

SUM-RATE MAXIMIZATION FOR TWO-WAY ACTIVE
CHANNELS

By
Pedram AbbasiSaei

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*To my wonderful family
In memory of my dear granny.*

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Abstract

In this thesis, the term passive channel refers to the conventional parallel wireless channel model, where there is no control over the gain of each individual subchannel. We define an active channel as a parallel channel where by injecting power into the subchannels, somewhere between a transmitter and a receiver, the gain of the subchannels are adjusted. We herein study the problem of joint power allocation and channel design over a reciprocal two-way active channel. Assuming the channel is reciprocal, we consider the sum-rate maximization problem under the assumption that the powers of transceivers as well as the channel power are limited. The goal is to jointly optimize the power of each subchannel as well as the allocated power by each transceiver to each subchannel. We use the KKT conditions to find the necessary conditions for the optimality. Then we devise a semi-closed-form solution for the problem by searching over the set of the solutions provided by the KKT conditions. We show that for a two-way active channel, the sum-rate maximization problem has a unique global solution. We prove that at the optimum the power should be allocated uniformly only to a subset of parallel subchannels.

Next we consider the sum-rate maximization problem for a reciprocal two-way active channel, when the total power of the network is limited. In our system model, the total power of the network is defined as the summation of the total power of each transceiver and the power of the active channel. We show that at the optimum, half of the total power of the network should be allocated to the active channel, while the remaining half should be distributed equally between the two transceivers.

Furthermore, we analyze the non-reciprocal two-way active channels. We consider

the sum-rate maximization problem over a non-reciprocal two-way active channel. To solve this problem, without loss of optimality, we break the maximization problem into two sub-problems. Then using the solution to the sum-rate maximization problem for the one-way reciprocal active channels, we find the optimal power distribution. Finally, in the simulation result section, we analyze several passive channels as well as several active channels. As shown in our numerical results two-way active channels outperform the passive channels in terms of sum-rate under the same total network power.

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Pedram AbbasiSaei

Chapter 1

Introduction

1.1 Overview

Over the past three decades, wireless communications have increasingly gained popularity. Wireless technology offers attractive options for many applications due to its flexibility, cost effectiveness, mobility, and ubiquitous connectivity. However, wireless networks cannot still achieve the same reliability and/or high data rate compared to their wired counterparts, due to its unique features such as fading, shadowing, and path loss. To address these problems, numerous ideas have been presented in the literature. For example, cooperative communication appears to be one of the most promising ideas.

Cooperative communications fundamentally changed the abstraction of a wireless link and proposed increasing potential advantages for wireless communication networks. As an example, cooperative communications have been recently considered as a state-of-the-art features of 3GPP LTE-Advanced (LTE-A). The basic idea behind cooperative communications can be traced back to the revolutionary studies on the information theoretic properties of the relay channels in 1980s. In late 1990s, a new

form of spatial diversity was proposed, where diversity gain was achieved via the cooperation of mobiles. The idea that each user has a partner user which is responsible for transmitting not only its own information but also the information of its partner, can make a groundbreaking development in wireless communications.

In conventional wireless channels, there is no control over the gain of each subchannel. In this thesis, we refer to this type of conventional channels as passive channels. In this thesis, we work on a system model that utilizes active channels. Based on its definition, an active channel is a kind of energy-limited wireless parallel channel that the square gain of each subchannel is controlled by injecting power into the signal path. Relay channels and Raman amplifiers are two examples of active channels. Raman gain arises from the transfer of power from one optical beam to another that is downshifted in frequency by the energy of an optical phonon. Since using the relays, power is injected into the signal path somewhere between the transmitter and receiver, relay channels can be considered in the context of the active channels. In this thesis, we consider the sum-rate maximization problem over a two-way active channel while the total power of the active channel is limited by a constant. Other constraints of the problem control the transmit power of each transceiver. The optimal solution to the sum-rate maximization problem is obtained for equal noise powers over different subchannels. Finally, we study the non-reciprocal two-way active channel and its properties.

1.2 Cooperative Communication

It is proved that the transmit diversity has many advantages, while its application may not be practical due to size, power, cost, or hardware restrictions. Recently, in

order to use transmit diversity in multi-user environment with single-antenna mobiles, cooperative communication techniques have been introduced. These promising techniques enable the users to share their antenna with other users of the network to create a virtual multiple-antenna transmitter [1,2].

In a cooperative communication system, each user transmits its own information and plays the role of a relay for other users. Such a user cooperation scheme will lead to trade-off between reliability and the transmit power. The cooperation of users results in a more reliable communication link between the transceivers.

A main concern in cooperative communication is that in a cooperative communication network, where each node transmits its own information and some of the other users information, the transmission rate may be decreased. It is worth mentioning that to design a cooperative communication network, some other important issues such as cooperation assignment, the total interference of the network, fairness, and transceivers requirements should be considered [3].

Compared to point-to-point wireless communication, cooperative wireless communication has many advantages. Cooperative communication may result in wider coverage area, larger capacity region, and enhanced communication reliability, compared to the point-to-point communication [20]. Utilization of terminals distributed in space can enhance the performance of wireless networks significantly [21,22]. As an example, a pair of neighboring nodes with channel state information (CSI), can cooperatively beamform the signal towards a destination. The cooperation of these two neighboring nodes will result in higher total capacity compared to the case that there is no cooperation between the nodes. In [21], the authors develop low-complexity cooperative diversity protocols which combat fading induced by multipath propagation in wireless

networks. Results of cooperation among users have been studied extensively in the literature [23–25].

Extending cooperation to more than one relay is currently a research trend. Space-time coding among participating nodes is a possible scenario [26]. Space-time code design is difficult in practice, due to the distributed and ad-hoc nature of cooperative links, as opposed to codes designed for co-located multiple-input multiple-output (MIMO) systems [27–29]. Antenna selection for MIMO systems is a way to realize the potential benefits of cooperation between multiple relays [31, 32]. In particular, some distributed single-relay selection algorithms are proposed that provides no performance loss from the perspective of diversity-multiplexing gain tradeoff.

1.3 Water-filling Power Allocation

Water-filling power allocation scheme is the optimal solution to the sum-rate maximization problem in a set of parallel communication subchannels [11, 12]. Let us consider a set of N parallel subchannels. The subchannel noises are assumed to be independent. It is shown in [13] that the sum-rate of this parallel channel is given by

$$\sum_{i=1}^N \log(1 + p_i \beta_i), \quad (1.3.1)$$

where p_i and β_i are, respectively, the power assigned to, and the signal-to-noise ratio (SNR) of the i th subchannel. Assuming that the channel state information (CSI) is available (i.e., β_i 's are known), the sum-rate maximization problem amounts to finding the optimal power allocation scheme that achieves the maximum sum-rate when the power constraint $\sum_{i=1}^N p_i \leq P_T$ is satisfied. The total available power at the

transmitter is denoted by P_T . Therefore, the optimization problem can be written as

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^N \log(1 + p_i \beta_i) \\ \text{subject to} \quad & \sum_{i=1}^N p_i \leq P_T \\ & p_i \geq 0 \quad \text{and} \quad 1 < i < N, \end{aligned} \tag{1.3.2}$$

where $\mathbf{p} \triangleq [p_1, p_2, \dots, p_N]^T$ is the vector of powers of different subchannels. It can be easily proved that at the optimum, the first constraint in (1.3.2) is satisfied with equality. Otherwise, if at the optimum, any of the first three constraints is not satisfied with equality, we can scale up the corresponding power vector such that that constraint is satisfied with equality. The new power vector achieves a higher sum-rate compared to the optimal power vector, thereby contradicting the optimality.

The solution to this optimization problem can be found with the aid of Lagrangian method. The main parameter in the solution is the so-called water level, which is chosen to satisfy the inequality constraint in (1.3.2) with equality. The parameter $\frac{1}{\lambda}$ shows the water level, where λ is the Lagrange multiplier corresponding to the optimization problem in (1.3.2) and k shows the number of subchannels. Water-filling algorithm is often referred to the simple strategy of pouring water into a vessel with its surface defined by the inverse channel gain. When the inverse subchannel gain is small, more power is transmitted in the corresponding subcarrier and when the inverse gain increases, the transmitted power in the corresponding subcarrier is significantly decreased [11, 14–16].

1.4 Motivation

In water-filling scheme, channels with better qualities receive relatively more power. As mentioned earlier, the water-filling approach is the optimal solution to the sum-rate maximization problem in parallel passive channels where there is no control on the gain of different subchannels. Obviously, the maximum achievable sum-rate of a passive channel depends on the source transmit power as well as on the quality of individual subchannels. It is obvious that based on the definition of an active channel, where the powers of different subchannels can be properly *designed* or optimally *adjusted* in order to achieve higher sum-rate, the classical water-filling approach is not an optimal solution to the sum-rate maximization problem over active channels. The sum-rate maximization problem for one-way active channels has been studied in [17–19], where it is shown that the achievable sum-rate of a one-way active channel can be larger than that of a passive channel. This improvement motivated us to generalize the idea of active channel and extend it to a two-way scenario.

1.5 Objective

We consider the sum-rate maximization problem over a reciprocal two-way active channel. Below, we bring a summary of our objectives in this thesis

- The main objective is to solve the sum-rate maximization problem for a two-way active channel, where the powers allocated by transceivers to each subchannel as well as the powers of each subchannel are considered as the optimization variables of the problem. To be more accurate, for each subchannel, the subchannel power is defined as the square gain of the subchannel and the total

channel power is defined as the summation of all subchannels powers. We consider three constraints for the optimization problem. The first constraint limits the total power consumed at the first transceiver and the second constraint restricts the total power consumed by the second transceiver. The third constraint controls the total power used in active subchannels. Considering these constraints, our goal is to jointly optimize the power of each subchannel as well as the power allocated by each transceiver to each subchannel in order to achieve the maximum sum-rate.

- The second objective is to find the optimal power distribution among the two transceivers, when the total power of two transceivers is given.
- Then we find the maximum sum-rate as well as the optimal power distribution among the two transceivers as well as the channel. The main constraint of this problem restricts the total power of the network.
- Finally, we solve the sum-rate maximization problem for non-reciprocal two-way active channels. In this problem, there are three constraints, controlling the powers of first transceiver, the power of the second transceiver, and the total power of the channel in both sides of transmission, respectively.

1.6 Methodology

In this section, we briefly review our methodology toward the aforementioned problems.

- The approach toward the first problem, which is the sum-rate maximization over a reciprocal two-way active channel, is to use the Karush-Kuhn-Tucker

(KKT) conditions in order to find the necessary conditions for the optimality. The KKT conditions are first order necessary conditions for an optimal solution in nonlinear programming, provided that some regularity conditions are satisfied. We use the KKT conditions to devise a semi-closed-form solution for the aforementioned optimization problem.

- In the second problem, to find the optimal power distribution among the two transceivers, we use the results of the solution to the first problem and prove that at the optimum the power should be distributed equally between the two transceivers.
- In the third problem, to find the optimal power distribution, when the total power of the network is given, we apply Lagrangian method to the problem.
- To solve the sum-rate maximization problem for the non-reciprocal two-way active channel, without loss of optimality, first we break the maximization problem into two sub-problems. Then, using the result of sum-rate maximization problem for the reciprocal one-way active channels, we calculate the optimal power distribution which lead to the maximum achievable sum-rate.

1.7 Summary of Results

In this section, we briefly review the summary of the results and contributions in this thesis.

- To solve the first problem, we prove that the sum-rate maximization problem over a two-way active channel has a unique global solution. We prove that at the

optimum, the transceivers may not necessarily provide power for all subchannels and only a subset of parallel subchannels may receive power. In order to design a two-way active channel which achieves the maximum sum-rate, first we need to find the optimal number of subchannels and then we distribute the total power among two transceivers and the channel.

- The solution to the second problem, which is the optimal power distribution among the two transceiver will result in equal power for each transceiver.
- In the third problem, to find the optimal power distribution when the total power of the network is given, we prove that at the optimum, half of the total power should be allocated to the active channel while the other half is equally distributed between the two transceivers.
- In the last problem, to solve the sum-rate maximization problem for a non-reciprocal two-way active channel, we derive a closed-form solution for the optimal power of the active channel allocated to each side of the transmission. Then, we find a semi-closed-form solution to the sum-rate optimization problem.

1.8 Outline of Thesis

The rest of this thesis is organized as follows. In Section 2.1, we first conduct a literature survey on parallel passive channels. In the 2.2, we analyze the power control techniques for parallel passive channels, where the best power allocation techniques are considered for OFDM systems as well as for MIMO channels. In Section 2.3, we define the active channels and review the sum-rate maximization problem for existing

examples of active channels. We review MIMO active channels including single user and multi user active channels. Also, we consider the sum-rate maximization problem for one-way active channels. Finally, we present our research contribution.

In Chapter 3, we study the sum-rate maximization problem for a two-way active channels. In Section 3.1, we present the system model. In Section 3.2, we describe the Karush-Kuhn-Tucker (KKT) conditions for our optimization problem and derive the necessary conditions for any optimal point. In Section 3.3, we focus on the solution of the problem. In Section 3.4, we analyze the two-way active channel considering constraints over the total network power. In Section 3.5, we analyze the non-reciprocal active channel and we calculate the optimal power allocation for the non-reciprocal two-way active channel and in Section 3.6, simulation results are presented.

In Chapter 4, we present the conclusion and the potential future work in this area of research.

Chapter 2

Literature Review

As we discussed in Chapter 1, cooperative communications has the potential to provide a more efficient throughput and reliability to wireless systems compared to direct communication. In this chapter, we first provide an overview of parallel passive channels. Then, we review power allocation techniques for passive Channels. Then, we review the idea of active channels and consider several types of active channels. We analyze the sum-rate maximization problem over MIMO active channels as well as the one-way active channels. It has been shown that the one-way active channels achieve higher sum-rate compared to passive channels with the same level of power. Finally, we present our research contribution.

2.1 Parallel Passive Channels

A parallel channel is defined as a link between a transmitter and a receiver where the transmitter/receiver is capable of communicating through different subchannels with interference between the subchannels. It is obvious that one of the main performance limiting parameters in each subchannel is the noise power. Many communication

channels, such as channels with inter-symbol interference, fading channels and multi-antenna systems, can be considered in the family of parallel Gaussian channels [4]. For example, in the case of fading channels, each subchannel corresponds to a fading state and in the case of ISI channels, each parallel subchannel corresponds to a specific frequency. Wireless parallel channels have been studied extensively in the literature, such as [5–10].

In conventional wireless parallel channels, there is no control over the gain of each individual subchannel. In this thesis we refer to this type of channels as passive channels. As mentioned before, for a set of N parallel subchannels, the sum-rate of the passive parallel channel is given by

$$\sum_{i=1}^N \log(1 + p_i \beta_i), \quad (2.1.1)$$

where p_i and β_i are, respectively, the power assigned to, and the signal-to-noise ratio (SNR) of the i th subchannel. Assuming that the channel state information (CSI) is available (i.e., β_i 's are known), the sum-rate maximization problem amounts to finding the optimal power allocation scheme that achieves the maximum sum-rate when the power constraint $\sum_{i=1}^N p_i \leq P_T$ is satisfied. The total available power at the transmitter is denoted by P_T . The optimal solution to the sum-rate maximization problem in a set of passive parallel subchannels under total power constraint is water-filling power allocation scheme.

The main parameter in the water filling power allocation scheme is the so-called water level, which is chosen to satisfy the power constraint such that the summation of the powers injected to all subchannels reach the total power budget. Water-filling algorithm is often referred to the simple strategy of pouring water into a vessel with its surface defined by the inverse channel gain. When the inverse subchannel gain is

small, more power is transmitted in the corresponding subcarrier and when the inverse gain increases, the transmitted power in the corresponding subcarrier is significantly decreased. In the next section, we overview power control techniques for passive channels.

2.2 Power Allocation Techniques for Passive Channels

Optimal power allocation strategies for wireless parallel channels have been at the center of attention. Here we briefly review these strategies.

OFDM Systems

Orthogonal frequency division multiplexing (OFDM) is a method of encoding digital data on multiple carrier frequencies. Applications of OFDM scheme can combat severe channel conditions such as attenuation of high frequency or narrow band interference. Assuming that channel state information at the transceiver (CSIT) is available, the water-filling power allocation strategy has been proved to be efficient in terms of sum-rate [7, 33, 34].

When the CSIT is not completely available, other modified versions of water-filling scheme are proposed in the literature for example in [35–39]. In [36], a power allocation scheme is proposed that minimizes the BER, assuming that the CSIT is available. In [37], an optimal power allocation scheme has been presented that maximizes the spectral efficiency. In [39], the authors generalize the idea of [38], assuming Nakagami-m fading over each subchannel and show that the water-filling scheme of [38] achieves

the maximum sum-rate. Unlike [38], in [39], the authors maximize the value of sum-rate not its upperbound.

Another example of wireless parallel channels is orthogonal frequency division multiple access (OFDMA) scheme, where each user has its own subchannel. The resource allocation for this system includes power and subcarrier allocation, which have been studied extensively in the literature [40–44]. It has been proved in [41, 42] that for downlink scenarios, sum-rate of the system achieves its upperbound when each subcarrier is allocated only to the user with the best channel gain for that subcarrier. Furthermore in [43], a numerical method for characterizing the achievable rate region is proposed for Gaussian multiple-access channel with intersymbol interference under the frequency division multiple access restriction. In [44], the authors focus on joint subcarrier and power allocation in the uplink of an OFDMA system and maximize the sum-rate capacity in the uplink.

MIMO Channels

Applications of MIMO channel have been at the center of attention of the research community . It is proved that having multiple antennas at transmitter and receiver improves the capacity of the communication link [7, 45, 46].

Resource allocation in MIMO communication systems has been a recent research trend. Power allocation schemes to achieve the capacity of the channel or to minimize the bit error rate (BER) or the mean squared error (MSE) in MIMO systems have been studied extensively in the literature [33, 47–53].

In [46], the authors aimed to jointly design a precoder and decoder for a MIMO system, using a weighted minimum mean-square error (MMSE) approach while the

total transmit power is limited. In [52, 54], the goal is to jointly design a precoder and a decoder to achieve the maximum sum-rate as well as minimizing the un-weighted MMSE while the quality of service (QoS) is satisfied in each subchannel. (see also, [33, 49–51, 53, 55, 56]).

There exists several researches to solve sum-rate maximization problem for MMO channels. In [38], the authors prove that the statistical water-filling scheme, which is the typical water-filling approach over the statistical parameters of channel instead of instantaneous parameters, will result in achieving the upperbound for correlated MIMO channel with partial CSIT. There are some different methods in the literature to calculate the water level in water filling algorithm. For example, the water level can be calculated through an iterative algorithm. There are some closed-form methods that will result in the exact value of water level [33, 52, 57]. In [58], the authors consider vector communication through a MIMO channel with a set of QoS requirements. The authors in [58], aim to optimally design joint transmit/receive linear processing to satisfy the QoS requirement with minimum transmit power and prove that the solution of the problem is a multi-level water-filling scheme. The authors in [59], form a unified viewpoint to bridge the gap between the family of water-filling solutions and their efficient implementations in practice.

2.3 Sum-rate Maximization for Active Channels

Active channel is defined as a parallel channel where by injecting power into the subchannels, somewhere between a transmitter and a receiver, the gain of the subchannels are adjusted. There are several examples of active channels in the literature. In this

section, we review these active channels and the solutions to the sum-rate maximization problem for active channels.

2.3.1 MIMO Active Channel

The joint optimization of channel and power allocation in MIMO communications will result in a new class of energy-constrained channels. This class of channels fall into the category of active channels. In [60–62], the authors find an upperbound on capacity of MIMO channels and show that this upperbound may be used to direct the adaptive antenna array configuration.

Single User:

In [61,62], the authors study single-user MIMO active channels with equal noise power over different subchannels. The capacity of a multi-antenna Gaussian channel is studied in [62] and it is shown that the location of the antenna can be chosen optimally to maximize the sum-rate of a MIMO channel under transceiver power and channel energy constraints. For sufficiently large SNRs, it has been proved that the maximum sum-rate is achievable when the power is distributed among all subchannels uniformly. In [62], the MIMO channel is transformed into a set of subchannels, where each of them is corresponding to one eigen mode. Square of the corresponding eigenvalues of channel matrix shows the strength of each subchannel. Relocating antennas will result in another version of channel matrix. Therefore, the sum-rate maximization problem can be transformed to the problem of finding the optimal position of transmitter/receiver antenna such that the channel matrix produces equal eigenvalues.

The capacity of a MIMO channel under the transmit power constraint and channel norm constraint is studied in [61]. It has been proved that the maximum sum-rate is

achieved when the channel has equal singular values for all non-zero eigen modes.

Multi User :

There have been some investigation on the multi-user MIMO active channels in the literature. In [60], the authors consider a multi-user MIMO channel while channel energy is limited and the noise powers are assumed to be equal over different subchannels. The goal of [60] is to find the maximum sum-rate over all possible channel states, assuming the total energy of the channel is limited. It has been proved that when the user channels are mutually orthogonal, for sufficiently large SNRs, the sum-rate of the network achieves the upperbound. Also, it has been shown that at optimum, the channel energy and the transmit power are equally distributed among non-zero MIMO eigen modes.

2.3.2 One-way Active Channel

In this subsection, we review the sum-rate maximization problem for two types of one-way active channels. The first type is an active channel with equal subchannel noise powers while in the second type subchannel noise powers are assumed to be unequal. The one-way active channel has been studied extensively in [17–19, 63]

Equal Subchannel Noise Powers

In [17], the authors consider the sum-rate maximization problem for parallel active channels under two main constraints. The first constraint is on the total transmit power and the second one controls the total power of the subchannels. They aim to jointly optimize the powers of each subchannel and the transmit power of the source over each individual subchannel, such that the sum-rate is maximized. They showed that this maximization problem is not convex. Therefore, using the KKT conditions,

they develop a semi closed-form solution to the problem. It has been proved that at the optimum, some of the subchannels receive zero power. This means that to achieve the maximum value of sum-rate, some of the subchannels should be turned off. It also has been proved that to reach the maximum sum-rate, the total channel power must be allocated uniformly to non-zero subchannels. They also showed that if the sum of the powers available to the source and to the channel is limited, to maximize the sum-rate, half of the total available power should be allocated to the active channel and the other half should be allocated to the source.

Unequal Subchannel Noise Powers

In [18, 19], the authors study the sum-rate maximization problem for one-way active parallel channels under two main constraints, assuming unequal subchannel noise powers. The first constraint limits the total power of the transmitter and the second constraint controls the power of the active channel. Under these constraints, the goal is to jointly optimize the power of each individual subchannel as well as the transmitted power allocated to each subchannel such that the sum-rate is maximized. Subchannel powers and the source power over different subchannels are considered as the variables of the optimization problem. The authors, use the KKT conditions and show that the KKT conditions can be used to determine the number of subchannels that can be active for the source power constraint to be feasible. They propose a computationally efficient method to calculate the feasible number of active subchannels. It is proved that only a subset of subchannels may receive power from the source, and at the optimum, the source power should be distributed uniformly among the active subchannels. Simulation results show that active channels outperform passive channels in terms of sum-rate, considering the same number of subchannels and the

same amount of total network power.

2.4 Research Contribution

In this thesis, we consider the sum-rate maximization problem over a two-way active channel and we present the solution to four problems. First, we analyze the sum-rate maximization problem for a two-way active channel, where the powers allocated by transceivers to each subchannel as well as the powers of each subchannel are considered as the optimization variables of the problem. We jointly optimize the power of each subchannel as well as the power allocated by each transceiver to each subchannel in order to achieve the maximum sum-rate. We prove that the sum-rate maximization problem over a two-way active channel has a unique global solution. We prove that at the optimum, the transceivers may not necessarily provide power for all subchannels and only a subset of parallel subchannels may receive power.

The second problem is to find the optimal power distribution among the two transceivers, when the total power of two transceivers is given. The solution to the second problem, which is the optimal power distribution among the two transceiver will result in equal power for each transceiver.

Then we find the maximum sum-rate as well as the optimal power distribution among the two transceivers as well as the channel. The main constraint of this problem restricts the total power of the network. We prove that at the optimum, half of the total power should be allocated to the active channel while the other half is equally distributed between the two transceivers.

Finally, we solve the sum-rate maximization problem for non-reciprocal two-way active channels. We derive a closed-form solution for the optimal power of the active channel allocated to each side of the transmission. Then, we find a semi-closed-form solution to the sum-rate optimization problem.

Chapter 3

Sum-rate Maximization for Two-way Active Channel

In this chapter, we consider the sum-rate maximization problem for two-way active channels. First, we present system model as well as the sum-rate maximization problem for two-way scenario. Then, we review the KKT conditions, which is the main tool to approach the optimization problem. There are several theorems that lead to the solution. Also, we consider the sum-rate maximization problem under some total constraints and then we analyze the non-reciprocal two-way active channels. Finally, present the simulation results.

3.1 System Model and Sum-rate Maximization

We consider a two-way active parallel channel, where two transceivers communicate with each other through N orthogonal parallel subchannels. A two-way parallel channel is referred to as active meaning that by “injecting” power into each subchannel, somewhere between the two transceivers, we can control the gain of that subchannel. We assume that the subchannel noise powers are the same, and without loss

of generality, are all equal to 1. We further assume that the channels are reciprocal, meaning that for every subchannel, the gain, when the message is sent from one transceiver to the other one is equal to the gain of that subchannel when the message is sent in the opposite direction. Later, in Section 3.5, we analyze the case of active channel with non-reciprocal subchannels. Since the subchannels are orthogonal, transmission in each subchannel does not produce interference in the other subchannels. Also, assuming that each transceiver knows its transmitted signal perfectly, the self-interference can be completely canceled. we define $\bar{q} = 1$, if $q = 2$, and $\bar{q} = 2$, if $q = 1$. Let \tilde{p}_{qi} denote the transmit power of Transceiver q over the i th subchannel whose squared gain is represented by \tilde{h}_i . Without loss of generality, we assume that the n th subchannel squared gain is real valued as the subchannel phases do not affect the subchannel SNRs. The signal r_q received at Transceiver q can be written as

$$r_q = \sqrt{\tilde{p}_{\bar{q}i}\tilde{h}_i}s_{\bar{q}i} + n_{qi}, \quad (3.1.1)$$

where $s_{\bar{q}i}$ is the signal transmitted by Transceiver \bar{q} over the i th subchannel and n_{qi} is the received noise at Transceiver q . This model is applicable, for example, to two-way multi-carrier relay systems, when the relay noises at different subcarriers are negligible. As mentioned before, the total transmit power of Transceiver q is defined as the summation of the transceiver powers allocated to all subchannels at that transceiver, i.e., the transmit power of Transceiver q is given by $\sum_{i=1}^N \tilde{p}_{qi}$, for $q \in \{1, 2\}$. The total power of the active channel is defined as the sum of the powers of different subchannels, i.e., the total power of the active channel is given by $\sum_{i=1}^N \tilde{h}_i$. The total transmit power of the first transceiver, that of the second transceiver, and the total power of the channel are assumed to be limited by P_1^{\max} , P_2^{\max} , and P_h^{\max} , respectively. We denote the rate of the i th subchannel at Transceiver q as R_{qi} and

write this rate as

$$R_{qi} = \log_2(1 + \tilde{p}_{\tilde{q}i}\tilde{h}_i). \quad (3.1.2)$$

Then, the problem of sum-rate maximization for the two-way active channel can be written as

$$\begin{aligned} & \max_{\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}}} \sum_{i=1}^N \log_2(1 + \tilde{p}_{1i}\tilde{h}_i) + \sum_{i=1}^N \log_2(1 + \tilde{p}_{2i}\tilde{h}_i) \\ \text{subject to} & \quad \mathbf{1}^T \tilde{\mathbf{p}}_1 \leq P_1^{\max}, \quad \mathbf{1}^T \tilde{\mathbf{p}}_2 \leq P_2^{\max}, \quad \mathbf{1}^T \tilde{\mathbf{h}} \leq P_h^{\max} \\ & \quad \tilde{\mathbf{p}}_1 \succcurlyeq 0, \quad \tilde{\mathbf{p}}_2 \succcurlyeq 0, \quad \tilde{\mathbf{h}} \succcurlyeq 0 \end{aligned} \quad (3.1.3)$$

where $\tilde{\mathbf{p}}_1 \triangleq [\tilde{p}_{11} \ \tilde{p}_{12} \ \dots \ \tilde{p}_{1N}]^T$ and $\tilde{\mathbf{p}}_2 \triangleq [\tilde{p}_{21} \ \tilde{p}_{22} \ \dots \ \tilde{p}_{2N}]^T$ are the $N \times 1$ vectors of transmit powers of Transceivers 1 and 2 over different subchannels, respectively, and $\tilde{\mathbf{h}} \triangleq [\tilde{h}_1 \ \tilde{h}_2 \ \dots \ \tilde{h}_N]^T$ represents the $N \times 1$ vector of subchannel powers. The first two constraints in (3.1.3) control the total power consumed by the two transceivers, while the third constraint limits the power of the active channel.

The optimization problem in (3.1.3) is non-convex and cannot be solved using convex optimization techniques. To solve the optimization problem in (3.1.3), we propose an algorithm whereby we break down the optimization problem in (3.1.3) into a finite number of sub-problems and find the solution to each sub-problem, if such a solution exists¹. Then, we find the global solution to the optimization problem in (3.1.3) by searching among the solutions to these subproblems. We prove that this global solution exists, and it is unique. To do this, let us partition the feasible set of the optimization problem (3.1.3), denoted as \mathcal{F} , into N different sub-sets. That is, we

¹Note that, as we show later, each sub-problem may or may not have a solution.

define the set \mathcal{F}_n as

$$\mathcal{F}_n = \{\Theta = (\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}}) \in \mathcal{F} \mid \tilde{\mathbf{h}} \text{ has only } n \text{ non-zero elements}\}, \quad (3.1.4)$$

$$\text{for } n = 1, \dots, N. \quad (3.1.5)$$

It is easy to see that the sets $\{\mathcal{F}_n\}_{n=1}^N$ satisfy the following conditions:

$$\bigcup_{n=1}^N \mathcal{F}_n = \mathcal{F} \quad (3.1.6)$$

$$\mathcal{F}_n \cap \mathcal{F}_m = \emptyset \quad \text{for } n \neq m. \quad (3.1.7)$$

That the intersection of every two distinct subsets is empty, and that the union of all subsets is equal to the feasible set \mathcal{F} , imply that $\{\mathcal{F}_n\}_{n=1}^N$ is a valid partitioning of \mathcal{F} . Using this partitioning, we can write the optimization problem in (3.1.3) as

$$\begin{aligned} \max_{n \in \mathcal{N}} \quad & \max_{\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}} \in \mathcal{F}_n} \sum_{i=1}^n \log_2(1 + \tilde{p}_{1i} \tilde{h}_i) + \sum_{i=1}^n \log_2(1 + \tilde{p}_{2i} \tilde{h}_i) \\ \text{subject to} \quad & \mathbf{1}^T \tilde{\mathbf{p}}_1 \leq P_1^{\max}, \quad \mathbf{1}^T \tilde{\mathbf{p}}_2 \leq P_2^{\max}, \quad \mathbf{1}^T \tilde{\mathbf{h}} \leq P_h^{\max} \\ & \tilde{\mathbf{p}}_1 \succ 0, \quad \tilde{\mathbf{p}}_2 \succ 0, \quad \tilde{\mathbf{h}} \succ 0 \end{aligned} \quad (3.1.8)$$

where $\mathcal{N} \triangleq \{1, 2, \dots, N\}$. It is quite straightforward to show that if for a certain subchannel, $\tilde{h}_i = 0$ holds true, the corresponding transceiver powers should also be zero, i.e., $\tilde{p}_{1i} = \tilde{p}_{2i} = 0$. Hence, if there are n non-zero subchannels, without loss of optimality, we can assume that the transceivers allocate power only to those n subchannels. Subsequently, we can express the optimization problem in (3.1.8) as

$$\begin{aligned} \max_{n \in \mathcal{N}} \quad & \max_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{h}} \sum_{i=1}^n \log_2(1 + p_{1i} h_i) + \sum_{i=1}^n \log_2(1 + p_{2i} h_i) \\ \text{subject to} \quad & \mathbf{1}^T \mathbf{p}_1 \leq P_1^{\max}, \quad \mathbf{1}^T \mathbf{p}_2 \leq P_2^{\max}, \quad \mathbf{1}^T \mathbf{h} \leq P_h^{\max} \\ & \mathbf{p}_1 \succ 0, \quad \mathbf{p}_2 \succ 0, \quad \mathbf{h} \succ 0. \end{aligned} \quad (3.1.9)$$

Here, h_i is the i th entry of \mathbf{h} , the $n \times 1$ vector \mathbf{h} contains the n non-zero elements of $\tilde{\mathbf{h}}$, while \mathbf{p}_1 and \mathbf{p}_2 are $n \times 1$ vectors which contain the corresponding n non-zero entries of $\tilde{\mathbf{p}}_1$ and $\tilde{\mathbf{p}}_2$, and p_{1i} and p_{2i} are the i th entries of \mathbf{p}_1 and \mathbf{p}_2 , respectively. Note that in (3.1.9), the dimension of the inner maximization problem is the variable of the outer maximization. Indeed, the outer maximization searches among all values of n to find the optimal value of the number of subchannels in the active channel that are switched on. Note that at the optimum, the first three constraints should be satisfied with equality. Otherwise, if at the optimum, any of the first three constraints is not satisfied with equality, we can scale up the corresponding power vector such that constraint is satisfied with equality. The new power vector achieves a higher sum-rate, thereby contradicting the optimality.

3.2 KKT Conditions

In this section, we use the Karush-Kuhn-Tucker (KKT) conditions to develop necessary conditions for any local maximizer of the inner maximization problem in (3.1.9). The KKT conditions provide first-order necessary conditions for any local optimum point provided that some regularity conditions are satisfied [76]. It is worth mentioning that the KKT conditions are necessary conditions for the optimality if and only if the duality gap is zero. Since the constraints in (3.1.9) are linear, the linear constraint qualification is satisfied, and therefore, the duality gap is zero. Now considering the inner maximization problem in (3.1.9), for any fixed n , the Lagrangian

for this maximization problem can be written as

$$\begin{aligned}
\mathcal{L}_n(\mathbf{p}_1, \mathbf{p}_2, \mathbf{h}, \lambda_1, \lambda_2, \lambda_3, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\mu}_3) &\triangleq - \sum_{i=1}^n \log_2(1 + p_{1i}h_i) + \log_2(1 + p_{2i}h_i) \\
&+ \lambda_1(\mathbf{1}^T \mathbf{p}_1 - P_1^{\max}) \\
&+ \lambda_2(\mathbf{1}^T \mathbf{p}_2 - P_2^{\max}) \\
&+ \lambda_3(\mathbf{1}^T \mathbf{h} - P_h^{\max}) \\
&- \boldsymbol{\mu}_1^T \mathbf{p}_1 - \boldsymbol{\mu}_2^T \mathbf{p}_2 - \boldsymbol{\mu}_3^T \mathbf{h}. \tag{3.2.1}
\end{aligned}$$

where $\lambda_1, \lambda_2, \lambda_3, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2,$ and $\boldsymbol{\mu}_3$ are the Lagrange multiplier coefficients/vectors.

Applying the KKT conditions, any local optimum of the inner maximization problem in (3.1.9) is required to satisfy the following conditions:

Primal feasibility:

$$\mathbf{1}^T \mathbf{p}_1 = P_1^{\max} \tag{3.2.2}$$

$$\mathbf{1}^T \mathbf{p}_2 = P_2^{\max} \tag{3.2.3}$$

$$\mathbf{1}^T \mathbf{h} = P_h^{\max} \tag{3.2.4}$$

$$\mathbf{p}_1 \succcurlyeq 0, \quad \mathbf{p}_2 \succcurlyeq 0, \quad \mathbf{h} \succcurlyeq 0. \tag{3.2.5}$$

Dual feasibility:

$$\boldsymbol{\mu}_1 \succeq 0, \quad \boldsymbol{\mu}_2 \succeq 0, \quad \boldsymbol{\mu}_3 \succeq 0. \tag{3.2.6}$$

Complementary slackness:

$$\lambda_1(\mathbf{1}^T \mathbf{p}_1 - P_1^{\max}) = 0 \tag{3.2.7}$$

$$\lambda_2(\mathbf{1}^T \mathbf{p}_2 - P_2^{\max}) = 0 \tag{3.2.8}$$

$$\lambda_3(\mathbf{1}^T \mathbf{h} - P_h^{\max}) = 0 \tag{3.2.9}$$

$$\boldsymbol{\mu}_1 \odot \mathbf{p}_1 = 0, \quad \boldsymbol{\mu}_2 \odot \mathbf{p}_2 = 0, \quad \boldsymbol{\mu}_3 \odot \mathbf{h} = 0. \tag{3.2.10}$$

Stationary conditions:

$$\frac{\partial \mathcal{L}_n(\mathbf{P}_1, \mathbf{P}_2, \mathbf{h}, \lambda_1, \lambda_2, \lambda_3, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\mu}_3)}{\partial p_{1i}} = 0 \quad (3.2.11)$$

$$\frac{\partial \mathcal{L}_n(\mathbf{P}_1, \mathbf{P}_2, \mathbf{h}, \lambda_1, \lambda_2, \lambda_3, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\mu}_3)}{\partial p_{2i}} = 0 \quad (3.2.12)$$

$$\frac{\partial \mathcal{L}_n(\mathbf{P}_1, \mathbf{P}_2, \mathbf{h}, \lambda_1, \lambda_2, \lambda_3, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\mu}_3)}{\partial h_i} = 0 \quad (3.2.13)$$

Since we have assumed that for $i = 1, 2, \dots, n$, the optimization variables h_i , p_{1i} , and p_{2i} are positive, then the primal feasibility and complementary slackness conditions imply that $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_2 = \boldsymbol{\mu}_3 = \mathbf{0}$ holds true.

3.3 Solution

In this section, we show how the KKT conditions can be used to solve the inner maximization problem in (3.1.9). To do so, the following theorem plays a key role.

Theorem 1: *For any value of $n \in \{1, 2, \dots, N\}$, the KKT conditions yield a unique solution, which is locally² optimum for the inner maximization problem in (3.1.9).*

Proof: To prove this theorem, we first use the stationary conditions to find the local maximizer of the inner maximization problem in (3.1.9). Using (3.2.11)-(3.2.13), we obtain

$$\frac{1}{\ln 2} \times \frac{-h_i}{1 + p_{1i}h_i} + \lambda_1 = 0, \quad \text{for } i = 1, 2, \dots, n \quad (3.3.1)$$

$$\frac{1}{\ln 2} \times \frac{-h_i}{1 + p_{2i}h_i} + \lambda_2 = 0, \quad \text{for } i = 1, 2, \dots, n \quad (3.3.2)$$

$$\frac{1}{\ln 2} \left(\frac{-p_{1,i}}{1 + p_{1i}h_i} + \frac{-p_{2i}}{1 + p_{2i}h_i} \right) + \lambda_3 = 0 \quad \text{for } i = 1, 2, \dots, n. \quad (3.3.3)$$

²Note that a local optimum may or may not be a global optimum.

Using (3.3.1) and defining $\tilde{\lambda}_1 \triangleq \frac{1}{\lambda_1 \ln 2}$, we can write

$$p_{1i} = \tilde{\lambda}_1 - \frac{1}{h_i}, \quad \text{for } i = 1, 2, \dots, n. \quad (3.3.4)$$

Using (3.3.4) along with the fact that at each local optimum³, $\mathbf{1}^T \mathbf{p}_1 = P_1^{\max}$ must hold true, we can write

$$P_1^{\max} = \sum_{k=1}^n p_{1k} = \sum_{k=1}^n \left(\tilde{\lambda}_1 - \frac{1}{h_k} \right) = n\tilde{\lambda}_1 - \sum_{k=1}^n \frac{1}{h_k} \quad (3.3.5)$$

and hence, we obtain $\tilde{\lambda}_1$, as

$$\tilde{\lambda}_1 = \frac{P_1^{\max} + \sum_{k=1}^n \frac{1}{h_k}}{n}. \quad (3.3.6)$$

Substituting (3.3.6) in (3.3.4), we obtain the locally optimal value of p_{1i} , in terms of $\{h_i\}_{i=1}^n$, as

$$p_{1i} = \frac{P_1^{\max} + \sum_{k=1}^n \frac{1}{h_k}}{n} - \frac{1}{h_i}, \quad \text{for } i = 1, 2, \dots, n. \quad (3.3.7)$$

Similarly, we can obtain the locally optimal value for p_{2i} , in terms of $\{h_i\}_{i=1}^n$, as

$$p_{2i} = \frac{P_2^{\max} + \sum_{k=1}^n \frac{1}{h_k}}{n} - \frac{1}{h_i}, \quad \text{for } i = 1, 2, \dots, n. \quad (3.3.8)$$

Substituting (3.3.7) and (3.3.8) in (3.3.3), we obtain the following relationship between $\{h_i\}_{i=1}^n$:

$$2h_i = -\lambda_3 + \frac{P_1^{\max} + \sum_{k=1}^n \frac{1}{h_k}}{n} + \frac{P_2^{\max} + \sum_{k=1}^n \frac{1}{h_k}}{n}. \quad (3.3.9)$$

³Note that in our problem KKT conditions are necessary for local optimality.

Since the right hand side of (3.3.9) is independent of i , we conclude that h_i 's are equal to each other, i.e., $h_1 = h_2 = \dots = h_n$. Now, we can use the constraint $\mathbf{1}^T \mathbf{h} = P_h^{\max}$ to obtain the locally optimal values of $\{h_i\}_{i=1}^n$ as

$$h_1^{(n)} = h_2^{(n)} = \dots = h_n^{(n)} = \frac{P_h^{\max}}{n} \quad (3.3.10)$$

where $h_i^{(n)}$ is the solution to the KKT conditions for the inner maximization in (3.1.9) and the superscript (n) is used to emphasis that while solving the inner maximization in (3.1.9), the dimension of \mathbf{h} is n . Replacing (3.3.10) in (3.3.7) and (3.3.8), we can find the corresponding locally optimal values of p_{1i} and p_{2i} , denoted respectively by $p_{1i}^{(n)}$ and $p_{2i}^{(n)}$, as

$$p_{1i}^{(n)} = \frac{P_1^{\max} + \frac{n^2}{P_h^{\max}}}{n} - \frac{n}{P_h^{\max}} = \frac{P_1^{\max}}{n} \quad \text{for } i = 1, 2, \dots, n \quad (3.3.11)$$

$$p_{2i}^{(n)} = \frac{P_2^{\max} + \frac{n^2}{P_h^{\max}}}{n} - \frac{n}{P_h^{\max}} = \frac{P_2^{\max}}{n}, \quad \text{for } i = 1, 2, \dots, n \quad (3.3.12)$$

With the unique solution given in (3.3.10), (3.3.11), and (3.3.12), the proof is now complete. ■

This proof implies that the solution to the KKT conditions (which is locally optimal for the inner maximization in (3.1.9)) requires a uniform distribution of the power at the two Transceivers and among the subchannels.

For any fixed value of n , let us substitute the optimal values $h_i^{(n)}$, $p_{1i}^{(n)}$ and $p_{2i}^{(n)}$, as derived in (3.3.10), (3.3.11) and (3.3.12), respectively, in the objective function of the optimization problem in (3.1.9). Doing so, for any n , the locally maximum value of the objective function of (3.1.9) is given by function $f(n)$, which is defined as

$$f(n) \triangleq n \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1 \right). \quad (3.3.13)$$

Note that, for any value of n , the value of $f(n)$, is the locally maximum rate (i.e., it is obtained by solving KKT conditions as shown above) that can be achieved with n non-zero active subchannels.

Now we analyze the function $f(n)$, find its maximum value for all values of n , and prove that this maximum is unique and that this maximum is the maximum value for the sum-rate of the two-way active channel. Let us find the solution to the following problem:

$$\max_{n \in \mathcal{N}} f(n). \quad (3.3.14)$$

To do so, we relax the discrete variable n and replace it with a continuous variable x , where $1 \leq x \leq N$. It can be shown that for any positive values of α and β , the function $s(x; \alpha, \beta) \triangleq x \log(1 + \alpha\beta/x^2)$ has a unique real-valued maximizer for $x > 0$. As it is clearly shown in (3.3.13), $f(x)$ is the summation of $s(x; P_1^{\max}, P_h^{\max})$ and $s(x; P_2^{\max}, P_h^{\max})$. We now prove that $f(x)$ has a unique real-valued maximizer.

Theorem 2: *The function $f(x)$ defined as*

$$f(x) = x \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{x^2} + 1 \right) + x \log_2 \left(\frac{P_2^{\max} P_h^{\max}}{x^2} + 1 \right), \quad (3.3.15)$$

has one and only one real-valued maximizer, for $x > 0$.

Proof: See the appendix.

Now using an iterative algorithm, for example bisection technique, we can find the unique maximizer of the function $f(x)$. Let x° denote this maximizer. Note that in (3.3.14), we are looking for an integer value of $n \in \mathcal{N}$, which results in the largest value for $f(n)$. We deduce that if $x^\circ \geq N$, then $f(N)$ is the largest value among $\{f(n)\}_{n=1}^N$. If $x^\circ < N$, either the smallest integer greater than x° or the largest

integer smaller than x° is the solution to (3.3.14). In this case, we calculate the value of $f(n)$ for these two integers and find the largest value for $f(n)$. Let us define

$$n^\circ \triangleq \arg \max_{n \in \mathcal{N}} f(n). \quad (3.3.16)$$

Now we prove that for $n > n^\circ$, the inner maximization problem in (3.1.9) does not have a global optimum, meaning that for $n > n^\circ$, one cannot find a point in the feasible set, which results in the largest possible value for the objective function.

Theorem 3: *For any $n > n^\circ$, the local optimum of the inner maximization problem in (3.1.9) is not the global optimum for this maximization problem. Also, the inner maximization problem in (3.1.9) does not have a solution.*

Proof: Consider a subchannel power distribution as below:

$$\begin{cases} h_i = \frac{P_h^{\max}}{n^\circ} - \frac{n - n^\circ}{n^\circ} \epsilon, & \text{for } i = 1, \dots, n^\circ, \\ h_i = \epsilon, & \text{for } i = n^\circ + 1, \dots, n. \end{cases} \quad (3.3.17)$$

where, as defined in (3.3.16), n° is the integer value for which $f(n)$ achieves its maximum. It is easy to verify that,

$$\sum_{i=1}^n h_i = P_h^{\max}. \quad (3.3.18)$$

Now we introduce the transmit powers of Transceivers 1 and 2 over all $n > n^\circ$ subchannels as

$$p_{1i} = \frac{h_i}{P_h^{\max}} P_1^{\max}, \quad \text{for } i = 1, \dots, n, \quad (3.3.19)$$

$$p_{2i} = \frac{h_i}{P_h^{\max}} P_2^{\max}, \quad \text{for } i = 1, \dots, n. \quad (3.3.20)$$

It is quite straightforward to see that

$$\sum_{i=1}^n p_{1i} = P_1^{\max}, \quad (3.3.21)$$

$$\sum_{i=1}^n p_{2i} = P_2^{\max}. \quad (3.3.22)$$

It follows from (3.3.18), (3.3.21), and (3.3.22) that (3.3.17), (3.3.19), and (3.3.20) form a feasible point of the optimization problem in (3.1.9). Now, we calculate the sum-rate corresponding to this specific distribution by substituting (3.3.19) and (3.3.20) into the objective function of (3.1.9) and let ϵ approach zero. Doing so, we can write

$$\begin{aligned} & \lim_{\epsilon \rightarrow 0} \sum_{i=1}^n \log_2 \left(1 + \left(\frac{h_i}{P_h^{\max}} P_1^{\max} \right) h_i \right) + \log_2 \left(1 + \left(\frac{h_i}{P_h^{\max}} P_2^{\max} \right) h_i \right) = \\ & \lim_{\epsilon \rightarrow 0} \sum_{i=1}^{n^\circ} \log_2 \left(1 + \left(\frac{h_i}{P_h^{\max}} P_1^{\max} \right) h_i \right) + \log_2 \left(1 + \left(\frac{h_i}{P_h^{\max}} P_2^{\max} \right) h_i \right) + \\ & \lim_{\epsilon \rightarrow 0} \sum_{i=n^\circ+1}^n \log_2 \left(1 + \left(\frac{h_i}{P_h^{\max}} P_1^{\max} \right) h_i \right) + \log_2 \left(1 + \left(\frac{h_i}{P_h^{\max}} P_2^{\max} \right) h_i \right) \end{aligned} \quad (3.3.23)$$

Note that $\lim_{\epsilon \rightarrow 0} h_i = \frac{P_h^{\max}}{n^\circ}$, for $i = 1, \dots, n^\circ$ and it is equal to zero otherwise. Therefore, the last expression in (3.3.23) is equal to zero, and thus, we can write (3.3.23) as

$$f(n^\circ) = n^\circ \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^{\circ 2}} + 1 \right) + n^\circ \log_2 \left(\frac{P_2^{\max} P_h^{\max}}{n^{\circ 2}} + 1 \right). \quad (3.3.24)$$

Note that (3.3.24) implies that for the power distributions in (3.3.17), (3.3.19), and (3.3.20), when ϵ approaches zero, the sum-rate will approach the maximum value of the function $f(n)$, i.e., $f(n^\circ)$. Note that $f(n^\circ)$ is the upper bound of the achievable rate for $n > n^\circ$, but this upper bound is not achievable by any feasible point in the feasible set of the inner maximization problem in (3.1.9), rather this bound can be approached arbitrarily closely, hence, it can be concluded that for any $n > n^\circ$, the inner maximization problem in (3.1.9) does not have a solution. \blacksquare

Now we have the requirements to state and prove the main theorem of this paper. To do so, let us add one more definition. As mentioned before, \mathcal{F}_n is set of all elements in

\mathcal{F} , such as $\Theta = (\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})$ that contains three vectors, each with the same n nonzero elements. In other words, each element in \mathcal{F}_n contains three vectors, each with only $N - n$ zero elements. Suppose that, for each element, such as $\Theta \in \mathcal{F}_n$, we delete the zero elements of all three vectors, and produce a new triple vector, such as $\tilde{\Theta}_n$, which contains three vectors, each with dimension n . Now, we define $\tilde{\mathcal{F}}_n$ as

$$\tilde{\mathcal{F}}_n \triangleq \{\tilde{\Theta}_n | \Theta \in \mathcal{F}_n\}, \quad \text{for } n = 1, \dots, N. \quad (3.3.25)$$

We also define the function of the sum-rate function $R(\cdot)$ over the domain $\tilde{\mathcal{F}}_n$, as

$$\forall \tilde{\Theta}_n \in \tilde{\mathcal{F}}_n; \quad R(\tilde{\Theta}_n) \triangleq \sum_{i=1}^n \log_2(1 + p_{1i}h_i) + \sum_{i=1}^n \log_2(1 + p_{2i}h_i) \quad (3.3.26)$$

where p_{1i} , p_{2i} , and h_i are the i th elements of the first, second, and the third vector of $\tilde{\Theta}_n$, respectively. This function maps each element of $\tilde{\mathcal{F}}_n$ to its corresponding sum-rate achieved by that distribution.

Theorem 4: *The maximum of the function $f(n)$ for $n \in \mathcal{N}$, i.e., $f(n^\circ)$, is the global maximum of the optimization problem in (3.1.9).*

Proof: We define “plus-supremum” of any $\tilde{\mathcal{F}}_n$, as an element $\tilde{\Gamma}_k$ of another subset $\tilde{\mathcal{F}}_k$, ($k \neq n$) if

$$\text{i) } \sup\{\mathcal{A}\} = R(\tilde{\Gamma}_k) \quad \text{where } \mathcal{A} = \{R(\tilde{\Theta}_n) | \forall \tilde{\Theta}_n \in \tilde{\mathcal{F}}_n\} \quad (3.3.27)$$

$$\text{ii) } \nexists \tilde{\Theta}_n \in \tilde{\mathcal{F}}_n \quad \text{such that} \quad R(\tilde{\Theta}_n) \geq R(\tilde{\Gamma}_k) \quad (3.3.28)$$

It is worth mentioning that the plus-supremum does not necessarily exist for the feasible set of each inner optimization problem in (3.1.9). Theorem 3 proves that, for $n > n^\circ$, there exists a plus-supremum for the feasible set of each inner maximization problem with dimension n , and its corresponding rate is equal to $f(n^\circ)$.

Now if we prove that for $n \leq n^\circ$, the value of the plus-supremum for each $\tilde{\mathcal{F}}_n$, if

it exists, is less than or equal to $f(n^\circ)$, the proof is complete. To prove this, we use contradiction. Suppose that for certain values of n , where $n \leq n^\circ$, there is a plus-supremum for $\tilde{\mathcal{F}}_n$. We choose the largest plus-supremum among all these plus-supremums, and suppose, without loss of generality, that this largest plus-supremum corresponds to the inner maximization with dimension n_s and the set $\tilde{\mathcal{F}}_{n_s}$. Based on the definition, plus-supremum of a set does not belong to that set, so we assume that the plus-supremum of $\tilde{\mathcal{F}}_{n_s}$ is $\tilde{\Gamma}_{n_p}$ which belongs to $\tilde{\mathcal{F}}_{n_p}$, where $n_p \neq n_s$.

Now based on the value of n_p , two scenarios can happen. First scenario is the case where $n_p > n^\circ$ and in the second scenario $n_p \leq n^\circ$. In the first scenario, Theorem 3 introduces that all the plus-supremums, and proves that

$$R(\tilde{\Gamma}_{n_p}) = f(n^\circ). \quad (3.3.29)$$

Now consider the second scenario where $n_p \leq n_s$, since this plus-supremum and its neighborhood⁴ belong to $\tilde{\mathcal{F}}_{n_p}$, therefore KKT conditions imply that

$$R(\tilde{\Gamma}_{n_p}) \leq f(n_p) \quad (3.3.30)$$

where $f(n_p)$ is the locally optimum rate corresponding to $\tilde{\mathcal{F}}_{n_p}$. Hence, the maximum rate corresponding to $\tilde{\Gamma}_{n_p}$ should be less than or equal to the local optimum of that feasible set⁵, i.e., $f(n_p)$, which is indeed less than or equal to $f(n^\circ)$. This means that $f(n^\circ)$ is not only the global optimum of the inner maximization problem with dimension n° , but also is the global optimum of the main optimization problem in (3.1.9). ■

⁴We use the commonly used definition of neighborhood in a metric space.

⁵Since the plus-supremum belongs to feasible set of the maximization problem with dimension n_p , then its corresponding rate should be less than or equal to the local optimum of that feasible set, otherwise it is in contradiction with the definition of local optimum

The above theorem concludes our derivation of the optimal solution to the sum-rate maximization for two-way active channels under separate transceivers' power budget and channel power budgets. Indeed, we can now conclude that n° is the optimal number of active subchannels and that the transceivers' power budget and the channel power budget should be uniformly distributed among any set of n° subchannels.

3.4 Total Power Constraints

In this section, we consider a two-way active channel with more general constraints over the the transceiver and subchannel powers. Theorem 5 analyzes a two-way active channel with two different types of power constraints. In the first scenario, we assume that sum of the powers of the two transceivers are limited to a total transceiver power budget. In the second scenario, the total power of the whole network, which is defined as the sum of the transceiver powers and subchannel powers, is limited by a total power budget.

Theorem 5: *(a). If the total available power for the two transceivers is limited, i.e., if $P_1^{\max} + P_2^{\max} \leq P_{T_1}$, for a given positive P_{T_1} , then to achieve the maximum sum-rate of the active channel, the power should be distributed equally between the two transceivers, i.e., $P_1^{\max} = P_2^{\max} = \frac{1}{2}P_{T_1}$.*

(b). If the total available power for the channel and the two transceivers is limited, i.e., if $P_1^{\max} + P_2^{\max} + P_h^{\max} \leq P_{T_2}$, for a given positive P_{T_2} , then to achieve the maximum sum-rate of the active channel, half of the total power should be allocated to the active channel and the other half should be distributed equally between the two

transceivers, i.e., $P_1^{\max} = P_2^{\max} = \frac{1}{2}P_h^{\max} = \frac{1}{4}P_{T_2}$.

Proof: (a). Without loss of optimality, we claim that at the optimum, $P_1^{\max} + P_2^{\max} \leq P_{T_1}$ must be satisfied with equality. Otherwise, if at the optimum, the summation of the transceiver powers is not equal to P_{T_1} , we can scale up the powers such that $P_1^{\max} + P_2^{\max} = P_{T_1}$ holds true. This scaling will increase the sum-rate of the active channel, thus contradicting the assumption that the first distribution is sum-rate optimal. Assuming P_1^{\max} and P_2^{\max} are to be determined optimally, the sum-rate maximization problem, based on our discussion in the previous section, is simplified as

$$\begin{aligned} & \max_{P_1^{\max}, P_2^{\max}} \max_{n \in \mathcal{N}} \quad n \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1 \right) \\ & \text{subject to} \quad P_1^{\max} + P_2^{\max} = P_{T_1} \\ & \quad \quad \quad P_1^{\max} \geq 0, \quad P_2^{\max} \geq 0. \end{aligned} \quad (3.4.1)$$

Indeed, the inner maximization in (3.4.1) is equivalent to the sum-rate maximization of a two-way active channels problem for given P_1^{\max} and P_2^{\max} , as shown in the previous section. Since for any fixed n , the maximization is over P_1^{\max} and P_2^{\max} , using the constraint $P_2^{\max} = P_{T_1} - P_1^{\max}$, we can write the optimization problem in (3.4.1) as

$$\begin{aligned} & \max_{n \in \mathcal{N}} \max_{P_1^{\max}} \quad n \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right) + n \log_2 \left(\frac{(P_{T_1} - P_1^{\max}) P_h^{\max}}{n^2} + 1 \right). \quad (3.4.2) \\ & \text{subject to} \quad 0 \leq P_1^{\max} \leq P_{T_1} \end{aligned}$$

For any fixed n , the first derivative of the objective function in (3.4.2) with respect to P_1^{\max} is given as

$$\frac{\frac{P_h^{\max}}{n}}{(\ln 2) n \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right)} + \frac{-\frac{P_h^{\max}}{n}}{(\ln 2) n \log_2 \left(\frac{(P_{T_1} - P_1^{\max}) P_h^{\max}}{n^2} + 1 \right)}. \quad (3.4.3)$$

For any fixed value of n , equating (3.4.3) to zero yields the optimum value of the transceiver powers as

$$P_1^{\max} = P_2^{\max} = \frac{1}{2}P_{T_1}. \quad (3.4.4)$$

Note that power distribution in (3.4.4) satisfies the constraint in (3.4.2). The proof of part (a) is now complete.

(b). To prove part (b), without loss of optimality, we can show that at the optimum, the inequality will be satisfied with equality i.e., $P_1^{\max} + P_2^{\max} + P_h^{\max} = P_{T_2}$. In the second scenario, the objective function stays the same in part (a), while the constraint is different. That is we aim to solve the following optimization problem:

$$\begin{aligned} \max_{n \in \mathcal{N}} \quad & \max_{P_1^{\max}, P_2^{\max}, P_h^{\max}} n \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1 \right), \\ \text{subject to} \quad & P_1^{\max} + P_2^{\max} + P_h^{\max} = P_{T_2} \\ & P_1^{\max} \geq 0, \quad P_2^{\max} \geq 0, \quad P_h^{\max} \geq 0. \end{aligned} \quad (3.4.5)$$

For any fixed n , the Lagrangian function of the optimization problem can be expressed as

$$\begin{aligned} \tilde{\mathcal{L}}_n(P_1^{\max}, P_2^{\max}, P_h^{\max}) &= \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1 \right) \\ &\quad - \lambda (P_1^{\max} + P_2^{\max} + P_h^{\max} - P_{T_2}) \end{aligned} \quad (3.4.6)$$

Now equating all partial derivatives of the Lagrangian function to zero, we obtain the following set of equations:

$$\frac{\partial \tilde{\mathcal{L}}_n(P_1^{\max}, P_2^{\max}, P_h^{\max})}{\partial P_1^{\max}} = \frac{\frac{P_h^{\max}}{n}}{(\ln 2)n \log_2\left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1\right)} + \lambda = 0 \quad (3.4.7)$$

$$\frac{\partial \tilde{\mathcal{L}}_n(P_1^{\max}, P_2^{\max}, P_h^{\max})}{\partial P_2^{\max}} = \frac{\frac{P_h^{\max}}{n}}{(\ln 2)n \log_2\left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1\right)} + \lambda = 0 \quad (3.4.8)$$

$$\begin{aligned} \frac{\partial \tilde{\mathcal{L}}_n(P_1^{\max}, P_2^{\max}, P_h^{\max})}{\partial P_1^{\max}} &= \frac{\frac{P_1^{\max}}{n}}{(\ln 2)n \log_2\left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1\right)} \\ &+ \frac{\frac{P_2^{\max}}{n}}{(\ln 2)n \log_2\left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1\right)} + \lambda = 0. \end{aligned} \quad (3.4.9)$$

We can simplify (3.4.7) and (3.4.8) as

$$(\ln 2)n \log_2\left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1\right) = -\frac{P_h^{\max}}{n\lambda}, \quad (3.4.10)$$

$$(\ln 2)n \log_2\left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1\right) = -\frac{P_h^{\max}}{n\lambda}. \quad (3.4.11)$$

It follows from (3.4.7) and (3.4.8) that P_1^{\max} and P_2^{\max} are equal at the optimum.

Moreover, using (3.4.10) and (3.4.11), we can write (3.4.9) as

$$\frac{\frac{P_1^{\max}}{n}}{\left(-\frac{P_h^{\max}}{n\lambda}\right)} + \frac{\frac{P_2^{\max}}{n}}{\left(-\frac{P_h^{\max}}{n\lambda}\right)} + \lambda = 0 \quad (3.4.12)$$

or, equivalently, as

$$\frac{P_1^{\max}}{P_h^{\max}} + \frac{P_2^{\max}}{P_h^{\max}} = 1. \quad (3.4.13)$$

Now it is easy to conclude that $P_1^{\max} + P_2^{\max} = P_h^{\max}$ and since the sum of these three powers is constant, the optimal power distribution among the two transceivers and

the channel is given as

$$P_1^{\max} = P_2^{\max} = \frac{1}{2}P_h^{\max} = \frac{1}{4}P_{T_2}. \quad (3.4.14)$$

■

Note that the power distribution in (3.4.14) satisfies the constraints in (3.4.5), which ensures that the powers are non-negative.

Since for any fixed value of n , the power distribution in (3.4.14) is optimal, Theorem 5 shows that the optimal distribution of powers between the subchannels and the transceivers, when the total power is limited, is independent of the number of available active subchannels. Therefore, in order to design an active channel with maximum sum-rate, using Theorem 5, we distribute the total power among the two transceivers and the active channel. Then, as discussed previously, by substituting the optimal power distribution in (3.4.14) in the objective function of (3.4.5), we calculate the optimal sum-rate as a function of n as

$$g(n) = 2n \log_2 \left(\frac{(P_{T_2})^2}{8n^2} + 1 \right). \quad (3.4.15)$$

where P_{T_2} is the total power for the transceivers and the channel. As we proved earlier, the optimal value of n is unique and it is the value of n which maximizes the value of the function in (3.4.15). To find the optimal value of n , we first obtain the unique maximizer of $g(x)$, for $x \in [1 N]$, using an iterative algorithm such as Newton-Raphson technique. Let us assume that the maximizer of $g(x)$ is denoted by x° . Note that we are looking for an integer value of $n \in \mathcal{N}$, which results in the largest value for $g(n)$. We deduce that if $x^\circ \geq N$, then $g(N)$ is the largest value among $\{g(n)\}_{n=1}^N$. If $x^\circ < N$, either the smallest integer greater than x° or the largest integer smaller than x° is the maximizer of (3.4.15). In this case, we calculate the value of $g(n)$ for

these two integers and find the largest value between these two values of $g(n)$.

3.4.1 Comparison with one-way active channels

To compare a two-way active channel with one-way active channels, we need to calculate the sum-rate corresponding to two one-way active channels with overall power limit equal to P_T^{\max} . Due to symmetry, half of P_T^{\max} should be allocated to each one-way active channel. Also, we allocate half of the subchannels to each side of transmission, so $\frac{N}{2}$ is the maximum number of available subchannels for each side of transmission. In [17], the optimal sum-rate for a one-way active channel is calculated as function of n as

$$n \log_2 \left(\frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right) \quad (3.4.16)$$

where P_1^{\max} is the power of transceiver and P_h^{\max} is the power allocated to the active channel. Using (3.4.16), we can write the the total sum-rate for two one-way active channels as

$$h(n) = n \log_2 \left(\frac{P_1^{\max} P_{h1}^{\max}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} P_{h2}^{\max}}{n^2} + 1 \right). \quad (3.4.17)$$

where P_1^{\max} and P_2^{\max} are the allocated powers to the transceivers in the first and second one-way active channels, respectively, and P_{h1}^{\max} and P_{h2}^{\max} are the powers allocated to the first and second active channels, respectively. As we mentioned before, half of P_{T_2} is allocated to each side of transmission, and as proved in [17], for each one-way active channel, the total available power should be distributed equally between the channel and the transceivers, therefore, we choose

$$P_1^{\max} = P_2^{\max} = P_{h1}^{\max} = P_{h2}^{\max} = \frac{P_{T_2}}{4} \quad (3.4.18)$$

Substituting (3.4.18) in (3.4.17), we can write $h(n)$ as

$$h(n) = 2n \log_2 \left(\frac{(P_{T_2})^2}{16n^2} + 1 \right). \quad (3.4.19)$$

Note that the optimum sum-rate achieved by two one-way active channel with total power equal to P_T^{\max} , is the maximum of $h(n)$, for $n = 1, 2, \dots, \lfloor \frac{N}{2} \rfloor$. Obviously $h(n) < g(n)$, for $n \in \mathcal{N}$, which means that the achievable sum-rate for a two-way active channel is always greater than the total sum-rate of two one-way active channel. Since a two-way active channel uses the channel more efficiently, and as a result, it will lead to a higher sum-rate compared to two one-way active channels, it is more reasonable to use one two-way active channel, rather than two one-way active channels, to exchange information between two transceivers. We will discuss this result later with numerical examples in the simulation section.

3.5 Non-Reciprocal Active Channel

In this section, we analyze a two-way active channel where subchannel reciprocity is not assumed. Under such a non-reciprocity assumption, there are two squared gain parameters for each subchannel that have to be optimized. The first one is the squared gain of the subchannel when the signal is transmitted from the first transceiver while there exists another subchannel squared gain coefficient corresponding to the other direction of the transmission. In this scenario, we assume that the sum of the sub-channel powers in two directions is limited to P_h^{\max} . Then, the corresponding sum-rate

optimization problem can be written as

$$\begin{aligned}
& \max_{n, \mathbf{p}_1, \mathbf{p}_2, \mathbf{h}_1, \mathbf{h}_2} && \sum_{i=1}^n \log_2(1 + p_{1i}h_{1i}) + \log_2(1 + p_{2i}h_{2i}) \\
& \text{subject to} && \mathbf{1}^T \mathbf{p}_1 \leq P_1^{\max} \\
& && \mathbf{1}^T \mathbf{p}_2 \leq P_2^{\max} \\
& && \mathbf{1}^T \mathbf{h}_1 + \mathbf{1}^T \mathbf{h}_2 \leq P_h^{\max} \\
& && \mathbf{p}_1 \succ 0, \quad \mathbf{p}_2 \succ 0, \quad \mathbf{h}_1 \succ 0, \quad \mathbf{h}_2 \succ 0.
\end{aligned} \tag{3.5.1}$$

Here, n is the number of subchannels which receive non-zero power, $\mathbf{p}_1 \triangleq [p_{11} \ p_{12} \ \dots \ p_{1n}]^T$ and $\mathbf{p}_2 \triangleq [p_{21} \ p_{22} \ \dots \ p_{2n}]^T$ are the $n \times 1$ vectors of the transmitted powers of Transceivers 1 and 2, respectively, and $\mathbf{h}_1 \triangleq [h_{11} \ h_{12} \ \dots \ h_{1n}]^T$ and $\mathbf{h}_2 \triangleq [h_{21} \ h_{22} \ \dots \ h_{2n}]^T$ are the $n \times 1$ vectors of subchannel powers from Transceiver 1 (2) to Transceiver 2 (1), where h_{ji} is the squared gain of the i th subchannel from Transceiver j to Transceiver \bar{j} , for $j \in \{1, 2\}$. Without loss of generality, we assume that $P_1^{\max} \geq P_2^{\max}$ holds true.

One can easily show that the first three inequalities in (3.5.1) must be satisfied with equality. Hence, we can rewrite the optimization problem in (3.5.1) as

$$\begin{aligned}
& \max_n \max_{P_{h_1}, P_{h_2}} \max_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{h}_1, \mathbf{h}_2} && \sum_{i=1}^n \log_2(1 + p_{1i}h_{1i}) + \log_2(1 + p_{2i}h_{2i}) \\
& \text{subject to} && \mathbf{1}^T \mathbf{p}_1 = P_1^{\max} \\
& && \mathbf{1}^T \mathbf{p}_2 = P_2^{\max} \\
& && \mathbf{1}^T \mathbf{h}_1 = P_{h_1} \\
& && \mathbf{1}^T \mathbf{h}_2 = P_{h_2} \\
& && P_{h_1} + P_{h_2} = P_h^{\max} \\
& && \mathbf{p}_1 \succ 0, \quad \mathbf{p}_2 \succ 0, \quad \mathbf{h}_1 \succ 0, \quad \mathbf{h}_2 \succ 0, \quad P_{h_1} \geq 0, \quad P_{h_2} \geq 0.
\end{aligned} \tag{3.5.2}$$

Now, we solve the inner maximization problem for fixed values of P_{h_1} and P_{h_2} , and

then, maximize the sum-rate over these two variables. For fixed values of P_{h_1} and P_{h_2} , we can break down the optimization problem into two sub-problems, for $i \in \{1, 2\}$, as

$$\begin{aligned} & \max_{\mathbf{p}_i, \mathbf{h}_i} \quad \sum_{j=1}^n \log_2(1 + p_{ij}h_{ij}) \\ \text{subject to} \quad & \mathbf{1}^T \mathbf{p}_i = P_i^{\max} \\ & \mathbf{1}^T \mathbf{h}_i = P_{h_i} \\ & \mathbf{p}_i \succ 0, \mathbf{h}_i \succ 0 \quad . \end{aligned} \quad (3.5.3)$$

It is proved in [17] that for fixed n , the maximum value of the objective function of the optimization problem in (3.5.3) is equal to

$$n \log_2 \left(\frac{P_i^{\max} P_{h_i}}{n^2} + 1 \right) \quad (3.5.4)$$

and the corresponding optimal values of the powers are given by

$$\mathbf{p}_i^{(n)} = \mathbf{1} \frac{P_i^{\max}}{n} \quad (3.5.5)$$

$$\mathbf{h}_i^{(n)} = \mathbf{1} \frac{P_{h_i}}{n} \quad (3.5.6)$$

where $\mathbf{1}$ is an $n \times 1$ all-one vector. Now using (3.5.4), the optimization problem in (3.5.2) can be written as

$$\begin{aligned} & \max_n \max_{P_{h_1}, P_{h_2}} \quad n \log_2 \left(\frac{P_1^{\max} P_{h_1}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} P_{h_2}}{n^2} + 1 \right) \\ \text{subject to} \quad & P_{h_1} + P_{h_2} = P_h^{\max} \\ & P_{h_1} \geq 0, P_{h_2} \geq 0. \end{aligned} \quad (3.5.7)$$

Since $P_{h_2} = P_h^{\max} - P_{h_1}$, the optimization problem in (3.5.7) is equivalent to

$$\begin{aligned} & \max_n \max_{P_{h_1}} \quad n \log_2 \left(\frac{P_1^{\max} P_{h_1}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} (P_h^{\max} - P_{h_1})}{n^2} + 1 \right) \\ \text{subject to} \quad & 0 \leq P_{h_1} \leq P_h^{\max}. \end{aligned} \quad (3.5.8)$$

or to the following one

$$\begin{aligned} \max_n \quad \max_{P_{h_1}} \quad & n \log_2 \left(\frac{P_1^{\max} P_{h_1}}{n^2} + 1 \right) \left(\frac{P_2^{\max} (P_h^{\max} - P_{h_1})}{n^2} + 1 \right) \\ \text{subject to} \quad & 0 \leq P_{h_1} \leq P_h^{\max}. \end{aligned} \quad (3.5.9)$$

Since $\log(\cdot)$ is an increasing function, for any fixed n , to solve the inner maximization problem in (3.5.9), we need to find the maximum of

$$\left(\frac{P_1^{\max} P_{h_1}}{n^2} + 1 \right) \left(\frac{P_2^{\max} (P_h^{\max} - P_{h_1})}{n^2} + 1 \right). \quad (3.5.10)$$

when $P_{h_1} \in [0, P_h^{\max}]$. The global maximum of (3.5.10) happens when P_{h_1} is chosen as

$$P_{h_1} = \frac{1}{2} \left(\frac{n^2 (P_1^{\max} - P_2^{\max})}{P_1^{\max} P_2^{\max}} + P_h^{\max} \right). \quad (3.5.11)$$

If the value of P_{h_1} in (3.5.11) belongs to the interval $[0, P_h^{\max}]$, then this value is the optimal value of P_{h_1} , for any given value of n . Note that if $P_1^{\max} = P_2^{\max}$, then the value of P_{h_1} in (3.5.11) becomes equal to $0.5P_h^{\max}$, which belongs to the interval $[0, P_h^{\max}]$, and thus, this value is the optimal value of P_{h_1} , for *any* value of n . In the case where $P_1^{\max} > P_2^{\max}$ holds true, for any given value of n , the value of P_{h_1} in (3.5.11) cannot be smaller than 0, and if for a given value of n , the value of P_{h_1} in (3.5.11) is larger than P_h^{\max} , then the optimal value of P_{h_1} is P_h^{\max} , for that value of n . As a result, for any given n , the optimal value of P_{h_1} is given by

$$P_{h_1} = \begin{cases} 0.5P_h^{\max}, & \text{for any } n, \text{ when } P_1^{\max} = P_2^{\max} \\ \frac{1}{2} \left(\frac{n^2 (P_1^{\max} - P_2^{\max})}{P_1^{\max} P_2^{\max}} + P_h^{\max} \right), & \text{for } n \leq \sqrt{\frac{P_h^{\max} P_1^{\max} P_2^{\max}}{P_1^{\max} - P_2^{\max}}}, \text{ when } P_1^{\max} > P_2^{\max} \\ P_h^{\max}, & \text{for } n > \sqrt{\frac{P_h^{\max} P_1^{\max} P_2^{\max}}{P_1^{\max} - P_2^{\max}}}, \text{ when } P_1^{\max} > P_2^{\max}. \end{cases} \quad (3.5.12)$$

3.5.1 Case $P_1^{max} = P_2^{max}$

In this case, the objective function in (3.5.9), can be written, for any n , as

$$\tilde{f}(n) \triangleq n \log_2 \left(\frac{P_1^{max} P_h^{max}}{2n^2} + 1 \right)^2 \quad (3.5.13)$$

Using Theorem 2, it can be easily shown that the function $\tilde{f}(x)$ in (3.5.13) has a unique real-valued maximizer. Now using an iterative algorithm, for example, a bisection technique, we can find the unique maximizer of the function $\tilde{f}(x)$ and denote that maximizer as \tilde{x}° . Note that we are looking for an integer value of $n \in \mathcal{N}$, which results in the largest value for $\tilde{f}(n)$. We deduce that if $\tilde{x}^\circ \geq N$, then $\tilde{f}(N)$ is the largest value among $\{\tilde{f}(n)\}_{n=1}^N$. If $\tilde{x}^\circ < N$, either the smallest integer greater than \tilde{x}° or the largest integer smaller than \tilde{x}° is the optimal value of n . In this case, we calculate the value of $\tilde{f}(n)$ for these two integers and introduce the optimal value of n as the one of these two integer numbers that results in a larger value for $\tilde{f}(n)$.

3.5.2 Case $P_1^{\max} > P_2^{\max}$

For $n \leq \sqrt{\frac{P_h^{\max} P_1^{\max} P_2^{\max}}{P_1^{\max} - P_2^{\max}}}$ and when $P_1^{\max} > P_2^{\max}$, the objective function in (3.5.9) can be written as

$$\begin{aligned}
& n \log_2 \left(\frac{P_1^{\max} P_{h_1}}{n^2} + 1 \right) + n \log_2 \left(\frac{P_2^{\max} (P_h^{\max} - P_{h_1})}{n^2} + 1 \right) = \\
& n \log_2 \left(0.5 \left(\frac{P_1^{\max} - P_2^{\max}}{P_2^{\max}} + \frac{P_1^{\max} P_h^{\max}}{n^2} \right) + 1 \right) + \\
& n \log_2 \left(0.5 \left(\frac{P_2^{\max} - P_1^{\max}}{P_1^{\max}} + \frac{P_2^{\max} P_h^{\max}}{n^2} \right) + 1 \right) = \\
& n \log_2 \left(0.5 \left(\frac{P_1^{\max}}{P_2^{\max}} + \frac{P_1^{\max} P_h^{\max}}{n^2} + 1 \right) \right) + \\
& n \log_2 \left(0.5 \left(\frac{P_2^{\max}}{P_1^{\max}} + \frac{P_2^{\max} P_h^{\max}}{n^2} + 1 \right) \right) = \\
& n \log_2 \left(0.25 P_1^{\max} P_2^{\max} \left(\frac{1}{P_1^{\max}} + \frac{P_h^{\max}}{n^2} + \frac{1}{P_2^{\max}} \right)^2 \right) \tag{3.5.14}
\end{aligned}$$

For $n > \sqrt{\frac{P_h^{\max} P_1^{\max} P_2^{\max}}{P_1^{\max} - P_2^{\max}}}$ and when $P_1^{\max} > P_2^{\max}$, the objective function in (3.5.9) can be written as

$$\check{f}(n) n \log_2 \left(\frac{P_2^{\max} P_h^{\max}}{n^2} + 1 \right) \tag{3.5.15}$$

It can be easily shown that in this case, the function $\check{f}(\cdot)$ has a unique maximizer which can be obtained using an approach similar the technique proposed earlier. We omit the details for the sake brevity.

3.6 Simulation Results

Using computer simulations, we compare the performance of several two-way active channels with the performance of several passive channels, in terms of maximum

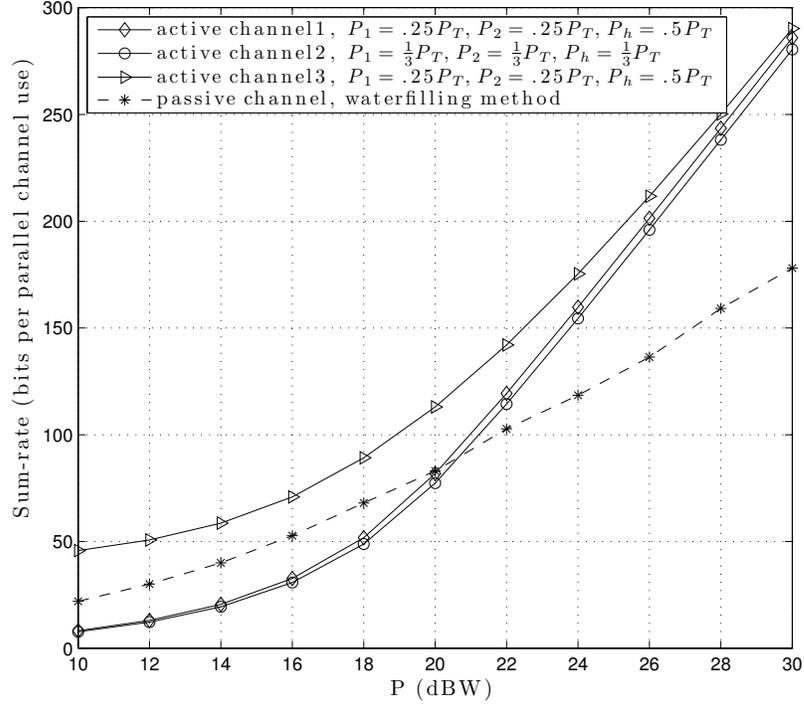


Figure 3.1: Maximum sum-rate versus total consumed power, $N=16$.

achievable sum-rate. For the passive channels, we assume that we have no control over the channel coefficients. Indeed, we model each subchannel of a passive channel as zero-mean complex Gaussian random variables with unit variance, i.e., $h_i \sim \mathcal{CN}(0, 1)$, for $i = 1, 2, \dots, N$. We apply water-filling scheme to find the maximum sum-rate for passive channels.

In Fig. 3.1, assuming $N = 16$, we plot the maximum sum-rate of three two-way active channels as well as that of a passive channel versus the *consumed power*, which is denoted as P . For the passive channel, we consider that the whole consumed power P is allocated to the transceivers. Since each subchannel of the passive channels is modeled as a zero-mean unit variance complex Gaussian random variable, the average total power of all subchannels of the passive channel is equal to

N . For the two-way active channel, we consider two scenarios. In the first scenario, we assume the total power available to the active channel is equal to the total power consumed by the transceivers in the passive channel, i.e., $P_{T_2} = P$. Active channels 1 and 2 represent the first scenario. In active channel 1, half of the total power P is allocated to the channel, while the remaining half is equally divided between the two transceivers. In active channel 2, the total power is uniformly distributed among the two transceivers and the channel. In other words, one third of the total power $P_{T_2} = P$ is allocated to each transceiver and one third of the total power is assigned to the channel. In the second scenario, the total power available to the active channel is assumed to equal to the power consumed by the transceiver powers of the passive channel P plus the average total power of all subchannels of the passive channel which is equal to N , as mentined above. That is $P_{T_2} = P + N$. Active channel 3 is an example of the second scenario. In active channel 3, half of the total available power $P_{T_2} = P + N$ is allocated to the channel and the remaining half is equally distributed between the two transceivers. Active channel 3, which has the same amount of total network power as the passive channel does, outperforms the passive channel for all values of P . Active channel 3 also outperforms both active channels in the first scenario, i.e., active channels 1 and 2. This superior performance of active channel 3 over the passive channel is the direct result of the fact that this active channel has more degrees of freedom compared to the passive channel with the same total available power. From Fig. 3.1, it can be seen that for sufficiently large values of P , i.e., $P > 20$ dBW, active channels 1 and 2 also outperform the passive channel in terms of the sum-rate. Note that compared to the passive channel (whose total power is $P + N$) the active channels 2 and 3 have less power available to them, i.e.,

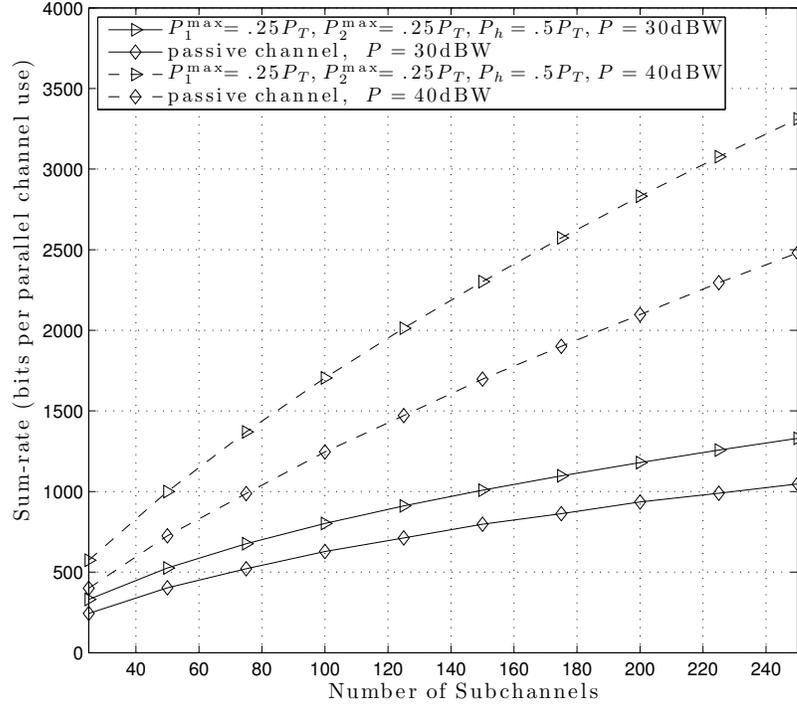


Figure 3.2: Maximum sum-rate versus number of subchannels.

as $P_{T_2} = P$. As a result for low values of P , the passive channel outperforms active channel 1 and 2 due to the fact the intrinsic power of the passive channel is larger than the amount of the power allocated to the channel in active channels 1 and 2.

Also shown in Fig. 3.1, active channel 1 is slightly better than active channel 2, since the total power is distributed optimally between the two transceivers and the channel. Also, it is worth mentioning that the maximum sum-rate of the active channel in the first scenario, i.e., sum-rates achieved by active channels 1 and 2 approaches the sum-rate of the active channel 3, as P increases. This is due to the fact that by increasing P , the additional power N available to active channel 3 is negligible compared to P . In Fig. 3.2, we consider an active channel and a passive channel for two different values of P , versus the number of subchannels. The total

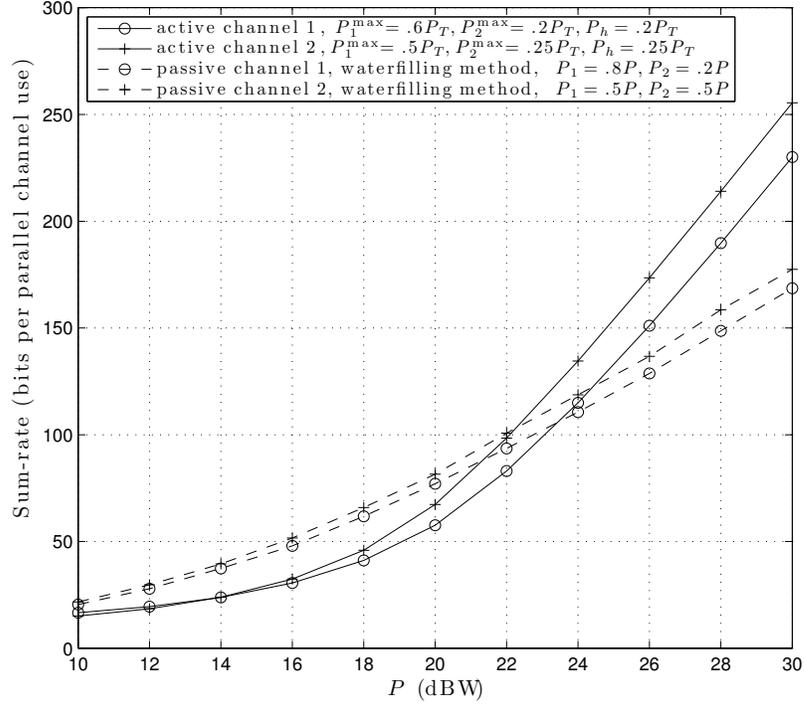


Figure 3.3: Maximum sum-rate versus number of subchannels, non-reciprocal case.

power of the active channel is distributed optimally, such that half of the power is allocated to the channel and the remaining half is distributed uniformly between the two transceivers. As the number of subchannels N increases, the sum-rates of both active and passive channels increase. As can be seen from this figure, for both values of P , the active channel outperforms the passive channel in terms of sum-rate, for any number of subchannels.

In Figs. 3.3 and 3.4, we present the simulation results of the sum-rate maximization problem over the non-reciprocal active channels and passive channels. In Fig. 3.3, assuming $N = 16$, we plot the sum-rates, achieved by two different active channels as well as two different passive channels. In active channel 1, the power distribution

is chosen as

$$P_1^{max} = 0.6P_T, \quad P_2^{max} = 0.2P_T, \quad P_h^{max} = 0.2P_T \quad (3.6.1)$$

where P_T is the total power available to the active channel, The power distribution among the two transceivers and the channel for active channel 2 is given by

$$P_1^{max} = 0.5P_T, \quad P_2^{max} = 0.25P_T, \quad P_h^{max} = 0.25P_T. \quad (3.6.2)$$

We also consider two passive channels. For the passive channel 1, 80% of the total consumed power P is allocated to the first transceiver and the remaining 20% is allocated to the second transceiver. In the passive channel 2, the total power is distributed equally between the two transceivers. To ensure that we compared all passive and active channel with the same total available power, we choose $P_T = P + N$. As shown in Fig. 3.3, as we increase P , the sum-rates achieved by all channels are increasing. Furthermore, for sufficiently large values of P , both active channels outperform both passive channels in terms of achieved sum-rate.

In Fig. 3.4, we consider an active channels with a total available power $P_T = P + N$ and a passive channel with a total consumed power P , for two different values of P . For the passive channel, half of the total consumed power is allocated to the first transceiver and the remaining half is allocated to the second transceiver. For passive channel 1, $P = 50$ dBW and for passive channel 2, $P = 40$ dBW. The power distribution for the active channels is chosen as

$$P_1^{max} = 0.25P_T, \quad P_2^{max} = 0.25P_T, \quad P_h^{max} = 0.5P_T. \quad (3.6.3)$$

As we increase the number of subchannels N , the sum-rate achieved by both active channels and both passive channels increase. As can be seen from this figure, for both values of P , each active channel outperforms the corresponding passive channel.

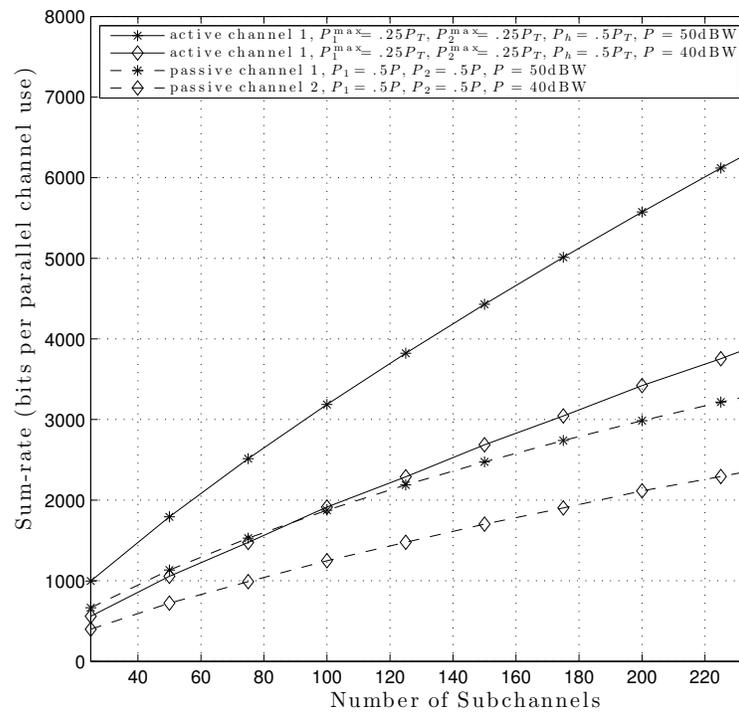


Figure 3.4: Maximum sum-rate versus total consumed power, non-reciprocal case.

Chapter 4

Conclusions And Future work

In this thesis, the term passive channel refers to the conventional parallel wireless channel model, where there is no control over the gain of each individual subchannel. We defined an active channel as a parallel channel where by injecting power into the subchannels, somewhere between a transmitter and a receiver, the gain of the subchannels are adjusted. We considered the sum-rate maximization problem over a two-way active channel and we presented the solution to four main problems. First, we considered the sum-rate maximization problem for a two-way active channel. In this problem, the powers allocated by transceivers to each subchannel as well as the powers of each subchannel are considered as the optimization variables of the problem. Our goal is to jointly optimize the power of each subchannel as well as the power allocated by each transceiver to each subchannel in order to achieve the maximum sum-rate. We showed that the sum-rate maximization problem has a unique global solution. Also that at the optimum, the transceivers may not necessarily provide power for all subchannels and only a subset of parallel subchannels may receive power. The second problem is to find the optimal power distribution among the two transceivers, when the total power of two transceivers is given. We showed that the solution to

the second problem will result in equal power for each transceiver.

Then we found the maximum sum-rate as well as the optimal power distribution among the two transceivers as well as the channel. The main constraint of this problem restricts the total power of the network. We proved that at the optimum, half of the total power should be allocated to the active channel while the other half is equally distributed between the two transceivers.

Then, we solved the sum-rate maximization problem for non-reciprocal two-way active channels. We derived a closed-form solution for the optimal power of the active channel allocated to each side of the transmission. Then, we found a semi-closed-form solution to the sum-rate optimization problem.

Finally, in the simulation result section, we analyzed several passive channels as well as several active channels. As shown in our numerical results two-way active channels outperform the passive channels in terms of sum-rate under the same total network power.

4.1 Future work

This work can be continued in several directions as listed below

- Deriving the optimal power allocation for two-way active channels with unequal subchannel noise powers.
- Deriving the achievable rate region for two-way active channels.
- Deriving the optimal power allocation for multi-way active channels.
- Deriving the optimal power allocation for active multi-user channels.
- Deriving the optimal power allocation for active Broadcast channels.

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