k}^{*}. \qquad (2.4)$$

The amplified noise from the relays can be reformulated as

$$\begin{aligned}
&\mathrm{E}((\sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} \mathbf{n}_{r,m})(\sum_{m=1}^{M} \mathbf{h}_{2,mk}^{T} \mathbf{W}_{m} \mathbf{n}_{r,m})^{H}) \\
&= \sum_{m=1}^{M} \sigma_{r,m}^{2} \left\| \mathbf{h}_{2,mk}^{T} \mathbf{W} \right\|^{2}. \qquad (2.5)
\end{aligned}$$

Finally, the SNR expression for user k can be expressed as:

$$\mathrm{SNR}_k = \frac{P_0 \sum_{m=1}^{M} \left| \mathbf{h}_{2,mk}^T \mathbf{W}_m \mathbf{h}_{1,km} \right|^2}{P_0 \mathbf{I}_{\text{int}} + \sum_{m=1}^{M} \sigma_{r,m}^2 \left\| \mathbf{h}_{2,mk}^T \mathbf{W}_m \right\|^2 + \sigma_{d,k}^2}. \tag{2.6}$$

The power usage at antenna i of relay m, denoted as $P_{m,i}$, is given by

$$P_{m,i} = \mathrm{E}\{|[\mathbf{W}_m \mathbf{y}_{r,m}]_i|^2\} = \left[\mathbf{W}_m \left(P_0 \sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H + \sigma_{r,m}^2 \mathbf{I} \right) \mathbf{W}_m^H \right]_{ii}. \tag{2.7}$$

2.2 MUP2P under Per-Antenna Power Minimization

Let γ_k be the SNR target at destination k, our goal is to design $\{\mathbf{W}_m\}$ to minimize the antenna power consumption at each relay, while satisfying the received SNR target at each destination. This problem can be expressed as

$$\min_{\{\mathbf{W}_m\}} \max_{m,i} \; P_{m,i} \tag{2.8}$$

$$\text{subject to } \; \mathrm{SNR}_k \geq \gamma_k, \; \forall k. \tag{2.9}$$

The above min-max power problem is equivalent to the following problem

$$\min_{\{\mathbf{W}_m\}, P_r} \; P_r \tag{2.10}$$

$$\text{subject to } \; P_{m,i} \leq P_r, \forall i, m \tag{2.11}$$

$$\mathrm{SNR}_k \geq \gamma_k, \; \forall k$$

where P_r is the common per-antenna power budget to be minimized.

2.2.1 Vectorization of $\{\mathbf{W}_m\}$

To simplify the derivation, we first rewrite the SNR expression in (2.6) by vectorizing the processing matrices $\{\mathbf{W}_m\}$. Using the property $\text{vec}(\mathbf{ABC}) = (\mathbf{A} \otimes \mathbf{C}^T)\text{vec}(\mathbf{B}^T)$ for matrices $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$, we have

$$\mathbf{h}_{2,mk}^T \mathbf{W}_m \mathbf{h}_{1,km} = (\mathbf{h}_{2,mk} \otimes \mathbf{h}_{1,km})^T \text{vec}\left(\mathbf{W}_m^T\right) = \left(\mathbf{h}_{k(m)}^H \mathbf{w}_m\right)^* \tag{2.12}$$

where $\mathbf{h}_{k(m)} \triangleq \mathbf{h}_{2,mk} \otimes \mathbf{h}_{1,km}$ is the channel vector of the kth source-destination pair through relay m, and $\mathbf{w}_m \triangleq \text{vec}\left(\mathbf{W}_m^H\right)$ is the vectorized relay processing matrix for relay m.

Using (2.12), the desired signal power in the SNR expression can be rewritten as

$$\begin{aligned} & P_0 \sum_{m=1}^{M} |\mathbf{h}_{2,mk}^T \mathbf{W}_m \mathbf{h}_{1,km}|^2 \\ &= P_0 \left(\sum_{m=1}^{M} (\mathbf{h}_{2,mk} \otimes \mathbf{h}_{1,km})^T \text{vec}(\mathbf{W}_m^T) \right) \left(\sum_{m=1}^{M} (\mathbf{h}_{2,mk} \otimes \mathbf{h}_{1,km})^T \text{vec}(\mathbf{W}_m^T) \right)^* \\ &= P_0 \left(\sum_{m=1}^{M} \mathbf{h}_{k(m)}^H \mathbf{w}_m \right)^* \cdot \left(\sum_{m=1}^{M} \mathbf{h}_{k(m)}^H \mathbf{w}_m \right) \\ &= P_0 \left| \mathbf{h}_k^H \mathbf{w} \right|^2 \end{aligned} \tag{2.13}$$

where $\mathbf{h}_k \triangleq [\mathbf{h}_{k(1)}^H \cdots \mathbf{h}_{k(M)}^H]^H$ is the channel vector of the kth source-destination pair through all relays, and $\mathbf{w} \triangleq [\mathbf{w}_1^H, \cdots, \mathbf{w}_M^H]^H$ is the vectorized relay processing matrices for all relays.

The interference from other sources in (2.4) can be reformulated as

$$\begin{aligned}
\mathbf{I}_{\text{int}} &= \sum_{l\neq k}\left(\sum_{m=1}^{M}\mathbf{h}_{2,mk}^{T}\mathbf{W}_{m}\mathbf{h}_{1,lm}\right)\left(\sum_{m'=1}^{M}\left(\mathbf{h}_{2,m'k}^{T}\mathbf{W}_{m'}\mathbf{h}_{1,lm'}\right)^{*}\right)\\
&= P_0\cdot\sum_{l\neq k}\left(\sum_{m=1}^{M}(\mathbf{h}_{2,mk}\otimes\mathbf{h}_{1,lm})^{T}\text{vec}(\mathbf{W}_{m}^{T})\right)\left(\sum_{m'=1}^{M}(\mathbf{h}_{2,m'k}\otimes\mathbf{h}_{1,lm'})^{T}\text{vec}(\mathbf{W}_{m'})^{T}\right)^{*}\\
&= P_0\cdot\sum_{l\neq k}\left(\left(\sum_{m=1}^{M}\mathbf{h}_{k,l(m)}^{H}\mathbf{w}_{m}\right)^{*}\cdot\left(\sum_{m'=1}^{M}\mathbf{h}_{k,l(m')}^{H}\mathbf{w}_{m'}\right)\right) \qquad (2.14)\\
&= P_0\cdot\sum_{l\neq k}(\mathbf{h}_{k,l}^{H}\mathbf{w})^{*}\cdot(\mathbf{h}_{k,l}^{H}\mathbf{w})\\
&= P_0|\sum_{l\neq k}\mathbf{h}_{k,l}^{H}\mathbf{w}|^{2}\\
&= P_0\left\|\mathbf{G}_{k^-}^{H}\mathbf{w}\right\|^{2} \qquad (2.15)
\end{aligned}$$

where to arrive at (2.14), we use the similar derivation as in (2.12), with $\mathbf{h}_{k,l(m)} \triangleq \mathbf{h}_{2,mk}\otimes\mathbf{h}_{1,lm}$ being the interference channel vector of source l to destination k through relay m; in addition, $\mathbf{h}_{k,l} \triangleq [\mathbf{h}_{k,l(1)}^{H}\cdots\mathbf{h}_{k,l(M)}^{H}]^{H}$ is the interference channel vector of source l to destination k through all relays, and $\mathbf{G}_{k^-} \triangleq [\mathbf{h}_{k,1}\cdots\mathbf{h}_{k,k-1},\mathbf{h}_{k,k+1}\cdots\ \mathbf{h}_{k,K}]$ is the $N^2\times(K-1)$ interference channel matrix for the kth source-destination pair.

Similarly, we have

$$\text{vec}\left(\mathbf{h}_{2,mk}^{T}\mathbf{W}_{\mathbf{m}}\right) = (\mathbf{h}_{2,mk}\otimes\mathbf{I})^{T}\text{vec}\left(\mathbf{W}_{m}^{T}\right) = \left((\mathbf{h}_{2,mk}\otimes\mathbf{I})^{H}\mathbf{w}_{m}\right)^{*}. \qquad (2.16)$$

Using (2.16), the amplified noise from the relays can be rewritten as

$$\begin{aligned}
&\sigma_{r,m}^2 \sum_{m=1}^{M} \left\| \mathbf{h}_{2,mk}^T \mathbf{W}_m \right\|^2 \\
&= \sum_{m=1}^{M} (\mathbf{h}_{2,mk}^T \mathbf{W}_m \mathbf{n}_{r,m})(\mathbf{h}_{2,mk}^T \mathbf{W}_m \mathbf{n}_{r,m})^H \\
&= \sum_{m=1}^{M} \sigma_{r,m}^2 ((\mathbf{h}_{2,mk} \otimes \mathbf{I})^H \mathbf{w}_m)^* (((\mathbf{h}_{2,mk} \otimes \mathbf{I})^H \mathbf{w}_m)^*)^H \\
&= \sigma_{r,m}^2 \sum_{m=1}^{M} \left\| ((\mathbf{h}_{2,mk} \otimes \mathbf{I})^H \mathbf{w}_m)^* \right\|^2 \\
&= \sigma_{r,m}^2 \sum_{m=1}^{M} \left\| ((\mathbf{h}_{2,mk} \otimes \mathbf{I})^H \mathbf{w}_m) \right\|^2 \\
&= \sigma_{r,m}^2 \sum_{m=1}^{M} (\mathbf{w}_m^H \mathbf{h}_{2,mk} \otimes \mathbf{I})((\mathbf{h}_{2,mk} \otimes \mathbf{I})^H \mathbf{w}_m) && (2.17) \\
&= \sum_{m=1}^{M} \mathbf{w}_m^H (\mathbf{h}_{2,mk} \mathbf{h}_{2,mk}^H \otimes \mathbf{I} \sigma_{r,m}^2)) \mathbf{w}_m && (2.18) \\
&= \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{F}_{k(m)} \mathbf{w}_m \\
&= \mathbf{w}^H \mathbf{F}_k \mathbf{w} \\
&= \left\| \mathbf{F}_k^{\frac{1}{2}} \mathbf{w} \right\|^2 && (2.19)
\end{aligned}$$

where we use the property $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$ to obtain (2.18) from (2.17), also $\mathbf{F}_{k(m)} \triangleq \mathbf{h}_{2,mk} \mathbf{h}_{2,mk}^H \otimes \mathbf{I} \sigma_{r,m}^2$ is defined as the $N^2 \times N^2$ amplified noise covariance matrix from relay m for the kth source-destination pair, and $\mathbf{F}_k \triangleq \text{diag}(\mathbf{F}_{k(1)} \cdots \mathbf{F}_{k(M)})$ is the $M \times M$ block diagonal amplified noise covariance matrix from all relays for the kth pair.

Finally, from (2.13), (2.15), and (2.19), the SNR expression in (2.6) can be rewritten as

$$\mathrm{SNR}_k = \frac{P_0 \left|\mathbf{h}_k^H \mathbf{w}\right|^2}{P_0 \left\|\mathbf{G}_{k^-}^H \mathbf{w}\right\|^2 + \left\|\mathbf{F}_k^{\frac{1}{2}} \mathbf{w}\right\|^2 + \sigma_{d,k}^2}. \tag{2.20}$$

For $P_{m,i}$ in (2.7), we denote $\mathbf{W}_m^H = [\mathbf{w}_{m,1}, \cdots \mathbf{w}_{m,N}]$. Then, $P_{m,i}$ can be rewritten as

$$\begin{aligned} P_{m,i} &= [\mathbf{W}_m \mathbf{W}_m^H \sigma_{r,m}^2 + P_0 \mathbf{W}_m \sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H \mathbf{W}_m^H]_{ii} \\ &= \mathbf{w}_{m,i}^H (\sigma_{r,m}^2 + P_0 \sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H) \mathbf{w}_{m,i} \\ &= \mathbf{w}_{m,i}^H \mathbf{D}_m \mathbf{w}_{m,i} \end{aligned} \tag{2.21}$$

where $\mathbf{D}_m \triangleq \sigma_{r,m}^2 \mathbf{I} + P_0 \sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H$.

2.2.2 Necessary Condition on Feasibility

The existence of $\mathbf{W}_m$ while satisfying the SNR constraints in (2.9) depends on the transmission power P_0, the SNR targets $\{\gamma_k\}$, and channel conditions characterized by $\{\mathbf{h}_{1,km}\}$ and $\{\mathbf{h}_{2,mk}\}$. The following is the derivation of feasibility condition for problem (2.10).

The upper bound of kth user's SNR is given as:

$$\begin{aligned}\mathrm{SNR}_{k,up} &= \frac{\left|\mathbf{h}_k^H\mathbf{w}\right|^2 P_0}{\left\|\mathbf{G}_{k^-}^H\mathbf{w}\right\|^2 P_0 + \left\|\mathbf{F}_k^{\frac{1}{2}}\mathbf{w}\right\|^2} \\ &= \frac{\mathbf{w}^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}P_0}{\mathbf{w}^H\mathbf{G}_{k^-}\mathbf{G}_{k^-}^H\mathbf{w}P_0 + \mathbf{w}^H\mathbf{F}_k\mathbf{w}} \\ &= \frac{\mathbf{w}^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}P_0}{\mathbf{w}^H(\mathbf{G}_{k^-}\mathbf{G}_{k^-}^H P_0 + \mathbf{F}_k)\mathbf{w}} \\ &= \frac{\mathbf{w}^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}P_0}{\mathbf{w}^H\mathbf{R}_{g,k}\mathbf{w}} > \gamma_k,\ \forall k \end{aligned} \tag{2.22}$$

where $\mathbf{R}_{g,k} \triangleq \mathbf{G}_{k^-}\mathbf{G}_{k^-}^H P_0 + \mathbf{F}_k$. When $\mathbf{w} = \mathbf{R}_{g,k}^{\dagger}\mathbf{h}_k$, the LHS of (2.22) is maximized, and its maximum value is $P_0\mathbf{h}_k^H\mathbf{R}_{g,k}^{\dagger}\mathbf{h}_k$.

A necessary condition for the multi-pair multi-antenna relay beamforming problem (2.10) to be feasible is that the source transmit power P_0, SNR targets $\{\gamma_k\}$, and channel vectors $\{\mathbf{h}_{1,km}\}$ and $\{\mathbf{h}_{2,mk}\}$ should satisfy

$$P_0\mathbf{h}_k^H\mathbf{R}_{g,k}^{\dagger}\mathbf{h}_k > \gamma_k,\ \forall k. \tag{2.23}$$

2.3 MUP2P Design through Dual Approach

2.3.1 Lagrange Function

We develop the solution to the non-convex power minimization problem (2.10) through its dual domain. With the SNR and per-antenna power expressions in vectorized $\{\mathbf{W}_m\}$ in (2.20) and (2.21), respectively, the Lagrangian of the optimization problem

(2.10) is given by

$$\begin{aligned} L(P_r, \mathbf{w}, \boldsymbol{\lambda}, \boldsymbol{\nu}) &= P_r + \sum_{m=1}^{M} \sum_{i=1}^{N} \lambda_{m,i} \{ \mathbf{w}_{m,i}^H \mathbf{D}_m \mathbf{w}_{m,i} - P_r \} \\ &\quad - \sum_{k=1}^{K} \nu_k \{ \left| \mathbf{h}_k^H \mathbf{w} \right|^2 \frac{P_0}{\gamma_k} - (\left\| \mathbf{G}_{k-}^H \mathbf{w} \right\|^2 P_0 + \left\| \mathbf{F}_k^{\frac{1}{2}} \mathbf{w} \right\|^2 + \sigma_{d,k}^2) \} \\ &= P_r - P_r \mathrm{tr}(\boldsymbol{\Lambda}) + \sum_{m=1}^{M} \mathbf{w}_m^H \left[\boldsymbol{\Lambda}_m \otimes \mathbf{D}_m \right] \mathbf{w}_m \\ &\quad - \sum_{k=1}^{K} \nu_k \left[\left| \mathbf{h}_k^H \mathbf{w} \right|^2 \frac{P_0}{\gamma_k} - (\left\| \mathbf{G}_{k-}^H \mathbf{w} \right\|^2 P_0 + \left\| \mathbf{F}_k^{\frac{1}{2}} \mathbf{w} \right\|^2 + \sigma_{d,k}^2) \right] \end{aligned} \tag{2.24}$$

where $\boldsymbol{\Lambda}_m \triangleq \mathrm{diag}(\lambda_{m1} \cdots \lambda_{mN})$, with $\lambda_{m,i}$ being the Lagrange multiplier related to the ith antenna power constraint of relay m, and $\boldsymbol{\nu} = [\nu_1, \cdots, \nu_K]^T$ with ν_k being the Lagrange multipliers associated with the SNR constraint for destination k.

Define $\mathbf{R}_m \triangleq \boldsymbol{\Lambda}_m \otimes \mathbf{D}_m$, $\mathbf{R} \triangleq \mathrm{diag}(\mathbf{R}_1, \cdots, \mathbf{R}_M)$, and $\boldsymbol{\Lambda} \triangleq \mathrm{diag}(\boldsymbol{\Lambda}_1 \cdots \boldsymbol{\Lambda}_M)$. The Lagrangian in (2.24) can be rewritten as

$$L(P_r, \mathbf{w}, \boldsymbol{\Lambda}, \boldsymbol{\nu}) = P_r(1 - tr(\boldsymbol{\Lambda})) + \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 + \mathbf{w}^H (\mathbf{R} + \sum_{k=1}^{K} \nu_k \left[\mathbf{R}_{g,k} - \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H \right]) \mathbf{w}. \tag{2.25}$$

2.3.2 Dual Problem Expression

The dual problem optimization is analyzed in [48]. With Lagrangian (2.25), the dual problem of the power minimization problem (2.10) is given by

$$\max_{\boldsymbol{\Lambda}, \boldsymbol{\nu}} \min_{P_r, \mathbf{w}} \; L(P_r, \mathbf{w}, \boldsymbol{\Lambda}, \boldsymbol{\nu}) \tag{2.26}$$

$$\text{subject to } \; \boldsymbol{\Lambda} \succeq 0, \boldsymbol{\nu} \succeq 0. \tag{2.27}$$

Examining the expression of $L(P_r, \mathbf{w}, \boldsymbol{\Lambda}, \boldsymbol{\nu})$ in (2.25), the above dual problem is equivalent to the following optimization problem with two new added constraints

$$\max_{\boldsymbol{\Lambda},\boldsymbol{\nu}} \min_{P_r,\mathbf{w}} \; L(P_r, \mathbf{w}, \boldsymbol{\Lambda}, \boldsymbol{\nu}) \tag{2.28}$$

$$\text{subject to} \;\; \boldsymbol{\Lambda} \succeq 0, \boldsymbol{\nu} \succeq 0$$

$$\text{tr}(\boldsymbol{\Lambda}) \leq 1, \;\; \boldsymbol{\Lambda} \text{ being diagonal} \tag{2.29}$$

$$\boldsymbol{\Sigma} \succeq \sum_{k=1}^{K} \nu_k \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H \tag{2.30}$$

where $\boldsymbol{\Sigma} \triangleq \mathbf{R} + \sum_{k=1}^{K} \nu_k \mathbf{R}_{g,k}$. The two added constraints (2.29) and (2.30) will not affect the optimal solution of (2.26). This is because, if any one of two constraints is not satisfied, the inner minimization in (2.28) will result in $L(P_r, \mathbf{w}, \boldsymbol{\Lambda}, \boldsymbol{\nu}) = -\infty$. This is obviously not the optimal solution for the dual problem (2.26). Therefore the optimal solution for the optimization problem (2.26) remains in the feasible set of the optimization problem (2.28) and is optimal for (2.28).

Solving the inner minimization of (2.28) over $\mathbf{w}$ and P_r, we have

$$\max_{\boldsymbol{\Lambda},\boldsymbol{\nu}} \;\; \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 \tag{2.31}$$

$$\text{subject to} \;\; \boldsymbol{\Lambda} \succeq 0, \boldsymbol{\nu} \succeq 0$$

$$\text{tr}(\boldsymbol{\Lambda}) \leq 1, \;\; \boldsymbol{\Lambda} \text{ being diagonal}$$

$$\boldsymbol{\Sigma} \succeq \sum_{k=1}^{K} \nu_k \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H.$$

To solve the above optimization problem, we first examine the constraint in (2.30). We will need the following lemma [15].

Lemma 1 ([15]) *Let* $\mathbf{A}$ *and* $\mathbf{B}$ *be* $n \times n$ *positive semi-definite matrices. Then,*

$$\mathbf{A} \succcurlyeq \mathbf{B} \Rightarrow 1 - \sigma_{\max}\left(\mathbf{A}^{\frac{\dagger}{2}}\mathbf{B}\mathbf{A}^{\frac{\dagger}{2}}\right) \geq 0 \tag{2.32}$$

where $\sigma_{\max}(\mathbf{A})$ means the maximum eigenvalue of matrix $\mathbf{A}$, and the equality of right hand side holds when $(\mathbf{A} - \mathbf{B})$ is not strictly positive definite.

Using Lemma 1, we have that the constraint (2.30) implies the constraint (2.34). Thus, any feasible solution to the optimization problem (2.31) is feasible to the following problem:

$$\begin{aligned} \max_{\mathbf{\Lambda},\boldsymbol{\nu}} \quad & \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 && (2.33) \\ \text{subject to} \quad & \mathbf{\Lambda} \succeq 0, \boldsymbol{\nu} \succeq 0 \\ & \mathrm{tr}(\mathbf{\Lambda}) \leq 1, \ \mathbf{\Lambda} \text{ being diagonal} \\ & \sigma_{\max}\left(\mathbf{\Sigma}^{\frac{\dagger}{2}}(\sum_{k=1}^{K} \nu_k \frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H)\mathbf{\Sigma}^{\frac{\dagger}{2}}\right) \leq 1. && (2.34) \end{aligned}$$

We now show that the optimization problem (2.33) is further equivalent to the following problem:

$$\begin{aligned} \max_{\mathbf{\Lambda}} \min_{\boldsymbol{\nu}} \quad & \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 && (2.35) \\ \text{subject to} \quad & \mathbf{\Lambda} \succeq 0, \boldsymbol{\nu} \succeq 0 \\ & \mathrm{tr}(\mathbf{\Lambda}) \leq 1, \ \mathbf{\Lambda} \text{ being diagonal} \\ & \sigma_{\max}\left(\mathbf{\Sigma}^{\frac{\dagger}{2}}(\sum_{k=1}^{K} \nu_k \frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H)\mathbf{\Sigma}^{\frac{\dagger}{2}}\right) \geq 1. && (2.36) \end{aligned}$$

Notice that from the optimization problem (2.33) to (2.35), we change the maximization to minimization over $\boldsymbol{\nu}$, and flip the inequality in (2.34) to (2.36). To see the equivalence of the two problems, we note that, for any given $\mathbf{\Lambda}$, to reach the optimality, both optimization problems (2.33) and (2.35) require the constraints (2.34) and (2.36) to be met with equality, *i.e.*,

$$\sigma_{\max}\left(\mathbf{\Sigma}^{\frac{\dagger}{2}}(\sum_{k=1}^{K}\nu_k\frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H)\mathbf{\Sigma}^{\frac{\dagger}{2}}\right)=1 \tag{2.37}$$

with the optimal $\boldsymbol{\nu}^o$ being the root of the above equation, which lead to the same solution.

We now show that the optimization problem (2.35) is equivalent to the following problem:

$$\max_{\mathbf{\Lambda}}\min_{\boldsymbol{\nu},\mathbf{w}} \quad \sum_{k=1}^{K}\nu_k\sigma_{d,k}^2 \tag{2.38}$$

$$\begin{aligned} \text{subject to} \quad & \mathbf{\Lambda}\succeq 0, \boldsymbol{\nu}\succeq 0 \\ & \text{tr}(\mathbf{\Lambda})\leq 1,\ \mathbf{\Lambda}\text{ being diagonal} \\ & \frac{P_0\sum_{k=1}^{K}\frac{\nu_k}{\gamma_k}\left|\mathbf{h}_k^H\mathbf{w}\right|^2}{\mathbf{w}^H\mathbf{\Sigma}\mathbf{w}}\geq 1. \end{aligned} \tag{2.39}$$

For a given $\mathbf{\Lambda}$, we look at the inner minimization of (2.38). It is easy to see that, at the optimality, the constraint (2.39) is attained with equality as follow

$$\max_{\mathbf{w}}\frac{P_0\sum_{k=1}^{K}\frac{\nu_k^o}{\gamma_k}\left|\mathbf{h}_k^H\mathbf{w}\right|^2}{\mathbf{w}^H\mathbf{\Sigma}^o\mathbf{w}}=1. \tag{2.40}$$

The above optimization is a generalized eigenvalue problem [49]. The optimal $\tilde{\mathbf{w}}$ for the problem (2.40), and thus for (2.38), has the following structure

$$\tilde{\mathbf{w}} = \boldsymbol{\Sigma}^{o\frac{\dagger}{2}} \mathcal{P}\left(\boldsymbol{\Sigma}^{o\frac{\dagger}{2}} \left[\sum_{k=1}^{K} \frac{\nu_k^o}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H \right] \boldsymbol{\Sigma}^{o\frac{\dagger}{2}} \right) \tag{2.41}$$

where $\mathcal{P}(\cdot)$ denotes the principal eigenvector of a matrix, $\boldsymbol{\nu}^o$ is the optimal value for $\boldsymbol{\nu}$, and $\boldsymbol{\Sigma}^o$ is $\boldsymbol{\Sigma}$ under the optimal $\boldsymbol{\Lambda}^o$ and $\boldsymbol{\nu}^o$.

Now, we show that the optimal solution for the optimization problems (2.35) is also optimal for the optimization problem (2.31). First, we note that maximizing the objective in (2.35) over $\boldsymbol{\nu}$ requires the constraint (2.30) being met with $\succeq$" instead of strictly "$\succ$", *i.e.,* $[\boldsymbol{\Sigma} - \sum_{k=1}^{K} \nu_k \frac{P_o}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H]$ is not strictly positive definite. Otherwise, we can always scale $\boldsymbol{\nu}$ to ensure that $[\boldsymbol{\Sigma} - \sum_{k=1}^{K} \nu_k \frac{P_o}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H]$ becomes semi-definite and "$\succeq$" instead of "$\succ$ holds for (2.30). By Lemma 1, it follows that, at the optimality of (2.35), the optimal $\boldsymbol{\nu}$ also satisfies (2.37). Thus, the optimal solution for the problem (2.31) is in the feasible solution set of the problems (2.35). Since the feasible solution set of the problem (2.35) contains that of the problem (2.31), the two problems have the same optimal solution.

Now we verify the optimal $\mathbf{w}$ to the two optimization problems (2.26) and (2.38) are identical. The inner minimization of (2.26) over $\mathbf{w}$ is obtained when the third term in (2.25) is equal to 0, *i.e.,*

$$\mathbf{w}^H \left[\boldsymbol{\Sigma} - \sum_{k=1}^{K} \nu_k \frac{P_o}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H \right] \mathbf{w} = 0.$$

As indicated earlier, at the optimum, the null space of $[\boldsymbol{\Sigma} - \sum_{k=1}^{K} \nu_k \frac{P_o}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H]$ is not empty, the solution $\mathbf{w}$ to the above is in the form as in (2.41). Note that we ignore another trivial solution $\mathbf{w} = \mathbf{0}$ which obviously will not satisfy the SNR constraint.

From the above, we have shown that the two optimization problems (2.26) and

(2.38) are equivalent.

From (2.40), the final solution $\mathbf{w}^o$ for the optimization problem (2.38) (thus (2.26)) will have the form

$$\mathbf{w}^o = \beta \tilde{\mathbf{w}} \tag{2.42}$$

where β is a scaling factor to ensure that the SNR constraint (2.9) is met, for all k. This means

$$\frac{\beta^2 \left|\mathbf{h}_k^H \tilde{\mathbf{w}}\right|^2 P_0}{\beta^2 P_0 \left\|\mathbf{G}_{k^-}^H \tilde{\mathbf{w}}\right\|^2 + \beta^2 \left\|\mathbf{F}_k^{\frac{1}{2}} \tilde{\mathbf{w}}\right\|^2 + \sigma_{d,k}^2} \geq \gamma_k, \forall k. \tag{2.43}$$

Thus, the optimal β is obtained as

$$\beta = \sqrt{\frac{\sigma_{d,k}^2}{\min\limits_k g_k(\tilde{\mathbf{w}})}} \tag{2.44}$$

where

$$g_k(\tilde{\mathbf{w}}) \triangleq \frac{P_0}{\gamma_k} \left|\mathbf{h}_k^H \tilde{\mathbf{w}}\right|^2 - P_0 \left\|\mathbf{G}_{k^-}^H \tilde{\mathbf{w}}\right\|^2 - \left\|\mathbf{F}_k^{\frac{1}{2}} \tilde{\mathbf{w}}\right\|^2. \tag{2.45}$$

Note that for the inequality in (2.43) to hold, it requires $\min_k g_k(\tilde{\mathbf{w}}) > 0$; otherwise, $\mathbf{w}^o$ cannot be obtained through this approach. Thus, we have the sufficient condition for the feasibility of the original problem (2.10). That is, there exist $\boldsymbol{\Lambda} \succeq 0$, and $\boldsymbol{\nu} \succeq 0$, such that

$$\min_k g_k(\tilde{\mathbf{w}}) > 0. \tag{2.46}$$

2.3.3 SDP Formulation for Dual Problem

To determine $\mathbf{w}^o$ in (2.42), we need to obtain the optimal $\mathbf{\Lambda}^o$ and $\boldsymbol{\nu}^o$, which can be obtained from the optimization problem (2.31). The dual problem (2.31) can be transformed into an SDP as

$$\begin{aligned} &\min_{\mathbf{x}} \ \boldsymbol{\sigma}^T\mathbf{x} \\ &\text{subject to } \mathbf{b}^T\mathbf{x} \preceq 1,\ \mathbf{x} \succeq 0, \\ &\sum_{m=1}^{M}\sum_{i=1}^{N} x_{(m-1)N+i}\mathbf{D}_{m,i} + \sum_{k=1}^{K} x_{MN+k}\mathbf{T}_k \preceq 0 \end{aligned} \tag{2.47}$$

where $\boldsymbol{\sigma} \triangleq [\mathbf{0}^T_{MN\times 1}, -\sigma^2_{d,k}\mathbf{1}^T_{K\times 1}]^T$, $\mathbf{b} \triangleq [\mathbf{1}^T_{MN\times 1}, \mathbf{0}^T_{K\times 1}]^T$, and $\mathbf{x} = [x_1, \cdots, x_{MN+K}]^T \triangleq [\lambda_1, \cdots \lambda_{MN}, \nu_1, \cdots \nu_K]^T$. The last constraint above corresponds to the constraint (2.30), where $\mathbf{D}_{m,i}$ is a block diagonal matrix with MN diagonal blocks of size $N \times N$, with the $(m-1)N+i$ diagonal block being $\mathbf{D}_m$ (defined below (2.21)) and the rest being zero. Finally, $\mathbf{T}_k$ is an $MN^2 \times MN^2$ matrix, defined as $\mathbf{T}_k \triangleq \frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H - \mathbf{R}_g$, for $k = 1, \cdots, K$.

The SDP can be efficiently solved using standard SDP software, such as SeDuMi [27]. Note the above SDP converts the optimization problem (2.10) with MN^2 variables and $MN+K$ constraints to $MN+K$ variables and three constraints. Also, as M,N, and K increases, the number of constraints is fixed.

2.3.4 Simulation Results

In this section we will study the performance of the dual approach for the per antenna power minimization problem. We assume the channel vectors $\mathbf{h}_{1,km}$ and $\mathbf{h}_{2,mk}$ are i.i.d. Gaussian with zero mean and unit variance. We set noise power at relays and destinations to be equal $\sigma^2_{r,m}$=$\sigma^2_{d,k} = 1$ W$, \forall m, k$. The source transmission power over

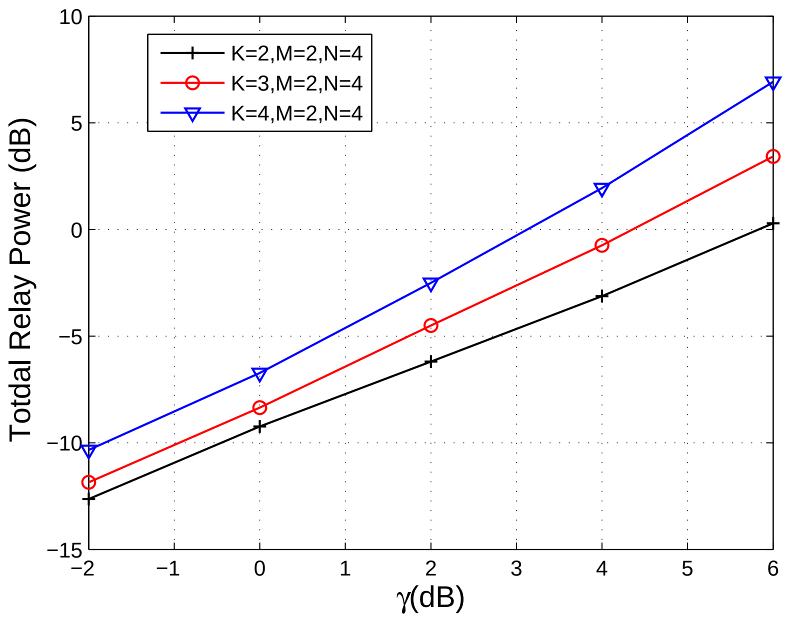

Figure 2.2: Total relay power usage for different K $(M = 2, N = 4)$

noise power is set to be $P_0/\sigma_{r,m}^2 = 0$ dB. The SNR target γ_k's are equal for all k, from -2 dB to 6 dB with 2 dB interval. The channel realization for each γ_k is 800.

Relay Power v.s. User Pair

We first study how K will affect the relay power under the dual approach. We set $M = 2$, $N = 4$, and K varies from 2 to 4. Fig. 2.2 shows how K will affect the total relay power. From one pair of source-destination, the relay will carry more interference when forwarding its signal with K increasing. Moreover, each relay needs to send signals to more destinations, so the relay needs more power to reach the same SNR requirement.

Relay Power v.s. Relay Antenna Number

Next we look into the relay power for different N. The setting is $K = 2, M = 2,$ and $N = 2, 4, 6$. Fig. 2.3 shows the effect of N on the total relay power. From the

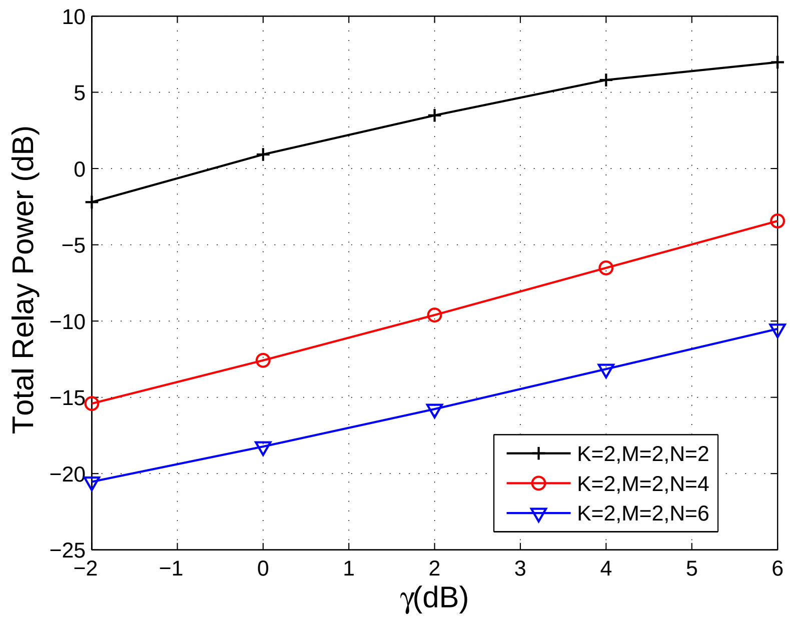

Figure 2.3: Total relay power usage for different N ($K = 2, M = 2$)

plot we can see that if there are more antennas in each relay, it will reduce the total relay power. This is because increasing N at the relays will help joint processing, and it can save power at the relays.

Centralized Relaying v.s. Distributed Relaying

Then we study the performance of the distributed relay network and the centralized relay network. We set $K = 2$, and MN is fixed to be 8 and 12, respectively. Fig. 2.4 and Fig. 2.5 show that the more centralized the relay network is, the less power it will use in total. This is due to the fact that the centralized structure can help joint processing among antennas, which can save the relay power to reach the same SNR. The maximum difference can be as high as 13 dB between the total centralized case and the total distributed case, as shown in Fig. 2.5.

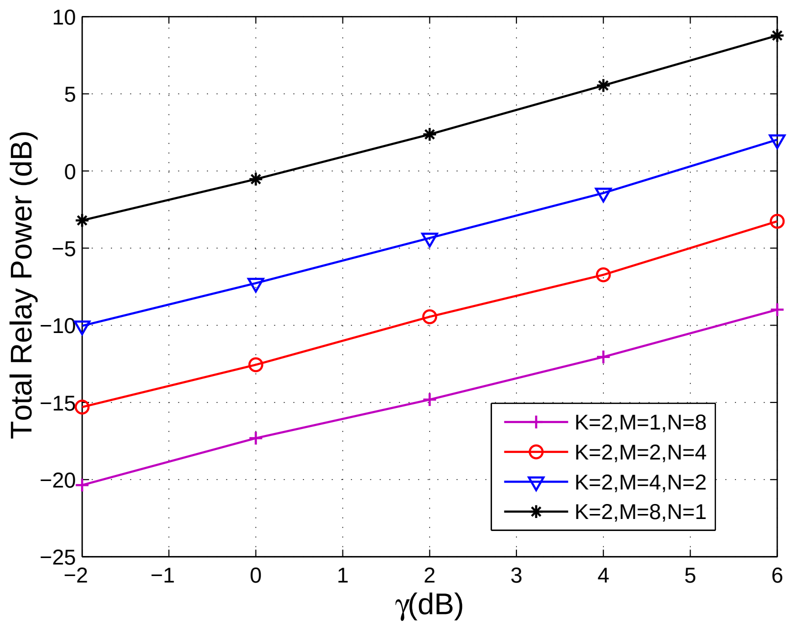

Figure 2.4: Centralized relaying v.s. Distributed relaying ($K = 2, MN = 8$)

Received SNR Distribution

Finally, we investigate the PDF of the SNR distribution at receivers' side with $M = 2$, $N = 2$, $K = 2$. Fig. 2.6 gives the SNR distribution under different SNR requirements for a given destination. We can see that about 60 percent of the receiver SNRs equal to the requirement, and the rest 40 percent exceed the requirement. Also, the SNR distribution starts from the SNR target, which means the SNR requirement is always satisfied.

Next we look at the the CDF of the SNR distribution. We fix $\gamma_k = 0$ dB and change K to see how the CDF curve shifts. Fig. 2.7 shows that if K increases, the receiver SNR is less likely to equal the SNR requirement, meanwhile it will have a shorter tail. In addition, we study the SNR difference in Fig. 2.8. We compute the maximum SNR difference for each realization, and plot its CDF. Form the figure we can see if K increases, its CDF will have a shorter tail, this corresponds to our

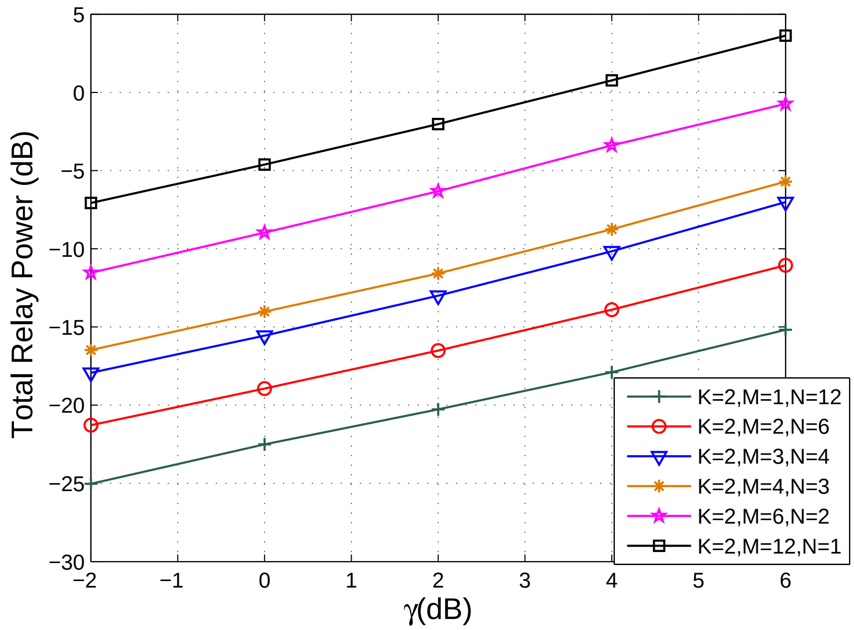

Figure 2.5: Centralized relaying v.s. Distributed relaying ($K = 2, MN = 12$)

observation in Fig. 2.7.

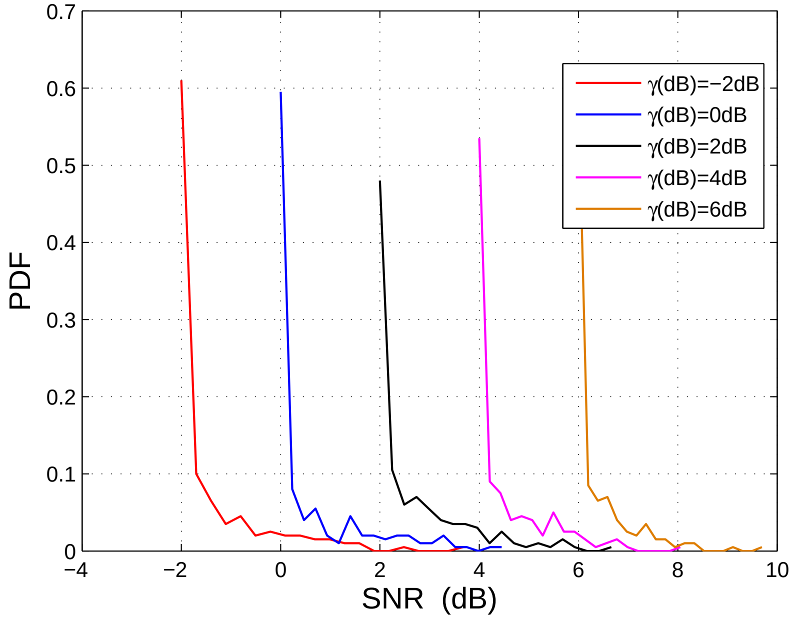

Figure 2.6: The PDF of SNR

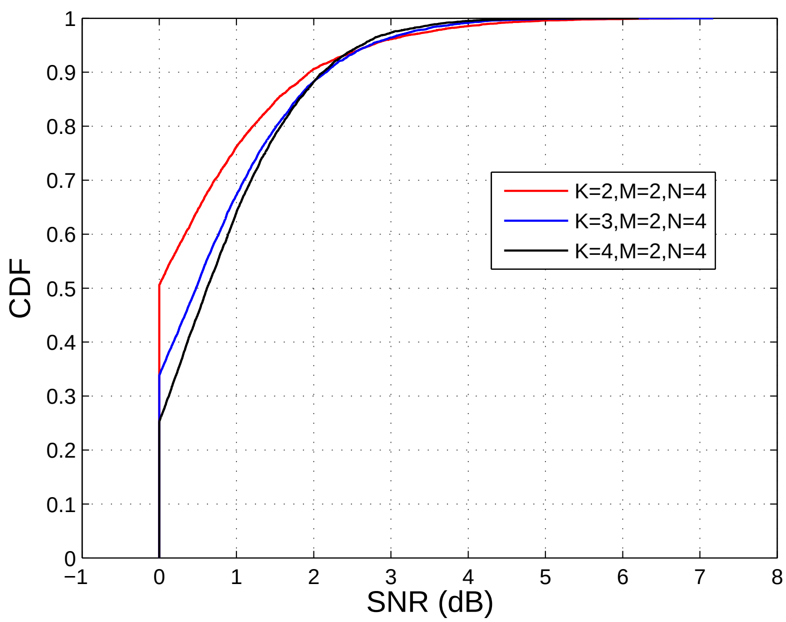

Figure 2.7: The CDF of SNR

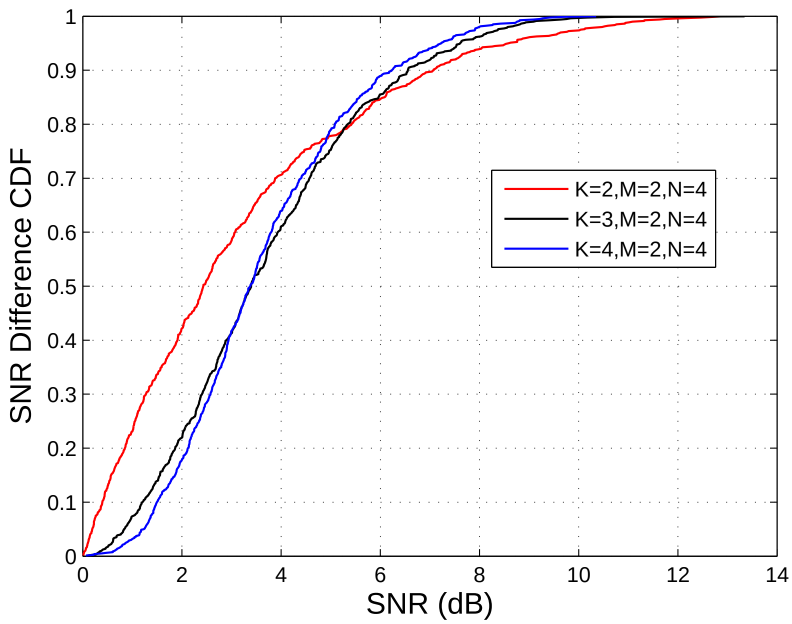

Figure 2.8: The CDF of SNR difference for different K ($\gamma_k = 0$dB)

2.4 MUP2P Design through SDR Approach

The power minimization problem (2.10) can be also solved using the traditional SDR approach. Here we explain the procedure briefly. Define $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$, the per-antenna power $P_{m,i}$ in (2.21) is rewritten as

$$\begin{aligned} P_{m,i} &= \mathbf{w}_{m,i}^H(\sigma_{r,m}^2 + P_0 \sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H)\mathbf{w}_{m,i} \\ &= \mathrm{tr}(\mathbf{w}_{m,i}^H \mathbf{D}_m \mathbf{w}_{m,i}) \\ &= \mathrm{tr}(\mathbf{D}_{m,i}\mathbf{X}) \end{aligned} \tag{2.48}$$

where $\mathbf{D}_{m,i}$ is defined after (2.47).

The SNR constraint in (2.9) can be rewritten as

$$\begin{aligned} &\mathrm{SNR}_k = \frac{\mathbf{w}^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w} P_0}{\mathbf{w}^H \mathbf{G}_{k^-} \mathbf{G}_{k^-}^H \mathbf{w} P_0 + \mathbf{w}^H \mathbf{F}_k \mathbf{w} + \sigma_{d,k}^2} \geq \gamma_k \\ &P_0 \mathbf{w}^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w} \geq \gamma_k (\mathbf{w}^H \mathbf{G}_{k^-} \mathbf{G}_{k^-}^H \mathbf{w} P_0 + \mathbf{w}^H \mathbf{F}_k \mathbf{w} + \sigma_{d,k}^2) \\ &\mathbf{w}^H (\frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H - [\mathbf{G}_{k^-} \mathbf{G}_{k^-}^H P_0 + \mathbf{F}_k])\mathbf{w} \geq \sigma_{d,k}^2. \end{aligned} \tag{2.49}$$

The LHS of (2.49) can be converted to trace form as:

$$\begin{aligned} &\mathbf{w}^H (\frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H - [\mathbf{G}_{k^-} \mathbf{G}_{k^-}^H P_0 + \mathbf{F}_k])\mathbf{w} \\ &= \mathrm{tr}(\mathbf{w}^{\mathrm{H}} \mathbf{T}_{\mathrm{k}} \mathbf{w}) \\ &= \mathrm{tr}(\mathbf{T}_{\mathrm{k}} \mathbf{w} \mathbf{w}^{\mathrm{H}}) \\ &= \mathrm{tr}(\mathbf{T}_{\mathrm{k}} \mathbf{X}) \end{aligned} \tag{2.50}$$

where $\mathbf{T}_k$ is defined after (2.47).

With antenna power and SNR expression both in trace format, the optimization problem (2.10) can be reformulated as the following

$$\min_{\mathbf{X}} \; P_r \tag{2.51}$$

$$\text{subject to} \;\; \text{tr}(\mathbf{T}_k\mathbf{X}) \geq \sigma_{d,k}^2, \forall k, \tag{2.52}$$

$$\text{tr}(\mathbf{D}_{m,i}\mathbf{X}) \leq P_r, \forall m, i \tag{2.53}$$

$$\text{rank}(\mathbf{X}) = 1, \;\; \mathbf{X} \succcurlyeq 0. \tag{2.54}$$

By removing the rank constraint in (2.54), the above non-convex optimization problem is relaxed to the following SDP problem

$$\min_{\mathbf{X}} \; P_r \tag{2.55}$$

$$\text{subject to} \;\; \text{tr}(\mathbf{T}_k\mathbf{X}) \geq \sigma_{d,k}^2, \forall k,$$

$$\text{tr}(\mathbf{D}_{m,i}\mathbf{X}) \leq P_r, \forall m, i$$

$$\mathbf{X} \succcurlyeq 0. \tag{2.56}$$

For a fixed P_r, the above problem is an SDP feasibility problem. Thus, we can solve the optimization problem (2.55) using a bi-section search on P_r as an outer loop over an SDP feasibility problem w.r.t. P_r. After obtaining the optimal $\mathbf{X}^o$ from (2.55), if $\text{rank}(\mathbf{X}^o) = 1$, the solution $\mathbf{w}$ can be extracted from $\mathbf{X}^o$ directly; otherwise, for $\text{rank}(\mathbf{X}^o) > 1$, we extract $\mathbf{w}$ through some randomization methods [28, 50]. The randomization algorithm is given below.

Algorithm 2.1 Randomization

1: Find the eigenvectors $\mathbf{v}_1,\ \mathbf{v}_2 \cdots \mathbf{v}_n$ of $\mathbf{X}$ and $\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_n]$.

2: Find the eigenvalues $\lambda_1,\ \lambda_2 \cdots \lambda_n$ of $\mathbf{X}$, and $\mathbf{\Lambda} = [\lambda_1,\ \lambda_2 \cdots \lambda_n]$.

3: Generate a list of zero-mean, unit-variance complex Gaussian ν_l.

4: $\mathbf{w}_l = \mathbf{V}\mathbf{\Lambda}^{\frac{1}{2}}\nu_l$.

5: Scale all the candidate $\mathbf{w}_l$ to satisfy the SNR constraint (2.52).

6: For a given scaled $\mathbf{w}_l$, we can get MN Pr using (2.53), then we choose the maximum one among these MN results, as the final result for one $\mathbf{w}_l$.

7: Repeat step 6 with another $\mathbf{w}'_l$, and get a new Pr'.

8: Find the minimum Pr from the previous results.

2.5 Comparison of Dual and SDR Approaches

2.5.1 Computational Complexity

The computational complexity of the proposed dual approach is much lower than that of the SDR approach. Both approaches need to solve an SDP. But with different size and structure, the dual approach has significantly lower complexity than the SDR approach. To see this, we will compare the complexity for both approaches in terms of the problem size and the number of SDPs one needs to solve.

Size of each SDP

The complexity in solving an SDP problem can be analyzed by examining the size of the problem [48], [51]. For the dual approach, the SDP problem only has $(MN + K)$ variables and three constraints. The worst-case complexity per iteration for the SDP problem (2.47) is $\mathcal{O}((MN+K)^2(MN^2)^2)$. For the SDR approach, the SDP feasibility problem (2.55) contains $(MN^2)^2$ variables and $(MN + K)$ constraints. With a given

P_r, the complexity per iteration is $\mathcal{O}((MN^2)^4((MN^2)^2 + MN + K)$. The number of iterations required to solve an SDP is known to be insensitive to the problem size, generally ranges between 5 to 50 [48], [51]. Therefore, in solving each SDP, the dual approach has a much lower complexity than SDR approach when $(MN^2)^2 > MN+K$.

The number of SDPs

In the dual approach, the solution $\mathbf{w}$ is directly obtained from the semi-closed form solution (2.41), which only needs to solve SDP problem (2.47) once. Since the length of $\mathbf{w}$ is MN^2, computing the matrix inverse and principle eigenvector in (2.41) incur complexity of $\mathcal{O}((MN^2)^3)$. Matrix multiplications incur complexity of $\mathcal{O}((MN^2)^2)$. Thus, the complexity in computing (2.41) is $\mathcal{O}((MN^2)^3)$. It is less than the complexity for solving the corresponding SDP problem. The overall complexity for the dual approach is dominated by the SDP complexity.

For the SDR approach, obtaining $\mathbf{w}$ needs to go through a bi-section search over P_r, each time solving an SDP feasibility problem (2.55). Consider an lower bound P_r^{low} and and upper bound P_r^{high} for P_r, and an error tolerance level ε used in the bi-section search. The number of iterations in the bi-section is $\log((P_r^{high} - P_r^{low})/\varepsilon)$. Moreover, when $\mathbf{X}$ is not rank-one, we need to use the randomization method to extract a rank-one solution, which will add additional complexity in the SDR approach.

Based on the above analysis, we see that the overall complexity of the proposed dual approach is significantly lower than the SDR approach. In simulation, we will demonstrate the complexity of each approach through the actual processing time.

2.5.2 Performance

Regarding the performance, it is known that the dual problem (2.26) and the relaxed SDP problem (2.55) provide the same lower bound to the original primary problem

(2.10) [28]. This means that, when the optimal solution is obtained by one approach, it can be obtained by both approaches at the same time. However, as discussed above, the dual approach provides a semi-definite solution with significantly lower computational complexity. In addition, for the SDR approach, even through a rank-one solution may exist, extracting the rank-one solution $\mathbf{w}$ from $\mathbf{X}$ is a non-trivial problem, and needs to be discussed case by case.

When the solution is non-optimal, it is in general difficult to analyze the relative performance of the two approaches. Simulations demonstrate that the SDR approach provides a better approximate solution in the sense that the gap between the power solution and the lower bound is statistically smaller.

Simulation Results

In this section we will study the performance of the dual approach and the SDR approach for the per-antenna power budget problem. We assume the channel vectors $\mathbf{h}_{1,km}$ and $\mathbf{h}_{2,mk}$ are i.i.d. Gaussian with unit variance. We set noise power at relays and destinations to be equal $\sigma_{r,m}^2 = \sigma_{d,k}^2 = 1$ W, $\forall m, k$. The source transmission power over noise power is set to be $P_0/\sigma_{r,m}^2 = 0$ dB and γ_k's are equal for all k.

Now we compare the SNR performance of the two methods. Fig. 2.9 is given below. This figure shows the first user's SNR for the dual method and the SDR approach. The SNR requirement is 0 dB. The SNR of the SDR approach has a shorter tail compared to that of the dual approach. This shows the SDR approach has slightly better performance than the dual approach. Fig. 2.10 compares the gap of the SDR approach and the dual method for $N = 1$ with different M. The observation is that the SDR approach has smaller gap when solution is sub-optimal. With M increasing, both solutions will be less likely to be 0 gap. Fig. 2.11 compares the performance gap for the SDR and the dual approaches for different K. We can see that for $K = 1$,

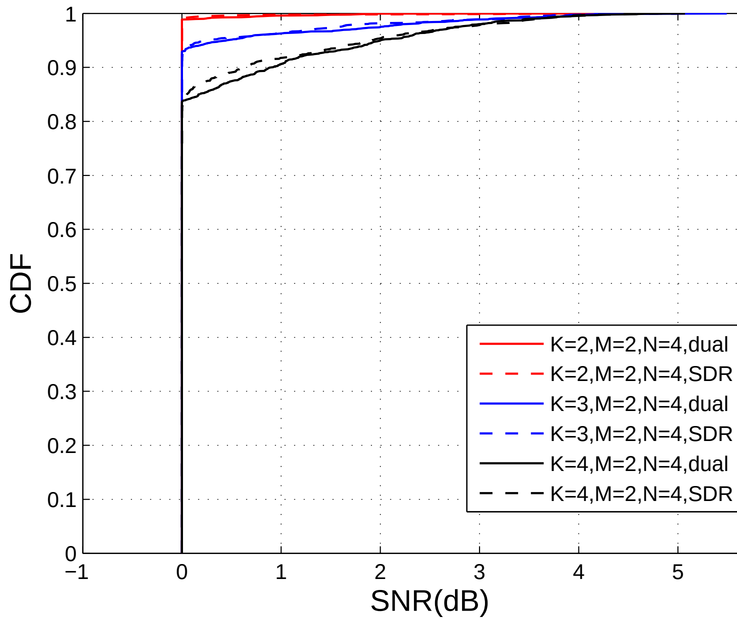

Figure 2.9: The CDF of SNR under the SDR and dual approaches (1st user)

both approaches achieve 100 percent optimal solution; for $K > 1$, they both achieve the same percent of the optimal solution, where the gap is 0 dB, and the rest results are sub-optimal. For the sub-optimal cases, the SDR approach always has a smaller gap, which means its performance is better.

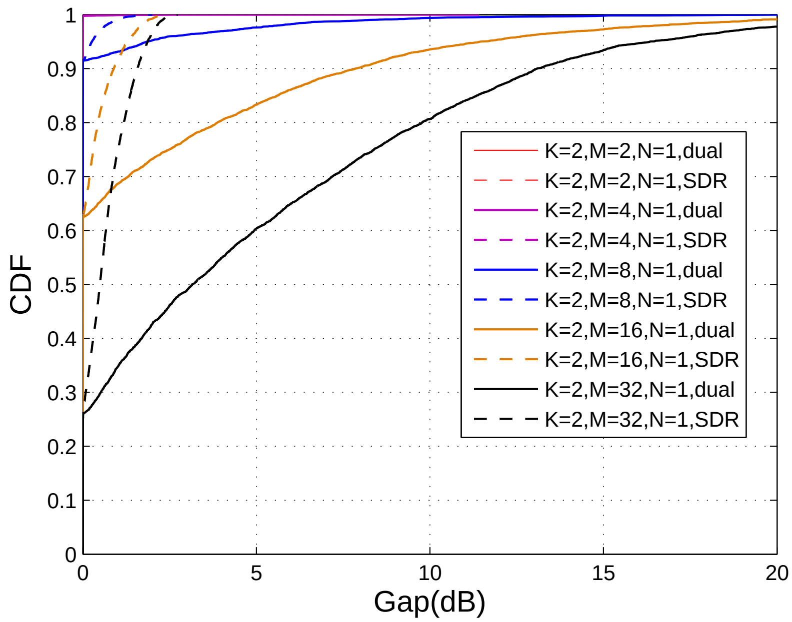

Figure 2.10: Gap CDF ($\gamma_k = 4$dB, $N = 1, K = 2$)

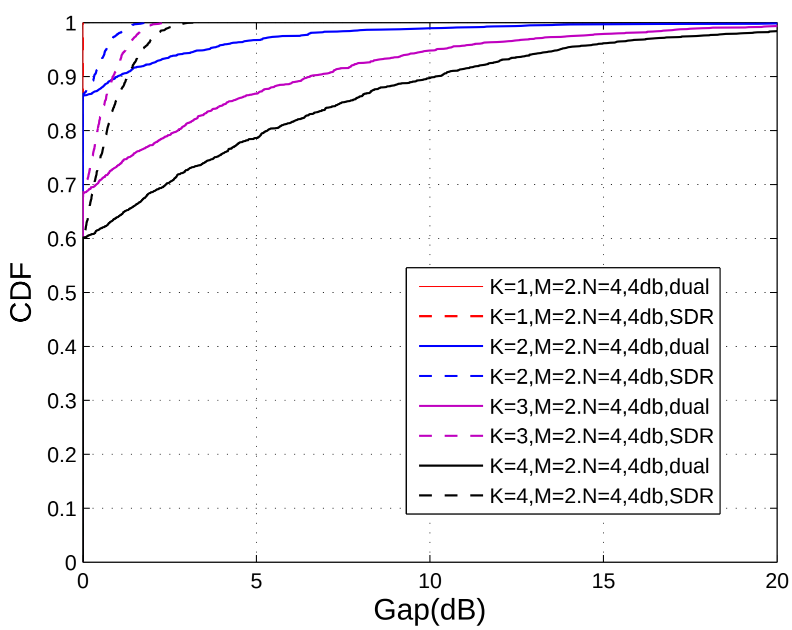

Figure 2.11: Gap CDF ($\gamma_k = 4$dB, $M = 2, N = 4$)

2.6 The Combined Method

2.6.1 Proposed Combined Method

Based on the above analysis, we realize that there is a trade-off in the performance and complexity of the dual and SDR approaches. For practical implementation, we suggest to combine the two approaches. Because dual approach has much lower complexity, we can first obtain the solution via dual approach. When the approximated solution by the dual approach is deemed to be not good, we will use SDR approach to obtain a better approximated solution.

To set the threshold for switching the two approach, we note that, for the SDR approach, an approximated bound of its performance to the optimal one exists in literature [28]. Let $\mathbf{w}^{\text{SNR}}$ denote the solution obtained by the SDR approach, and $P_r(\mathbf{w}^{\text{SNR}})$ denote the power obtained under the solution. Let $\mathbf{X}^o$ denote the solution to the SDP problem (2.55), and $P_r^{lw}(\mathbf{X}^o)$ the optimal objective in (2.55), *i.e.,* the lower bound of the original power minimization problem (2.10). The, the ratio $P_r(\mathbf{w}^{\text{SNR}})/P_r^{lw}(\mathbf{X}^o)$ gives the gap of the performance under the approximated solution to the lower bound of the performance. It is shown in [28] that this ratio is bounded as

$$\frac{P_r(\mathbf{w}^{\text{SNR}})}{P_r^{lw}(\mathbf{X}^o)} < \mathcal{O}(K + MN - 1).$$

We can use the bound to to measure the quality of the obtained solution and set the threshold to determine when to use the SDR approach. Specifically, we use a threshold η to decide when to use the dual method or the SDR approach.

$$\eta = \alpha(MN + K - 1) \tag{2.57}$$

where α is a scalar to control the balance between complexity and performance.

The combined method is given as

1. Compute $\mathbf{w}^o$ using (2.42).

2. Let $P_r(\mathbf{w}^o)$ be the per-antenna power in (2.10) under $\mathbf{w}^o$, and $P_r^{lw}(\mathbf{w}^o)$ be the optimal value of (2.26), *i.e.,* the lower bound of the original problem (2.10). Compute dual gap $G^{\mathrm{d}} \triangleq P_r(\mathbf{w}^o)/P_r^{lw}(\mathbf{w}^o)$, the solution of combined method $\mathbf{w}^{\mathrm{com}} = \mathbf{w}^o$. If $G^{\mathrm{d}} \leq \eta$, then $\mathbf{w}^o$ is the final solution. Otherwise, go to Step 3.

3. Use the SDR approach to produce a solution $\mathbf{w}^{\mathrm{SNR}}$ and SDR gap $G^{\mathrm{SDR}} \triangleq P_r(\mathbf{w}^{\mathrm{SNR}})/P_r^{lw}(\mathbf{X}^o)$, we compare G^{d} with G^{SDR}, if $G^{\mathrm{d}} \geq G^{\mathrm{SNR}}$, $\mathbf{w}^{\mathrm{com}} = \mathbf{w}^{\mathrm{SNR}}$, otherwise $\mathbf{w}^{\mathrm{com}} = \mathbf{w}^o$.

2.6.2 Simulation Results

In this section we will study the performance of the dual approach, the SDR approach and the combined method for the per-antenna power minimization problem. We assume the channel vectors $\mathbf{h}_{1,km}$ and $\mathbf{h}_{2,mk}$ are i.i.d. Gaussian with unit variance. We set noise power at relays and destinations to be equal $\sigma_{r,m}^2 = \sigma_{d,k}^2 = 1$ W, $\forall m, k$. The source transmission power over noise power is set to be $P_0/\sigma_{r,m}^2 = 0$ dB and γ_k's are equal for all k.

Gap comparison for the dual, SDR and combined approaches

We compare the performance of the dual approach, the SDR approach, and the combined method. Similar to the gap G^{d} and G^{SDR} defined in 2.6.1 for the dual and the SDR approach, the gap for the combined method is defined as $G^{\mathrm{com}} \triangleq P_r(\mathbf{w}^{\mathrm{com}})/P_r^{lw}(\mathbf{w}^o)$, where $\mathbf{w}^{\mathrm{com}}$ is given in 2.6.1.

As mentioned earlier, the dual approach and the SDR approach attain the same lower bound. In Fig. 2.12, we plot the CDF of G^{d}, G^{SDR} and G^{com} under the three approaches, respectively. We set $M = 2$, $N = 6$, $K = 2, 4, 8$, and $\gamma_k = 4$dB, $\forall k$. The same set of 2000 channel realizations are used for each method. The threshold $\eta = 10\log_{10}(MN + K - 1) - 5$ dB. The gap being 0 dB indicates the optimal solution is obtained. We can see that the percentage of 0 dB gap in three approaches are identical, verifying that the optimal solutions are obtained by these approaches at the same time. When the solution is suboptimal, we observe that the tail distribution of G^{SDR} is tighter than that of G^{d}. Therefore, the SDR approach produces a tighter approximate solution than the dual approach in this case. The corresponding average processing time of each method is shown in Fig. 2.13. We see that the dual approach uses significantly shorter time than the SDR approach to compute the solution. From Figs. 2.12 and 2.13, we see that the combined method can effectively trade-off the performance and complexity, with the performance between the two approaches and the processing time slightly worse than that of the dual approach. The threshold for the combined method can be adjusted to trade-off the performance and the complexity. Also, when we change the threshold of the combined method, its gap performance will be affected. We now reduce the threshold η to $10\log_{10}K$ dB from $10\log_{10}(MN + K - 1) - 5$ dB. Since the threshold has been smaller, the gap of the combined method will also be closer to that of the SDR approach. Fig. 2.14 compares the gap of the three methods. The combined method has the performance in between, and its performance is better than the previous case with the threshold $10\log_{10}(MN + K - 1) - 5$ dB.

Fig. 2.15 compares the computation time for three methods. Since we have performance improvement for reducing the threshold, the cost is to increase the computation time. This figure reflects this property.

Just now we reduce the threshold for better performance, and now we will increase the threshold for lower computational complexity. Fig. 2.16 compares the gap of three methods for $M = 2, N = 6$ under different K. The main difference is when we increase the threshold, the combined method's performance is approaching that of the dual method. Fig. 2.17 compares the computation time for the three methods. The threshold is set to be $10\log_{10}(MN + K - 1)$ dB. It shows that with the sacrifice of the gap performance, the complexity of the combined method is now very close to that of the dual approach, which saves tons of computation time.

Power usage v.s. the number of source-destination pairs

Using the combined method, we study the resulting total power over all relays vs. γ_k under various K in Fig. 2.18. We set $M = 2$, $N = 4$. The threshold $\eta = 10\text{log}_{10}(MN + K - 1) - 5$ dB, and channel realization for each γ_k is 800. As K increases, the interference from other sources at the relays increases. We see the effect of such interference on the total relay power consumption, especially at the high SNR target, when $\gamma_k = 6$ dB, if K increases from 3 to 4, the total relay power increases approximately 5 dB.

Relay power v.s. the number of antennas per relay

Next we investigate the effect of N on the total relay power. We set $K = 2, M = 2$, and $N = 2, 4, 6$, and channel realization for each γ_k is 800. Fig. 2.19 shows that N greatly influences the power usage over each antenna. This is due to the power gain achieved by beamforming using more antennas at each relay, thus requires much less power per antenna for the same SNR target. Every time we increase the antenna numbers by 2, we can see a significant saving of power up to 12 dB.

Distributed processing v.s. centralized processing

Now we study the performance of distributed relay network beamforming and centralized relay network beamforming. We set $K = 2$, and fix $MN = 12$. The channel realization for each γ_k is 800. As $N = 12$, $M = 1$ is the full centralized case, as M increases, the network setup is moving towards a more distributed case with $M = 12$ and $N = 1$ being fully distributed network beamforming. Fig. 2.20 shows that the more centralized the relay network is, the less power it will use in total. This is due to the fact that the centralized structure allows signals to be processed centrally for better beamforming gain, while the distributed case can only process signals by each relay. The maximum difference can be as high as 13 dB between the total centralized case and the total distributed case.

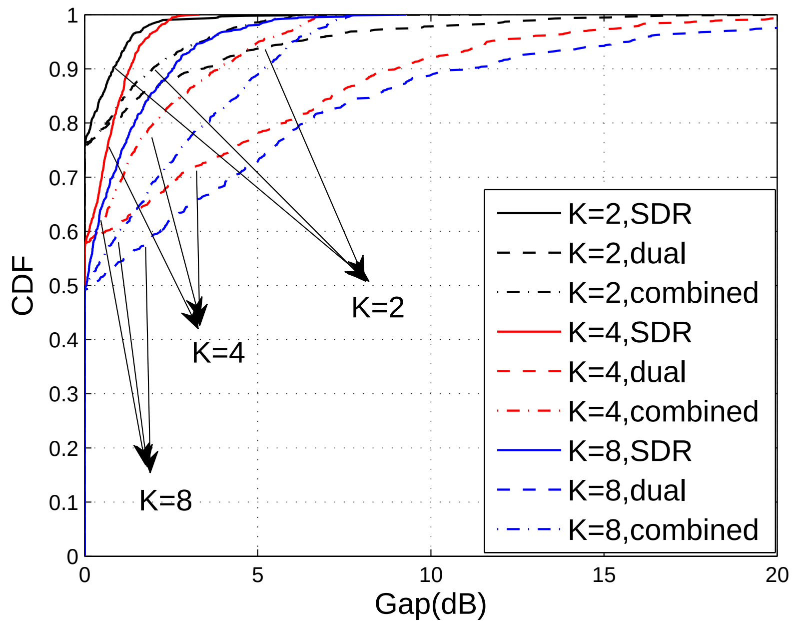

Figure 2.12: Gap CDF ($M = 2, N = 6$, $\gamma_k = 4$dB, $\eta = 10\log_{10}(MN + K - 1) - 5$dB)

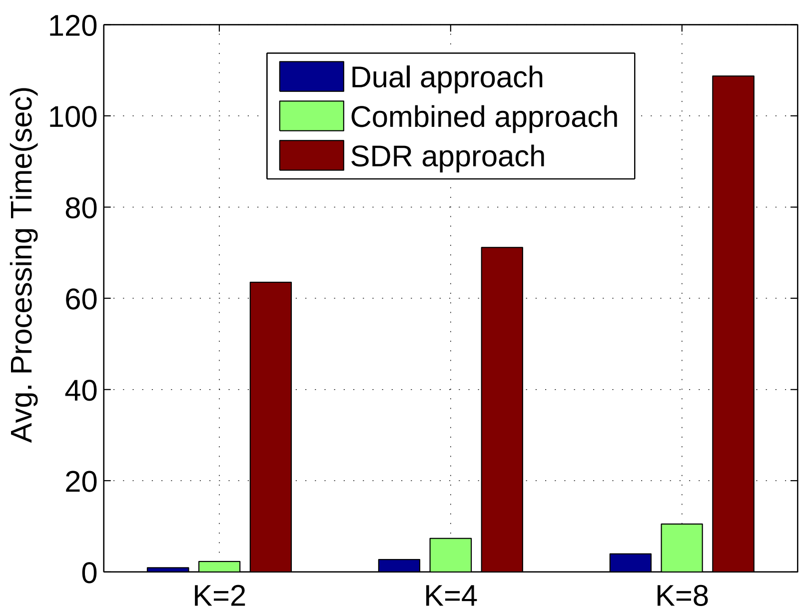

Figure 2.13: Average processing time ($M = 2, N = 6$, $\gamma_k = 4$dB)

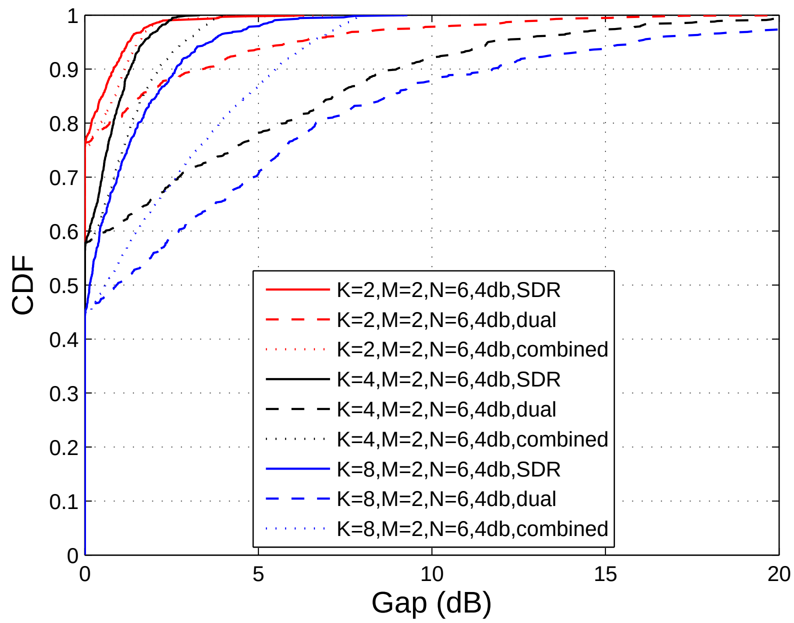

Figure 2.14: Gap CDF ($M = 2, N = 6$, $\gamma_k = 4$dB, $\eta = K$ dB)

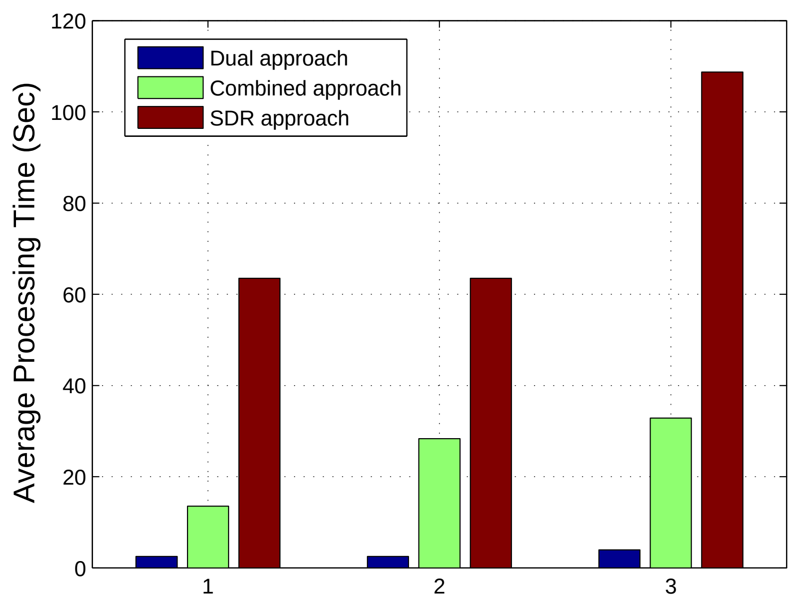

Figure 2.15: Average processing time ($M = 2, N = 6$, $K = 2, 4, 8$, $\gamma_k = 4$dB, $\eta = 10\log_{10} K$ dB)

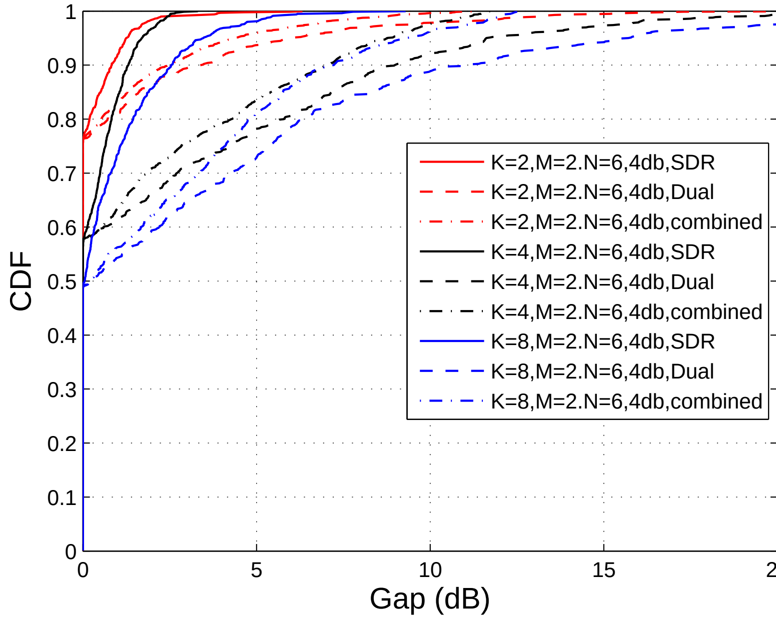

Figure 2.16: Gap CDF ($M = 2, N = 6$, $\gamma_k = 4$dB, $\eta = 10\log_{10}(MN + K - 1)$ dB)

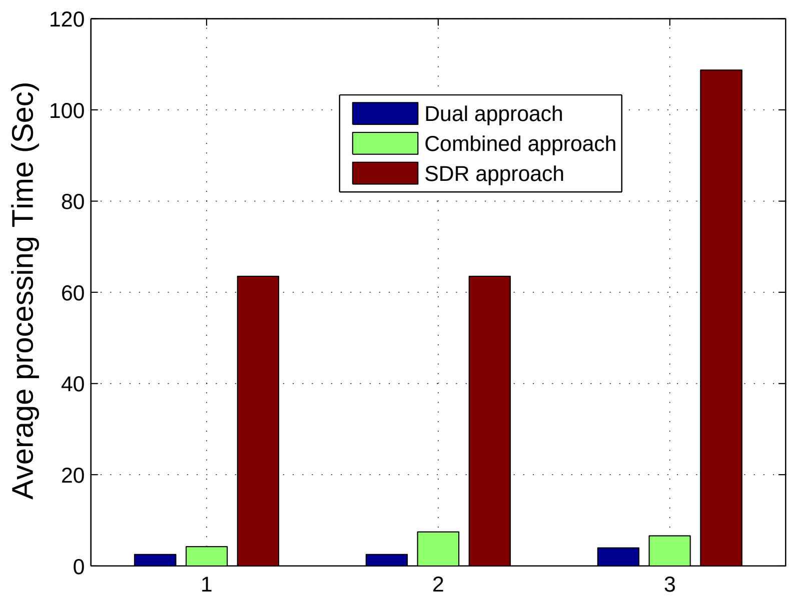

Figure 2.17: Average processing time ($M = 2, N = 6$, $K = 2, 4, 8$, $\gamma_k = 4$dB, $\eta = 10\log_{10}(MN + K - 1)$ dB)

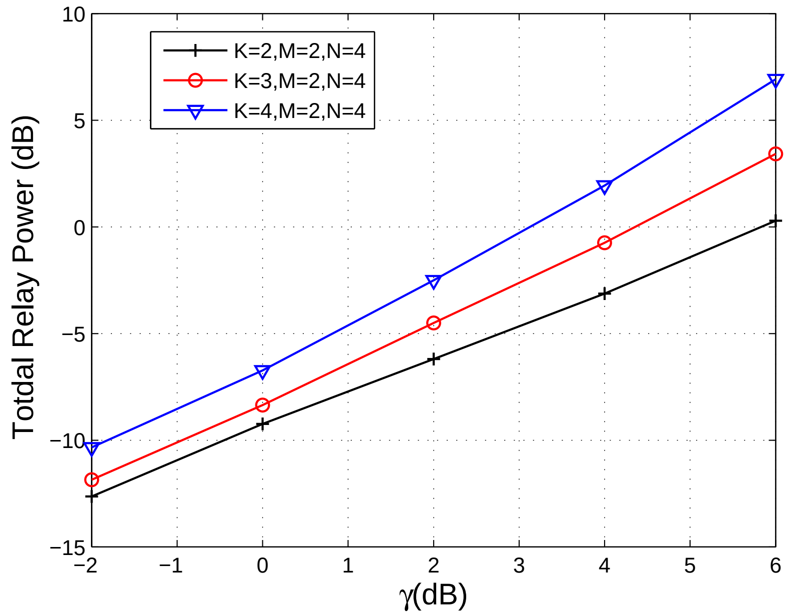

Figure 2.18: Total relay power vs γ_k for different K ($M = 2, N = 4$)

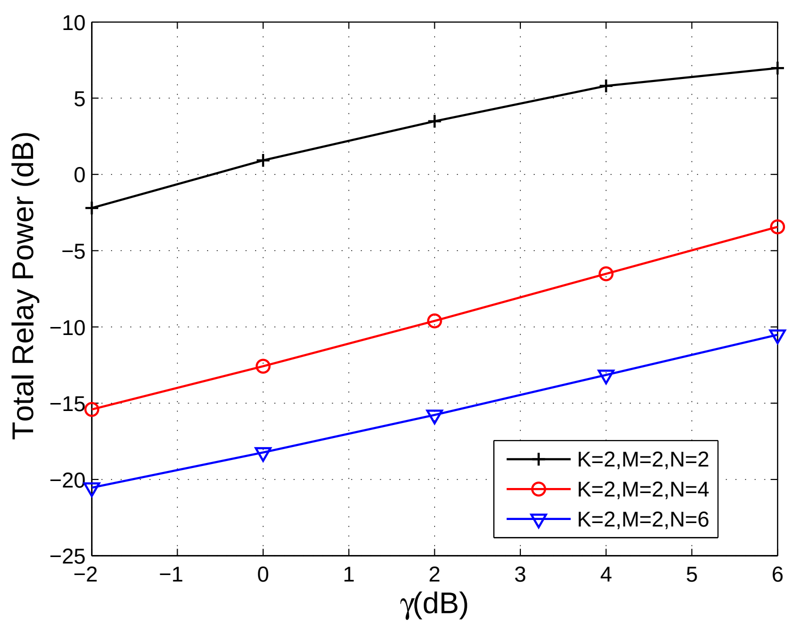

Figure 2.19: Total relay power vs γ_k for different N ($K = 2, M = 2$)

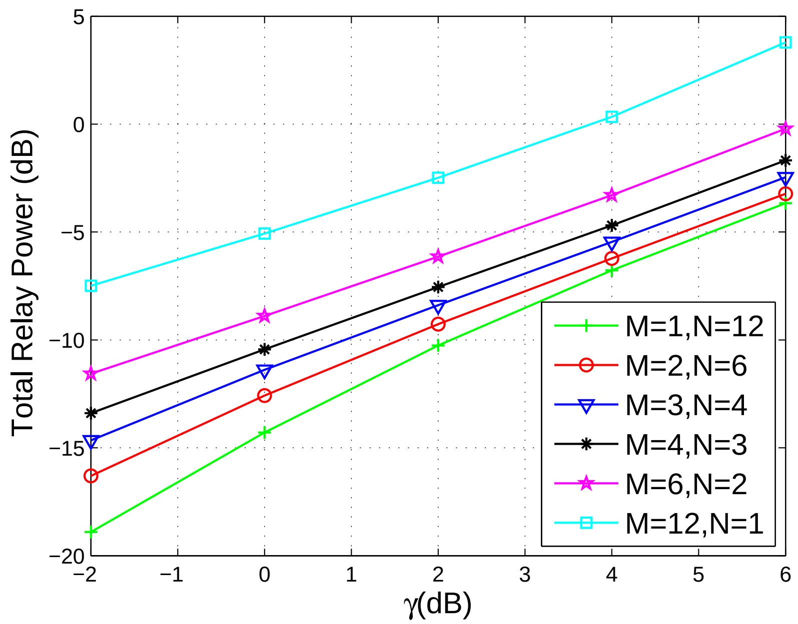

Figure 2.20: Centralized relaying v.s. Distributed relaying ($K = 2, MN = 12$)

Chapter 3

MUP2P under Total Relay Power Minimization

3.1 Problem Formulation

In this chapter, we consider a different power objective, where our goal is to minimize the total power consumed over all relays. The total relay power minimization problem emphasizes on budgeting the network-wide relay power. Specifically, we assume the system setting is the same as the system described in Chapter 2, with K source-destination pairs communicating through M multi-antenna AF relays. The objective is to minimize the total relay power while meeting the received SNR target at each destination. We will see that the dual approach and the SDR approach we studied in section 2.3 and 2.4 can be easily modified to solve the total relay power minimization problem.

Let P_T denote the total relay power budget. Using $P_{m,i}$ in (2.7), the total relay power minimization problem is given by

$$\min_{\{\mathbf{W}_m\},P_T} \quad P_T \tag{3.1}$$

$$\text{subject to} \quad \text{SNR}_k \geq \gamma_k, \ \forall k$$

$$\sum_{m=1}^{M}\sum_{i=1}^{N} \mathbf{w}_{m,i}^H \mathbf{D}_m \mathbf{w}_{m,i} \leq P_T. \tag{3.2}$$

3.1.1 Dual Approach

Define λ^{tot} as the Lagrange multiplier related to the total relay power constraint, and ν_k^{tot} is the Lagrange multiplier associated with the SNR constraint for destination k. The Lagrange Function of problem (3.1) is given below:

$$\begin{aligned} L^{\text{tot}}(P_T, \mathbf{w}, \lambda^{\text{tot}}, \boldsymbol{\nu}^{\text{tot}}) &= P_T + \lambda^{\text{tot}} \left[\sum_{m=1}^{M}\sum_{i=1}^{N} \mathbf{w}_{m,i}^H \mathbf{D}_m \mathbf{w}_{m,i} - P_T \right] \\ &- \sum_{k=1}^{K} \nu_k^{\text{tot}} \left[\left| \mathbf{h}_k^H \mathbf{w} \right|^2 \frac{P_0}{\gamma_k} - (\left\| \mathbf{G}_{k^-}^H \mathbf{w} \right\|^2 P_0 + \left\| \mathbf{F}_k^{\frac{1}{2}} \mathbf{w} \right\|^2 + \sigma_{d,k}^2) \right] \\ &= \sum_{k=1}^{K} \nu_k^{\text{tot}} \sigma_{d,k}^2 + P_T(1 - \lambda^{\text{tot}}) \\ &+ \mathbf{w}^H \left[\mathbf{V} + \sum_{k=1}^{K} \nu_k^{\text{tot}} (\mathbf{R}_{g,k} - \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H) \right] \mathbf{w} \end{aligned} \tag{3.3}$$

where $\mathbf{V}_m \triangleq \mathbf{D}_m \otimes \lambda \mathbf{I}$ and $\mathbf{V} \triangleq \text{diag}(\mathbf{V}_1 \cdots \mathbf{V}_M)$.

3.1.2 Dual Problem Expression

The dual problem of the optimization problem (3.1) is given as:

$$\max_{\lambda^{\text{tot}},\boldsymbol{\nu}^{\text{tot}}} \min_{P_T,\mathbf{w}} \; L^{\text{tot}}(P_T, \mathbf{w}\, \lambda^{\text{tot}}, \boldsymbol{\nu}^{\text{tot}}) \tag{3.4}$$

$$\text{subject to} \;\; \lambda^{\text{tot}} \geq 0, \boldsymbol{\nu}^{\text{tot}} \succeq 0. \tag{3.5}$$

With the expression of $L^{\text{tot}}(P_T, \mathbf{w}, \lambda^{\text{tot}}, \boldsymbol{\nu}^{\text{tot}})$, we can show that the above dual problem is equivalent to the following problem with two new added constraints:

$$\max_{\Lambda^{\text{tot}},\boldsymbol{\nu}^{\text{tot}}} \min_{P_T,\mathbf{w}} \; L^{\text{tot}}(P_T, \mathbf{w}, \lambda^{\text{tot}}, \boldsymbol{\nu}^{\text{tot}}) \tag{3.6}$$

$$\text{subject to} \;\; 0 \leq \lambda^{\text{tot}} \leq 1, \tag{3.7}$$

$$\boldsymbol{\nu}^{\text{tot}} \geq 0, \tag{3.8}$$

$$\boldsymbol{\Sigma}^{\text{tot}} \succeq \sum_{k=1}^{K} \nu_k^{\text{tot}} \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H. \tag{3.9}$$

Similar with the previous argument, the two added constraints (3.7) and (3.9) will not affect the optimal solution of (3.4).

After inner minimization of $\mathbf{w}$ and P_T, the expression becomes

$$\max_{\lambda^{\text{tot}},\boldsymbol{\nu}^{\text{tot}}} \;\; \sum_{k=1}^{K} \nu_k^{\text{tot}} \sigma_{d,k}^2 \tag{3.10}$$

$$\text{subject to} \;\; 0 \leq \lambda^{\text{tot}} \leq 1,$$

$$\boldsymbol{\nu}^{\text{tot}} \geq 0,$$

$$\boldsymbol{\Sigma}^{\text{tot}} \succeq \sum_{k=1}^{K} \nu_k^{\text{tot}} \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H.$$

where $\boldsymbol{\Sigma}^{\text{tot}} = \mathbf{V} + \sum_{k=1}^{K} \nu_k (P_0 \mathbf{G}_{k^-} \mathbf{G}_{k^-}^H + \mathbf{F}_k)$.

Similarly, by Lemmas 1, we can replace the constraint in (3.9) by (3.12), thus the optimization problem (3.10) is equivalent to the following problem:

$$\max_{\lambda^{\mathrm{tot}},\boldsymbol{\nu}^{\mathrm{tot}}} \quad \sum_{k=1}^{K} \nu_k^{\mathrm{tot}} \sigma_{d,k}^2 \tag{3.11}$$

$$\text{subject to} \quad 0 \leq \lambda^{\mathrm{tot}} \leq 1,$$

$$\boldsymbol{\nu}^{\mathrm{tot}} \succeq 0,$$

$$\sigma_{\max}\left(\boldsymbol{\Sigma}^{\mathrm{tot}\,\frac{\dagger}{2}}(\sum_{k=1}^{K} \nu_k^{\mathrm{tot}} \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H) \boldsymbol{\Sigma}^{\mathrm{tot}\,\frac{\dagger}{2}}\right) \leq 1. \tag{3.12}$$

And this problem is further equivalent to the following problem:

$$\max_{\lambda^{\mathrm{tot}}} \min_{\boldsymbol{\nu}^{\mathrm{tot}}} \quad \sum_{k=1}^{K} \nu_k^{\mathrm{tot}} \sigma_{d,k}^2 \tag{3.13}$$

$$\text{subject to} \quad 0 \leq \lambda^{\mathrm{tot}} \leq 1,$$

$$\boldsymbol{\nu}^{\mathrm{tot}} \succeq 0,$$

$$\sigma_{\max}\left(\boldsymbol{\Sigma}^{\mathrm{tot}\,\frac{\dagger}{2}}(\sum_{k=1}^{K} \nu_k^{\mathrm{tot}} \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H) \boldsymbol{\Sigma}^{\mathrm{tot}\,\frac{\dagger}{2}}\right) \geq 1. \tag{3.14}$$

where we change the maximization to minimization over $\boldsymbol{\nu}^{\mathrm{tot}}$, and inverse the inequality in (3.12) to (3.14). To show the equivalence, we note that, for any given λ^{tot}, both optimization problems (3.11) and (3.13) are equivalent. This is because when at optimality, they both require the constraints (3.12) and (3.14) to be met with equality, *i.e.*,

$$\sigma_{\max}\left(\boldsymbol{\Sigma}^{\mathrm{tot}\,\frac{\dagger}{2}}(\sum_{k=1}^{K} \nu_k^{\mathrm{tot}} \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H) \boldsymbol{\Sigma}^{\mathrm{tot}\,\frac{\dagger}{2}}\right) = 1. \tag{3.15}$$

We now show that the optimization problem (3.13) is equivalent to the following problem:

$$\max_{\lambda^{\text{tot}}} \min_{\boldsymbol{\nu}^{\text{tot}},\mathbf{w}} \sum_{k=1}^{K} \nu_k^{\text{tot}} \sigma_{d,k}^2 \tag{3.16}$$

$$\text{subject to } 0 \leq \lambda^{\text{tot}} \leq 1,$$

$$\boldsymbol{\nu}^{\text{tot}} \succeq 0,$$

$$\frac{P_0 \sum_{k=1}^{K} \frac{\nu_k^{\text{tot}}}{\gamma_k} \left|\mathbf{h}_k^H \mathbf{w}\right|^2}{\mathbf{w}^H \boldsymbol{\Sigma}^{\text{tot}} \mathbf{w}} \geq 1. \tag{3.17}$$

For a given λ^{tot}, we look at the inner minimization of (3.16). At the optimality, the constraint (3.17) is attained with equality as following

$$\max_{\mathbf{w}} \frac{P_0 \sum_{k=1}^{K} \frac{\nu_k^{\text{tot}^{\text{o}}}}{\gamma_k} \left|\mathbf{h}_k^H \mathbf{w}\right|^2}{\mathbf{w}^H \boldsymbol{\Sigma}^{\text{tot}^{\text{o}}} \mathbf{w}} = 1. \tag{3.18}$$

The above optimization is a generalized eigenvalue problem which we have studied in section 2.3. The optimal $\tilde{\mathbf{w}}$ for the problem (3.18), and thus for (3.16), has the following structure

$$\mathbf{w}^{\text{tot}} = \beta^{\text{tot}} \tilde{\mathbf{w}}^{\text{tot}} \tag{3.19}$$

where $\tilde{\mathbf{w}}^{\text{tot}}$ is given in (2.41) by replacing ν_k^o to $\nu_k^{\text{tot}^{\text{o}}}$, and $\boldsymbol{\Sigma}^o$ to $\boldsymbol{\Sigma}^{\text{tot}^{\text{o}}}$, with $\boldsymbol{\Sigma}^{\text{tot}^{\text{o}}}$ being $\boldsymbol{\Sigma}^{\text{tot}}$ under the optimal $\boldsymbol{\nu}^{\text{tot}^{\text{o}}}$ and $\lambda^{\text{tot}^{\text{o}}}$; In addition, β^{tot} is given in (2.44) by replacing $\tilde{\mathbf{w}}$ to $\tilde{\mathbf{w}}^{\text{tot}}$.

3.1.3 SDP Formulation for Dual Problem

The optimal $\boldsymbol{\nu}^{\text{tot}^o}$ and λ^{tot^o} can be obtained by solving the following SDP:

$$\min_{\tilde{\mathbf{x}}} \; \tilde{\boldsymbol{\sigma}}^T \tilde{\mathbf{x}} \tag{3.20}$$
$$\text{subject to } \; \tilde{\mathbf{b}}^T \tilde{\mathbf{x}} - 1 \preceq 0, \; \tilde{\mathbf{x}} \succeq 0, \; \sum_{i=1}^{1+K} \tilde{\mathbf{x}}_i \mathbf{G}_i \preceq 0$$

where $\tilde{\boldsymbol{\sigma}} \triangleq [0, -\sigma_{d,k}^2 \mathbf{1}_{K\times 1}^T]^T$, $\tilde{\mathbf{b}} \triangleq [1, \mathbf{0}_{K\times 1}^T]^T$, $\tilde{\mathbf{x}} \triangleq [\lambda^{\text{tot}}, \nu_1^{\text{tot}}, \cdots \nu_K^{\text{tot}}]^T$; and $\mathbf{G}_i$ is an $MN^2 \times MN^2$ matrix, defined as $\mathbf{G}_1 \triangleq -\mathbf{V}$, and $\mathbf{G}_i \triangleq \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H - \mathbf{R}_{g,k}$, for $k = i - 1$ and $i = 2 \cdots K + 1$. This problem can be solved using standard solver tools, such as SeDuMi [27].

3.2 SDR Approach

The total power minimization problem (3.1) can again be solved using the SDR approach.

Define $\mathbf{X}^{\text{tot}} \triangleq \mathbf{w}^{\text{tot}} (\mathbf{w}^{\text{tot}})^H$, the total relay power P_T in (3.2) is rewritten as

$$P_T = \sum_{m=1}^{M} \sum_{i=1}^{N} \text{tr}(\mathbf{D}_{m,i} \mathbf{X}^{\text{tot}}) \tag{3.21}$$

where $\mathbf{D}_{m,i}$ is defined after (2.47).

The SNR constraint in trace form is give in (2.50). Then the total power optimization problem is rewritten as:

$$\min_{\mathbf{X}^{\text{tot}}} \sum_{m=1}^{M}\sum_{i=1}^{N} \text{tr}(\mathbf{D}_{m,i}\mathbf{X}^{\text{tot}}) \tag{3.22}$$

$$\text{subject to} \quad \text{tr}(\mathbf{T}_k\mathbf{X}^{\text{tot}}) \geq \sigma_{d,k}^2, \text{for } k = 1, 2 \cdots K, \tag{3.23}$$

$$\text{rank}(\mathbf{X}^{\text{tot}}) = 1, \ \mathbf{X}^{\text{tot}} \succcurlyeq 0 \tag{3.24}$$

which is relaxed to the following SDP

$$\min_{\mathbf{X}^{\text{tot}}} \sum_{m=1}^{M}\sum_{i=1}^{N} \text{tr}(\mathbf{D}_{m,i}\mathbf{X}^{\text{tot}}) \tag{3.25}$$

$$\text{subject to} \quad \text{tr}(\mathbf{T}_k\mathbf{X}^{\text{tot}}) \geq \sigma_{d,k}^2, \text{for } k = 1, 2 \cdots K,$$

$$\mathbf{X}^{\text{tot}} \succcurlyeq 0. \tag{3.26}$$

Finally, we can similarly adopt the combined method described in section 2.6 for this total power minimization problem to trade-off performance and complexity.

3.3 Simulation Results

In this section, We study the performance of the total power minimization problem for three proposed approaches. The setting of parameters are similar with the previous one. We assume the channel vectors $\mathbf{h}_{1,km}$ and $\mathbf{h}_{2,mk}$ are i.i.d. Gaussian with zero mean and unit variance. We set noise power at relays and destinations to be equal $\sigma_{r,m}^2$=$\sigma_{d,k}^2 = 1$ W, $\forall m, k$. The source transmission power over noise power is set to be $P_0/\sigma_{r,m}^2 = 0$ dB.

3.3.1 Gap Comparison for Dual and SDR Approaches

We again look at the gap performance of the three approaches for the total power minimization problem. The gap definition is similarly with the per-antenna power problem. In the total power minimization problem, the property that the dual approach and the SDR approach attain the same lower bound also holds. In Fig. 3.1, we plot the CDF of $G^{\mathrm{d}}, G^{\mathrm{SDR}}$ and G^{com} under the three methods, respectively. We set $M = 2$, $N = 6$, $K = 2, 4, 8$, and $\gamma_k = 4$dB, $\forall k$. The same set of 2000 channel realizations are used for each method. We set the threshold for the combined method is $10 \log_{10}(MN + K - 1) - 5$ dB. The corresponding average processing time of each method is shown in Fig. 3.2. The observation is similar with the per-antenna power problem, the dual approach has a much lower complexity while the SDR approach has better performance.

We see that for the total power minimization, the percentage of optimal solutions is higher than that in the per-antenna power case. In particular, the dual approach can find the optimal solution over 90 percentage of cases, and the combined method will use the dual approach in most cases. Thus, the performance gap as well as the complexity in the combined method is very close to those of the dual approach. It also implies for the total power problem, the dual approach is a more favorable approach.

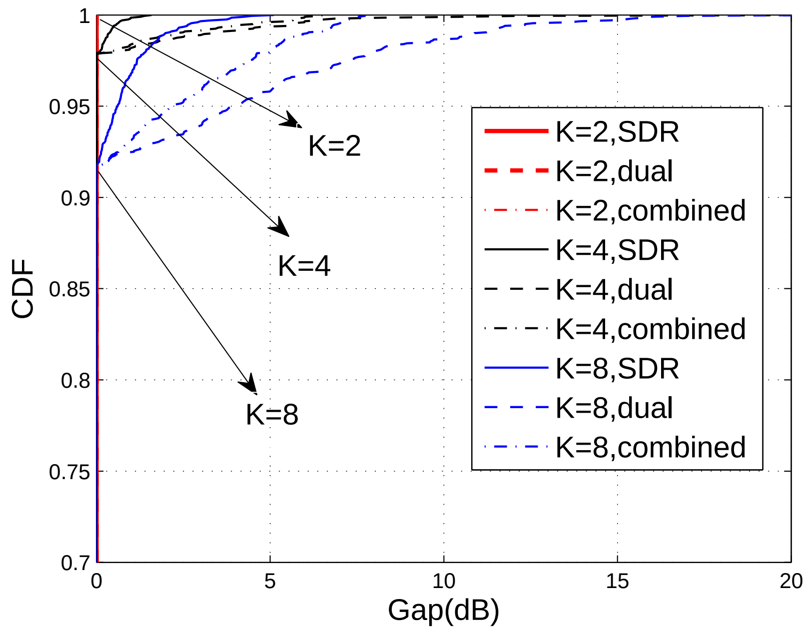

Figure 3.1: Gap CDF ($M = 2, N = 6$, $\gamma_k = 4$dB, $\eta = 10\log_{10}(MN + K - 1) - 5$dB)

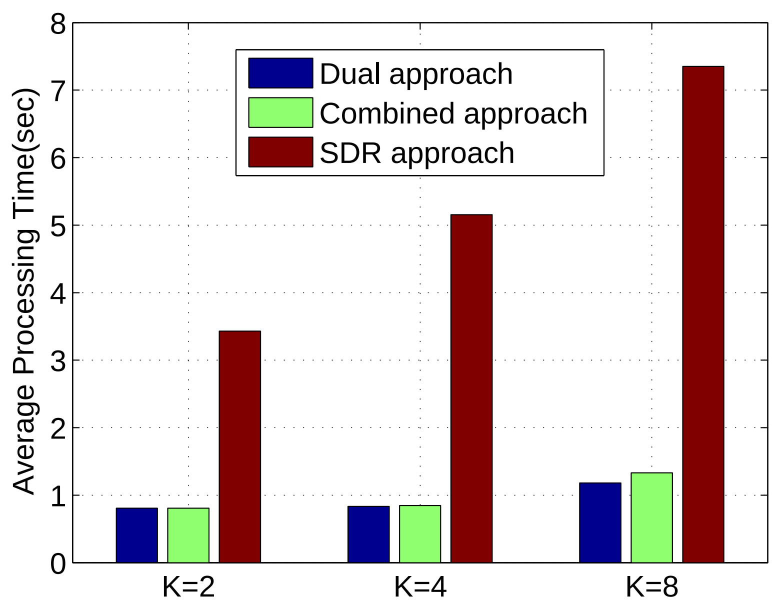

Figure 3.2: Average processing time ($M = 2, N = 6$, $\gamma_k = 4$dB, $\eta = 10\log_{10}(MN + K - 1) - 5$dB)

Relay power v.s. the number of source-destination pairs

We again compare the maximum antenna power for the two cases (per antenna power and total power optimization), both with the combined method. We set $M = 4, N = 2$, and $K = 2, 3, 4$ respectively. The threshold $\eta = 10\log_{10}(MN + K - 1) - 5$ dB. For the per antenna power minimization problem, its objective is to minimize the maximum antenna power. Thus, the resulting maximum antenna power is lower than that from the total power minimization problem. Fig. 3.3 shows this phenomenon. The maximum antenna power of the total power problem is about 2 dB higher than that of the per antenna problem. Also when K increases, more power is required at each antenna to reach the SNR target. This is due to the increase of interfering sources. Next we look into the total relay power consumed under the two problems. In this case, the total power optimization leads to the lowest total power usage. Fig. 3.4 shows the total power usage vs. γ_k. This reversed relationship in Fig. 3.3 and Fig. 3.4 shows the difference in the essence of the optimization objectives in the two problems. Again when K increases, the total relay power will increase to reach the same SNR target.

Relay power v.s. relay number

We study how the number of relays M will affect the relay power. We set $N = 1, K = 2$, and $M = 2, 4, 6$. Fig. 3.5 shows the average maximum antenna power under the two optimization problems, and Fig. 3.6 gives the total relay power usage for the two problems. Similar behaviors can be observed for the relative performance of the two problems. From these two figures we can see when M increases, both the maximum antenna power and the total relay power will reduce, this is because with the total number of antenna increasing, the beamforming gain will increase.

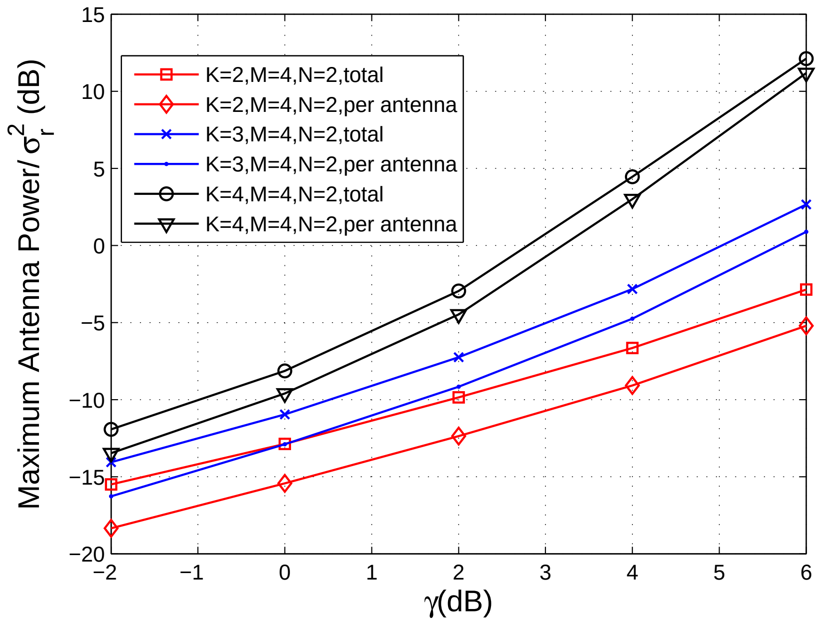

Figure 3.3: Max ant power for different K with both combined methods ($M = 4$,$N = 2$, per antenna power and total power)

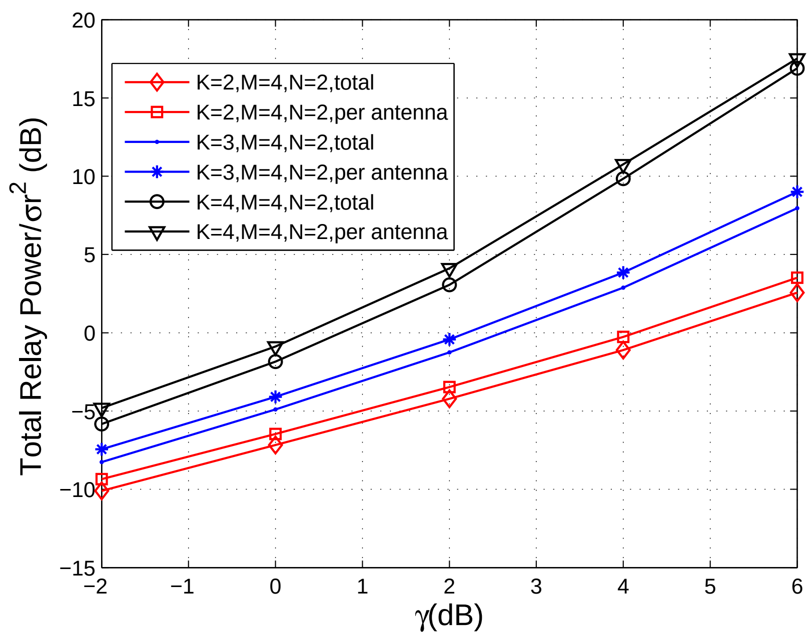

Figure 3.4: Total relay power for different K with both combined methods ($M = 4$,$N = 2$, per antenna power and total power)

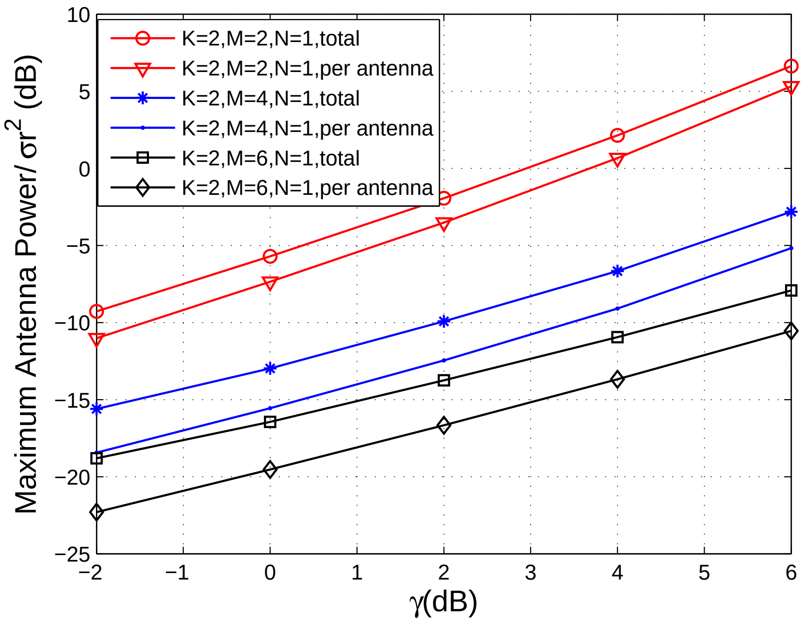

Figure 3.5: Max ant power for different M with both combined methods ($K = 2$,$N = 1$, per antenna power and total power)

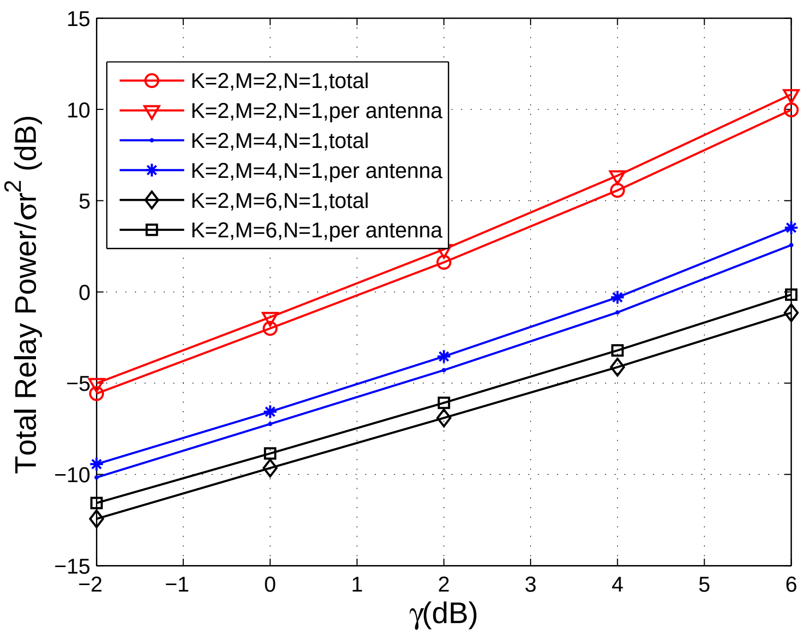

Figure 3.6: Total relay power for different M with both combined methods ($K = 2$,$N = 1$,per antenna power and total power)

Chapter 4

Conclusion

In this work, we considered the design of distributed multi-antenna multi-relay beamforming in a MUP2P AF relay network to minimize the per-antenna relay power usage. We developed an approximate solution through the Lagrange dual domain, and obtained a semi-closed form solution for each relay processing matrix. We also considered the SDR approach. Compared with the traditional SDR approach, the proposed solution has significantly lower computational complexity. The advantage of such a solution is apparent when the optimal solution can be obtained by both approaches. Since the SDR approach has better performance when the solution is suboptimal, we proposed a combined method to trade-off performance and complexity. Simulations showed the effectiveness of the combined method. We then proposed the dual approach for the total power minimization problem to minimize the total relay power usage for given users' SNR targets. After deriving the solution for the same problem via the SDR approach, we compared the dual's performance with the SDR approach, and we proposed a combined method for the total power minimization. We compared the two combined methods to see the performance difference in per antenna power and total relay power problems, and analyzed the reason for this difference.

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