

JOINTLY OPTIMAL PRE- AND POST-CHANNEL
EQUALIZATION AND DISTRIBUTED BEAMFORMING
IN ASYNCHRONOUS BI-DIRECTIONAL RELAY
NETWORKS

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To my loving parents.

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List of Acronyms

AF	Amplify-and-Forward
CRLB	Cramer-Rao Lower Band
CSI	Channel State Information
EF	Estimate-and-Forward
EPA	Equal Power Allocation
FF	Filter-and-Forward
FIR	Finite Impulse Response
ISI	Inter-Symbol-Interference
IBI	Inter-Block-Interference
MABC	Multiple Access Broadcast
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MSE	Mean Squared Error
OFDM	Orthogonal Frequency Division Multiplexing
SDP	Semi Definite Programming
SNR	Signal to Noise Ratio
SQP	Sequential Quadratic Programming
TDBC	Time Division Broadcast
DF	Decode-and-Forward

Qos	Quality of Service
LMMSE	Linear Minimum Mean Square Estimation
ANC	Analogue Network Coding
SINR	Signal-to-Interference-and-Noise Ratio
ZF	Zero-Forcing
MMSE	Minimum Mean Squared Error
SI	Self-Interference
CCI	Co-Channel Interference
LS	Least Squared
TENCE	Tensor-based Channel Estimation
SIC	Self-Interference Cancellation
BER	Bit Error Rate
DFE	Decision Feedback Estimation
MLSE	Maximum Likelihood Sequence Estimation

Abstract

We consider an asynchronous bi-directional relay network (consisting of two single-antenna transceivers and multiple single-antenna relays) where the transceiver-relay paths are subject to different relaying and/or propagation delays. In such a network, the end-to-end link can be viewed as a multi-path channel which can cause inter-symbol-interference (ISI) in the signals received by the two transceivers. Assuming a block transmission/reception scheme, we consider both pre- and post-channel equalization at both transceivers to combat the inter-block-interference (IBI) induced due to ISI. Considering amplify-and-forward (AF) relays, we study the problem of optimal design of pre- and post-channel linear equalizers and power loading at the two transceivers as well as the relay network beamforming. To do so, assuming a limited total transmit power budget, we minimize the total mean square error (MSE) of the linearly estimated signals at both transceivers by optimally obtaining the transceivers' powers and relay beamforming weights as well as pre- and post-channel linear equalizers at the two transceivers. We rigorously prove that this minimization leads to a certain relay selection scheme, where only a subset of the relays will be turned on and the rest switched off. We also provide semi-closed form solutions to the design parameters.

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Chapter 1

Introduction

1.1 Overview

Recently, wireless ad-hoc networks have been the center focus of numerous studies due to their extensive commercial and military applications. Such networks suffer, to a great extent, from severe signal fading inherent to multi-hop communication which in turn leads to improper reception of transmitted signals. Employing diversity techniques is a potential solution to mitigate the effects of multipath fading. For instance, using systems with multiple transmit and receive antennas, otherwise known as multiple-input-multiple-output (MIMO) systems, provides spatial diversity and/or multiplexing gains as well as interference mitigation and cancellation capacity in wireless networks. Using multi-antenna diversity, one can decrease the probability of receiving a poor signal at the destination, resulting in more reliable communication. However, the hardware complexity of multiple-antenna communication networks is higher than that of the single-antenna communication schemes. This trade-off between communication reliability and increased complexity should be taken into account for design and implementation of multi-antenna networks.

Alternatively, cooperative communication is a low-cost approach to providing spatial diversity while offering significant improvement in reception reliability, energy efficiency, network capacity and band-width efficiency [1]. In recent years, there has been significant interest in multi-user cooperative diversity [2–4]. This type of communication is based on user cooperation and utilizes spatial diversity of multiuser systems, eliminating the need for multiple antennas at each terminal [5]. In cooperative communications, different users establish multiple paths between a source and a destination by means of relaying the messages from the source towards the destination. This enables the nodes of the cooperative network to share their communication resources and exploit spatial diversity to maximize source-destination data rate. Cooperation in turn offers a trade-off between communication reliability and power consumption for each user. One can argue that, in a cooperative network, users need more transmit power since each user not only transmits its own data, but it should also relay the information from the other users. However in reality, the base line transmit power of each user is reduced due to the diversity, and thus, the transmit power of the total network will be reduced if all factors stay constant. It is worth mentioning that while transmission rate of cooperative communication links will be lowered since each user transmits its own data as well as information of other users, the spectral efficiency of each user increases through cooperation.

Additionally, inter-symbol-interference (ISI) is inevitable in cooperative relay networks due to different propagation delays of multiple links between the two transceivers. To mitigate the effect of ISI, various approaches have been studied in recent research efforts. Among these methods, equalization techniques have proven to be efficient approaches to mitigate the adverse effects of ISI.

In this thesis, we examine a communication network where all the relay nodes collaborate with each other to establish a two-way communication between two transceivers. In our studied network, two transceivers exchange their messages with the help of multiple relay nodes. To tackle the ISI caused by different relaying path delays, we propose joint pre- and post channel equalization at the two front-ends of the two transceivers. To this end, we aim to minimize the total mean-squared error (MSE) of the linearly estimated signals at both transceivers' under a total transmit power constraint to optimally obtain the relay weight vectors, transceivers transmit powers as well as pre- and post-channel equalizers at the two transceivers.

1.2 Relaying Networks

Relay networks have recently been the center focus of many studies on cooperative communications [2], because, in addition to exploiting the cooperative spatial diversity of different users, these networks can also extend the coverage of wireless communication systems [2–4, 6]. In relay-assisted wireless networks, one or multiple relay nodes collaborate with each other to establish a connection between the two transceivers (or between a source and a destination). In fact, in these networks, different users share their resources to assist each other in conveying the messages through the network. It can be noted that, in the relay networks, there may not be a direct communication link between the transceivers (or the source and the destination) due to shadowing or non-existence of a line-of-sight link. Therefore, the relay nodes process their received signals based on a certain scheme, and forward the resulting signals to the destination. Numerous relaying schemes have been proposed in the

literature. Examples are amplify-and-forward (AF), decode-and-forward (DF), filter-and-forward (FF) and estimate-and-forward (EF). Each of these techniques requires a different processing of the received signals at the relay nodes.

In the AF scheme, which has been extensively studied in the literature [1-4], each relay amplifies and adjusts the phase of its received signal, and then transmits the so-obtained signal to the receiver. Among various relaying protocols, the AF technique is of particular interest in relatively low-noise relays due to its simplicity, and also due to the fact that the relays do not need to perform detection on their received signals [4, 7–12].

The DF protocol is of interest when the noise power at the relays is relatively high, and amplifying signals leads to amplifying the noise [12]. In this relaying scheme, the relays decode and re-encode their received signals, and then, forward them to the destination. This approach also complicates the relay design and increases the power consumption [13].

Another relaying strategy is called filter-and-forward (FF), where each relay is equipped with a finite impulse response (FIR) filter to equalize its received signal in a distributed manner [14–17].

The estimate-and-forward (EF) technique is another relaying approach in which the received signal at the relays is transformed to obtain the estimated version of the transmitted signal. This estimate is then forwarded to the destination.

Cooperative relay networks can be divided into two main categories, namely half-duplex and full-duplex relaying schemes. In a half-duplex relaying scheme, data transmission and reception of the relays are performed in two different time slots, while in a full-duplex relaying scheme, the relay nodes transmit and receive the information

in the same time slot and in the same frequency band.

The full-duplex relaying scheme has a higher spectral efficiency compared to half-duplex relaying [18] due to the pre-log factor of 0.5 in the sum-rate expression [4]. On the other hand, compared to half-duplex relaying, the full-duplex relaying scheme is more difficult to implement due to the difference of the power levels of transmitted and received signals [19].

1.3 One-Way Relaying Scheme

Several distributed beamforming approaches have been presented for one-way relay networks, where a source transmits data to a destination with the help of single or multiple nodes. In a one-way relaying scheme, transmission is performed in two phases. In the first phase, the source transmits the symbols to the relays. In the second phase, the relays then forward the processed versions of their received signals to the receiver. In these networks, the transmission flow is in one direction, i.e, from the source to the destination. The relays can utilize any of the different relaying schemes to process their received signals at the relays. Among different relaying schemes, the AF protocol is more desirable for a network with low-noise relays and also offers more simplicity in comparison with other relaying techniques.

1.4 Two-Way Relaying Scheme

In contrast to one-way relaying of data from a source to a destination, in bi-directional relay networks, several relay nodes participate in a cooperative communication scheme

to establish a two-way communication between two transceivers [10, 14, 16, 20, 20–24, 24–43]. The concept of a two-way communication channel was introduced in 1961 by Shannon [44] where he studied the bi-directional communication between two transceivers at the same time. The cooperation of relays in these networks can improve reliability. The main idea of two-way relaying networks is to let each relay retransmit the processed signals it receives from the two transceivers, and then each transceiver node recovers the information transmitted by the other transceiver node after self-interference cancellation.

Essentially, there are three different protocols in two-way relay networks: the conventional approach of two successive one-way relaying; the time division broadcast (TDBC) relaying scheme; and the multiple access broadcast (MABC) relaying scheme. In the conventional approach, two symbols are conveyed in four steps with a one-way relaying scheme employed in each direction. This approach is not bandwidth efficient. The TDBC relaying approach conveys two symbols between two transceivers in three time slots. Obviously, the TDBC approach has a significantly higher throughput in comparison with the traditional four step approach. In the third approach, which is the MABC relaying scheme, the number of steps (time slots) required to exchange two symbols between two transceivers is reduced to two. In the first time slot, the transceivers transmit their information symbols to the relays, and then in the second time slot, the relays broadcast properly processed versions of their received signals to the transceivers.

Based on the above protocols, different relaying schemes have been proposed and studied in the literature [24, 45–56].

1.5 Motivation and Problem Statement

In the majority of earlier published results on two-way cooperative communications, the authors assumed that the relays and transceivers are time-synchronous. In other words, they ignore the fact that the signals traveling through different relaying paths are subject to different delays. The assumption of perfectly time-synchronous relay nodes or identical propagation delays for different relaying paths can be valid only at sufficiently low data rates [9, 10, 31, 35, 36, 45, 46, 52, 54, 57].

In fact, in bi-directional relay networks, two factors contribute to the overall propagation delay of the transmitted signal. First, the signal transmitted by any transceiver arrives at different relays with different delays. Second, the signal transmitted by different relays also arrives at any specific transceiver with different delays. Hence, the propagation delays from one transceiver to the other are different for different relaying paths. These different relaying delays lead to frequency selectivity of the end-to-end channel even if the relay-transceiver channels are frequency flat. The frequency selectivity of the end-to-end channel in turn gives rise to ISI in the received signal at the transceivers at sufficiently high data rates. If not taken into account, this ISI can adversely affect the overall performance of the communication network.

In order to combat ISI at the transceivers in one- or two way relay networks with frequency selective channels, essentially two different competing approaches exist. In the first approach, often referred as the FF method, FIR filters are deployed at the relays. This allows for the end-to-end channels to be equalized in a distributed manner [14–17]. This approach can be considered as a single-carrier equalization scheme. To reap the benefits of the FF relaying protocol, the FIR filters at the relays, as well as the transmit powers at the transceivers, should be optimally designed. The

second approach is based on a multi-carrier equalization scheme. In this approach all nodes are equipped with orthogonal-frequency division multiplexing (OFDM) schemes to “diagonalize” the end-to-end frequency selective channel into multiple orthogonal parallel frequency flat sub-channels [34, 40, 58–60]. In the OFDM-based method, to optimize the overall performance of the system, it is necessary to optimally allocate the power across different subcarriers at the transceivers and at the relays.

Even though these two schemes, either deploying FIR filters at relays or using OFDM schemes, combat ISI, they pose complex processing requirements at the relays which may be unnecessary, specifically in scenarios with frequency flat relay-transceiver channels. Another well studied method to tackle ISI is to employ pre- or post-channel equalizers at the transceivers. In the post-channel equalization method, as it is suggested by its name, a channel equalizer is employed at the receiver to eliminate the effect of the ISI prior to signal detection [23]. On the other hand, in pre-channel equalization techniques, a signal processing operation is performed on the modulated signal prior to transmission and at the transmitter end. Although the aforementioned equalization methods have been presented in the literature, to the best of our knowledge, joint pre- and post-channel equalization has not been considered to combat ISI in cooperative networks. Indeed, our motivation is to optimally design pre- and post-channel equalization in an asynchronous two-way relay network.

In this thesis, we examine an asynchronous AF two-way relay network with a frequency selective end-to-end channel. We aim to keep the relay processing simple by using AF relaying protocol. In addition, we employ a joint linear pre- and post channel equalization scheme at the two front-ends of both transceivers to combat ISI caused by different relaying path delays. The main goal is to improve the performance

of the communication network by minimizing total MSE of the linearly estimated signals. To achieve this goal, we optimally design the relay beamforming weight vectors, transceiver transmit powers and the pre- and post-channel equalizers at the two transceivers under a total power budget.

1.6 Objective and Methodology

Considering an asynchronous two-way relay network in which the relays simply amplify and then forward their received signals to the transceivers, to tackle the ISI caused by different propagation path delays from each transceiver to the relays and vice versa, a novel channel equalization scheme is proposed in this thesis. We utilize a joint pre- and post-channel linear equalization approach to mitigate the ISI.

First, we model the transceivers' received signals, the total received noise at each transceiver as well as the end-to-end channel impulse response. We then present an optimization problem to obtain the optimal values of the relay beamforming weight vectors, the transmit powers at both transceivers as well as pre- and post-channel equalizers to minimize the total MSE of the linearly estimated signals at the two transceivers, subject to a total transmit power budget. We show that this approach leads to a relay selection scheme, where only the set of relays which contribute to one tap of the end-to-end channel impulse response is active and the rest of the relays are inactive. Assuming a certain tap of the end-to-end channel impulse response is non-zero while the rest are zero, we derive a semi-closed form solution for the beamforming weight vector of the corresponding relays and the respective minimum total MSE of the symbol estimates. The total MSEs are calculated for all possible non-zero taps of the end-to-end channel impulse response. The non-zero tap, which yields the smallest

total MSE, determines the relays which have to be turned on. Finally, we evaluate the performance of our proposed algorithm through computer simulation and compare it to that of an equal power allocation (EPA) algorithm.

1.7 Outline of Dissertation

In this thesis, we focus on minimizing the total MSE of the linear estimates of the transmitted symbols at the two transceivers under a total transmit power constraint to optimally obtain the beamforming weight vector, transceivers' transmit powers as well as pre- and post-channel equalization matrices for an asynchronous AF bi-directional relay network. The remainder of this thesis is organized as follows:

In Chapter 2, we review recent relevant published results in one-way and two-way relay networks, either synchronous or asynchronous. In Chapter 3, we study the joint pre- and post-channel equalization scheme in an asynchronous AF two-way relay network. First, we model the received signals at the transceivers, the end-to-end channel impulse response, and the total received noise at each transceiver. We then optimally obtain the transmit powers at the transceivers, the beamforming weight vector as well as pre- and post-channel equalizers at the two front-ends through minimizing the total MSE of linearly estimated signals at the transceivers under a total transmit power constraint. Furthermore, we present the optimal design parameters in semi-closed-form solutions. Numerical results and discussions are presented in Chapter 4, while in Chapter 5, we present our concluding remarks as well as potential future extensions to our work.

1.8 Research Contribution

As a result of this thesis, we have submitted the following papers:

- F.Eshaghian and S. ShahbazPanahi, “Jointly Optimal Pre- and Post-channel Equalization and Distributed Beamforming in Asynchronous Bi-Directional Relay Networks,” *submitted to IEEE Transaction on Signal Processing*, July 2015.
- F.Eshaghian and S. ShahbazPanahi, “Optimal Equalization and Network Beamforming in Asynchronous Two-way Relay Networks,” *submitted to 2015 Asilomar Conference on Signals, Systems, and Computers*, May 2015.

1.9 Notation

Matrices and vectors are denoted by bold upper and lower-case letters, respectively. $E\{\cdot\}$ and $tr(\cdot)$ denote the expectation operator and trace of a matrix. Transpose, the complex conjugate and Hermitian transpose are denoted by $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$, respectively. We represent the l_1 and l_2 norms as $\|\cdot\|_1$ and $\|\cdot\|_2$. The $N \times N$ identity matrix and the $N \times M$ all-zero matrix are denoted as \mathbf{I}_N and $\mathbf{0}_{N \times M}$. $diag(\mathbf{v})$ yields a diagonal matrix whose diagonal entries are the elements of the vector \mathbf{v} . We use $(\cdot)^{-1}$, $(\cdot)^{-T}$ and $(\cdot)^{-H}$ to represent the inverse, inverse of transpose, and the inverse of Hermitian transpose of a matrix. The (i, j) -th element of a matrix is denoted as $[\cdot]_{ij}$.

Chapter 2

Literature Review

In this section, we briefly review the relevant work on distributed beamforming and power allocation in one-way and two-way relay networks. Moreover, various approaches to combat inter-symbol-interference (ISI) in such networks are introduced. We also review some relevant studies which rely on relay selection schemes.

2.1 One-way Relaying

In a typical one-way relay network, one or multiple relay nodes cooperate to establish one-way communication between a source and a destination [5,11,16,59,61–67]. Under the half-duplex mode of operation, the communication occurs during two time slots. In the first time slot, the transmitter broadcasts the symbols to the relay node(s), and then in the second time slot, each relay processes its received signals and forwards the processed signals to the receiver.

In [11], the authors consider a one-way relay network using amplify-and-forward (AF) protocol. In the AF relaying approach, the phase and the amplitude of the signals received at the relays are properly adjusted, and then the so-obtained signals

are forwarded to the transceivers. Since, in AF strategy, relays do not need to decode their received signals, this method is of particular interest in the networks when, at the relays, the noise power is relatively lower compared to the signal power. A distributed beamforming strategy is proposed to maximize the communication capacity considering individual relay power constraints. It is assumed that perfect channel information is available at the nodes. Relays utilize both channel direction information to create a beam at the receiver and also channel strength information to adjust their transmit powers. The obtained results showed that the optimal value of each relay's transmit power not only depends on its own channels, but also on the quality of all the other channels.

In [61], two different distributed beamforming designs are proposed for a one-way relay network with a transmitter, a receiver and several relay nodes with the assumption that the second-order statistics of the channel coefficients are known. In the first approach, the beamforming weights are obtained through minimization of the total transmit power subject to the quality of service (QoS) constraints at the receiver. In the second approach, the beamformers are designed through maximization of the receiver signal-to-noise ratio (SNR) subject to two different types of power constraints, namely, a total transmit power constraint and individual relay power constraints. It is shown that the SNR maximization problem subject to total transmit power has a closed-form solution, while the problem with individual relay power leads to a sequential quadratic programming (SQP) optimization problem. Using a semi-definite relaxation, the later optimization problem can be turned into a convex feasibility semi-definite programming (SDP), and then can be solved employing the interior point method. The simulation results show that, as the uncertainty in the channel state

information is increased, satisfying the quality of service constraint becomes much more difficult.

A relay network with one transmitter, one receiver and multiple relay nodes with frequency selective channels is examined in [16], where the so-called filter-and-forward (FF) relaying protocol is employed. In this relaying protocol, the relays are equipped with finite impulse response (FIR) filters. Three different beamforming problems have been considered. At first, subject to QoS constraints, the problem of total relay transmit power minimization is examined. Furthermore, the QoS at the receiver is maximized assuming two different sets of constraints, namely, total and individual power constraints at the relays.

In [59], an asynchronous one-way relay network is considered, where different propagation delays in the relaying paths cause ISI at the destinations. The authors employed orthogonal frequency division multiplexing (OFDM) only at the source and the destination to eliminate ISI. In fact, each relay simply performs the amplify-and-forward operation by multiplying its received signals by a complex weight. Thus, this network is modeled as multi-path channel. Unlike conventional multi-path channel models where no control on channel impulse response exists, in this model, the channel impulse response can be carefully adjusted by optimal design of the relays complex weights. The authors use a max-min fair design approach where the smallest of the subcarrier SNRs is maximized subject to constraints on the source and relay total transmit power. The numerical results showed that the asynchronous outperforms the synchronous scheme.

In [5], the authors have proposed various power allocation strategies for source and the relay(s) by minimizing the average transmit power for different cooperative

networks. These power allocation strategies are designed based on various optimization criteria, network topologies and channel state information (CSI) assumptions.

2.2 Two-way Relaying

In a two-way (bi-directional) relay network, two transceivers exchange data with the cooperation of single or multiple relay nodes. Among different relaying techniques such as decode-and-forward (DF), filter-and-forward (FF), or amplify and forward (AF) methods, where all can be utilized to process and forward the information at the relays, AF relaying has been widely studied in [4, 7–10, 22, 36].

Among different protocols used for establishing two-way relay-assisted communication, the multiple access broadcast (MABC) relaying strategy offers a bandwidth efficient bi-directional relaying scheme, and thus it has been well studied in the literature [9, 10, 21, 31, 32, 35–37, 45, 46, 48, 52, 54, 54–57, 68–76]. In this relaying protocol, the transmission of information symbols between two transceivers is accomplished in two time slots. In the first time slot, the transceivers transmit their information symbols to the relays and then, in the second time slot, the relays broadcast their properly processed information signals to the two transceivers.

2.2.1 Synchronous Two-way Relay Networks

In many works on two-way relay-assisted networks, the authors assume synchronous communication between relays and the transceivers. In this scenario, it is assumed

that all relaying paths going through each relay have an identical propagation delay, so that the transmitted and relayed signals are simultaneously received by transceiver.

In [9], the authors investigate the effect of channel estimation error on the performance of the receiver of a MIMO two-way AF relay network. In this paper, the authors analyzed the linear minimum mean square estimation (LMMSE) of composite and individual channels and showed that orthogonal pilot symbols minimize the individual and composite mean square errors.

In [10, 31, 36], the optimal value of beamforming coefficients as well as optimal transmit powers for a bi-directional AF relay network, is obtained using two different optimization criteria. In the first scenario, the total transmit power is minimized under two constraints on the transceivers' received signal-to-noise ratios (SNRs). An SNR balancing problem is next examined in which the smaller of the two transceivers' SNRs is maximized under a total transmit power constraint. It has been shown that both techniques have a unique solution which leads to a power allocation scheme where half of the maximum power budget is allocated to both transceivers and the other half is shared among the relay nodes. In [35], a semi-closed form solution is presented for the SNR balancing problem. Furthermore, a suboptimal solution is presented with a close performance to the optimal beamformers.

In [45], two single-antenna transceivers exchange the information with the help of a multi-antenna relay node. In the first time slot, the sum of signals from both source nodes is received at the relay and then in the second time slot, the assisting relay linearly transforms the received signal and forwards it to the two transceivers. In order to cancel self-interference of the signal received at both source nodes from the relay node, each source node applies the principle of analogue network coding

(ANC) and then decodes the desired message. The authors of this paper [45] present a capacity maximizing relay beamforming structure and an efficient algorithm to obtain the optimal beamforming matrix based on convex optimization techniques.

Assuming a multiuser two-way relay network, the authors of [46] consider a network where multiple pairs of partners communicate with each other in using a common sharing relay. In this paper, the joint power control and receiver optimization problem is investigated.

In [52], an iterative algorithm has been studied to obtain an optimal rate region in a two-way relay channel where two transceivers employ multiple AF relays. The proposed iterative algorithm in each step is equivalent to solving a power minimization problem subject to minimum signal-to-interference-and-noise ratio (SINR) constraints.

A multiuser two-way AF relaying scheme is proposed in [54] with multiple-input and MIMO relay transceiver processing. To optimize the relay processing, zero-forcing (ZF) and minimum-mean-square error (MMSE) schemes under the relay power constraints are investigated. The authors compare different transmit and beamforming methods including eigen-beamforming, antenna selection, random beamforming, and modified equal gain beamforming. In order to provide fairness to all users as well as to maximize the system SNR, different global and local power control techniques are designed. It has been proved that this system can efficiently combat both self-interference (SI) and co-channel interference (CCI).

In [57], a two-way relay network with amplify-and-forward MIMO relays and MIMO transceivers is studied. In this paper, the authors examine channel estimation

schemes (such as a simple least squared (LS) based scheme) to estimate the end-to-end channel as well as a tensor-based channel estimation (TENICE) scheme that improves the accuracy of the estimation by using a novel structure in the compound channel structure.

The problem of resynchronization of asynchronous cooperative communication systems has also been studied in [73–76]. In [74], considering an asynchronous AF cooperative network, to estimate the unknown timing and channel parameters, a framework is proposed which consists of a LS estimator as the initial estimation and then an iterative maximum-likelihood (ML) estimator to refine the LS estimates. Furthermore, in order to identify the system uncertainties resulting from estimation, an analysis based on Cramer-Rao bound (CRB) is presented. Moreover, the authors design efficient timing synchronization algorithms using the parameter estimates in their analysis at the relays and destinations. The results show the proposed framework approaches to the performance of a synchronized case with perfect channel state information.

The problem of timing synchronization for a DF cooperative communication system with a single source, a single destination and multiple relays is studied in [75]. In this paper, in order to estimate the multiple delays associated with different relays, a ML estimator with exhaustive search over the estimation range is employed. Since the complexity of the ML estimator exponentially increases with the number of relays, a correlation-timing estimator is considered to save computational complexity.

2.2.2 Asynchronous Two-way Relay Networks

In most of the results published in two-way relay networks, the authors assume that relays and the transceivers are time-synchronized. However, the fact that the propagation delays for various relay-transceiver paths can be different leads to time-asynchronous communication as well as frequency selective end-to-end channel impulse responses. In such scenarios, even if the relay-transceiver channel is frequency flat, ISI is inevitable at the transceivers. Therefore, different methods have been introduced in the literature to tackle ISI [23, 37, 40, 59, 77–85].

In [37], an asynchronous bi-directional multi-carrier relay network is considered with two single-antenna transceivers and multiple single-antenna relays. The authors proposed an optimization framework to obtain the achievable SNR and rate regions through optimal subcarrier transmit power allocation at the two transceivers and distributed beamforming at the relays. An asynchronous two-relay cooperation network with DF and AF relaying protocols is considered in [85]. The authors derived the outage probability in the high-SNR and then evaluated the impact of the relative delay between two relays on this outage probability. They showed that the outage probability performance becomes independent from the relative delay for a sufficiently high relative delay. In addition, the authors conducted an optimization approach in the high SNR regime to obtain optimal power distribution among the nodes of the network through minimizing the outage probability.

The work in [40] also examines a similar network to that in [37]. In order to combat the ISI caused by the multipath channels, the OFDM scheme is employed at both transceivers. Based on min-max fair design, the authors have proposed two different algorithms to obtain the subcarrier power loading at the two transceivers

as well as the relay beamforming weights. Moreover, in [84], the authors studied a similar problem and obtained a semi-closed-form solution for the relay beamforming weights and related maximum balanced SNR. In [23], for the aforementioned relay network, the authors used a single-carrier post-channel equalizer at two transceivers to combat ISI. The optimal transmit powers and the post-channel equalizers at the two transceivers as well as the relay beamforming weight vector are obtained by minimizing the total MSE of linearly estimated signals at the two transceivers under limited transmit power budget. It has been shown that the optimization problem led to a relay selection scheme and also the optimal relay beamforming weight vector has a semi-closed-form solution.

In [26], assuming a multi-carrier asynchronous bidirectional AF relay network, the OFDM scheme is used to equalize the frequency selective channel. The authors aimed to maximize the sum-rate subject to a total power constraint, through jointly optimal relay beamforming weights and transceiver subcarrier power loading. This study identified that this problem leads to a relay selection scheme where only the relays which contribute to one tap of end-to-end channel impulse response have to be utilized. A semi-closed-form solution for the optimal relay beamforming vectors and the subcarrier powers at the transceivers is obtained. Furthermore, a simple search method is employed to find the optimal tap.

2.3 Relay Selection

Relay selection has attracted a great deal of attention in the literature as an effective method to improve the performance of wireless cooperative networks.

The authors of [86] and [87] introduced different methods for relay selection in order to minimize the error rate and to optimize the outage probability of communication networks. In [88], a wireless communication network with a single source, a single destination and multiple uniformly distributed relay nodes is considered. The authors attempted to minimize the total transmission time of a fixed amount of data by selecting a set of cooperating relays.

The authors in [89] introduced a common and practical paradigm in cooperative communication systems as a dynamically selected "best" relay to decode and forward information from a source to a destination. Such proposed systems use two phases, called the relay selection phase and also the data transmission phase. In the first phase (relay selection), the system uses transmission time and energy in order to select the best relay. In the transmission phase, the spatial diversity benefits of the selection is used to transmit data. A closed-form expression for overall throughput and energy consumption is derived. The authors also studied the time and energy trade-off between the selection and data transmission phases.

In [90], the authors generalized the idea of a single-relay selection by introducing multiple relay selection schemes in a one-way AF relay network. Considering that the power used at the transmitter and relay nodes of this communication network is limited, the authors derived the achievable diversity of the existing single-relay selection schemes. Also, the SNR-optimal multiple relay selection schemes as well as suboptimal multiple relay selection schemes were discussed. It was shown that these schemes achieve low error, rate and full diversity and the number of cooperating relays varies subject to channel conditions.

2.4 Research Outcomes

In this thesis, we consider a two-way relay network consisting of two single-antenna transceivers and multiple single-antenna relays employing AF relaying protocol. This network is assumed to be asynchronous meaning, that the relaying paths are subject to different delays. In such a network, the end-to-end channel can be viewed as a multi-path channel which can cause ISI at the received signals at the two transceivers. While several methods have been studied to combat ISI, we propose a novel approach, namely a joint pre- and post-channel equalization scheme at the two front-ends of the transceivers. To this end, we study the problem of optimal design of pre- and post-channel equalizers, relay weight vectors, and transmit powers at the two transceivers. To do so, we minimize the total MSE of the linearly estimated signals at the two transceivers subject to a total transmit power constraint by optimally obtaining the network beamforming, power loading and the pre- and post-channel equalization blocks at both transceivers. We rigorously prove that this minimization leads to a relay selection scheme such that only those relays that contribute to the optimal tap (the non-zero tap obtained by minimization problem) of the end-to-end channel impulse response will be turned on and the rest of the relays are inactive. To find the optimal tap (the only non-zero tap) of the end-to-end channel impulse response, we present a simple algorithm. We also obtain semi-closed-form solutions for the the relay beamforming weight vector, for the optimal transmission powers, and for the optimal pre- and post-channel equalization matrices at the two transceivers.

Chapter 3

Jointly Optimal Pre- and Post-Channel Equalization and Distributed Beamforming

3.1 Data model

In this thesis, we consider a two-way amplify-and-forward relay network which consists of L relay nodes and two transceivers. Since the signals going through different relays arrive at the two transceivers at different times, the end-to-end channel can be viewed as a multi-path link, and thus can cause inter-symbol-interference (ISI) at sufficiently high data rates. Using pre- and post-channel block equalization is one way to combat such an ISI. In the sequel, we explain this equalization scheme in detail. As seen in Fig.1, at each transceiver, the information symbols go through a serial-to-parallel

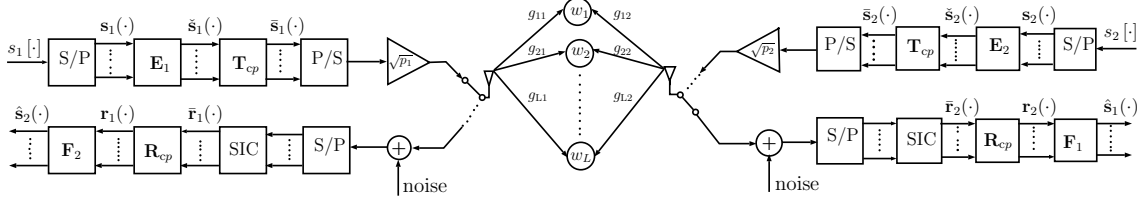


Figure 3.1: System block diagram.

conversion block (denoted as “S/P”), which converts serial symbols into blocks of N_s symbols. At Transceiver q , the i -th block of information symbols is represented as

$$\mathbf{s}_q(i) \triangleq [s_q[iN_s] \ s_q[iN_s + 1] \ \cdots \ s_q[(i + 1)N_s - 1]]^T \quad (3.1.1)$$

where $s_q[k]$ represents the k -th symbol transmitted by Transceiver q , for $q \in \{1, 2\}$. We assume $E\{|s_q[k]|^2\} = 1$ and $E\{s_q[k]\} = 0$, for $q \in \{1, 2\}$.

In order to equalize the end-to-end channel, one can resort to a joint linear pre- and post-channel equalization scheme, where the channel equalization is performed at both transmit and receive front-ends of the the two transceivers. In such a scheme, the blocks of information symbols that are to be transmitted by the two transceivers are pre-coded (pre-equalized) via multiplying them with a pre-channel equalization matrix. On the receiving side, the blocks of received data undergo a linear transformation (i.e, post-channel equalization) that yields a linear estimate of the transmitted block of information symbols. In our two-way relaying scheme, two $N_s \times N_s$ block *pre-channel* equalizers, denoted as \mathbf{E}_1 and \mathbf{E}_2 , are implemented at Transceivers 1 and 2, respectively. At the output of the pre-channel block equalizer at Transceiver q , the pre-equalized (pre-coded) block of symbols is given by $\check{\mathbf{s}}_q(i) \triangleq \mathbf{E}_q \mathbf{s}_q(i)$, for $q = 1, 2$.

In order to mitigate the effect of inter-block-interference (IBI) between adjacent blocks, a cyclic prefix insertion matrix is added to $\check{\mathbf{s}}_q(i)$ by multiplying $\check{\mathbf{s}}_q(i)$ with the

matrix $\mathbf{T}_{cp} = [\mathbf{I}_{cp}^T \mathbf{I}_{N_s}^T]^T$, where \mathbf{I}_{cp} is the matrix of the last N rows of the $N_s \times N_s$ identity matrix \mathbf{I}_{N_s} , and N is the length of the vector of the taps of the equivalent discrete-time end-to-end channel impulse response¹. At Transceiver q , the i -th transmitted block $\bar{\mathbf{s}}_q(i)$ after cyclic prefix insertion is given by

$$\begin{aligned} \bar{\mathbf{s}}_q(i) &= [\bar{s}_q[iN_t] \quad \bar{s}_q[iN_t + 1] \quad \cdots \quad \bar{s}_q[(i+1)N_t - 1]]^T \\ &\triangleq \mathbf{T}_{cp} \check{\mathbf{s}}_q(i) \\ &= [s_q[(i+1)N_s - N] \quad \cdots \quad s_q[(i+1)N_s - 1] \quad s_q[iN_s] \quad \cdots \quad s_q[(i+1)N_s - 1]]^T \end{aligned} \quad (3.1.2)$$

where $N_t = N + N_s$ is the length of the transmitted blocks and $\bar{s}_q[iN_t + k]$ is the k -th entry of $\bar{\mathbf{s}}_q(i)$, for $k = 0, 1, \dots, N_t - 1$, for $q \in \{1, 2\}$. The data block $\bar{\mathbf{s}}_q(i)$ goes through the parallel-to-serial block (denoted as ‘‘P/S’’) and is converted to serial symbols. Next, at Transceiver q , the serial symbols are amplified by $\sqrt{p_q}$, where p_q is the transmit power of this transceiver. The amplified symbols are then transmitted over the multi-path relay channel.

At the other side of the channel, the noise-corrupted version of the transmitted block received by Transceiver q is passed through a serial-to-parallel conversion block, thereby turning into blocks of length N_t . The signal block then goes through the self-interference cancellation² (SIC) block of Transceiver q . As a result, the i -th signal received block at output of the SIC block can be written as

$$\begin{aligned} \bar{\mathbf{r}}_q(i) &\triangleq \mathbf{H}_0(\mathbf{w})\bar{\mathbf{s}}_q(i) + \mathbf{H}_1(\mathbf{w})\bar{\mathbf{s}}_q(i-1) + \bar{\boldsymbol{\gamma}}_q(i) \\ &= \sqrt{p_q}\mathbf{H}_0(\mathbf{w})\mathbf{T}_{cp}\mathbf{E}_{\bar{q}}\mathbf{s}_{\bar{q}}(i) + \sqrt{p_q}\mathbf{H}_1(\mathbf{w})\mathbf{T}_{cp}\mathbf{E}_{\bar{q}}\mathbf{s}_{\bar{q}}(i-1) + \bar{\boldsymbol{\gamma}}_q(i) \end{aligned} \quad (3.1.3)$$

¹We will elaborate on the end-to-end channel model in the next section.

²The SIC block at Transceiver q subtracts, from the received signal, the self-signal that the relays transmit back to this transceiver. Hence, the self-signal that the relays transmit back to this transmitter is eliminated from the received signal. Note that for each transceiver, the self-signal goes through the multi-path channel between the transceiver and the relays.

where $\bar{q} = 1$, for $q = 2$, and $\bar{q} = 2$, when $q = 1$, whereas $\bar{\gamma}_q(i)$ is the total received noise at Transceiver q which consists of transceiver measurement noise and relay noises that are forwarded to this transceiver³. We have used the following definitions:

$$\mathbf{H}_0(\mathbf{w}) \triangleq \begin{bmatrix} h[0] & 0 & 0 & \cdots & 0 \\ \vdots & h[0] & 0 & \cdots & 0 \\ h[N-1] & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & h[N-1] & \cdots & h[0] \end{bmatrix}$$

$$\mathbf{H}_1(\mathbf{w}) \triangleq \begin{bmatrix} 0 & \cdots & h[N-1] & \cdots & h[1] \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & h[N-1] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}. \quad (3.1.4)$$

Here, $h[\cdot]$ represents the discrete-time equivalent impulse response corresponding to the end-to-end channel between Transceivers 1 and 2, and \mathbf{w} is the $L \times 1$ vector of the relay beamforming weights. In the next subsection, we show how $h[\cdot]$ is related to \mathbf{w} .

After self-interference cancellation, the first N entries of the received signal are discarded by pre-multiplying it with the cyclic removal matrix, denoted as $\mathbf{R}_{cp} \triangleq [\mathbf{0}_{N_s \times N} \ \mathbf{I}_{N_s}]$. It also can be proved that $\mathbf{R}_{cp}\mathbf{H}_1(\mathbf{w}) = 0$, hence IBI-inducing matrix $\mathbf{H}_1(\mathbf{w})$ is removed by the cyclic removal operation. Therefore, we can write

$$\begin{aligned} \mathbf{r}_q(i) &\triangleq \mathbf{R}_{cp}\bar{\mathbf{r}}_q(i) = \sqrt{p_{\bar{q}}}\mathbf{R}_{cp}\mathbf{H}_0(\mathbf{w})\mathbf{T}_{cp}\mathbf{E}_{\bar{q}}\mathbf{s}_{\bar{q}}(i) + \mathbf{R}_{cp}\bar{\gamma}_q(i) \\ &= \sqrt{p_{\bar{q}}}\tilde{\mathbf{H}}(\mathbf{w})\mathbf{E}_{\bar{q}}\mathbf{s}_{\bar{q}}(i) + \gamma_q(i) \end{aligned} \quad (3.1.5)$$

³In the next subsection, we present our model for the noise vector $\bar{\gamma}_q(i)$.

where $\boldsymbol{\gamma}(i) \triangleq \mathbf{R}_{cp} \tilde{\boldsymbol{\gamma}}_q(i)$ and $\tilde{\mathbf{H}}(\mathbf{w}) \triangleq \mathbf{R}_{cp} \mathbf{H}_0(\mathbf{w}) \mathbf{T}_{cp}$ is an $N_s \times N_s$ circulant matrix whose (k, l) -th entry is defined by $\tilde{h}[(k-l) \bmod N_s]$, where $\tilde{h}[n] \triangleq h[n]$, for $n = 0, 1, \dots, N-1$ and $\tilde{h}[n] = 0$, for $n = N, N+1, \dots, N_s-1$, i.e., $\tilde{h}[n]$ is the zero-padded version of $h[n]$ with $N - N_s$ zeros added to $h[n]$. Note that the number of the symbols per block N_s must be larger than, or equal to the length of the end-to-end channel N .

To mitigate the ISI caused by the frequency selectivity of the end-to-end channel at the output vector of the cyclic prefix removal matrix at both transceivers, two $N_s \times N_s$ post-channel block equalizers, denoted as \mathbf{F}_1 and \mathbf{F}_2 , are then used at Transceivers 1 and 2, respectively. The linear estimate of the information symbol block, transmitted by Transceiver \bar{q} , is obtained at the output of the corresponding post-channel block equalizer \mathbf{F}_q as

$$\begin{aligned} \hat{\mathbf{s}}_{\bar{q}}(i) &\triangleq \mathbf{F}_q \mathbf{r}_q(i) \\ &= \sqrt{p_{\bar{q}}} \mathbf{F}_q \tilde{\mathbf{H}}(\mathbf{w}) \mathbf{E}_{\bar{q}} \mathbf{s}_{\bar{q}}(i) + \mathbf{F}_q \boldsymbol{\gamma}_q(i) \end{aligned} \quad (3.1.6)$$

where $\hat{\mathbf{s}}_{\bar{q}}(i)$ is $N_s \times 1$ vector of the linear estimate of symbols transmitted by Transceiver \bar{q} .

3.1.1 End-to-End Channel Modeling

Assuming the channel between each relay and each transceiver is reciprocal and frequency flat, the linear time-invariant channel between the two transceivers can be represented by its channel impulse response, denoted as $h[\cdot]$. Indeed, the impulse response $h[\cdot]$ represents the linear time-invariant (LTI) channel between Transceivers 1 and 2. The end-to-end channel from Transceiver 1 to 2 can be viewed as a multi-path

channel whose discrete-time finite impulse response is given by

$$h[n] = \sum_{l=1}^L b_l \delta[n - \check{n}_l], \quad \text{for } 0 \leq n \leq N - 1 \quad (3.1.7)$$

where

$$b_l \triangleq g_{lq} g_{l\bar{q}}, \quad \text{for } q \in \{1, 2\}. \quad (3.1.8)$$

Here, \check{n}_l is the discrete-time propagation delay of the l -th relaying path which originates from Transceiver 1, goes through the l -th relay, and terminates at Transceiver 2. Assuming that τ_l denotes the propagation delay of the l -th signal path between Transceivers 1 and 2, corresponding to the l -th relay, \check{n}_l satisfies $(\check{n}_l - 1)T_s < \tau_l \leq \check{n}_l T_s$, where T_s represents the symbol period. Let N be the length of the equivalent discrete-time channel impulse response $h[\cdot]$, that is $N = 1 + \max_{1 \leq l \leq L} \check{n}_l$. Here, N is the maximum length of discrete-time end-to-end channel impulse response. Assuming a rectangular pulse shape with duration T_s , the l -th relay contributes to the n -th tap of $h[\cdot]$ only if $(n - 1)T_s < \tau_l \leq nT_s$. Hence, the contribution of different relay paths to the end-to-end channel impulse response can be determined by $N \times L$ matrix \mathbf{B} whose $(n + 1, l)$ -th element, for $n = 0, 1, \dots, N - 1$ and $l = 1, 2, \dots, L$, is defined as

$$B(n + 1, l) = \begin{cases} g_{lq} g_{l\bar{q}}, & (n - 1)T_s < \tau_l \leq nT_s \\ 0, & \text{otherwise.} \end{cases} \quad (3.1.9)$$

Indeed, the contribution of the l -th relay to the n -th tap of $h[\cdot]$, for $n = 0, 1, \dots, N - 1$, and $l = 1, 2, \dots, L$ can be described by $\mathbf{B}(n + 1, l)w_l$, where w_l is the complex beamforming weight of this relay. Hence, the vector of taps of the end-to-end channel impulse response, denoted as $\mathbf{h}(\mathbf{w})$, can be written as

$$\mathbf{h}(\mathbf{w}) = \mathbf{B}\mathbf{w} \quad (3.1.10)$$

where $\mathbf{h}(\mathbf{w}) \triangleq [h[0] \ h[1] \ \dots \ h[N-1]]^T$ is the $N \times 1$ vector of the discrete-time end-to-end channel taps and $\mathbf{w} \triangleq [w_1 \ w_2 \ \dots \ w_L]^T$ represents the $L \times 1$ vector of the complex relay weights.

3.1.2 Received Noise Modeling

Let τ'_{lq} represent the propagation delay between the l -th relay and Transceiver q and n'_{lq} is an integer value which satisfies $\frac{\tau'_{lq}}{T_s} \leq n'_{lq} < \frac{\tau'_{lq}}{T_s} + 1$. We denote the spatially and temporally white noise at the l -th relay as $v_l[n]$ which is assumed to be zero-mean with variance σ^2 . This noise is amplified by w_l and arrives at Transceiver q with delay n'_{lq} . The n -th sample of the relay noises received at Transceiver q , denoted as $\xi_q[n]$, can be modeled as

$$\xi_q[n] = \sum_{l=1}^L w_l g_{lq} v_l[n - n'_{lq}] = \mathbf{v}_{n,q}^T \mathbf{G}_q \mathbf{w} \quad (3.1.11)$$

where

$$\mathbf{v}_{n,q} = [v_1[n - n'_{1q}] \ v_2[n - n'_{2q}] \ \dots \ v_L[n - n'_{Lq}]]^T \quad (3.1.12)$$

$$\mathbf{G}_q = \text{diag}\{g_{1q}, g_{2q}, \dots, g_{Lq}\}. \quad (3.1.13)$$

The n -th sample of the total noise received at Transceiver q , denoted as $\bar{\gamma}_q[n]$, can be written as

$$\bar{\gamma}_q[n] = \xi_q[n] + \gamma'_q[n] \quad (3.1.14)$$

where $\gamma'_q[n]$ is the n -th sample of the measurement noise at Transceiver q . We can use vector notation to rewrite (3.1.14) as

$$\bar{\gamma}_q(i) = \boldsymbol{\xi}_q(i) + \gamma'_q(i) \quad (3.1.15)$$

where the following definitions are considered

$$\begin{aligned}\bar{\boldsymbol{\gamma}}_q(i) &\triangleq [\bar{\gamma}_q[iN_t] \quad \bar{\gamma}_q[iN_t + 1] \quad \dots \quad \bar{\gamma}_q[iN_t + N_t - 1]]^T \\ \boldsymbol{\xi}_q(i) &\triangleq [\xi_q[iN_t] \quad \xi_q[iN_t + 1] \quad \dots \quad \xi_q[iN_t + N_t - 1]]^T \\ \boldsymbol{\gamma}'_q(i) &\triangleq [\gamma'_q[iN_t] \quad \gamma'_q[iN_t + 1] \quad \dots \quad \gamma'_q[iN_t + N_t - 1]]^T\end{aligned}$$

Hence, the total noise received at Transceiver q can be written as

$$\bar{\boldsymbol{\gamma}}_q(i) = \boldsymbol{\Upsilon}_q(i)\mathbf{G}_q\mathbf{w} + \boldsymbol{\gamma}'_q(i) \quad (3.1.16)$$

where $\boldsymbol{\Upsilon}_q(i) \triangleq [\mathbf{v}_{(iN_t),q} \quad \mathbf{v}_{(iN_t+1),q} \quad \dots \quad \mathbf{v}_{(iN_t+N_t-1),q}]^T$ represents an $N_t \times L$ matrix whose l -th column is the l -th relay noise corresponding to the i -th received block after it goes through the delay between the l -th relay and Transceiver q . Using (3.1.16), we can write the covariance matrix of the noise vector $\boldsymbol{\gamma}_q(i) = \mathbf{R}_{cp}\bar{\boldsymbol{\gamma}}_q(i)$ as

$$E\{\boldsymbol{\gamma}_q(i)\boldsymbol{\gamma}_q^H(i)\} = \sigma^2(\mathbf{w}^H\mathbf{G}_q^H\mathbf{G}_q\mathbf{w} + 1)\mathbf{I}_{N_s} \quad (3.1.17)$$

where we have used the assumptions that the relay noise process $v_l[n]$ is temporally uncorrelated for $l = 1, 2, \dots, L$ and that the transceiver noise process $\gamma'_q[n]$ is also temporally uncorrelated for $q = 1, 2$. We will use our model for noise and in particular (3.1.17) to calculate the covariance matrix of the received block $\mathbf{r}_q(i)$ which is needed in our MSE minimization approach to jointly design the network beamformer, the pre- and post-channel block equalizers at the two transceivers, and the transmit powers of the two transceivers.

3.1.3 Total Transmit Power Derivations

We aim to find the power consumed in the whole network in terms of the relay weight vector \mathbf{w} and transceivers' transmit powers. The $N_t \times 1$ vector $\bar{\mathbf{x}}_l(i)$ of the i -th signal

block relayed by the l -th relay can be written as

$$\begin{aligned}\bar{\mathbf{x}}_l &\triangleq [\bar{x}_l[iN_t] \ \bar{x}_l[iN_t + 1] \ \dots \ \bar{x}_l[iN_t + N_t - 1]]^T \\ &= w_l (\sqrt{p_1}g_{l1}\bar{\mathbf{s}}_1[i] + \sqrt{p_2}g_{l2}\bar{\mathbf{s}}_2[i] + \mathbf{v}_l[i])\end{aligned}\quad (3.1.18)$$

where $\bar{x}_l[t]$ is the signal transmitted by the l -th relay at time t and the vector $\mathbf{v}_l(i) \triangleq [v_l[iN_t] \ v_l[iN_t+1] \ \dots \ v_l[iN_t + N_t - 1]]^T$ is the i -th block of noise at the l -th relay. We assume that $\mathbf{v}_l(\cdot)$ is a stationary zero-mean random vector with uncorrelated entries whose variances are equal to σ^2 . Using (3.1.18), the average transmit power of the l -th relay is then obtained as⁴

$$\begin{aligned}\tilde{p}_l &\triangleq \frac{1}{N_t} E\{\bar{\mathbf{x}}_l^H(i)\bar{\mathbf{x}}_l(i)\} \\ &= |w_l|^2 (|g_{l1}|^2 p_1 + |g_{l2}|^2 p_2 + \sigma^2).\end{aligned}\quad (3.1.19)$$

Using (3.1.19), the total transmit power of the network can be obtained as

$$\begin{aligned}p_{total} &\triangleq p_1 + p_2 + \sum_{i=1}^L \tilde{p}_i \\ &= p_1 + p_2 + \sum_{i=1}^L |w_i|^2 (|g_{i1}|^2 p_1 + |g_{i2}|^2 p_2 + \sigma^2) \\ &= p_1 (1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2 (1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w}.\end{aligned}\quad (3.1.20)$$

In our design, the total power p_{total} is assumed to be less than, or equal to the maximum power p_{max} .

⁴Note that to obtain (3.1.19), it is assumed that the communication time frame is much longer than the maximum time difference between arrivals of transceiver signals at the relays.

3.2 Optimal Design

Our goal is to optimally obtain the block pre- and post-channel equalizers \mathbf{E}_1 , \mathbf{E}_2 , \mathbf{F}_1 and \mathbf{F}_2 , the relay beamforming weight vector \mathbf{w} , and transmit powers p_1 and p_2 , such that the total MSE in the linear estimates of the received symbols at the two transceivers is minimized under a total power constraint. We can write the $N_s \times 1$ vector of the symbol estimate errors at Transceiver q , corresponding to the i -th symbol block transmitted by Transceiver q , as

$$\mathbf{e}_q(i) \triangleq \hat{\mathbf{s}}_q(i) - \mathbf{s}_q(i). \quad (3.2.1)$$

Note that as p_q is defined as the transmit power of Transceiver q , we can write

$$\begin{aligned} p_q &= \frac{1}{N_s} E\{(\sqrt{p_q} \mathbf{E}_q \mathbf{s}_q(i))^H (\sqrt{p_q} \mathbf{E}_q \mathbf{s}_q(i))\} = \frac{1}{N_s} E\{p_q \mathbf{s}_q^H(i) \mathbf{E}_q^H \mathbf{E}_q \mathbf{s}_q(i)\} \\ &= \frac{1}{N_s} E\{tr [p_q \mathbf{E}_q \mathbf{s}_q(i) \mathbf{s}_q^H(i) \mathbf{E}_q^H]\} = \frac{1}{N_s} tr [p_q \mathbf{E}_q E\{\mathbf{s}_q(i) \mathbf{s}_q^H(i)\} \mathbf{E}_q^H] \\ &= \frac{p_q}{N_s} tr [\mathbf{E}_q^H \mathbf{E}_q] \end{aligned} \quad (3.2.2)$$

Hence, the squared Frobenius norm of \mathbf{E}_q has to be equal to N_s (i.e., $\|\mathbf{E}_q\|_F^2 = tr [\mathbf{E}_q^H \mathbf{E}_q] = N_s$). In order to obtain: (1) jointly optimal pre- and post-channel block equalizers; (2) transmit powers at both transceivers; and (3) the relay beamforming weight vector, the problem of minimizing the total MSE under the total available power constraint can be written as

$$\begin{aligned} &\min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \min_{\mathbf{E}_1, \mathbf{E}_2} \min_{\mathbf{F}_1, \mathbf{F}_2} \sum_{q=1}^2 E\{\|\mathbf{e}_q(i)\|^2\} \\ &\text{subject to } p_{total} \leq p_{max} \quad \text{and} \quad \|\mathbf{E}_q\|_F^2 = N_s, \text{ for } q \in \{1, 2\} \end{aligned} \quad (3.2.3)$$

where p_{max} represents the maximum available power in this network. Using a total power constraint is widely used and has been well justified. For the sake of brevity, we do not repeat these and refer our reader to [26] for detailed justification for this type of constraint. The expectation in (3.2.3) is taken with respect to random symbols and noise. Let us first consider the inner minimization problem in (11) as

$$\begin{aligned} \min_{\mathbf{F}_1, \mathbf{F}_2} \quad & \sum_{q=1}^2 E\{\mathbf{e}_q^H(i)\mathbf{e}_q(i)\} \\ \text{subject to} \quad & \|\mathbf{E}_q\|_F^2 = N_s, \text{ for } q \in \{1, 2\}. \end{aligned} \quad (3.2.4)$$

Using the assumptions that $E\{\mathbf{s}_q(i)\} = \mathbf{0}$ and $E\{\boldsymbol{\gamma}_q(i)\} = \mathbf{0}$ along with (3.1.5) and (3.2.1), the MSE at Transceiver \bar{q} (corresponding to estimate error in $\mathbf{s}_q(i)$) can be written as

$$\begin{aligned} MSE_q(\mathbf{w}, \mathbf{F}_{\bar{q}}, \mathbf{E}_q, p_q) &\triangleq E\{\mathbf{e}_q^H(i)\mathbf{e}_q(i)\} \\ &= E\{[\hat{\mathbf{s}}_q^H(i) - \mathbf{s}_q^H(i)][\hat{\mathbf{s}}_q(i) - \mathbf{s}_q(i)]\} \\ &= E\{[\mathbf{r}_{\bar{q}}^H(i)\mathbf{F}_{\bar{q}}^H - \mathbf{s}_q^H(i)][\mathbf{F}_{\bar{q}}\mathbf{r}_{\bar{q}}(i) - \mathbf{s}_q(i)]\} \\ &= E\{\mathbf{r}_{\bar{q}}^H(i)\mathbf{F}_{\bar{q}}^H\mathbf{F}_{\bar{q}}\mathbf{r}_{\bar{q}}(i) - \mathbf{r}_{\bar{q}}^H(i)\mathbf{F}_{\bar{q}}^H\mathbf{s}_q(i) - \mathbf{s}_q^H(i)\mathbf{F}_{\bar{q}}\mathbf{r}_{\bar{q}}(i) + \mathbf{s}_q^H(i)\mathbf{s}_q(i)\} \\ &= tr[E\{\mathbf{F}_{\bar{q}}\mathbf{r}_{\bar{q}}(i)\mathbf{r}_{\bar{q}}^H(i)\mathbf{F}_{\bar{q}}^H\}] - tr[E\{\mathbf{r}_{\bar{q}}^H(i)\mathbf{F}_{\bar{q}}^H\mathbf{s}_q(i)\}] \\ &\quad - tr[E\{\mathbf{s}_q^H(i)\mathbf{F}_{\bar{q}}\mathbf{r}_{\bar{q}}(i)\}] + tr[E\{\mathbf{s}_q(i)\mathbf{s}_q^H(i)\}] \\ &= tr[\mathbf{F}_{\bar{q}}\mathbf{R}_{\bar{q}}(\mathbf{w})\mathbf{F}_{\bar{q}}^H] - \sqrt{p_q}tr[\mathbf{F}_{\bar{q}}\tilde{\mathbf{H}}(\mathbf{w})\mathbf{E}_q + \mathbf{E}_q^H\tilde{\mathbf{H}}^H(\mathbf{w})\mathbf{F}_{\bar{q}}^H] + N_s \end{aligned} \quad (3.2.5)$$

where $\mathbf{R}_q(\mathbf{w}) \triangleq E\{\mathbf{r}_q(i)\mathbf{r}_q^H(i)\}$ represents the correlation matrix of the received signal block \mathbf{r}_q at Transceiver q . Using (3.1.5) and (3.1.17), along with the assumption that different entries of $\mathbf{s}_{\bar{q}}(i)$ and $\boldsymbol{\gamma}(i)$ are uncorrelated, $\mathbf{R}_q(\mathbf{w})$ can be written as

$$\mathbf{R}_q(\mathbf{w}) \triangleq E\{\mathbf{r}_q(i)\mathbf{r}_q^H(i)\} = p_{\bar{q}}\tilde{\mathbf{H}}(\mathbf{w})\mathbf{E}_{\bar{q}}\mathbf{E}_{\bar{q}}^H\tilde{\mathbf{H}}^H(\mathbf{w}) + \sigma^2(\mathbf{w}^H\mathbf{G}_q^H\mathbf{G}_q\mathbf{w} + 1)\mathbf{I}_{N_s} \quad (3.2.6)$$

where we have used the assumptions of $E\{|\mathbf{s}_q(i)|^2\} = \mathbf{I}_{N_s}$.

The optimal value of \mathbf{F}_q can be obtained by differentiating (3.2.5) with respect to $\mathbf{F}_{\bar{q}}$ and equating the derivative to zero. Using the fact that for any given relay beamforming weight \mathbf{w} , we can write $\mathbf{R}_q^H(\mathbf{w}) = \mathbf{R}_q(\mathbf{w})$, the optimal value of \mathbf{F}_q is obtained as

$$\mathbf{F}_q^{opt}(\mathbf{w}) = \sqrt{p_{\bar{q}}} \mathbf{E}_{\bar{q}}^H \tilde{\mathbf{H}}^H(\mathbf{w}) \mathbf{R}_q^{-1}(\mathbf{w}). \quad (3.2.7)$$

Using (3.2.5) and (3.2.7), we can write

$$\sum_{q=1}^2 \text{MSE}_q(\mathbf{w}, \mathbf{F}_{\bar{q}}^{opt}, \mathbf{E}_q, p_1, p_2) = \sum_{q=1}^2 \left(N_s - p_q \text{tr} \{ \mathbf{E}_q^H \tilde{\mathbf{H}}^H(\mathbf{w}) \mathbf{R}_{\bar{q}}^{-1}(\mathbf{w}) \tilde{\mathbf{H}}(\mathbf{w}) \mathbf{E}_q \} \right). \quad (3.2.8)$$

Using (3.2.8) the minimization over \mathbf{E}_1 and \mathbf{E}_2 in (3.2.4) can be written as⁵

$$\begin{aligned} \min_{\mathbf{E}_1, \mathbf{E}_2} \quad & \sum_{q=1}^2 \left(N_s - p_{\bar{q}} \text{tr} \{ \mathbf{E}_{\bar{q}}^H \tilde{\mathbf{H}}^H(\mathbf{w}) \mathbf{R}_q^{-1}(\mathbf{w}) \tilde{\mathbf{H}}(\mathbf{w}) \mathbf{E}_{\bar{q}} \} \right) \\ \text{subject to} \quad & \|\mathbf{E}_q\|_F^2 = N_s, \text{ for } q \in \{1, 2\}. \end{aligned} \quad (3.2.9)$$

We note that the $N_s \times N_s$ circulant matrix $\tilde{\mathbf{H}}(\mathbf{w})$ can be decomposed as

$$\tilde{\mathbf{H}}(\mathbf{w}) = \mathbf{F}^H \mathbf{D}(\mathbf{w}) \mathbf{F}. \quad (3.2.10)$$

Here, \mathbf{F} is the $N_s \times N_s$ DFT matrix whose (k, k') -th element is defined as

$$F(k, k') = N_s^{-\frac{1}{2}} e^{-j2\pi(k-1)(k'-1)/N_s}$$

for $k = 1, \dots, N_s$ and $k' = 1, \dots, N_s$, and

$$\mathbf{D}(\mathbf{w}) = \text{diag} \{ H(e^{j0}), H(e^{j\frac{2\pi}{N_s}}), \dots, H(e^{j\frac{2\pi(N_s-1)}{N_s}}) \}$$

⁵Note that $\sum_{q=1}^2 \text{MSE}_q(\mathbf{w}, \mathbf{F}_{\bar{q}}^{opt}, \mathbf{E}_q, p_1, p_2) = \sum_{q=1}^2 \text{MSE}_{\bar{q}}(\mathbf{w}, \mathbf{F}_q^{opt}, \mathbf{E}_{\bar{q}}, p_1, p_2)$.

is also an $N_s \times N_s$ diagonal matrix whose diagonal entries are the frequency response of the end-to-end channel at integer multiples of $\frac{1}{N_s}$, i.e.; $H(e^{j2\pi f}) \triangleq \sum_{n=0}^{N-1} h[n]e^{-j2\pi fn}$ is the frequency response of end-to-end channel at the normalized frequency f . The k -th Vandermonde column vector of \mathbf{F}^H can be expressed as

$$\mathbf{f}_k = \frac{1}{\sqrt{N_s}} \left[1 \quad e^{\frac{j2\pi(k-1)}{N_s}} \quad \dots \quad e^{\frac{j2\pi(N_s-1)(k-1)}{N_s}} \right]^T$$

for $k = 1, 2, \dots, N_s$. (3.2.11)

Matrix $\mathbf{D}(\mathbf{w})$ can then be written as

$$\begin{aligned} \mathbf{D}(\mathbf{w}) &= \text{diag}\{H(e^{j0}), H(e^{j\frac{2\pi}{N_s}}), \dots, H(e^{j\frac{2\pi(N_s-1)}{N_s}})\} \\ &= \sqrt{N_s} \text{diag}\{\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}), \mathbf{f}_2^H \tilde{\mathbf{h}}(\mathbf{w}), \dots, \mathbf{f}_{N_s}^H \tilde{\mathbf{h}}(\mathbf{w})\} \end{aligned}$$
(3.2.12)

where $\tilde{\mathbf{h}}(\mathbf{w})$ is the zero-padded version of $\mathbf{h}(\mathbf{w})$, i.e., $\tilde{\mathbf{h}}(\mathbf{w}) = [\mathbf{h}^T(\mathbf{w}) \quad \mathbf{0}_{1 \times (N_s - N)}]^T$.

We now use (3.2.10) to rewrite (3.2.9) as

$$\begin{aligned} \min_{\mathbf{E}_1, \mathbf{E}_2} \quad & \sum_{q=1}^2 \left(N_s - p_{\bar{q}} \text{tr}\{\mathbf{E}_{\bar{q}}^H \mathbf{F}^H \mathbf{D}^H(\mathbf{w}) (\mathbf{F}\mathbf{R}_q(\mathbf{w})\mathbf{F}^H)^{-1} \mathbf{D}(\mathbf{w})\mathbf{F}\mathbf{E}_{\bar{q}}\} \right) \\ \text{subject to} \quad & \|\mathbf{E}_q\|_F^2 = N_s, \text{ for } q \in \{1, 2\} \end{aligned}$$
(3.2.13)

where using (3.2.6) and (3.2.10), we can write $(\mathbf{F}\mathbf{R}_q(\mathbf{w})\mathbf{F}^H)^{-1}$ as

$$\begin{aligned} (\mathbf{F}\mathbf{R}_q(\mathbf{w})\mathbf{F}^H)^{-1} &= (\mathbf{F}\{p_{\bar{q}}\mathbf{F}^H \mathbf{D}(\mathbf{w})\mathbf{F}\mathbf{E}_{\bar{q}}\mathbf{E}_{\bar{q}}^H \mathbf{F}^H \mathbf{D}^H(\mathbf{w})\mathbf{F} + \sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1)\mathbf{I}_{N_s}\}\mathbf{F}^H)^{-1} \\ &= (p_{\bar{q}}\mathbf{D}(\mathbf{w})\mathbf{F}\mathbf{E}_{\bar{q}}\mathbf{E}_{\bar{q}}^H \mathbf{F}^H \mathbf{D}^H(\mathbf{w}) + \sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1)\mathbf{I}_{N_s})^{-1}. \end{aligned}$$
(3.2.14)

Using (3.2.14), we can write the objective function in (3.2.13) as

$$\begin{aligned} \sum_{q=1}^2 \left(N_s - p_{\bar{q}} \text{tr}\{\mathbf{E}_{\bar{q}}^H \mathbf{F}^H \mathbf{D}^H(\mathbf{w}) (\mathbf{F}\mathbf{R}_q(\mathbf{w})\mathbf{F}^H)^{-1} \mathbf{D}(\mathbf{w})\mathbf{F}\mathbf{E}_{\bar{q}}\} \right) &= \\ \sum_{q=1}^2 N_s - p_{\bar{q}} \text{tr} \left\{ (p_{\bar{q}}\mathbf{I}_{N_s} + \delta_q(\mathbf{w})\mathbf{E}_{\bar{q}}^{-1}\mathbf{F}^{-1}\mathbf{D}^{-1}(\mathbf{w})\mathbf{D}^{-H}(\mathbf{w})\mathbf{F}^{-H}\mathbf{E}_{\bar{q}}^{-H})^{-1} \right\}. \end{aligned}$$
(3.2.15)

Defining $\mathbf{C}_{\bar{q}} \triangleq \mathbf{F}\mathbf{E}_{\bar{q}}$ and $\delta_q(\mathbf{w}) \triangleq \sigma^2(\|\mathbf{G}_q\mathbf{w}\|^2 + 1)$ and using (3.2.15), we can rewrite (3.2.13) as

$$\min_{\mathbf{C}_1, \mathbf{C}_2} \sum_{q=1}^2 \left(N_s - p_{\bar{q}} \text{tr} \{ \mathbf{C}_{\bar{q}}^H \mathbf{D}^H(\mathbf{w}) (P_{\bar{q}} \mathbf{D}(\mathbf{w}) \mathbf{C}_{\bar{q}} \mathbf{C}_{\bar{q}}^H \mathbf{D}^H(\mathbf{w}) + \delta_q(\mathbf{w}) \mathbf{I}_{N_s})^{-1} \mathbf{D}(\mathbf{w}) \mathbf{C}_{\bar{q}} \} \right)$$

subject to $\|\mathbf{C}_q\|_F^2 = N_s$, for $q \in \{1, 2\}$. (3.2.16)

Further simplifying (3.2.16) leads us to the following optimization problem:⁶

$$\min_{\mathbf{C}_1, \mathbf{C}_2} \sum_{q=1}^2 (N_s - p_{\bar{q}} \text{tr} \{ (p_{\bar{q}} \mathbf{I}_{N_s} + \delta_q(\mathbf{w}) \mathbf{C}_{\bar{q}}^{-1} \mathbf{D}^{-1}(\mathbf{w}) \mathbf{D}^{-H}(\mathbf{w}) \mathbf{C}_{\bar{q}}^{-H})^{-1} \})$$

subject to $\text{tr} \{ \mathbf{C}_q^H \mathbf{C}_q \} = N_s$, for $q \in \{1, 2\}$. (3.2.17)

Assuming $\mathbf{C}_{\bar{q}}^{-1} \mathbf{D}^{-1}(\mathbf{w}) = \mathbf{T}_{\bar{q}}$, we can then write (3.2.17) as⁷

$$\min_{\mathbf{T}_1, \mathbf{T}_2} \sum_{q=1}^2 (N_s - p_{\bar{q}} \text{tr} \{ (p_{\bar{q}} \mathbf{I}_{N_s} + \delta_q(\mathbf{w}) \mathbf{T}_{\bar{q}} \mathbf{T}_{\bar{q}}^H)^{-1} \})$$

subject to $\text{tr} \{ \mathbf{D}^{-1}(\mathbf{w}) \mathbf{T}_q^{-1} \mathbf{T}_q^{-H} \mathbf{D}^{-H}(\mathbf{w}) \} = N_s$, for $q \in \{1, 2\}$. (3.2.18)

The constrained optimization problem in (3.2.17) can be solved using the Lagrangian multiplier method. We define the Lagrangian as

$$\begin{aligned} \mathcal{L}(\mathbf{T}_1, \mathbf{T}_2, \mu_1, \mu_2) &= \sum_{q=1}^2 (N_s - p_{\bar{q}} \text{tr} \{ (p_{\bar{q}} \mathbf{I}_{N_s} + \delta_q(\mathbf{w}) \mathbf{T}_{\bar{q}} \mathbf{T}_{\bar{q}}^H)^{-1} \}) \\ &+ \sum_{q=1}^2 \mu_q (\text{tr} \{ \mathbf{D}^{-1}(\mathbf{w}) \mathbf{T}_{\bar{q}}^{-1} \mathbf{T}_{\bar{q}}^{-H} \mathbf{D}^{-H}(\mathbf{w}) \} - N_s). \end{aligned} \quad (3.2.19)$$

In order to take the derivative of (3.2.19) with respect to $\mathbf{T}_{\bar{q}}$, we define $\mathbf{X}_q \triangleq p_{\bar{q}} \mathbf{I}_{N_s} +$

⁶Note that at the optimum, \mathbf{C}_q is invertible as \mathbf{E}_q has to be invertible, otherwise the pre-channel equalizer could result in loss of information or ambiguity. As \mathbf{C}_q must be invertible at the optimum, \mathbf{T}_q will also be invertible at the optimum.

⁷Note that at the optimum, the diagonal matrix $\mathbf{D}(\mathbf{w})$ will have to be invertible otherwise the transmitted symbols will not be identifiable even in the absence of noise.

$\delta_q(\mathbf{w})\mathbf{T}_{\bar{q}}\mathbf{T}_{\bar{q}}^H$ and $g(\mathbf{X}_q) \triangleq \text{tr}(\mathbf{X}_q^{-1})$. The derivation can then be written as

$$\begin{aligned} \frac{\partial}{\partial \mathbf{T}_{\bar{q}}} \mathcal{L}(\mathbf{T}_1, \mathbf{T}_2, \mu_1, \mu_2) &= -p_{\bar{q}} \frac{\partial g(\mathbf{X}_q)}{\partial \mathbf{T}_{\bar{q}}} + \\ &\mu_q \frac{\partial}{\partial \mathbf{T}_{\bar{q}}} \text{tr}(\mathbf{D}^{-1}(\mathbf{w})\mathbf{T}_{\bar{q}}^{-1}\mathbf{T}_{\bar{q}}^{-H}\mathbf{D}^{-H}(\mathbf{w})) \end{aligned} \quad (3.2.20)$$

To take the derivative of a matrix which is a function of another matrix, we resort to chain rule. To find the derivative of the function $\mathbf{g}(\mathbf{X}_q) = \text{tr}(\mathbf{X}_q^{-1})$ with respect to $\mathbf{T}_{\bar{q}}$, we use the following differentiation rule:

$$\left[\frac{\partial g(\mathbf{X}_q)}{\partial \mathbf{T}_{\bar{q}}} \right]_{ij} = \text{tr} \left(\left(\frac{\partial g(\mathbf{X}_q)}{\partial \mathbf{X}_q} \right)^T \frac{\partial \mathbf{X}_q}{\partial [\mathbf{T}_{\bar{q}}]_{ij}} \right) \quad (3.2.21)$$

$$\frac{\partial}{\partial \mathbf{X}_q} \text{tr}(\mathbf{X}_q^{-1}) = -(\mathbf{X}_q^{-2})^T. \quad (3.2.22)$$

Using (3.2.21) and (3.2.22), the first term in the right hand side of (3.2.20) can be written as

$$\begin{aligned} \left[\frac{\partial g(\mathbf{X}_q)}{\partial \mathbf{T}_{\bar{q}}} \right]_{ij} &= \frac{\partial \text{tr}(\mathbf{X}_q^{-1})}{\partial [\mathbf{T}_{\bar{q}}]_{ij}} = \text{tr} \left(-\mathbf{X}_q^{-2} \frac{\partial}{\partial [\mathbf{T}_{\bar{q}}]_{ij}} (\delta_q(\mathbf{w})\mathbf{T}_{\bar{q}}\mathbf{T}_{\bar{q}}^H) \right) \\ &= \text{tr}(-\mathbf{X}_q^{-2}\delta_q(\mathbf{w})\mathbf{J}_{ij}\mathbf{T}_{\bar{q}}^H) = \text{tr} \left(-\delta_q(\mathbf{w}) (p_{\bar{q}}\mathbf{I}_{N_s} + \delta_q(\mathbf{w})\mathbf{T}_{\bar{q}}\mathbf{T}_{\bar{q}}^H)^{-2} \mathbf{J}_{ij}\mathbf{T}_{\bar{q}}^H \right) \\ &= \left[-\delta_q(\mathbf{w})\mathbf{T}_{\bar{q}}^H (p_{\bar{q}}\mathbf{I}_{N_s} + \delta_q(\mathbf{w})\mathbf{T}_{\bar{q}}\mathbf{T}_{\bar{q}}^H)^{-2} \right]_{ji} \end{aligned} \quad (3.2.23)$$

where \mathbf{J}_{ij} is an $N_s \times N_s$ matrix whose total entries are zero except its (i, j) -th entry which is equal to 1. To obtain the second term in the right hand side of (3.2.20), using the following differentiation rule

$$\frac{\partial}{\partial \mathbf{X}} \text{tr}(\mathbf{A}_1\mathbf{X}^{-1}\mathbf{A}_2) = -(\mathbf{X}^{-1}\mathbf{A}_2\mathbf{A}_1\mathbf{X}^{-1})^T = -\mathbf{X}^{-T}\mathbf{A}_1^T\mathbf{A}_2^T\mathbf{X}^{-T} \quad (3.2.24)$$

with $\mathbf{X} = \mathbf{T}_{\bar{q}}$, $\mathbf{A}_1 = \mathbf{D}^{-1}(\mathbf{w})$ and $\mathbf{A}_2 = \mathbf{T}_{\bar{q}}^{-H}\mathbf{D}^{-H}(\mathbf{w})$, we can write the second term in the right hand side (3.2.20) as

$$\mu_q \frac{\partial}{\partial \mathbf{T}_{\bar{q}}} \text{tr}(\mathbf{D}^{-1}(\mathbf{w})\mathbf{T}_{\bar{q}}^{-1}\mathbf{T}_{\bar{q}}^{-H}\mathbf{D}^{-H}(\mathbf{w})) = -\mu_q \mathbf{T}_{\bar{q}}^{-T}\mathbf{D}^{-T}(\mathbf{w})\mathbf{D}^{*-1}(\mathbf{w})\mathbf{T}_{\bar{q}}^{*-1}\mathbf{T}_{\bar{q}}^{-T}. \quad (3.2.25)$$

Then:

Inserting (3.2.23) and (3.2.25) into the Lagrangian in (3.2.20) and equating the Lagrangian to zero yields

$$\left(p_{\bar{q}} \delta_q(\mathbf{w}) \mathbf{T}_{\bar{q}}^H (p_{\bar{q}} \mathbf{I}_{N_s} + \delta_q(\mathbf{w}) \mathbf{T}_{\bar{q}} \mathbf{T}_{\bar{q}}^H)^{-2} \right)^T = \mu_q \mathbf{T}_{\bar{q}}^{-T} \mathbf{D}^{-T}(\mathbf{w}) \mathbf{D}^{*-1}(\mathbf{w}) \mathbf{T}_{\bar{q}}^{*-1} \mathbf{T}_{\bar{q}}^{-T}. \quad (3.2.26)$$

Transposing both sides of (3.2.26) and then multiplying both sides from left and right with \mathbf{T}_q , we arrive at

$$p_{\bar{q}} \delta_q(\mathbf{w}) \mathbf{T}_{\bar{q}} \mathbf{T}_{\bar{q}}^H (p_{\bar{q}} \mathbf{I}_{N_s} + \delta_q(\mathbf{w}) \mathbf{T}_{\bar{q}} \mathbf{T}_{\bar{q}}^H)^{-2} \mathbf{T}_{\bar{q}} = \mu_q \mathbf{T}_{\bar{q}}^{-H} \mathbf{D}^{-H}(\mathbf{w}) \mathbf{D}^{-1}(\mathbf{w}). \quad (3.2.27)$$

Inverting both sides of (3.2.27), we obtain that

$$\frac{1}{p_{\bar{q}} \delta_q(\mathbf{w})} \mathbf{T}_{\bar{q}}^{-1} (p_{\bar{q}} \mathbf{I}_{N_s} + \delta_q(\mathbf{w}) \mathbf{T}_{\bar{q}} \mathbf{T}_{\bar{q}}^H)^2 \mathbf{T}_{\bar{q}}^{-H} \mathbf{T}_{\bar{q}}^{-1} = \frac{1}{\mu_q} \mathbf{D}(\mathbf{w}) \mathbf{D}^H(\mathbf{w}) \mathbf{T}_{\bar{q}}^H. \quad (3.2.28)$$

Expanding the left hand side of (3.2.28) and multiplying both sides from right with $\mathbf{T}_{\bar{q}}^{-H}$, we arrive at

$$(p_{\bar{q}} \mathbf{T}_{\bar{q}}^{-1} \mathbf{T}_{\bar{q}}^{-H} + \delta_q(\mathbf{w}) \mathbf{I}_{N_s})^2 = \frac{p_{\bar{q}} \delta_q(\mathbf{w})}{\mu_q} \mathbf{D}(\mathbf{w}) \mathbf{D}^H(\mathbf{w}). \quad (3.2.29)$$

Let us define

$$\tilde{\mathbf{D}}(\mathbf{w}) \triangleq \sqrt{N_s} \text{diag}\{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|\}_{k=1}^{N_s}, \quad (3.2.30)$$

that is $\mathbf{D}(\mathbf{w}) \mathbf{D}^H(\mathbf{w}) = \tilde{\mathbf{D}}^2(\mathbf{w})$. Thus, from (3.2.29), we can deduct that

$$\mathbf{T}_{\bar{q}}^{-1} \mathbf{T}_{\bar{q}}^{-H} = \frac{1}{p_{\bar{q}}} \left(\pm \sqrt{\frac{p_{\bar{q}} \delta_q(\mathbf{w})}{\mu_q}} \tilde{\mathbf{D}}(\mathbf{w}) - \delta_q(\mathbf{w}) \mathbf{I}_{N_s} \right). \quad (3.2.31)$$

We now obtain an expression for $\frac{p_{\bar{q}} \delta_q(\mathbf{w})}{\mu_q}$ at the optimum and use that in (3.2.31) to obtain the optimal value for the objective function of the optimization problem

(3.2.18) for any given \mathbf{w} . Recalling that $\mathbf{T}_q = \mathbf{C}_q^{-1}\mathbf{D}^{-1}(\mathbf{w})$ and that $\mathbf{C}_q = \mathbf{F}\mathbf{E}_q$, we can rewrite (3.2.31) as

$$\mathbf{E}_{\bar{q}}\mathbf{E}_{\bar{q}}^H = \frac{1}{p_{\bar{q}}}\mathbf{F}^{-1}\mathbf{D}^{-1}(\mathbf{w}) \left(\pm \sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \tilde{\mathbf{D}}(\mathbf{w}) - \delta_q(\mathbf{w})\mathbf{I}_{N_s} \right) \mathbf{D}^{-H}(\mathbf{w})\mathbf{F}^{-H}. \quad (3.2.32)$$

The constraints in (3.2.4) imply that

$$\begin{aligned} N_s &= \|\mathbf{E}_{\bar{q}}\|_F^2 = \text{tr} [\mathbf{E}_{\bar{q}}\mathbf{E}_{\bar{q}}^H] \\ &= \text{tr} \left(\frac{1}{p_{\bar{q}}}\mathbf{F}^{-1}\mathbf{D}^{-1}(\mathbf{w}) \left(\pm \sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \tilde{\mathbf{D}}(\mathbf{w}) - \delta_q(\mathbf{w})\mathbf{I}_{N_s} \right) \mathbf{D}^{-H}(\mathbf{w})\mathbf{F}^{-H} \right) \\ &= \text{tr} \left(\frac{1}{p_{\bar{q}}}\mathbf{D}^{-H}(\mathbf{w})\mathbf{D}^{-1}(\mathbf{w}) \left(\pm \sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \tilde{\mathbf{D}}(\mathbf{w}) - \delta_q(\mathbf{w})\mathbf{I}_{N_s} \right) \right). \end{aligned} \quad (3.2.33)$$

(3.2.33) obviously implies that the solution with the negative sign is not acceptable, hence we consider only the solution with the positive sign. It follows from (3.2.33) that at the optimum, we can write

$$\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q} = \left(\frac{p_{\bar{q}}N_s + \delta_q(\mathbf{w})\text{tr}\{\mathbf{D}^{-H}(\mathbf{w})\mathbf{D}^{-1}(\mathbf{w})\}}{\text{tr}(\tilde{\mathbf{D}}^{-1}(\mathbf{w}))} \right)^2 \quad (3.2.34)$$

We define $\mathbf{A}_q \triangleq (p_{\bar{q}}\mathbf{I}_{N_s} + \delta_q(\mathbf{w})\mathbf{E}_{\bar{q}}^{-1}\mathbf{F}^{-1}\mathbf{D}^{-1}(\mathbf{w})\mathbf{D}^{-H}(\mathbf{w})\mathbf{F}\mathbf{E}_{\bar{q}}^{-H})^{-1}$ and denote $\beta_i(\mathbf{A}_q)$

as the i -th eigenvalue of matrix \mathbf{A} . We can now write

$$\begin{aligned}
tr(\mathbf{A}_q) &= \sum_{i=1}^{N_s} \beta_i(\mathbf{A}_q) \\
&= \sum_{i=1}^{N_s} \beta_i^{-1}(\mathbf{A}_q^{-1}) \\
&= \sum_{i=1}^{N_s} \beta_i^{-1} (p_{\bar{q}} \mathbf{I}_{N_s} + \delta_q(\mathbf{w}) \mathbf{E}_{\bar{q}}^{-1} \mathbf{F}^{-1} \mathbf{D}^{-1}(\mathbf{w}) \mathbf{D}^{-H}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}}^{-H}) \\
&= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \delta_q(\mathbf{w}) \beta_i \left((\mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}})^{-1} (\mathbf{E}_{\bar{q}}^H \mathbf{F}^H \mathbf{D}^H(\mathbf{w}))^{-1} \right) \right)^{-1} \\
&= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \delta_q(\mathbf{w}) \beta_i \left((\mathbf{E}_{\bar{q}}^H \mathbf{F}^H \mathbf{D}^H(\mathbf{w})) (\mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}})^{-1} \right)^{-1} \right)^{-1} \\
&= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \frac{\delta_q(\mathbf{w})}{\beta_i \left((\mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}})^H (\mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}}) \right)} \right)^{-1} \tag{3.2.35}
\end{aligned}$$

where in the fourth equality we have used the fact that $\beta_i(\alpha \mathbf{I} + \mathbf{X}) = \alpha + \beta_i(\mathbf{X})$ and in the second and the last equalities, we use the following identity: $\beta_i(\mathbf{X}^{-1}) = \beta_i^{-1}(\mathbf{X})$. Now, using the fact that for any square matrix \mathbf{X} , the identity $\beta_i(\mathbf{X}^H \mathbf{X}) = \beta_i(\mathbf{X} \mathbf{X}^H)$ holds, we write (3.2.35) as

$$\begin{aligned}
tr(\mathbf{A}_q) &= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \frac{\delta_q(\mathbf{w})}{\beta_i \left((\mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}}) (\mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}})^H \right)} \right)^{-1} \\
&= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \frac{\delta_q(\mathbf{w})}{\beta_i \left(\mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{E}_{\bar{q}} \mathbf{E}_{\bar{q}}^H \mathbf{F}^H \mathbf{D}^H(\mathbf{w}) \right)} \right)^{-1}. \tag{3.2.36}
\end{aligned}$$

Using (3.2.33), we can simplify

$$\begin{aligned}
tr(\mathbf{A}_q) &= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \frac{\delta_q(\mathbf{w})}{\beta_i(\mathbf{D}(\mathbf{w})\mathbf{F}\mathbf{E}_{\bar{q}}\mathbf{E}_{\bar{q}}^H\mathbf{F}^H\mathbf{D}^H(\mathbf{w}))} \right)^{-1} \\
&= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\beta_i \left(\sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \tilde{\mathbf{D}}(\mathbf{w}) - \delta_q(\mathbf{w})\mathbf{I}_{N_s} \right)} \right)^{-1} \\
&= \sum_{i=1}^{N_s} \left(p_{\bar{q}} + \frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \beta_i(\tilde{\mathbf{D}}(\mathbf{w})) - \delta_q(\mathbf{w})} \right)^{-1} \\
&= \sum_{i=1}^{N_s} \left(\frac{p_{\bar{q}} \sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \beta_i(\tilde{\mathbf{D}}(\mathbf{w}))}{\sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \beta_i(\tilde{\mathbf{D}}(\mathbf{w})) - \delta_q(\mathbf{w})} \right)^{-1} \\
&= \sum_{i=1}^{N_s} \left(\frac{1}{p_{\bar{q}}} - \frac{\delta_q(\mathbf{w})}{p_{\bar{q}} \sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}} \beta_i(\tilde{\mathbf{D}}(\mathbf{w}))} \right) \\
&= \frac{N_s}{p_{\bar{q}}} - \frac{\delta_q(\mathbf{w})}{p_{\bar{q}} \sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}}} \sum_{i=1}^{N_s} \beta_i(\tilde{\mathbf{D}}^{-1}(\mathbf{w})) \\
&= \frac{N_s}{p_{\bar{q}}} - \frac{\delta_q(\mathbf{w})}{p_{\bar{q}} \sqrt{\frac{p_{\bar{q}}\delta_q(\mathbf{w})}{\mu_q}}} tr(\tilde{\mathbf{D}}^{-1}(\mathbf{w})) \tag{3.2.37}
\end{aligned}$$

where in the third equality we have used the fact that $\beta_i(\alpha\mathbf{I} + \mathbf{X}) = \alpha + \beta_i(\mathbf{X})$. We

now use (3.2.34) to write (3.2.37) as

$$\begin{aligned}
tr(\mathbf{A}_q) &= \frac{N_s}{p_{\bar{q}}} - \frac{\delta_q(\mathbf{w})}{\frac{p_{\bar{q}}^2 N_s + p_{\bar{q}} \delta_q(\mathbf{w}) tr(\mathbf{D}^{-H}(\mathbf{w}) \mathbf{D}^{-1}(\mathbf{w}))}{tr(\tilde{\mathbf{D}}^{-1}(\mathbf{w}))}} tr(\tilde{\mathbf{D}}^{-1}(\mathbf{w})) \\
&= \frac{N_s}{p_{\bar{q}}} - \frac{\delta_q(\mathbf{w}) \left(tr(\tilde{\mathbf{D}}^{-1}(\mathbf{w})) \right)^2}{p_{\bar{q}}^2 N_s + p_{\bar{q}} \delta_q(\mathbf{w}) tr(\mathbf{D}^{-H}(\mathbf{w}) \mathbf{D}^{-1}(\mathbf{w}))}. \tag{3.2.38}
\end{aligned}$$

Using (3.2.38), for any given \mathbf{w} , we can now write the optimal value of the objective function (3.2.9) as

$$\begin{aligned}
&\sum_{q=1}^2 N_s - p_{\bar{q}} \left(\frac{N_s}{p_{\bar{q}}} - \frac{\delta_q(\mathbf{w}) \left(tr(\tilde{\mathbf{D}}^{-1}(\mathbf{w})) \right)^2}{p_{\bar{q}}^2 N_s + p_{\bar{q}} \delta_q(\mathbf{w}) tr(\mathbf{D}^{-H}(\mathbf{w}) \mathbf{D}^{-1}(\mathbf{w}))} \right) \\
&= \sum_{q=1}^2 \left(\frac{\delta_q(\mathbf{w}) \left(tr(\tilde{\mathbf{D}}^{-1}(\mathbf{w})) \right)^2}{p_{\bar{q}} N_s + \delta_q(\mathbf{w}) tr(\mathbf{D}^{-H}(\mathbf{w}) \mathbf{D}^{-1}(\mathbf{w}))} \right). \tag{3.2.39}
\end{aligned}$$

Using (3.2.12), we can write (3.2.39) as

$$\begin{aligned}
&\sum_{q=1}^2 \frac{\delta_q(\mathbf{w}) \left(tr \left\{ \frac{1}{\sqrt{N_s}} \text{diag} \left\{ \frac{1}{\{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|\}} \right\}_{k=1}^{N_s}} \right) \right)^2}{p_{\bar{q}} N_s + \delta_q(\mathbf{w}) tr \left\{ \text{diag} \left\{ \frac{1}{N_s |\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2} \right\}_{k=1}^{N_s}} \right)} \\
&= \sum_{q=1}^2 \frac{\frac{\delta_q(\mathbf{w})}{N_s} \left(\sum_{k=1}^{N_s} \frac{1}{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|} \right)^2}{p_{\bar{q}} N_s + \frac{\delta_q(\mathbf{w})}{N_s} \sum_{k=1}^{N_s} \frac{1}{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2}}. \tag{3.2.40}
\end{aligned}$$

Using the fact that $\delta_q(\mathbf{w}) = \sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1)$, the optimization problem (3.2.3) can

be rewritten as

$$\min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \left(\sum_{k=1}^{N_s} \frac{1}{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|} \right)^2}{p_{\bar{q}} N_s^2 + \sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \sum_{k=1}^{N_s} \frac{1}{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2}}$$

subject to $p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{\max}$ (3.2.41)

or, equivalently, as

$$\min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \left(\sum_{k=1}^{N_s} \phi_k(\mathbf{w}) \right)^2}{p_{\bar{q}} N_s^2 + \sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \sum_{k=1}^{N_s} \phi_k^2(\mathbf{w})}$$

subject to $p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq P_{\max}$ (3.2.42)

where we have used the following definition:

$$\phi_k(\mathbf{w}) \triangleq \frac{1}{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|} . \quad (3.2.43)$$

Denoting the ℓ_1 and ℓ_2 norms of any vector \mathbf{a} as $\|\mathbf{a}\|_1$ and $\|\mathbf{a}\|_2$, respectively, we rewrite (3.2.42) as

$$\min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \|\phi(\mathbf{w})\|_1^2}{p_{\bar{q}} N_s^2 + \sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \|\phi(\mathbf{w})\|_2^2}$$

subject to $p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{\max}$ (3.2.44)

where

$$\phi(\mathbf{w}) \triangleq [\phi_1(\mathbf{w}) \ \phi_2(\mathbf{w}) \ \dots \ \phi_{N_s}(\mathbf{w})]^T . \quad (3.2.45)$$

In order to further simplify (3.2.44), we use the fact that

$$\|\phi(\mathbf{w})\|_1 \leq \sqrt{N_s} \|\phi(\mathbf{w})\|_2 \quad (3.2.46)$$

and the equality holds if and only if all the entries of the vector $\phi(\mathbf{w})$ are equal. Based on (3.2.46), we replace $\|\phi(\mathbf{w})\|_1$ in the objective function of (3.2.42) with its corresponding upper bound; i.e. with $\sqrt{N_s}\|\phi(\mathbf{w})\|_2$ and solve the following optimization problem:

$$\begin{aligned} \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \quad & \sum_{q=1}^2 \frac{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) N_s \|\phi(\mathbf{w})\|_2^2}{p_{\bar{q}} N_s^2 + \sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \|\phi(\mathbf{w})\|_2^2} \\ \text{subject to} \quad & p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{\max}. \end{aligned} \quad (3.2.47)$$

Note that solving (3.2.47) provides an upper bound to (3.2.44). However, we later show that this upper bound is tight and hence there is no loss of optimality by solving (3.2.47) instead of (3.2.44). To solve (3.2.47), we rewrite it as

$$\begin{aligned} \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \quad & \sum_{q=1}^2 \frac{1}{\frac{P_{\bar{q}} N_s}{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \sum_{k=1}^{N_s} \phi_k^2(\mathbf{w})} + \frac{1}{N_s}} \\ \text{subject to} \quad & p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{\max}. \end{aligned} \quad (3.2.48)$$

Using (3.2.43), we can write the optimization problem (3.2.48) as

$$\begin{aligned} \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \quad & \sum_{q=1}^2 \frac{1}{\frac{p_{\bar{q}} N_s}{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1) \sum_{k=1}^{N_s} \frac{1}{|\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2}} + \frac{1}{N_s}} \\ \text{subject to} \quad & p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{\max} \end{aligned} \quad (3.2.49)$$

or, equivalently, as

$$\min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{1}{\frac{1}{\sum_{k=1}^{N_s} \frac{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1)}{p_{\bar{q}} N_s |\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2}} + \frac{1}{N_s}}$$

subject to $p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{max}$. (3.2.50)

We rewrite (3.2.50) as

$$\min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{1}{\frac{1}{\sum_{k=1}^{N_s} \psi_{k,q}(\mathbf{w})} + \frac{1}{N_s}}$$

subject to $p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{max}$ (3.2.51)

where we have assumed the following definition:

$$\psi_{k,q}(\mathbf{w}) \triangleq \frac{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1)}{p_{\bar{q}} N_s |\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2}, \quad \text{for } q = 1, 2, \quad k \in \{1, 2, \dots, N_s\}. \quad (3.2.52)$$

To simplify the optimization problem (3.2.51), we use the fact that the arithmetic mean of any set of the positive numbers $\{\alpha_k\}_{k=1}^{N_s}$ is greater or equal to their harmonic mean; i.e.,

$$\frac{1}{N_s} \sum_{k=1}^{N_s} \alpha_k \geq \frac{1}{\frac{1}{N_s} \sum_{k=1}^{N_s} \frac{1}{\alpha_k}}. \quad (3.2.53)$$

The equality in (3.2.53) holds, if and only if $\{\alpha_k\}_{k=1}^{N_s}$ are all equal. Using (3.2.53) along with the fact that each $\psi_{k,q}(\mathbf{w})$ as defined in (3.2.52) is positive, the following inequality holds true, for $q = 1, 2$:

$$\sum_{k=1}^{N_s} \psi_{k,q}(\mathbf{w}) \geq \frac{N_s^2}{\sum_{k=1}^{N_s} \frac{1}{\psi_{k,q}(\mathbf{w})}} \quad (3.2.54)$$

where the equality holds, for a given q , if and only if we can find a set of \mathbf{w} vectors for which $\{\psi_{k,q}(\mathbf{w})\}_{k=1}^{N_s}$ are all equal to each other. We replace the summation $\sum_{k=1}^{N_s} \psi_{k,q}(\mathbf{w})$ in the objective function of (3.2.51) with its corresponding lower bound in (3.2.54). To ensure that these lower bounds are achieved at the same time, we restrict \mathbf{w} to be such that $\{\psi_{k,q}(\mathbf{w})\}_{k=1}^{N_s}$ are all equal to each other for a given q . Let \mathcal{W}_q represent the set of the values of \mathbf{w} such that all $\{\psi_{k,q}(\mathbf{w})\}_{k=1}^{N_s}$ are equal for any transceiver index q . That is

$$\mathcal{W}_q = \left\{ \mathbf{w} \mid |\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})| = |\mathbf{f}_{k'}^H \tilde{\mathbf{h}}(\mathbf{w})|, \forall k \neq k' \right\} \text{ for } q = 1, 2 \cdot \quad (3.2.55)$$

From (3.2.55), it can be observed that \mathcal{W}_q does not depend on q , and hence $\mathcal{W}_1 = \mathcal{W}_2 \triangleq \mathcal{W}_q$ holds true. Although it may not be inferred at this time that $\psi_{k,1}(\mathbf{w})$ is equal to $\psi_{k,2}(\mathbf{w})$, we soon prove that indeed $\psi_{k,1}(\mathbf{w})$ is equal to $\psi_{k,2}(\mathbf{w})$, for $k = 1, 2, \dots, N_s$. Note that as shown in [40], \mathcal{W}_q can be written as $\mathcal{W} = \bigcup_{n=0}^{N-1} \mathcal{U}_n$, where \mathcal{U}_n is the set of the relay weight vectors \mathbf{w} such that only the n -th tap of the end-to-end channel impulse response is non-zero and the remaining taps are zero⁸. Any weight vector $\mathbf{w} \in \mathcal{U}_n$ has non-zero entries only for those relays which contribute to the n -th tap of the end-to-end channel impulse response and its other entries are zero. Note that $\mathcal{U}_n \cap \mathcal{U}_{n'} = \emptyset$, for $n \neq n'$, as each relay contributes only to one of the taps of the end-to-end channel impulse response. Therefore, without any loss of

⁸It has been shown in [40] that for any $\mathbf{w} \in \mathcal{W}$, the end-to-end channel has one single tap. We now provide a shorter proof for this statement: when $\mathbf{w} \in \mathcal{W}$, it then follows from the definition of the set \mathcal{W} in (3.2.55) that the end-to-end FIR channel impulse response, given by (3.1.7), will have a flat amplitude response, and thus it can have only one tap. The reason is that any all-pass FIR system has a single tap due to two facts: i) each of its zeros has to be a reflection of one of its poles across the boundary of the unit circle in the complex plane and ii) the poles of any FIR system are at the origin of the complex plane. Thus, all the zeros of any all-pass FIR filter have to be at infinity, meaning that it can have only one tap.

optimality, the optimization problem (3.2.51) can be rewritten as

$$\begin{aligned}
& \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{1}{\frac{\sum_{k=1}^{N_s} \psi_{k,q}(\mathbf{w})}{N_s^2} + \frac{1}{N_s}} \\
& \text{subject to } p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{\max} \\
& \text{and } \mathbf{w} \in \bigcup_{n=0}^{N-1} \mathcal{U}_n. \tag{3.2.56}
\end{aligned}$$

Using (3.2.52), the optimization problem (3.2.56) can be represented as

$$\begin{aligned}
& \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{1}{\frac{P_{\bar{q}} \sum_{k=1}^{N_s} |\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2}{N_s \sigma^2 (\|\mathbf{G}_q \mathbf{w}\|^2 + 1)} + \frac{1}{N_s}} \\
& \text{subject to } p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{\max} \\
& \text{and } \mathbf{w} \in \bigcup_{n=0}^{N-1} \mathcal{U}_n. \tag{3.2.57}
\end{aligned}$$

Note that due to Parseval's theorem, we can write

$$\sum_{k=1}^{N_s} |\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w})|^2 = \|\tilde{\mathbf{h}}(\mathbf{w})\|^2 = \|\mathbf{h}(\mathbf{w})\|^2 = \|\mathbf{B}\mathbf{w}\|^2 = \mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w} \tag{3.2.58}$$

where we have used the fact that since the vector $\tilde{\mathbf{h}}(\mathbf{w})$ is the zero-padded version of the vector $\mathbf{h}(\mathbf{w})$, norm of $\tilde{\mathbf{h}}(\mathbf{w})$ is equal to the norm of $\mathbf{h}(\mathbf{w})$. Using (3.2.58), the

optimization problem (3.2.57) can be rewritten as

$$\begin{aligned}
& \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{N_s}{\frac{p_q \mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w}}{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1)} + 1} \\
& \text{subject to } p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{max} \\
& \text{and } \mathbf{w} \in \bigcup_{n=0}^{N-1} \mathcal{U}_n. \tag{3.2.59}
\end{aligned}$$

To solve the optimization problem (3.2.59), we use the fact that the sets $\{\mathcal{U}_n\}_{n=0}^{N-1}$ are mutually exclusive, hence the optimal \mathbf{w} belongs to only one of these sets. The set \mathcal{U}_n in which the optimal \mathbf{w} resides can be found by noting that the optimization problem (3.2.59) can be turned into a set of maximum N subproblems, each of which assumes that \mathbf{w} belongs to one of the sets $\{\mathcal{U}_n\}_{n=0}^{N-1}$. Each of these subproblems can be solved separately to obtain the corresponding minimum value of objective function (i.e., the total MSE). This approach results in N candidate values for the optimal \mathbf{w} . The optimal value of \mathbf{w} can then be found by determining which of these candidates results in the lowest possible value for the total MSE. More specifically, the optimization problem (3.2.59) is equivalent to solving the following minimization problem:

$$\begin{aligned}
& \min_{0 \leq n \leq N-1} \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \sum_{q=1}^2 \frac{N_s}{\frac{p_q \mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w}}{\sigma^2(\|\mathbf{G}_q \mathbf{w}\|^2 + 1)} + 1} \\
& \text{subject to } p_1(1 + \|\mathbf{G}_1 \mathbf{w}\|^2) + p_2(1 + \|\mathbf{G}_2 \mathbf{w}\|^2) + \sigma^2 \mathbf{w}^H \mathbf{w} \leq p_{max} \\
& \text{and } \mathbf{w} \in \mathcal{U}_n. \tag{3.2.60}
\end{aligned}$$

Let L_n denote the number of the relays which contribute to the n -th tap of the end-to-end channel impulse response and let \mathbf{w}_n represent the $L_n \times 1$ vector of the weights

of those relays which contribute to the n -th tap of the end-to-end channel impulse response. If $\mathbf{w} \in \mathcal{U}_n$, then we can write

$$\mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w} = \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n \quad (3.2.61)$$

where \mathbf{b}_n^H is an $1 \times L_n$ vector which captures the non-zero entries⁹ of the $(n+1)$ -th row of matrix \mathbf{B} . As mentioned above, in order to solve (3.2.59), we can solve N separate optimization problems (the same as inner minimization in (3.2.60)), thereby choosing the value of n which results in the minimum value for the objective function. Indeed, the total estimation error of the received signals can be different for different indexes of the non-zero taps of the end-to-end channel impulse response. Therefore, we need to turn on those relays which contribute to the tap of the end-to-end channel impulse response that results in a minimum total mean squared error of the estimated signals at both transceivers. In other words, the optimum value of n is determined such that the total MSE is minimized. Using (3.2.61), we can rewrite the optimization problem (3.2.60) as

$$\begin{aligned} \min_{0 \leq n \leq N-1} \quad & \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \quad \min_{\mathbf{w}_n} \quad \sum_{q=1}^2 \frac{N_s}{\frac{p_{\bar{q}} \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n}{\sigma^2 (\|\mathbf{G}_q^{(n)} \mathbf{w}_n\|^2 + 1)} + 1} \\ & \text{subject to } p_1 (1 + \|\mathbf{G}_1^{(n)} \mathbf{w}_n\|^2) + p_2 (1 + \|\mathbf{G}_2^{(n)} \mathbf{w}_n\|^2) + \sigma^2 \mathbf{w}_n^H \mathbf{w}_n \leq p_{\max}. \end{aligned} \quad (3.2.62)$$

Here, $\mathbf{G}_q^{(n)}$ is an $L_n \times L_n$ diagonal matrix whose diagonal entries are a subset of the diagonal entries of \mathbf{G}_q which correspond to the relays that contribute to n -th tap of the end-to-end channel impulse response. Let us define $\alpha_q(\mathbf{w}_n) \triangleq \left(\frac{p_{\bar{q}} \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n}{\sigma^2 (\|\mathbf{G}_q^{(n)} \mathbf{w}_n\|^2 + 1)} + 1 \right)$.

⁹Note that if the $(n+1)$ -th row of matrix \mathbf{B} does not have any non-zero entries, then the n -th tap of the end-to-end channel impulse response is zero, meaning that the optimal \mathbf{w} does not belong to \mathcal{U}_n as \mathcal{U}_n is empty.

Without loss of optimality, we can assume $\alpha_1(\mathbf{w}_n) = \alpha_2(\mathbf{w}_n)$ holds true at the optimum. Otherwise, if, for example, $\alpha_2(\mathbf{w}_n) \geq \alpha_1(\mathbf{w}_n)$ holds true at the optimum, we can reduce the power p_1 such that $\alpha_2(\mathbf{w}_n) = \alpha_1(\mathbf{w}_n)$ holds true, without violating the constraint in (3.2.62). The optimization problem (3.2.62) can now be rewritten as

$$\begin{aligned} \min_{0 \leq n \leq N-1} \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}_n} & \frac{2N_s}{\frac{p_1 \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n}{\sigma^2(\|\mathbf{G}_2^{(n)} \mathbf{w}_n\|^2 + 1)} + 1} \\ \text{subject to} & p_1(1 + \|\mathbf{G}_1^{(n)} \mathbf{w}_n\|^2) + p_2(1 + \|\mathbf{G}_2^{(n)} \mathbf{w}_n\|^2) + \sigma^2 \mathbf{w}_n^H \mathbf{w}_n \leq p_{\max} \\ \text{and} & p_1(1 + \|\mathbf{G}_1^{(n)} \mathbf{w}_n\|^2) = p_2(1 + \|\mathbf{G}_2^{(n)} \mathbf{w}_n\|^2) \end{aligned} \quad (3.2.63)$$

where the second constraint follows from the fact that for any n , $\alpha_2(\mathbf{w}_n) = \alpha_1(\mathbf{w}_n)$ holds true at the optimum. The optimization problem (3.2.63) can be equivalently written as

$$\begin{aligned} \min_{0 \leq n \leq N-1} \min_{p_1 \geq 0} \min_{\mathbf{w}_n} & \frac{2N_s}{\frac{p_1 \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n}{\sigma^2(\|\mathbf{G}_2^{(n)} \mathbf{w}_n\|^2 + 1)} + 1} \\ \text{subject to} & 2p_1(1 + \|\mathbf{G}_1^{(n)} \mathbf{w}_n\|^2) + \sigma^2 \mathbf{w}_n^H \mathbf{w}_n \leq p_{\max}. \end{aligned} \quad (3.2.64)$$

It can be readily shown that the constraint in (3.2.64) can be satisfied with equality. Therefore, the optimization problem (3.2.64) can be written as

$$\begin{aligned} \max_{0 \leq n \leq N-1} \max_{p_1 \geq 0} \max_{\mathbf{w}_n} & \frac{p_1 \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n}{\sigma^2(\|\mathbf{G}_2^{(n)} \mathbf{w}_n\|^2 + 1)} \\ \text{subject to} & p_1 = \frac{p_{\max} - \sigma^2 \mathbf{w}_n^H \mathbf{w}_n}{2(1 + \|\mathbf{G}_1^{(n)} \mathbf{w}_n\|^2)}. \end{aligned} \quad (3.2.65)$$

The constraint in (3.2.65) can now be used to eliminate p_1 , while noting $p_1 \geq 0$ implies that, at the optimum, $p_{\max}/\sigma^2 \geq \mathbf{w}_n^H \mathbf{w}_n$ holds true. Hence, we can rewrite

the optimization problem (3.2.65) as

$$\begin{aligned} & \max_{0 \leq n \leq N-1} \max_{\mathbf{w}_n} \frac{(p_{\max} - \sigma^2 \mathbf{w}_n^H \mathbf{w}_n) \mathbf{w}_n^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n}{2\sigma^2 \left(\mathbf{w}_n^H \mathbf{Q}_1^{(n)} \mathbf{w}_n + 1 \right) \left(\mathbf{w}_n^H \mathbf{Q}_2^{(n)} \mathbf{w}_n + 1 \right)} \\ & \text{subject to } \mathbf{w}_n^H \mathbf{w}_n \leq p_{\max} / \sigma^2 \end{aligned} \quad (3.2.66)$$

where $\mathbf{Q}_q^{(n)} \triangleq \left(\mathbf{G}_q^{(n)} \right)^H \mathbf{G}_q^{(n)}$, for $q = 1, 2$. In light of the results of [38], the inner maximization in (3.2.66) aims to find \mathbf{w}_n such that in a synchronous relay sub-network where we activate only those relays contributing to the n -th tap of the end-to-end channel impulse response in the main network, the smaller of the two transceiver SNRs is maximized under a total power constraint of p_{\max} . This maximization is equivalent to the maximization of the balanced SNR at the two transceivers in the same subnetwork. Indeed, the objective function in (3.2.66) is the balanced SNR for a given \mathbf{w}_n . This max-min SNR fair design approach has been shown to be also equivalent to maximizing the sum-rate for this sub-network under the same total power constraint [21]. In fact, the optimization problem (3.2.64) is amenable to a semi-closed-form solution for the optimal \mathbf{w}_n , denoted as \mathbf{w}_n^o , which is given by

$$\mathbf{w}_n^o = k_n \sqrt{2\nu_n} \left(2\mu_n \mathbf{Q}_1^{(n)} + 2\mu_n \mathbf{Q}_2^{(n)} + \mathbf{I}_{L_n} \right)^{-1} \mathbf{b}_n. \quad (3.2.67)$$

Here, we define $\nu_n \triangleq 0.5p_{\max}/\sigma^2 - \mu_n$. The integer number L_n is the number of the relays which contribute to the n -th tap of the end-to-end channel. The parameter k_n is obtained as

$$k_n \triangleq \left(\mathbf{b}_n^H \left(\mathbf{I}_{L_n} + 2\mu_n \mathbf{Q}_1^{(n)} \right) \left(\mathbf{I}_{L_n} + 2\mu_n \mathbf{Q}_1^{(n)} + 2\nu_n \mathbf{Q}_2^{(n)} \right)^{-2} \mathbf{b}_n \right)^{-\frac{1}{2}} \quad (3.2.68)$$

and the parameter μ_n is obtained as the unique solution to the following equation:

$$\begin{aligned} & (p_{\max}/\sigma^2 - 4\mu_n) \mathbf{b}_n^H \left(2\mu_n \mathbf{Q}_1^{(n)} + (p_{\max}/\sigma^2 - 2\mu_n) \mathbf{Q}_2^{(n)} + \mathbf{I}_{L_n} \right)^{-1} \mathbf{b}_n - \\ & \mu_n (p_{\max}/\sigma^2 - 2\mu_n) \mathbf{b}_n^H \left(2\mu_n \mathbf{Q}_1^{(n)} + (p_{\max}/\sigma^2 - 2\mu_n) \mathbf{Q}_2^{(n)} + \mathbf{I}_{L_n} \right)^{-2} \left(2\mathbf{Q}_1^{(n)} - 2\mathbf{Q}_2^{(n)} \right) \mathbf{b}_n = 0 \end{aligned} \quad (3.2.69)$$

which satisfies $\mu_n \in [0 \ 0.5p_{\max}/\sigma^2]$. Using a simple bisection algorithm, we can obtain the value of μ_n in the interval $[0 \ 0.5p_{\max}/\sigma^2]$ such that the left hand side of (3.2.69) vanishes.

Once we have \mathbf{w}_n , for $n = 0, 1, \dots, N-1$, we can determine the optimal value of n by evaluating the objective function in (3.2.66) for each \mathbf{w}_n^o and by choosing that value of n which results in the largest value of this objective function. Hence, the optimal value of n is obtained as

$$n^o = \arg \max_{0 \leq n \leq N-1} \frac{(p_{\max} - \sigma^2 \|\mathbf{w}_n^o\|^2) |\mathbf{b}_n^H \mathbf{w}_n^o|^2}{2\sigma^2 \left(\mathbf{w}_n^{o,H} \mathbf{Q}_1^{(n)} \mathbf{w}_n^o + 1 \right) \left(\mathbf{w}_n^{o,H} \mathbf{Q}_2^{(n)} \mathbf{w}_n^o + 1 \right)}. \quad (3.2.70)$$

In other words, n^o specifies the set of the relays which contribute only to one tap of the end-to-end channel impulse response and also which leads to the minimum value of MSE among other relay sets which contributes to other taps of the end-to-end channel impulse response. If for a certain value of n , no relay contributes to the end-to-end channel impulse response, then $h[n] = 0$ for that value of n . In this case, the $(n+1)$ -th row of matrix \mathbf{B} will be zero, and that value of n is skipped (i.e., \mathcal{U}_n is empty). The maximum number of the feasible values of n is basically equal to the number of the relays, L . Actually, n belongs to the set $\{\tilde{n}_l\}_{l=1}^L$. Hence, we can restrict our search for the optimal value of n to this set.

We can now use the constraint in (3.2.65) to obtain the optimal value of p_1 as

$$p_1^o = \frac{p_{\max} - \sigma^2 \mathbf{w}_n^{o,H} \mathbf{w}_n^o}{2(1 + \|\mathbf{G}_1^{(n)} \mathbf{w}_n^o\|^2)}. \quad (3.2.71)$$

Using the second constraint in (3.2.63) along with (3.2.71), the optimal value of p_2 can be expressed as

$$p_2^o = \frac{p_{\max} - \sigma^2 \mathbf{w}_n^{o,H} \mathbf{w}_n^o}{2(1 + \|\mathbf{G}_2^{(n)} \mathbf{w}_n^o\|^2)}. \quad (3.2.72)$$

Let \mathbf{w}_{opt} represent the optimal relay weight vector. If the l -th relay is active, then the l -th entry of \mathbf{w}_{opt} is equal to the element of \mathbf{w}_n^o which corresponds to the l -th relay. If the l -th relay is not active, then the l -th element of \mathbf{w}_{opt} is zero. We now obtain the optimal values of \mathbf{E}_q and \mathbf{F}_q , for $q = 1, 2$. To obtain the optimal value of \mathbf{E}_q using (3.2.32), we need to obtain the values of $\mathbf{D}(\mathbf{w})$, $\tilde{\mathbf{D}}(\mathbf{w})$, $\delta_q(\mathbf{w})$, $\frac{p_{\bar{q}} \delta_q(\mathbf{w})}{\mu_q}$ for the optimal value of \mathbf{w} . Noting that \mathbf{w}_{opt} belongs to the set \mathcal{W} defined in (3.2.55), and hence $\mathbf{f}_k^H \tilde{\mathbf{h}}(\mathbf{w}_{opt}) = \mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt})$, for all k , we can use (3.2.12) to write

$$\mathbf{D}(\mathbf{w}_{opt}) = \sqrt{N_s} \left(\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt}) \right) \mathbf{I}_{N_s}. \quad (3.2.73)$$

Using (3.2.30), we arrive at

$$\tilde{\mathbf{D}}(\mathbf{w}_{opt}) = \sqrt{N_s} |\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt})| \mathbf{I}_{N_s}. \quad (3.2.74)$$

Moreover, substituting (3.2.73) and (3.2.74) into (3.2.32), we observe that $\mathbf{E}_q \mathbf{E}_q^H$ must be proportional to the identity matrix \mathbf{I}_{N_s} at the optimum, and since $tr\{\mathbf{E}_q \mathbf{E}_q^H\} = N_s$ must hold true, we conclude that at the optimum $\mathbf{E}_q \mathbf{E}_q^H = \mathbf{I}_{N_s}$ holds true. Hence, \mathbf{E}_q can be any unitary matrix. We choose the optimal value of \mathbf{E}_q as¹⁰

$$\mathbf{E}_q^o = \mathbf{I}_{N_s}. \quad (3.2.75)$$

Using (3.2.73) in (3.2.10), we obtain that

$$\tilde{\mathbf{H}}(\mathbf{w}_{opt}) = \sqrt{N_s} \left(\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt}) \right) \mathbf{I}_{N_s} \quad (3.2.76)$$

¹⁰Note that another solution is to choose \mathbf{E}_q to be equal to the DFT matrix \mathbf{F} , thereby turning the communication scheme into a multi-carrier.

It follows from (3.2.58) and (3.2.61) that

$$|\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt})|^2 = N_s^{-1} \mathbf{w}_{opt}^H \mathbf{B}^H \mathbf{B} \mathbf{w}_{opt} = N_s^{-1} \mathbf{w}_{n^o}^{o,H} \mathbf{b}_{n^o} \mathbf{b}_{n^o}^H \mathbf{w}_{n^o}^o. \quad (3.2.77)$$

Note also the optimal value of the post-channel equalizer at Transceiver q can be written as

$$\begin{aligned} \mathbf{F}_q^o &\triangleq \mathbf{F}_q^{opt}(\mathbf{w}_{opt}) = \sqrt{p_q^o} \mathbf{E}_q^{o,H} \tilde{\mathbf{H}}^H(\mathbf{w}_{opt}) \mathbf{R}_q^{-1}(\mathbf{w}_{opt}) \\ &= \sqrt{p_q^o} (\sqrt{N_s}) \left(\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt}) \right)^* \times (N_s p_q^o |\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt})|^2 + \sigma^2 \delta_q(\mathbf{w}_{opt}))^{-1} \mathbf{I}_{N_s} \end{aligned} \quad (3.2.78)$$

where we can obtain $|\mathbf{f}_1^H \tilde{\mathbf{h}}(\mathbf{w}_{opt})|^2$ as in (3.2.77).

Our proposed method is summarized as Algorithm 1.

Algorithm 1: Joint equalization, beamforming and power allocation method.

Step 1. Set $n = 0$.

Step 2. If no relay contributes to the n -th tap of the end-to-end channel impulse response, i.e., the $(n + 1)$ -th row of matrix \mathbf{B} is zero, go to step 9.

Step 3. Let the row vector \mathbf{b}_n^H capture the non-zero entries of the $(n + 1)$ -th row of matrix \mathbf{B} . Define $\mathbf{Q}_q^{(n)} \triangleq \left(\mathbf{G}_q^{(n)}\right)^H \mathbf{G}_q^{(n)}$, where $\mathbf{G}_q^{(n)}$ is an $L_n \times L_n$ diagonal matrix whose diagonal entries are a subset of the diagonal entries of \mathbf{G}_q which correspond to the relays that contribute to the n -th tap of the end-to-end channel impulse response.

Step 4. Use the bisection method to obtain μ_n in the interval $[0 \ 0.5p_{\max}/\sigma^2]$ such that

$$\begin{aligned} & (p_{\max}/\sigma^2 - 4\mu_n) \mathbf{b}_n^H \left(2\mu_n \mathbf{Q}_1^{(n)} + (p_{\max}/\sigma^2 - 2\mu_n) \mathbf{Q}_2^{(n)} + \mathbf{I}_{L_n} \right)^{-1} \mathbf{b}_n - \\ & \mu_n (p_{\max}/\sigma^2 - 2\mu_n) \mathbf{b}_n^H \left(2\mu_n \mathbf{Q}_1^{(n)} + (p_{\max}/\sigma^2 - 2\mu_n) \mathbf{Q}_2^{(n)} + \mathbf{I}_{L_n} \right)^{-2} \left(2\mathbf{Q}_1^{(n)} - 2\mathbf{Q}_2^{(n)} \right) \mathbf{b}_n = 0. \end{aligned}$$

Step 5. Calculate $\nu_n = 0.5p_{\max}/\sigma^2 - \mu_n$.

Step 6. Calculate k_n as

$$k_n \triangleq \left(\mathbf{b}_n^H \left(\mathbf{I}_{L_n} + 2\mu_n \mathbf{Q}_1^{(n)} \right) \left(\mathbf{I}_{L_n} + 2\mu_n \mathbf{Q}_1^{(n)} + 2\nu_n \mathbf{Q}_2^{(n)} \right)^{-2} \mathbf{b}_n \right)^{-\frac{1}{2}}.$$

Step 7. Calculate \mathbf{w}_n^o using the obtained values of k_n , μ_n and ν_n such that

$$\mathbf{w}_n^o = k_n \sqrt{2\nu_n} \left(2\mu_n \mathbf{Q}_1^{(n)} + 2\nu_n \mathbf{Q}_2^{(n)} + \mathbf{I}_{L_n} \right)^{-1} \mathbf{b}_n.$$

Step 8. Calculate the maximum balanced SNR, for \mathbf{w}_n^o , as

$$SNR_n(\mathbf{w}_n^o) = \frac{(p_{\max} - \sigma^2 \|\mathbf{w}_n^o\|^2) |\mathbf{b}_n^H \mathbf{w}_n^o|^2}{2\sigma^2 \left(\mathbf{w}_n^{o,H} \mathbf{Q}_1^{(n)} \mathbf{w}_n^o + 1 \right) \left(\mathbf{w}_n^{o,H} \mathbf{Q}_2^{(n)} \mathbf{w}_n^o + 1 \right)}.$$

Step 9. Set $n = n + 1$. If $n \geq N$, then go to the next step, otherwise go back to Step 2.

Step 10. Find the value of n which leads to the maximum $SNR_n(\mathbf{w}_n^o)$; i.e.,

$$n^o = \arg \max_{0 \leq n \leq N-1} SNR_n(\mathbf{w}_n^o).$$

Step 11. Let \mathbf{w}_{opt} represent the optimal relay weight vector. If the l -th relay is active, then the l -th entry of \mathbf{w}_{opt} is equal to the element of $\mathbf{w}_{n^o}^o$ which corresponds to the l -th relay. If the l -th relay is not active, then the l -th element of \mathbf{w}_{opt} is zero.

Step 12. Calculate the transmit power of Transceiver q for $q \in \{1, 2\}$ as

$$p_q = \frac{p_{max} - \sigma^2 \|\mathbf{w}_{opt}\|^2}{2(1 + \|\mathbf{G}_q \mathbf{w}_{opt}\|^2)}.$$

Chapter 4

Simulation Results

We consider an asynchronous bi-directional relay network with two single-antenna transceivers and $L = 60$ single-antenna relays. The signals transmitted by the transceivers are blocks containing $N_s = 64$ symbols. The frequency flat channel coefficients between transceivers and relays are considered to be complex Gaussian random variables whose means are zero and their variances are inversely proportional to the path loss. We assume that the path loss corresponding to the propagation from any transceiver to any relay or vice versa is proportional to the corresponding delay to the power of 3. The noises at the relays and also at the transceivers are assumed to be white Gaussian random processes with zero mean and unit variance. In each simulation run, the propagation delay from (to) a transceiver to (from) any relay is uniformly distributed in the interval $[T_s \quad 4T_s]$. As a result, since no relay contributes to the first two taps of the end-to-end channel impulse response, these two taps are zero and the delay of each relaying path is a random variable with triangular distribution in the interval $[2T_s \quad 8T_s]$.

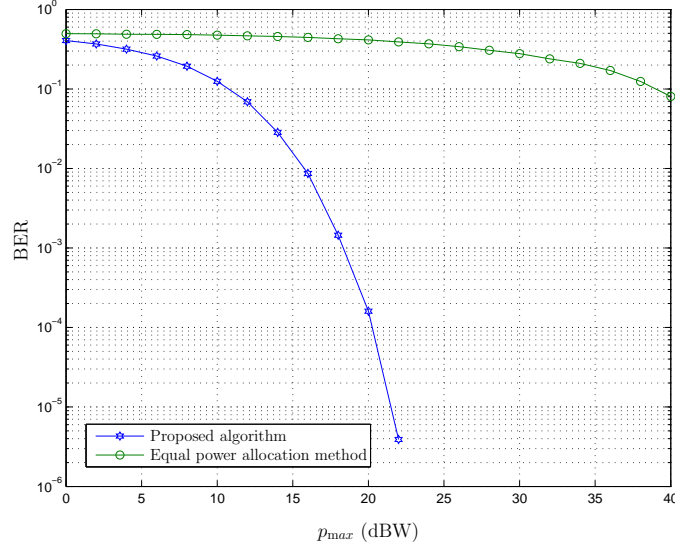


Figure 4.1: Bit error rate versus available transmit power p_{max} for different methods.

In Figure. 4.1, we illustrate the total bit error rate (BER) of our proposed algorithm versus the total transmit power p_{max} , for QPSK modulation. Herein, we compare our proposed algorithm with an equal power allocation (EPA) scheme, where the total transmit power is equally distributed among all network nodes. As can be seen from this figure, our proposed algorithm outperforms the EPA method.

Figure. 4.2 shows the average maximum balanced SNRs of the two transceivers for our proposed algorithm and for the EPA scheme. As demonstrated in this figure, the average maximum balanced SNRs at the two transceivers increase with increasing the maximum available total transmit power. This figure also shows that our proposed algorithm outperforms the EPA method.

Figure. 4.3 shows the total mean square error of the estimated received signals at the two transceivers for our proposed method and for the EPA scheme. Compared to Figure. 4.2, Figure. 4.3 shows that the total MSE is inversely proportional to the

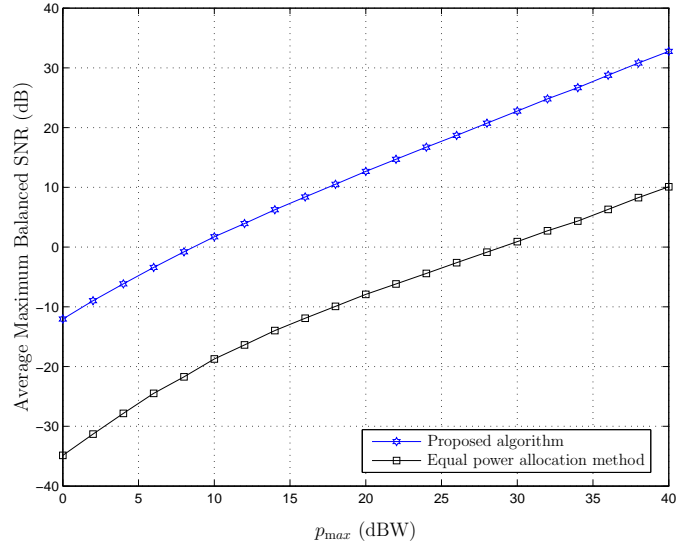


Figure 4.2: Average maximum balanced SNR versus the total available transmit power, p_{max} , for different methods.

maximum balanced SNR. This figure also shows that our proposed algorithm has a better performance compared to the EPA scheme.

In Figure. 4.4, we depict the sum-rate achieved by our algorithm versus the total available transmit power p_{max} , which is then compared with that of the EPA method. It can easily be seen that our proposed method offers a significantly higher sum-rate, as compared to the EPA scheme, for any given transmit power p_{max} .

Figure. 4.5 depicts the total MSE behavior versus the variances of the error in measurement of propagation delays for each relaying path. We model these errors using a zero mean Gaussian random variable. It can be seen that with an increase in the variance of the error, the total MSE doesn't change significantly when the standard deviation of the error is 0.447 second; i.e., the variance of the error is 0.2 sec^2 . Also, we can see from this figure that as long as the variance of the error is less

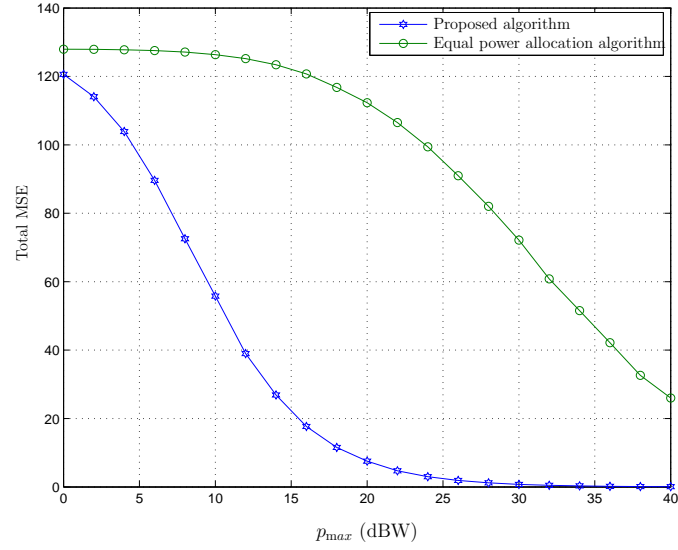


Figure 4.3: The total mean squared error of received signals at both transceivers curves versus the total available transmit power p_{max} , for different methods.

than 0.4 second, the performance loss in term of MSE is about 6 percent.

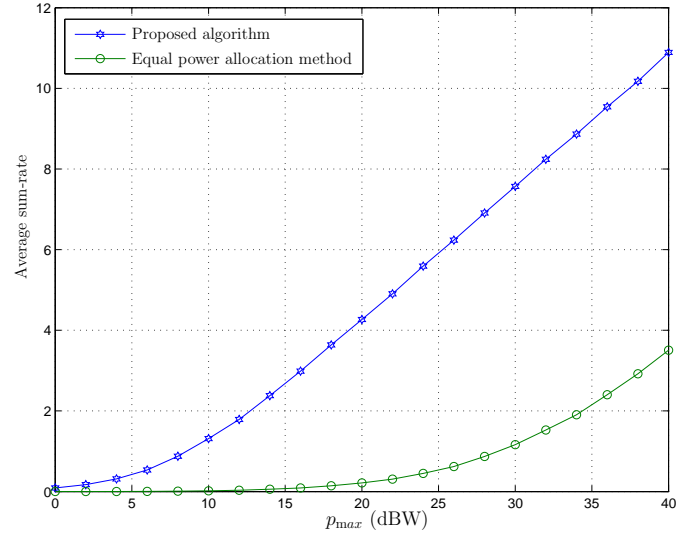


Figure 4.4: The sum-rate curves versus the total available transmit power p_{max} , for different methods.

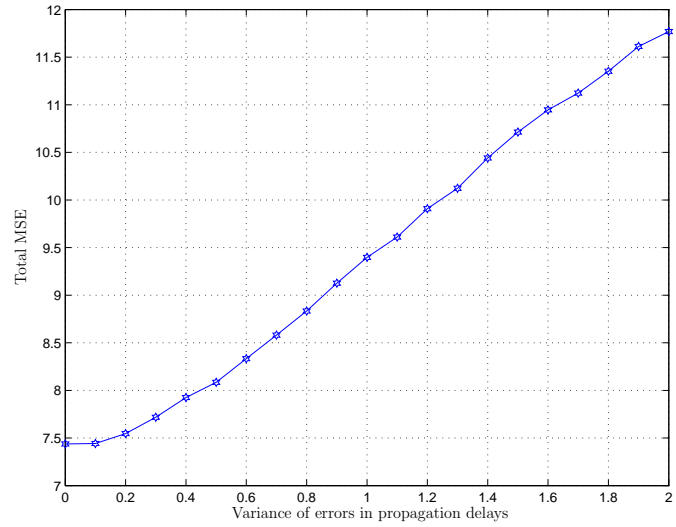


Figure 4.5: The total MSE behaviors versus different variances of error in propagation delays.

Chapter 5

Conclusions And Future work

We considered a two-way relay network consisting of two single transceivers and multiple single-antenna relays. The network we consider is assumed to be asynchronous, meaning that the transceiver-relay paths are subject to different relaying and/or propagation delays. In such a network, the end-to-end link can be viewed as a multipath channel which can cause inter-symbol-interference (ISI) in the signals received by the two transceivers. We model the end-to-end channel between the two transceivers as a linear time-invariant system whose impulse response can have as many taps as the number of the relays. In our model, each relay contributes to only one tap of the end-to-end channel impulse response while several relays can contribute to the same tap. Assuming a block transmission/reception scheme, we consider both pre- and post-channel equalization at both transceivers to combat the inter-block-interference (IBI) induced due to ISI. Considering amplify-and-forward relays, we study the problem of optimal design of pre- and post-channel linear equalizers and power loading at the two transceivers as well as the network beamforming at the relays. To do so, assuming a limited total transmit power budget, we minimize the total mean square

error (MSE) of the linearly estimated signals at both transceivers by optimally obtaining the transceivers' transmit powers and relay beamforming weights as well as block pre- and post-channel linear equalizers at the two transceivers. We rigorously prove that this minimization leads to all but one of the taps of the end-to-end channel impulse response being zero. As a result, only those relays which contribute to the non-zero tap (optimal tap) of the end-to-end channel impulse response will be turned on and the remainder of the relays will have to be switched off. We present a simple algorithm for determining which tap of the end-to-end channel impulse response has to be non-zero. This tap dictates which relays have to be active while the rest of the relays are turned off. We also provide semi-closed form solutions to the design parameters, namely the transceivers' transmit powers, the relay beamforming weights, and block pre- and post-channel equalization matrices at the two transceivers.

5.1 Future work

In this thesis, we extensively discussed the total MSE minimization of the linearly estimated signals under a total transmit power budget for an asynchronous AF two-way relay network with joint pre- and post-channel equalization scheme at both front-ends. Some of the possible extensions to the work presented in this thesis are listed below.

- In this thesis, we have tackled the problem of MSE minimization under a total power constraint. Conversely, one can consider the problem of minimization of the total transmit power subject to specific requirement on the MSEs of linearly

estimated signals at the transceivers. This problem can be formulated as

$$\begin{aligned} & \min_{\substack{p_1 \geq 0 \\ p_2 \geq 0}} \min_{\mathbf{w}} \min_{\mathbf{E}_1, \mathbf{E}_2} \min_{\mathbf{F}_1, \mathbf{F}_2} P_{total} \\ & \text{subject to } \text{MSE}_1(\mathbf{w}, p_1, \mathbf{E}_1, \mathbf{F}_2) \leq \varepsilon_1 \\ & \text{MSE}_2(\mathbf{w}, p_2, \mathbf{E}_2, \mathbf{F}_1) \leq \varepsilon_2 \end{aligned}$$

where p_1 and p_2 represent the transmit power at Transceivers 1 and 2, respectively. Pre-channel equalization blocks at Transceiver 1 and 2 are denoted as \mathbf{E}_1 and \mathbf{E}_2 , and also post-channel equalization blocks are shown as \mathbf{F}_1 and \mathbf{F}_2 at Transceivers 1 and 2. The relay beamforming weights are denoted as \mathbf{w} and P_{total} is the total transmit power.

- This thesis focused on a single-input, single-output communication scheme; i.e., we have assumed that both transceivers and all the relay nodes are equipped with a single-antenna. A possible extension to this work is to consider a MIMO communication network where each node is equipped with more than one transmit and receive antennas.
- We have chosen the total MSE of the linearly estimated signal as the performance criterion for our optimization problem. One can also consider the problem of sum-rate maximization in a similar setting. A comparison between the results obtained in our work and the sum-rate maximization problem can provide helpful insight into the design of cooperative communication systems.
- In our communication scheme, considering an asynchronous two-way relay network, we deploy joint pre- and post-channel block equalization at the two front-ends of the two transceivers. Instead of employing linear equalization blocks,

utilizing decision feedback equalization (DFE) or maximum likelihood sequence estimation (MLSE) equalization at the transceivers seems to be another challenging open area in this field.

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