

# **Spectrum Sharing for Two-way Relay Networks**

by

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## Abstract

In this thesis, spectrum sharing for two-way relaying scheme is proposed. We consider a communication network consisting of two transceiver nodes, one relay node, and one receiving node. The relay node enables a two-way communication between the two transceivers, and at the same time, sends its own information to the receiving node. We study the problem of the power allocation to the transceiver signal and the relay signal to maximize the data rate of link between the relay node and the receiving node subject to two constraints on the quality of service (QoS) at the two transceivers. We use the structure of the corresponding rate maximization problem to obtain the optimal power allocation scheme in a closed form.

Next, we consider a three-phase two-way relaying consisting of two transceiver nodes and two relay nodes. These two relays enable a bidirectional communication between the two transceivers and meanwhile communicate with each other. The two transceivers transmit their respective signals to the relays simultaneously in phase 1. In phases 2 and 3, the received signal at each relay is multiplied by a complex beamforming weight and is added to a scaled version of the unit-power information-bearing symbol intended for the other relay; then, each relay transmits its so-obtained signal to the two transceivers and the other relay. We study the problem of the optimal beamforming weights and the optimal value of the relay amplification factors for their own signals such that the smaller of the data rates of two relays is maximized subject to two constraints on the QoS at the two transceivers. We prove that at the optimum, the data rates of

two relays are equal. We show that the respective rate maximization problem can be written as a combination of a bisection technique and second-order cone convex feasibility problem.

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# **Chapter 1**

## **Introduction**



## 1.1 Overview

With the advancement of technology, our civilization is now being propelled towards the information society. As a result, wireless technologies, such as Wi-Fi access and smart phones, are playing a significantly indispensable role in our daily life. Due to the rapidly growing demands for wireless services, radio spectrum becomes extremely scarce.

In reality, most traditional networks are based on point-to-point communications, where only two sources are connected directly. However, channel fading results in a significant transmit power loss in the traditional wireless communication networks. Different types of diversity include time diversity, frequency diversity, and space diversity have been studied to address this problem. Enhancing the overall spectral efficiency of cooperative systems by using signal space diversity is proposed in [1, 2]. Although the transmit diversity has a majority of advantages, it may not always be practical because of size, cost, or hardware limitations [3].

Cooperative communication techniques are introduced in order to let the transmit diversity used in a multi-user environment, in which the mobiles are equipped with a single antenna. In a cooperative network, each user transmits its own information and at the same time, acts as a relay for other users. As a result, cooperative communication systems can be used to enhance power and spectral efficiency. Due to the fact that the relay networks is a critical branch of cooperative wireless communication schemes, a two-way relaying scheme is studied in this thesis due to its ability to improve the spectral efficiency. We study two problems:

- 1) In the first problem, we consider a two-phase spectrum sharing protocol consisting of two transceiver nodes, one relay node, and one receiving node. The relay node enables a bidirectional communication between the two transceivers, and at the same time, sends its own information to the receiving node. We study the problem of the power allocation to the transceiver signal and

the relay signal to maximize the data rate of link between the relay node and the receiving node under the constraints that the data rate of each transceiver is not smaller than a given threshold.

2) In the second problem, we consider a three-phase two-way relaying scheme consisting of two transceiver nodes and two relay nodes. These two relays enable a bidirectional communication between the two transceivers and meanwhile communicate with each other. We study the problem of the resource sharing to maximize the smaller of the data rates of two relays under the constraints that the data rate of each transceiver is not smaller than a given threshold.

## 1.2 Cooperative Communication

Compared to traditional communication networks, cooperative communication enables efficient spectrum utilization by allowing nodes to cooperate with each other during information transmission.

The cooperative transmission enables single-antenna users in a multi-user environment to share their communication resources with other users and creates a virtual multi-antenna transmitter [4, 5]. In a cooperative network, each user transmits its own information, and at the same time, acts as a relay for other users. Therefore, cooperative communication systems can be used to improve network connectivity, to enhance power and spectral efficiency, and to improve communication reliability.

Since cooperative communication allows each user to transmit some version of overheard information and its own information [4, 6], the transmission rate will be reduced. Several complicated issues, such as the loss of rate to the cooperative mobile, overall interference in the network, cooperation assignment, fairness of the system, and transceiver requirements must be

considered while designing a cooperative communication network [6].

Relay transmission is the main feature of cooperative communication. Recently, relay-assisted communication is becoming more accepted in a number of wireless systems, such as ad-hoc, mesh, and cellular networks [6–8]. This is because relay communication is an effective method to improve the spectral efficiency, to expand the coverage area, and to mitigate channel impairments [9, 10]. To take full advantage of these features, efficient wireless resource allocation is significantly important for relay communication systems. In particular, the problem formulation may differ significantly in optimization objectives, relaying protocols, transmit power constraints, and system architectures [11].

### **1.3 Relay Networks**

Recently, relay transmission has received considerable attention [12–20]. Relay networks have become a critical branch of cooperative wireless communication schemes, in which the relays cooperate with each other to establish a communication link between a source node and a destination node [21, 22]. Utilization of relay transmission brings significant performance benefits include arriving at spatial diversity through node cooperation, expanding coverage without the need to increase transmitter power, improving the reliability, and increasing the data rate [6, 23, 24].

Different types of relaying schemes include amplified-and-forward (AF) scheme, estimate-and-forward (EF) scheme, decode-and-forward (DF) scheme, and compress-and-forward (CF) scheme have been studied in the literatures. The AF scheme has attracted much attention because of its simplicity, as this scheme does not aim to decode the transmitted signal. Moreover,

compared to EF, DF, and CF techniques, the AF scheme requires less processing power at the relays. Different relay networks can be categorized as: one-way relaying schemes and two-way relaying schemes. We focus on introducing two-way relaying schemes in the next section.

## 1.4 Two-way Relaying

Compared to one-way relaying, two-way relaying enables relays to establish a bidirectional communication link between the two transceivers to exchange information simultaneously [25]. The simplest two-way relaying scheme consists of two transceiver nodes and one relay node, where the two transceivers exchange information with the help of the relay [26] in a half-duplex mode. Two-way relaying aims to enable the relay to retransmit a processed version of the signal it received from the two transceivers, and each transceiver retrieves transmitted information from the other transceiver subsequent to cancel out the self-interference created by its own transmission [25].

The two-way communication channel (without relay) was first introduced by Shannon, showing how to efficiently design message structures to enable simultaneous bidirectional communication at the highest possible data rate [27]. Two-way relaying was later studied in [28] from information theoretic point of view. Recently, two-way relaying has attracted considerable attention, because a two-way relaying scheme can establish a reliable bandwidth-efficient bidirectional communication link between the two transceivers. Different types of two-way relaying scheme include the four time-slot scheme, the three time-slot time division broadcast (TDBC) scheme, and the two time-slot multiple access broadcast (MABC) scheme.

- In the four time-slot scheme, two consequent one-way relaying schemes are used to convey

information in two directions. In phase 1, node 1 transmits its own signal to the relays. In phase 2, the relays transmit the signal they received to node 2. In phase 3, node 2 transmits its received signal to the relays. In phase 4, the relays retransmit their received signal from node 2 to node 1. Since the proposed protocol requires four phases to exchange two information symbols between the two transceivers, the spectral efficiency is relatively low.

- The TDBC scheme consists of three phases. In phases 1 and 2, the two transceivers transmit their signals to the relays, respectively. In phase 3, the relay nodes transmit a mixture of the signal they received to two transceivers. Compared to the traditional two-way relaying scheme, a TDBC scheme offers a higher bandwidth efficiency [18].
- The MABC scheme takes two phases to exchange two information symbols between the two transceivers. In phase 1, the two transceivers transmit their respective signals to the relays simultaneously. In phase 2, the relay nodes broadcast an amplified and phase-adjusted version of the signal they received in phase 1 to the two transceivers.

## 1.5 Motivation and Problem Statement

Radio spectrum becomes extremely scarce due to rapidly growing demands for wireless services. One way to overcome spectrum scarcity is to exploit spectrum between two or more networks. A joint spectrum sharing and two-way relaying scheme, which consists of two transceiver nodes, one relay node, and one receiving node, is proposed in Chapter 3 because spectrum sharing can provide a flexible manner to improve spectrum utilization for wireless systems [29] and two-way relaying can establish a reliable bandwidth-efficient bidirectional communication link between the two transceivers. The combination of these two techniques can help enhance

the spectral efficiency. The relay node enables a bidirectional communication between the two transceivers and meanwhile sends its own information to the receiving node. The two transceivers transmit their respective signals to the relay simultaneously in phase 1 and in phase 2, the relay node broadcasts a linear combination of the amplified and phase-adjusted version of the signal it received from the two transceivers and its own signal intended for the receiving node.

We will study the problem of the resource sharing for a three-phase two-way relaying scheme in Chapter 4. We consider a communication scheme consisting of two transceiver nodes and two relay nodes. These two relays enable a bidirectional communication between the two transceivers and meanwhile communicate with each other. In phase 1, the two transceivers transmit their respective signals to the relays simultaneously. Then, the received signal at each relay is multiplied by a complex beamforming weight and is added to a scaled version of the unit-power information-bearing symbol intended for the other relay. Indeed, in phases 2 and 3, each relay transmits a linear combination of the amplified and phase-adjusted version of the signal it received from the two transceivers and its own signal intended for the other relay.

## 1.6 Objective

We consider the data rate maximization problem in two-way relaying schemes. Below, we bring a summary of our objectives in this thesis

- For a two-phase spectrum sharing protocol, we aim to find the power allocation to the transceiver signal and the relay signal to maximize the data rate of the link between the relay node and the receiving node subject to two constraints on the QoS at the two

transceivers.

- For a three-phase two-way relaying scheme, we aim to find the optimal beamforming weights and the optimal value of the relay amplification factors for their own signals to maximize the smaller of the data rates of two relays subject to two constraints on the QoS at the two transceivers.

## 1.7 Methodology

In this section, we briefly review our methodology toward the aforementioned problems.

- In the first problem, we use the structure of the corresponding rate maximization problem to obtain the optimal power allocation scheme in a closed form.
- In the second problem, we show that the respective rate maximization problem can be written as a combination of a bisection technique and second-order cone convex feasibility problem.

## 1.8 Thesis Organization

This thesis is organized in five chapters. In Chapter 2, we present our literature review. In Chapter 3, we present our two time-slot spectrum sharing scheme for two-way relay networks. In Chapter 4, we present our three time-slot distributed beamforming scheme for two-way relay networks. Finally, the thesis is concluded in Chapter 5.

## 1.9 Notations

We represent the statistical expectation as  $E\{\cdot\}$ . We use lowercase and uppercase boldface letters to represent the vectors and matrices, respectively. Complex conjugate, transpose, and Hermitian transpose are respectively denoted as  $(\cdot)^*$ ,  $(\cdot)^T$ , and  $(\cdot)^H$ . We denote the identity matrix by  $\mathbf{I}$  and denote the all-zero vectors or matrices by  $\mathbf{0}$ . We use  $\text{diag}\{\mathbf{a}\}$  to represent a diagonal matrix whose diagonal entries are the elements of the vector  $\mathbf{a}$  and  $\text{diag}\{\mathbf{A}\}$  to represent a vector which contains the diagonal entries of the square matrix  $\mathbf{A}$ .



# Chapter 2

## Literature Review

As we discussed in Chapter 1, two-way relaying schemes are of practical importance due to their ability to improve spectrum utilization for wireless networks as compared to one-way relaying schemes. In this chapter, we first review a joint spectrum sharing and two-way relaying scheme. Then, we review power allocation problem based on a joint spectrum sharing and two-way relaying scheme. Next, we briefly review the concept of convex optimization. Then, we review the convex optimization used in distributed beamforming algorithms for relaying schemes. Finally, we present our research contribution.

## 2.1 Joint Spectrum Sharing and Two-way Relaying Scheme

Radio spectrum becomes extremely scarce due to rapidly growing demands for wireless services. One way to overcome spectrum scarcity is to exploit spectrum between two or more networks. As a result, joint spectrum sharing and two-way relaying scheme has attracted considerable attention [30–36] because of two reasons:

- Spectrum sharing can provide a flexible manner to improve spectrum utilization for wireless systems [29].
- A two-way relaying approach can be used to establish a reliable bandwidth-efficient bidirectional communication link between the two transceivers.

As a result, the combination of these two techniques can help improve spectrum utilization for wireless networks.

### 2.1.1 Power Allocation Problem

Spectrum sharing for two-way relaying scheme is proposed in [35, 37]. The proposed protocol consists of two transceiver nodes, one relay node, and one receiving node, in which the two transceivers exchange information with the help of the relay. In phase 1, the two transceivers transmit their respective signals to the relay simultaneously. In phase 2, the relay node uses a portion of its own total power to transmit a mixture of the signal it received from the two transceivers, and uses the remaining power to transmit an independent information to the receiving node. The following problems have been studied in [35, 37]:

- The authors study the problem of the outage probability of each transceiver under the constraint that its own data rate is smaller than a given target rate.

- Spectrum sharing region is characterized.
- The authors study the problem of the lower and upper ergodic capacity for each transceiver.
- They study the problem of the outage probability of the receiving node under the constraint that the data rate of the link between the relay node and the receiving node is smaller than a given target rate.
- They also study the problem of the ergodic capacity for the receiving node.

The results of [35, 37] show that a better performance of the primary system can be achieved under the proposed protocol, and at the same time, this proposed protocol provides opportunities for secondary transmission.

Although the two-way relaying scheme that we will study in Chapter 3 is similar to that considered in [35, 37], we study the problem of the power allocation to the transceiver signal and the relay signal such that the data rate of the link between the relay node and the receiving node is maximized under the constraints that the data rate of each transceiver is not smaller than a given threshold. We use the structure of the corresponding rate maximization problem to obtain the optimal power allocation scheme in a closed form.

## 2.2 Convex Optimization

A convex optimization problem refers to minimization of a convex objective function subject to convex and second-order constraints [38, 39]. The convex optimization problem is given by

[40]:

$$\begin{aligned}
 & \text{minimize} && f_0(\mathbf{x}) \\
 & \text{subject to} && f_1(\mathbf{x}) \leq b_1, \dots, f_m(\mathbf{x}) \leq b_m,
 \end{aligned} \tag{2.1}$$

where  $\mathbf{x} \in \mathbf{R}^n$  is the vector of the optimization variables and the functions  $\{f_i(\mathbf{x})\}_{i=0}^m: \mathbf{R}^n \rightarrow \mathbf{R}$  are convex, i.e., they satisfy

$$f_i(\lambda\mathbf{x} + (1 - \lambda)\mathbf{y}) \leq \lambda f_i(\mathbf{x}) + (1 - \lambda)f_i(\mathbf{y})$$

for all  $\mathbf{x}, \mathbf{y} \in \mathbf{R}^n$  and  $\lambda \in \mathbf{R}$  with  $0 \leq \lambda \leq 1$ .

Although there is no guarantee that an analytical for the solution of a given convex optimization problem existing, such a problem can be efficiently solved using interior-point methods and the highly accurate optimal solutions can be obtained. Nowadays, convex optimization problems have been widely used in numerous fields (such as automatic control systems, signal processing, and communications) to obtain the fast, efficient and reliable solutions.

## 2.3 Distributed Beamforming Problem

Recently, convex optimization used in distributed beamforming algorithms for relaying schemes have attracted considerable attention.

**One-way Relaying:**

In [41, 42], the authors assume that the second-order statistics of the channel coefficients are available at the receiver, they aim to find the optimal beamforming weight to maximize the receiver signal-to-noise ratio (SNR) under the constraints that the transmit power of each relay is not larger than a given threshold. Using a semi-definite relaxation approach, the optimization problem can be converted to a semi-definite programming (SDP) problem, which can be solved using interior-point methods.

In [43, 44], the authors consider a network consisting of  $d$  source nodes,  $r$  relay nodes, and  $d$  destination nodes. In phase 1, each source transmits its own signal to all relays. In phase 2, each relay transmits a mixture of the signal it received in phase 1 to the all destination nodes. The authors of [43, 44] study the problem of minimizing the total transmit power subject to the destination QoS constraints. Using a semi-definite relaxation approach, the optimization problem in [43] can be written as a SDP problem. Compared to the approach used in [43], the power minimization problem in [44] can be converted to a second-order convex cone programming (SOCP) problem, which aims to reduce the computational complexity.

**Two-way Relaying:**

Recently, distributed beamforming algorithms for two-way relaying schemes, which consist of two transceiver nodes and several relay nodes, have attracted considerable attention in [45–49]. In phase 1, the two transceivers transmit their respective signals to the relays simultaneously. In phase 2, the received signal at each relay is multiplied by a complex beamforming weight; then, each relay transmits its so-obtained signal to the two transceivers. The following problems have been studied in [45–49]:

- The authors study the problem of minimizing the total transmit power under the constraints that the data rate of each transceiver is not smaller than a given threshold, and the optimal solutions can be obtained by using an iterative steepest decent algorithm.
- They study the problem of SNR balancing subject to an individual power constraint on each node. For fixed transmit powers of two transceivers, the optimization problem can be written as a combination of a bisection technique and SOCP problem.
- They also study the problem of SNR balancing under the constraint that the total transmit power is not larger than a given threshold, and the optimal solutions can be obtained by using a simple iterative algorithm.

The results of [45–49] show that when the transceiver SNRs are identical, the equal total transmit power is respectively allocated to the two transceivers and the relays.

The relays in the two-way relaying scheme that considered in [45–49] enable a bidirectional communication between the two transceivers. Compared to the two-way relaying scheme that considered in [45–49], the two-way relaying scheme that we will study in Chapter 4 enables relays to exchange their own information with each other. We consider a communication network consisting of two transceiver nodes and two relay nodes. These two relays enable a bidirectional communication between the two transceivers; meanwhile, these two relays both have their own information which wish to exchange. We study the problem of the resource sharing to maximize the smaller of the data rates of two relays under the constraints that the data rate of each transceiver is not smaller than a given threshold. We show that the respective rate maximization problem can be written as a combination of a bisection technique and second-order cone convex feasibility problem.

## 2.4 Research Contribution

In this thesis, the first problem is to obtain the optimal power allocation to the transceiver signal and the relay signal to maximize the data rate of link between the relay node and the receiving node subject to two constraints on the QoS at the two transceivers. We use the structure of the corresponding rate maximization problem to obtain the optimal power allocation scheme in a closed form.

The second problem is to obtain the optimal beamforming weights and the optimal value of the relay amplification factors for their own signals such that the smaller of the data rates of two relays is maximized subject to two constraints on the QoS at the two transceivers. We prove that at the optimum, the data rates of two relays are equal. By using the beamforming weight vector as the design parameter, we show that the respective rate maximization problem can be converted to a combination of a bisection technique and second-order cone convex feasibility problem.

## **Chapter 3**

# **Two Time-slot Spectrum Sharing Scheme**

## **for Two-way Relay Networks**



### 3.1 System Model

We consider a communication scheme consisting of two transceiver nodes  $S_1$  and  $S_2$ , one relay node  $R$ , and one receiving node  $S_3$ . Due to the poor quality of the channel between the two transceivers, no direct communication link can be established between them, and therefore, they communicate with each other with the help of the relay. Operating in the half-duplex mode, the relay  $R$  enables a two-way communication between  $S_1$  and  $S_2$ ; meanwhile,  $R$ , as an information source, has its own information intended for the receiving node  $S_3$ . We assume that the transmit powers of the two transceivers  $S_1$  and  $S_2$  are respectively denoted as  $p_1$  and  $p_2$ . Also,  $p_s$  denotes the transmit power of relay  $R$ .

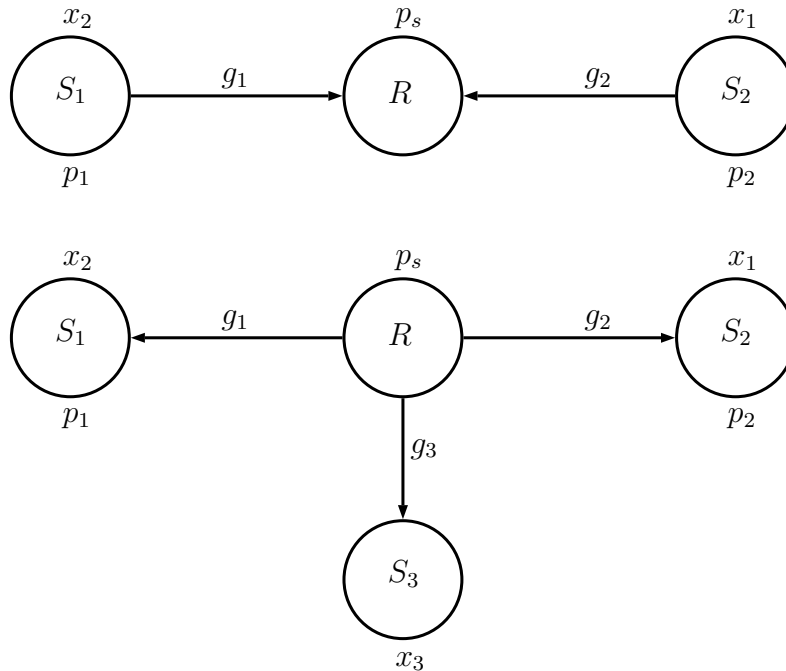


Figure 3.1: System model for the proposed two-phase protocol.

All channels are assumed to be frequency flat and reciprocal. The complex channel co-

efficient between  $S_i$  and  $R$  is denoted as  $g_i$ . Assuming that the perfect channel knowledge is available at the receiver side for each link. Moreover, we assume that  $S_1$  has the knowledge of  $g_2$ , that  $S_2$  has the knowledge of  $g_1$ , and that  $R$  has the knowledge of  $g_3$ . Finally, the additive white Gaussian noise at  $S_1$ ,  $S_2$ ,  $S_3$  and  $R$  are respectively denoted as  $n_1$ ,  $n_2$ ,  $n_3$  and  $n_R$ , and these are modeled as independent and identically distributed Gaussian random variables with zero mean and variance equal to  $\sigma^2$ .

As shown in Figure 3.1, the proposed scheme consists of two phases. In phase 1,  $S_1$  and  $S_2$  transmit their respective signals to relay  $R$  simultaneously. Hence, the signal  $y_R$  received at  $R$  in phase 1 is given by

$$y_R = \sqrt{p_1}g_1x_2 + \sqrt{p_2}g_2x_1 + n_R, \quad (3.1)$$

where  $x_1$  and  $x_2$  are the information symbol transmitted by the two transceivers  $S_2$  and  $S_1$ .

In phase 2,  $R$  uses a fraction  $\alpha$ , where  $0 \leq \alpha \leq 1$ , of its own total power  $p_s$  to transmit a linear combination of the amplified and phase-adjusted version of the signal it received in phase 1, and uses the remaining power to transmit an independent information to the receiving node  $S_3$ . As a result,  $\alpha$  is referred to as a power allocation factor. Consequently, the composite signal broadcasted by relay  $R$  in phase 2 can be expressed as

$$x_R = \frac{\sqrt{\alpha p_s} y_R}{\beta} + \sqrt{(1 - \alpha) p_s} x_3, \quad (3.2)$$

where  $\beta = \sqrt{p_1|g_1|^2 + p_2|g_2|^2 + \sigma^2}$  is the normalization factor in order to ensure that the power used to transmit the superimposed primary signals is always equal to  $\alpha p_s$ , and  $x_3$  is the unit-

power information-bearing symbol intended for  $S_3$ .

The signal  $y_1$  received at  $S_1$  in phase 2 can be written as

$$\begin{aligned}
y_1 &= g_1 x_R + n_1 \\
&= g_1 \left( \frac{\sqrt{\alpha p_s} y_R}{\beta} + \sqrt{(1-\alpha) p_s} x_3 \right) + n_1 \\
&= \frac{\sqrt{\alpha p_s} g_1}{\beta} (\sqrt{p_1} g_1 x_2 + \sqrt{p_2} g_2 x_1 + n_R) + \sqrt{(1-\alpha) p_s} g_1 x_3 + n_1 \\
&= \underbrace{\frac{\sqrt{\alpha p_s p_2} g_1 g_2 x_1}{\beta}}_{\text{desired signal}} + \underbrace{\frac{\sqrt{\alpha p_s p_1} g_1^2 x_2}{\beta}}_{\text{self-interference}} + \underbrace{\sqrt{(1-\alpha) p_s} g_1 x_3}_{\text{interference}} + \underbrace{\frac{\sqrt{\alpha p_s} g_1 n_R}{\beta}}_{\text{noise}} + n_1. \tag{3.3}
\end{aligned}$$

Since  $x_2$  is the symbol transmitted by  $S_1$  in phase 1, and the second term in (3.3) containing  $x_2$  is perfectly known to  $S_1$ , the second term in (3.3) can be subtracted from  $y_1$ . As a result, the residual signal  $\tilde{y}_1$  is defined as

$$\begin{aligned}
\tilde{y}_1 &= y_1 - \frac{\sqrt{\alpha p_s p_1} g_1^2 x_2}{\beta} \\
&= \underbrace{\frac{\sqrt{\alpha p_s p_2} g_1 g_2 x_1}{\beta}}_{\text{desired signal}} + \underbrace{\sqrt{(1-\alpha) p_s} g_1 x_3}_{\text{interference}} + \underbrace{\frac{\sqrt{\alpha p_s} g_1 n_R}{\beta}}_{\text{noise}} + n_1. \tag{3.4}
\end{aligned}$$

After removing the self-interference from  $y_1$ , the secondary signal  $x_3$  is treated as interference, and  $x_1$  is decoded directly. Consequently, the signal-to-interference-plus-noise ratio (SINR) achieved at  $S_1$  to decode  $x_1$  is given by

$$\text{SINR}_1 = \frac{\alpha \gamma_2 \gamma_s |g_1|^2 |g_2|^2}{(1-\alpha) \gamma_s |g_1|^2 (\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2) + \gamma_s |g_1|^2 + \gamma_1 |g_1|^2 + \gamma_2 |g_2|^2 + 1}, \tag{3.5}$$

where we have used the following definitions that  $\gamma_1 \triangleq \frac{p_1}{\sigma^2}$ ,  $\gamma_2 \triangleq \frac{p_2}{\sigma^2}$ , and  $\gamma_s \triangleq \frac{p_s}{\sigma^2}$ , and assumed that  $E[|x_1|^2] = 1$  and  $E[|x_3|^2] = 1$ .

In light of (3.5), the data rate achieved at  $S_1$  to decode  $x_1$  is given by

$$R_1 = \frac{1}{2} \log_2(1 + \text{SINR}_1), \quad (3.6)$$

where the factor  $\frac{1}{2}$  is used because two time slots are utilized in the overall transmission.

Similarly, the signal  $y_2$  received at  $S_2$  in phase 2 can be written as

$$\begin{aligned} y_2 &= g_2 x_R + n_2 \\ &= g_2 \left( \frac{\sqrt{\alpha p_s} y_R}{\beta} + \sqrt{(1-\alpha) p_s} x_3 \right) + n_2 \\ &= \frac{\sqrt{\alpha p_s} g_2}{\beta} (\sqrt{p_1} g_1 x_2 + \sqrt{p_2} g_2 x_1 + n_R) + \sqrt{(1-\alpha) p_s} g_2 x_3 + n_2 \\ &= \underbrace{\frac{\sqrt{\alpha p_s p_1} g_1 g_2 x_2}{\beta}}_{\text{desired signal}} + \underbrace{\frac{\sqrt{\alpha p_s p_2} g_2^2 x_1}{\beta}}_{\text{self-interference}} + \underbrace{\sqrt{(1-\alpha) p_s} g_2 x_3}_{\text{interference}} + \underbrace{\frac{\sqrt{\alpha p_s} g_2 n_R}{\beta}}_{\text{noise}} + n_2. \end{aligned} \quad (3.7)$$

Since  $x_1$  is the symbol transmitted by  $S_2$  in phase 1, and the second term in (3.7) containing  $x_1$  is perfectly known to  $S_2$ , the second term in (3.7) can be subtracted from  $y_2$ . As a result, the residual signal  $\tilde{y}_2$  is defined as

$$\begin{aligned} \tilde{y}_2 &= y_2 - \frac{\sqrt{\alpha p_s p_2} g_2^2 x_1}{\beta} \\ &= \underbrace{\frac{\sqrt{\alpha p_s p_1} g_1 g_2 x_2}{\beta}}_{\text{desired signal}} + \underbrace{\sqrt{(1-\alpha) p_s} g_2 x_3}_{\text{interference}} + \underbrace{\frac{\sqrt{\alpha p_s} g_2 n_R}{\beta}}_{\text{noise}} + n_2. \end{aligned} \quad (3.8)$$

After removing the self-interference from  $y_2$ , the secondary signal  $x_3$  is treated as the interference, and  $x_2$  is decoded directly. Consequently, the SINR achieved at  $S_2$  to decode  $x_2$  is given by

$$\text{SINR}_2 = \frac{\alpha\gamma_1\gamma_s|g_1|^2|g_2|^2}{(1-\alpha)\gamma_s|g_2|^2(\gamma_1|g_1|^2 + \gamma_2|g_2|^2) + \gamma_s|g_2|^2 + \gamma_1|g_1|^2 + \gamma_2|g_2|^2 + 1}, \quad (3.9)$$

where we have used the assumptions that  $E[|x_2|^2] = 1$ .

In light of (3.9), the data rate achieved at  $S_2$  to decode  $x_2$  is given by

$$R_2 = \frac{1}{2} \log_2(1 + \text{SINR}_2). \quad (3.10)$$

The signal  $y_3$  received at  $S_3$  in phase 2 is given by

$$\begin{aligned} y_3 &= g_3x_R + n_3 \\ &= g_3\left(\frac{\sqrt{\alpha p_s}y_R}{\beta} + \sqrt{(1-\alpha)p_s}x_3\right) + n_3 \\ &= \frac{\sqrt{\alpha p_s}g_3}{\beta}(\sqrt{p_1}g_1x_2 + \sqrt{p_2}g_2x_1 + n_R) + \sqrt{(1-\alpha)p_s}g_3x_3 + n_2 \\ &= \underbrace{\sqrt{(1-\alpha)p_s}g_3x_3}_{\text{desired signal}} + \underbrace{\frac{\sqrt{\alpha p_s p_2}g_2g_3x_1}{\beta} + \frac{\sqrt{\alpha p_s p_1}g_1g_3x_2}{\beta}}_{\text{interference}} + \underbrace{\frac{\sqrt{\alpha p_s}g_3n_R}{\beta}}_{\text{noise}} + n_3. \end{aligned} \quad (3.11)$$

Unlike the two transceivers  $S_1$  and  $S_2$ ,  $S_3$  has no knowledge about  $x_1$  or  $x_2$ ; therefore, the SINR achieved at  $S_3$  to decode its desired signal  $x_3$ , under the interference from the two primary

signals  $x_1$  and  $x_2$ , is given by

$$\text{SINR}_3 = \frac{(1 - \alpha)\gamma_s |g_3|^2}{\alpha\gamma_s |g_3|^2 + 1}. \quad (3.12)$$

In light of (3.12), the data rate achieved at  $S_3$  to decode  $x_3$  is given by

$$R_3 = \frac{1}{2} \log_2(1 + \text{SINR}_3), \quad (3.13)$$

where the factor  $\frac{1}{2}$  is used because the secondary transmission occurs only in one out of two time slots.

## 3.2 Secondary User's Data Rate Maximization

In this chapter, we aim to find the optimal power allocation factor  $\alpha$  such that  $R_3$  is maximized when the transmit powers of  $p_1$  and  $p_2$  of the two transceivers  $S_1$  and  $S_2$  and the transmit power  $p_s$  of relay  $R$  are given under the constraints that  $R_1$  and  $R_2$  are not smaller than the two given thresholds  $\eta_1$  and  $\eta_2$ , respectively. Mathematically, we solve the following optimization problem:

$$\begin{aligned} & \max_{0 \leq \alpha \leq 1} && R_3 \\ & \text{subject to} && R_1 \geq \eta_1 \\ & && R_2 \geq \eta_2 \end{aligned} \quad (3.14)$$

Substituting (3.6), (3.10), and (3.13) into (3.14), the optimization problem in (3.14) can be equivalently written as

$$\begin{aligned}
& \max_{\alpha} \quad \text{SINR}_3 \\
& \text{subject to} \quad \text{SINR}_1 \geq 2^{2\eta_1} - 1 \\
& \quad \quad \quad \text{SINR}_2 \geq 2^{2\eta_2} - 1 \\
& \quad \quad \quad 0 \leq \alpha \leq 1
\end{aligned} \tag{3.15}$$

Substituting (3.5), (3.9), and (3.12) into (3.15), the optimization problem in (3.15) can be mathematically expressed as

$$\begin{aligned}
& \max_{\alpha} \quad \frac{(1 - \alpha)\gamma_s|g_3|^2}{\alpha\gamma_s|g_3|^2 + 1} \\
& \text{subject to} \quad \frac{\alpha\gamma_2\gamma_s|g_1|^2|g_2|^2}{(1 - \alpha)\gamma_s|g_1|^2(\gamma_1|g_1|^2 + \gamma_2|g_2|^2) + \gamma_s|g_1|^2 + \gamma_1|g_1|^2 + \gamma_2|g_2|^2 + 1} \geq 2^{2\eta_1} - 1 \\
& \quad \quad \quad \frac{\alpha\gamma_1\gamma_s|g_1|^2|g_2|^2}{(1 - \alpha)\gamma_s|g_2|^2(\gamma_1|g_1|^2 + \gamma_2|g_2|^2) + \gamma_s|g_2|^2 + \gamma_1|g_1|^2 + \gamma_2|g_2|^2 + 1} \geq 2^{2\eta_2} - 1 \\
& \quad \quad \quad 0 \leq \alpha \leq 1
\end{aligned} \tag{3.16}$$

or, equivalently, as

$$\begin{aligned}
& \max_{\alpha} && \frac{(1 - \alpha)\gamma_s |g_3|^2}{\alpha\gamma_s |g_3|^2 + 1} \\
\text{subject to} &&& \alpha \geq \frac{(2^{2\eta_1} - 1)[(\gamma_s |g_1|^2 + 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2) + \gamma_s |g_1|^2 + 1]}{\gamma_s |g_1|^2 [\gamma_2 |g_2|^2 + (2^{2\eta_1} - 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2)]} \\
&&& \alpha \geq \frac{(2^{2\eta_2} - 1)[(\gamma_s |g_2|^2 + 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2) + \gamma_s |g_2|^2 + 1]}{\gamma_s |g_2|^2 [\gamma_1 |g_1|^2 + (2^{2\eta_2} - 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2)]} \\
&&& 0 \leq \alpha \leq 1
\end{aligned} \tag{3.17}$$

In light of the optimization problem (3.17), we can obtain the following objective function, that is

$$\text{SINR}_3 = \frac{(1 - \alpha)\gamma_s |g_3|^2}{\alpha\gamma_s |g_3|^2 + 1},$$

then differentiating this objective function, we would have

$$\begin{aligned}
\frac{\partial \text{SINR}_3}{\partial \alpha} &= \frac{-\gamma_s |g_3|^2 (\alpha\gamma_s |g_3|^2 + 1) - (1 - \alpha)\gamma_s^2 |g_3|^4}{(\alpha\gamma_s |g_3|^2 + 1)^2} \\
&= \frac{-\gamma_s |g_3|^2 - \gamma_s^2 |g_3|^4}{(\alpha\gamma_s |g_3|^2 + 1)^2} \\
&= -\frac{\gamma_s |g_3|^2 + \gamma_s^2 |g_3|^4}{(\alpha\gamma_s |g_3|^2 + 1)^2} < 0.
\end{aligned} \tag{3.18}$$

Apparently, in light of (3.18), we know that  $R_3$  is a decreasing function of  $\alpha$ . In other words, with the power allocation factor  $\alpha$  increasing, the data rate  $R_3$  decreases. Therefore, if we want to obtain the maximum  $R_3$ , we need to let the variable  $\alpha$  be as small as possible.



And at the same time, using the constraints in (3.17), we can obtain the following conditions on  $\alpha$ , that is

$$\alpha \geq \frac{(2^{2\eta_1} - 1)[(\gamma_s |g_1|^2 + 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2) + \gamma_s |g_1|^2 + 1]}{\gamma_s |g_1|^2 [\gamma_2 |g_2|^2 + (2^{2\eta_1} - 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2)]}, \quad (3.19)$$

$$\alpha \geq \frac{(2^{2\eta_2} - 1)[(\gamma_s |g_2|^2 + 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2) + \gamma_s |g_2|^2 + 1]}{\gamma_s |g_2|^2 [\gamma_1 |g_1|^2 + (2^{2\eta_2} - 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2)]}, \quad (3.20)$$

$$0 \leq \alpha \leq 1. \quad (3.21)$$

Let us define

$$\alpha_1 \triangleq \frac{(2^{2\eta_1} - 1)[(\gamma_s |g_1|^2 + 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2) + \gamma_s |g_1|^2 + 1]}{\gamma_s |g_1|^2 [\gamma_2 |g_2|^2 + (2^{2\eta_1} - 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2)]}, \quad (3.22)$$

$$\alpha_2 \triangleq \frac{(2^{2\eta_2} - 1)[(\gamma_s |g_2|^2 + 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2) + \gamma_s |g_2|^2 + 1]}{\gamma_s |g_2|^2 [\gamma_1 |g_1|^2 + (2^{2\eta_2} - 1)(\gamma_1 |g_1|^2 + \gamma_2 |g_2|^2)]}. \quad (3.23)$$

In light of (3.21), (3.22), and (3.23), we know that if  $\alpha_1$  or  $\alpha_2$  is larger than 1, then the optimization problem is not feasible for the given values of  $\eta_1$  and  $\eta_2$ . Otherwise, if  $\alpha_1$  and  $\alpha_2$

are both smaller than 1, we can obtain

$$\alpha = \max(\alpha_1, \alpha_2) \quad (3.24)$$

to satisfy the optimization problem (3.17).

We summarize the method for obtaining the optimal power allocation factor  $\alpha$  as Algorithm

I.

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**Algorithm 1** : Obtaining the optimal power allocation factor  $\alpha$  Scheme

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- 1: Set  $p_1, p_2, p_s, h_1, h_2, h_3, d_1, d_2, d_3, \eta_1, \eta_2$ , and  $\sigma^2$ .
  - 2: Calculate  $\alpha_1$  in (3.22) and  $\alpha_2$  in (3.23).
  - 3: **if**  $0 \leq \alpha_1 \leq 1$  and  $0 \leq \alpha_2 \leq 1$  **then**
  - 4:     **if**  $\alpha_1 < \alpha_2$  **then**
  - 5:          $\alpha = \alpha_2$
  - 6:     **else**
  - 7:          $\alpha = \alpha_1$
  - 8:     **end if**
  - 9:      $\alpha$
  - 10: **else**
  - 11: **end if**
  - 12: Output the optimal power allocation factor  $\alpha_{opt} = \alpha$ .
- 

### 3.3 Simulation Results

The distance between  $S_1$  and  $S_2$  is normalized to be unity, and the normalized distance between  $S_i$  and  $R$  is denoted as  $d_i$ , for  $i = 1, 2, 3$ . The complex channel coefficient between  $S_i$  and  $R$  is denoted as  $g_i$ , where  $g_i = d_i^{-\frac{v}{2}} h_i$ , with  $v > 0$  being the path-loss exponent and  $h_i$  capturing the Rayleigh fading effects; thus we have  $h_i \sim \mathcal{CN}(0, 1)$  and  $g_i \sim \mathcal{CN}(0, d_i^{-v})$ . We

choose  $v = 4$  and  $\sigma^2 = 1$ .

For Figures 3.2 and 3.3, we choose  $p_1 = p_2 = p_s = 10$  dBW and assume that  $\eta_1 = \eta_2 = \eta$ , which changes from 0.01 b/cu to 4.21 b/cu and that the distance  $d_3$ , between relay  $R$  and the secondary user  $S_3$ , changes from 0.5 to 1.5. Moreover, we assume that the distance  $d_i$ , where  $i = 1, 2$ , between the primary user  $S_i$  and relay  $R$  are known.

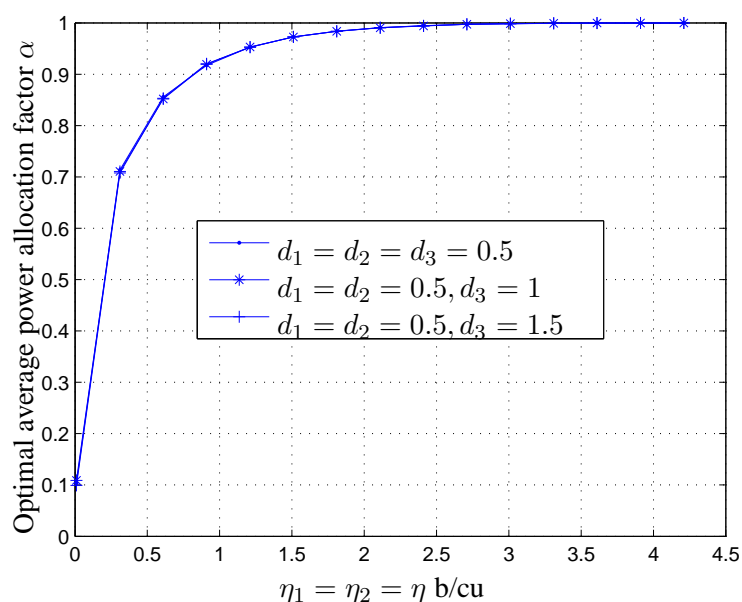


Figure 3.2: Optimal average power allocation factor  $\alpha$  versus  $\eta$  when  $p_1 = p_2 = p_s = 10$  dBW.

Figure 3.2 displays the optimal average power allocation factor  $\alpha$  versus  $\eta$  for three different values of  $d_3$ . From Figure 3.2, we observe that the optimal average power allocation factor  $\alpha$  is an increasing function of  $\eta$ . However, while  $\eta > 2.5$  b/cu, the optimal average power allocation factor  $\alpha$  become equal to 1, which means that all relay power has to be dedicated to meet the data rate constraints of primary users. Furthermore, from Figure 3.2, we observe that for three different values of  $d_3$ , the optimal average power allocation factor  $\alpha$  is identical, because  $\alpha_1$  in

(3.22) and  $\alpha_2$  in (3.23) only depend on  $g_1$  and  $g_2$  and not depend on  $g_3$  ( $d_3$ ).

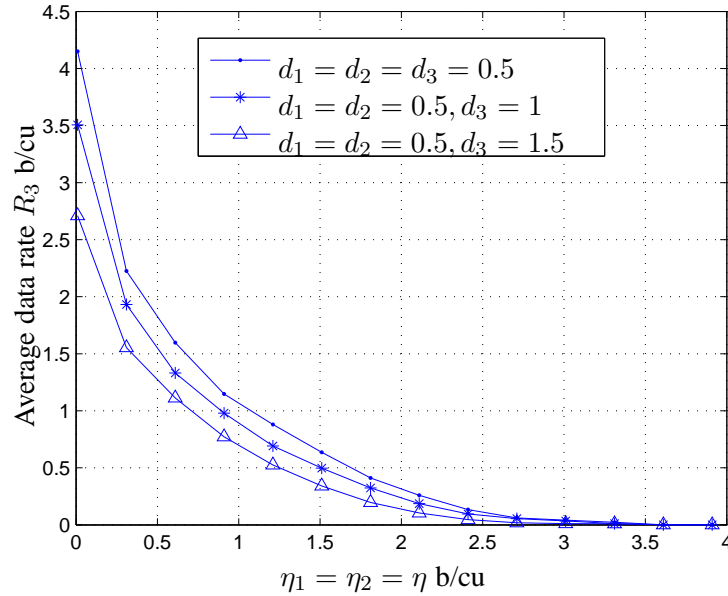


Figure 3.3: Average data rate  $R_3$  versus  $\eta$  when  $p_1 = p_2 = p_s = 10$  dBW.

Figure 3.3 displays the average data rate  $R_3$  versus  $\eta$  for three different values of  $d_3$ . From Figure 3.3, we observe that the average data rate  $R_3$  is not only a decreasing function of  $\eta$ , but also a decreasing function of  $d_3$ .  $\text{SINR}_3$  is significantly different for different values of  $g_3$  ( $d_3$ ). However, with  $\eta$  increasing, these three  $\text{SINR}_3$  curves gradually become almost identical. While  $\eta > 2.5$  b/cu, the average data rate  $R_3$  becomes equal to 0, which means that there is no relay power used to provide opportunities for secondary transmission. This is because that all relay power is allocated to meet the data rate constraints of primary users. By comparing Figures 3.2 and 3.3, the analytical result shows that the average data rate  $R_3$  is a decreasing function of  $\alpha$ .

## **3.4 Summary**

In this chapter, a two-phase spectrum sharing protocol based on analog network coding (ANC) strategy is proposed. In this proposed protocol, the relay node is allowed to share the spectrum by assisting the two transceivers in exchanging information with each other through two-way relaying scheme, and at the same time, performing secondary transmission. We optimally obtain the power allocation to the transceiver signal and the relay signal to maximize the data rate of the link between the relay node and the receiving node under the constraints that the data rate of each transceiver is not smaller than a given threshold. Analytical and simulation results show that with the power allocation factor increasing, less and less relay power is allocated to provide opportunities for secondary transmission.

## **Chapter 4**

### **Three Time-slot Distributed Beamforming**

### **Scheme for Two-way Relay Networks**

## 4.1 System Model

We consider a communication scheme consisting of two transceiver nodes  $S_1$  and  $S_2$  and two relay nodes  $R_1$  and  $R_2$ . Due to the poor quality of the channel between the two transceivers, no direct communication link can be established between them. For this reason, they exchange information with the help of the relays. Operating in the half-duplex mode, the two relays  $R_1$  and  $R_2$  enable a two-way communication between  $S_1$  and  $S_2$ ; meanwhile,  $R_1$  and  $R_2$  both have their own information which wish to exchange. We assume that the transmit powers for the two transceivers  $S_1$  and  $S_2$  are respectively denoted as  $p_1$  and  $p_2$ . Also,  $p_3$  and  $p_4$  denote the transmit powers of the two relays  $R_1$  and  $R_2$ , respectively.

All channels are assumed to be frequency flat and reciprocal. The complex channel coefficient between  $S_i$  and  $R_j$  is denoted as  $g_{ji}$  and the complex channel coefficient between  $R_1$  and  $R_2$  is denoted as  $f$ . Assuming that the perfect channel knowledge is available at the receiver side for each link. Moreover, we assume that  $S_1$  has the knowledge of  $g_{11}$  and  $g_{21}$ , that  $S_2$  has the knowledge of  $g_{12}$  and  $g_{22}$ , and that  $R_1$  and  $R_2$  have the knowledge of  $f$ . Finally, the additive white Gaussian noises at  $R_1$  and  $R_2$  are respectively denoted as  $n_{R1}$  and  $n_{R2}$ , and these are modeled as independent and identically distributed Gaussian random variables with zero mean and variance equal to  $\sigma^2$ .

As shown in Figure 4.1, the proposed scheme consists of three phases. In phase 1, the two transceivers  $S_1$  and  $S_2$  transmit their respective signals to relays  $R_1$  and  $R_2$  simultaneously. Therefore, the signals received at relays  $R_1$  and  $R_2$  can be mathematically expressed in vector

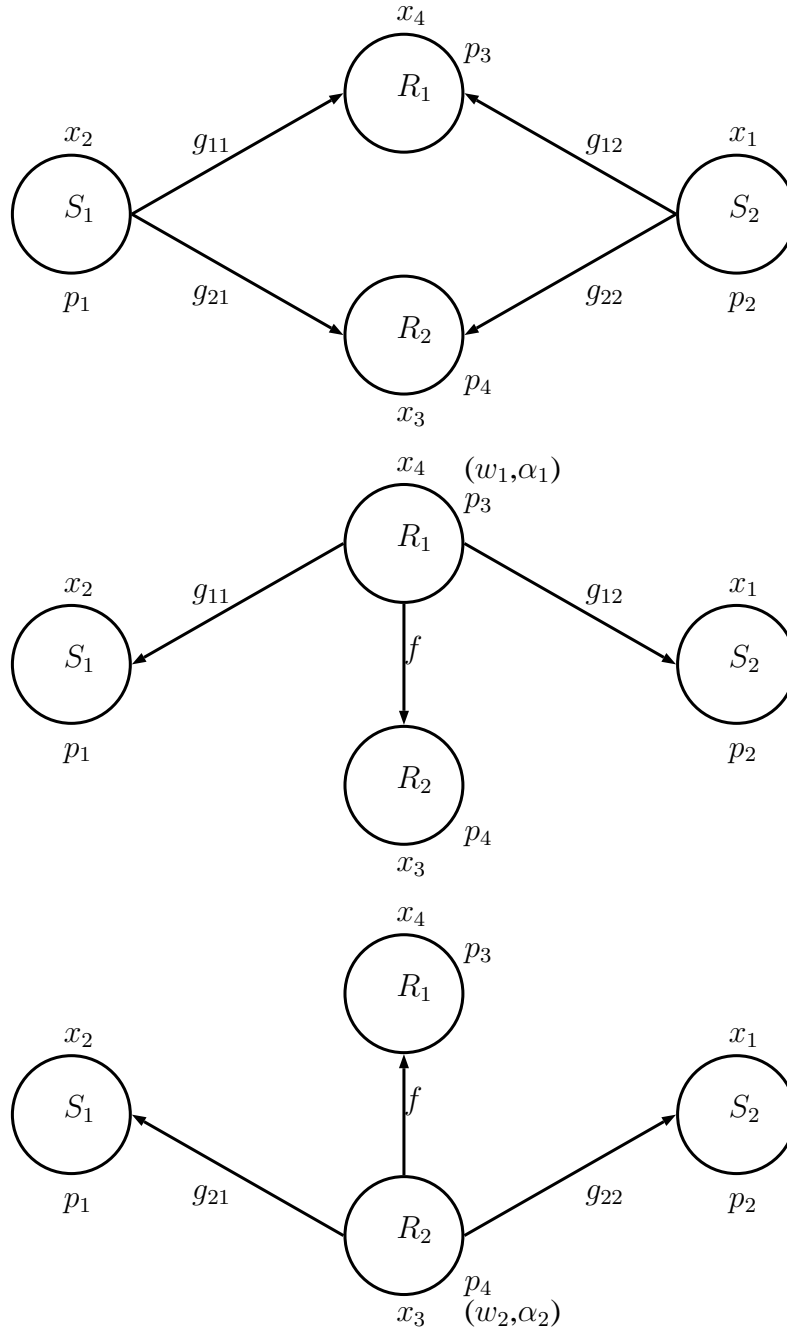


Figure 4.1: System model for the proposed three-phase two-way relaying scheme.



form as

$$\begin{aligned}
\mathbf{y}_R &= \begin{bmatrix} y_{R1} \\ y_{R2} \end{bmatrix} \\
&= \begin{bmatrix} \sqrt{p_1}g_{11}x_2 + \sqrt{p_2}g_{12}x_1 + n_{R1} \\ \sqrt{p_1}g_{21}x_2 + \sqrt{p_2}g_{22}x_1 + n_{R2} \end{bmatrix} \\
&= \sqrt{p_1}\mathbf{g}_1x_2 + \sqrt{p_2}\mathbf{g}_2x_1 + \mathbf{n}_R
\end{aligned} \tag{4.1}$$

where  $\mathbf{y}_R$  is the  $2 \times 1$  complex vector of the signals received at relays,  $x_1$  and  $x_2$  are the information symbols transmitted by the two transceivers  $S_2$  and  $S_1$ , respectively,  $\mathbf{n}_R \triangleq [n_{R1} \ n_{R2}]^T$  is the  $2 \times 1$  complex vector of the relay noises with distribution  $\mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$ , and  $\mathbf{g}_1 \triangleq [g_{11} \ g_{21}]^T$  and  $\mathbf{g}_2 \triangleq [g_{12} \ g_{22}]^T$  are the vectors of the complex channel coefficients between the relays and the transceivers. As mentioned earlier, each transceiver knows both complex channel vectors  $\mathbf{g}_1$  and  $\mathbf{g}_2$ .

In phase 2, the relay node  $R_1$  multiplies its received signal, i.e., the first entry of  $\mathbf{y}_R$  in (4.1) is multiplied by a complex beamforming weight  $w_1^*$  and is added to  $\alpha_1x_3$ . Here,  $x_3$  is the unit-power information-bearing symbol intended for the relay node  $R_2$  and  $\alpha_1$  is a coefficient that is to be optimally determined. Similarly, in phase 3, the relay node  $R_2$  multiplies its received signal, i.e., the second entry of  $\mathbf{y}_R$  in (4.1) is multiplied by a complex beamforming weight  $w_2^*$  and is added to  $\alpha_2x_4$ . Here,  $x_4$  is the unit-power information-bearing symbol intended for the relay node  $R_1$  and  $\alpha_2$  is a coefficient that is to be optimally determined. Consequently, the composite

signal broadcasted by  $R_1$  in phase 2 and by  $R_2$  in phase 3 can be expressed in vector form as

$$\begin{aligned}
 \mathbf{x}_R &= \begin{bmatrix} x_{R1} \\ x_{R2} \end{bmatrix} \\
 &= \begin{bmatrix} w_1^* y_{R1} + \alpha_1^* x_3 \\ w_2^* y_{R2} + \alpha_2^* x_4 \end{bmatrix} \\
 &= \mathbf{W} \mathbf{y}_R + \mathbf{A} \mathbf{x}
 \end{aligned} \tag{4.2}$$

where  $\mathbf{W} \triangleq \text{diag}([w_1^* \ w_2^*])$ ,  $\mathbf{A} \triangleq \text{diag}([\alpha_1^* \ \alpha_2^*])$ , and  $\mathbf{x} \triangleq [x_3 \ x_4]^T$ .

Adding the two received signals at the transceiver  $S_1$ , we obtain optimal combining of the two received signals appears to be very difficult to consider in our solution:

$$\begin{aligned}
 y_1 &= g_{11} x_{R1} + n_{11} + g_{21} x_{R2} + n_{21} \\
 &= \mathbf{g}_1^T \mathbf{x}_R + n_{11} + n_{21} \\
 &= \mathbf{g}_1^T (\mathbf{W} \mathbf{y}_R + \mathbf{A} \mathbf{x}) + n_{11} + n_{21} \\
 &= \mathbf{g}_1^T \mathbf{W} (\sqrt{p_1} \mathbf{g}_1 x_2 + \sqrt{p_2} \mathbf{g}_2 x_1 + \mathbf{n}_R) + \mathbf{g}_1^T \mathbf{A} \mathbf{x} + n_{11} + n_{21} \\
 &= \underbrace{\sqrt{p_2} \mathbf{g}_1^T \mathbf{W} \mathbf{g}_2 x_1}_{\text{desired signal}} + \underbrace{\sqrt{p_1} \mathbf{g}_1^T \mathbf{W} \mathbf{g}_1 x_2}_{\text{self-interference}} + \underbrace{\mathbf{g}_1^T \mathbf{A} \mathbf{x}}_{\text{interference}} + \underbrace{\mathbf{g}_1^T \mathbf{W} \mathbf{n}_R + n_{11} + n_{21}}_{\text{noise}},
 \end{aligned} \tag{4.3}$$

where  $n_{11}$  is the noise at  $S_1$  in phase 2 with distribution  $\mathcal{CN}(0, \sigma^2)$  and  $n_{21}$  is the noise at  $S_1$  in phase 3 with distribution  $\mathcal{CN}(0, \sigma^2)$ .

Similarly, adding the two received signals at the transceiver  $S_2$ , we obtain optimal combining of the two received signals appears to be very difficult to consider in our solution:

$$\begin{aligned}
y_2 &= g_{12}x_{R1} + n_{12} + g_{22}x_{R2} + n_{22} \\
&= \mathbf{g}_2^T \mathbf{x}_R + n_{12} + n_{22} \\
&= \mathbf{g}_2^T (\mathbf{W} \mathbf{y}_R + \mathbf{A} \mathbf{x}) + n_{12} + n_{22} \\
&= \mathbf{g}_2^T \mathbf{W} (\sqrt{p_1} \mathbf{g}_1 x_2 + \sqrt{p_2} \mathbf{g}_2 x_1 + \mathbf{n}_R) + \mathbf{g}_2^T \mathbf{A} \mathbf{x} + n_{12} + n_{22} \\
&= \underbrace{\sqrt{p_1} \mathbf{g}_2^T \mathbf{W} \mathbf{g}_1 x_2}_{\text{desired signal}} + \underbrace{\sqrt{p_2} \mathbf{g}_2^T \mathbf{W} \mathbf{g}_2 x_1}_{\text{self-interference}} + \underbrace{\mathbf{g}_2^T \mathbf{A} \mathbf{x}}_{\text{interference}} + \underbrace{\mathbf{g}_2^T \mathbf{W} \mathbf{n}_R + n_{12} + n_{22}}_{\text{noise}}, \tag{4.4}
\end{aligned}$$

where  $n_{12}$  is the noise at  $S_1$  in phase 2 with distribution  $\mathcal{CN}(0, \sigma^2)$  and  $n_{22}$  is the noise at  $S_1$  in phase 3 with distribution  $\mathcal{CN}(0, \sigma^2)$ .

Using the fact that  $\mathbf{a}^T \text{diag}(\mathbf{b}) = \mathbf{b}^T \text{diag}(\mathbf{a})$ , (4.3) and (4.4) can be respectively rewritten as

$$y_1 = \underbrace{\sqrt{p_2} \mathbf{w}^H \mathbf{G}_1 \mathbf{g}_2 x_1}_{\text{desired signal}} + \underbrace{\sqrt{p_1} \mathbf{w}^H \mathbf{G}_1 \mathbf{g}_1 x_2}_{\text{self-interference}} + \underbrace{\boldsymbol{\alpha}^H \mathbf{G}_1 \mathbf{x}}_{\text{interference}} + \underbrace{\mathbf{w}^H \mathbf{G}_1 \mathbf{n}_R + n_{11} + n_{21}}_{\text{noise}}, \tag{4.5}$$

$$y_2 = \underbrace{\sqrt{p_1} \mathbf{w}^H \mathbf{G}_2 \mathbf{g}_1 x_2}_{\text{desired signal}} + \underbrace{\sqrt{p_2} \mathbf{w}^H \mathbf{G}_2 \mathbf{g}_2 x_1}_{\text{self-interference}} + \underbrace{\boldsymbol{\alpha}^H \mathbf{G}_2 \mathbf{x}}_{\text{interference}} + \underbrace{\mathbf{w}^H \mathbf{G}_2 \mathbf{n}_R + n_{12} + n_{22}}_{\text{noise}}, \tag{4.6}$$

where  $\mathbf{G}_i \triangleq \text{diag}(\mathbf{g}_i)$ ,  $\mathbf{w} \triangleq \text{diag}(\mathbf{W}^H) \triangleq [w_1^* \ w_2^*]^H$ , and  $\boldsymbol{\alpha} \triangleq \text{diag}(\mathbf{A}^H) \triangleq [\alpha_1^* \ \alpha_2^*]^H$ .

Since  $x_2$  is the symbol transmitted by  $S_1$  in phase 1, and the second term in (4.5) containing

$x_2$  is perfectly known to  $S_1$ , the second term in (4.5) can be subtracted from  $y_1$ . As a result, the residual signal  $\tilde{y}_1$  is defined as

$$\begin{aligned}\tilde{y}_1 &= y_1 - \sqrt{p_1} \mathbf{w}^H \mathbf{G}_1 \mathbf{g}_1 x_2 \\ &= \underbrace{\sqrt{p_2} \mathbf{w}^H \mathbf{G}_1 \mathbf{g}_2 x_1}_{\text{desired signal}} + \underbrace{\boldsymbol{\alpha}^H \mathbf{G}_1 \mathbf{x}}_{\text{interference}} + \underbrace{\mathbf{w}^H \mathbf{G}_1 \mathbf{n}_R + n_{11} + n_{21}}_{\text{noise}}.\end{aligned}\quad (4.7)$$

After removing the self-interference from  $y_1$ , the secondary signal vector  $\mathbf{x}$  is treated as the interference, and  $x_1$  is decoded directly. Consequently, the SINR achieved at  $S_1$  to decode  $x_1$  is given by

$$\text{SINR}_1 = \frac{p_2 \mathbf{w}^H \mathbf{G}_1 \mathbf{g}_2 \mathbf{g}_2^H \mathbf{G}_1^H \mathbf{w}}{\boldsymbol{\alpha}^H \mathbf{G}_1 \mathbf{G}_1^H \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{G}_1 \mathbf{G}_1^H \mathbf{w} + 2\sigma^2}, \quad (4.8)$$

where  $E\{\mathbf{n}_R \mathbf{n}_R^H\} = \sigma^2 \mathbf{I}$ , and we have used the assumption that  $E[|x_1|^2] = 1$ .

Similarly, since  $x_1$  is the symbol transmitted by  $S_2$  in phase 1, and the second term in (4.6) containing  $x_1$  is perfectly known to  $S_2$ , the second term in (4.6) can be subtracted from  $y_2$ . As a result, the residual signal  $\tilde{y}_2$  is defined as

$$\begin{aligned}\tilde{y}_2 &= y_2 - \sqrt{p_2} \mathbf{w}^H \mathbf{G}_2 \mathbf{g}_2 x_1 \\ &= \underbrace{\sqrt{p_1} \mathbf{w}^H \mathbf{G}_2 \mathbf{g}_1 x_2}_{\text{desired signal}} + \underbrace{\boldsymbol{\alpha}^H \mathbf{G}_2 \mathbf{x}}_{\text{interference}} + \underbrace{\mathbf{w}^H \mathbf{G}_2 \mathbf{n}_R + n_{12} + n_{22}}_{\text{noise}}.\end{aligned}\quad (4.9)$$

After removing the self-interference from  $y_2$ , the secondary signal vector  $\mathbf{x}$  is treated as the interference, and  $x_2$  is decoded directly. Consequently, the SINR achieved at  $S_2$  to decode  $x_2$  is

given by

$$\text{SINR}_2 = \frac{p_1 \mathbf{w}^H \mathbf{G}_2 \mathbf{g}_1 \mathbf{g}_1^H \mathbf{G}_2^H \mathbf{w}}{\boldsymbol{\alpha}^H \mathbf{G}_2 \mathbf{G}_2^H \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{G}_2 \mathbf{G}_2^H \mathbf{w} + 2\sigma^2}, \quad (4.10)$$

where we have used the assumption that  $E[|x_2|^2] = 1$ .

Let  $\odot$  denotes the Schur-Hadamard matrix product. Assuming  $\mathbf{D}_1 \triangleq \mathbf{G}_1 \mathbf{G}_1^H$ ,  $\mathbf{D}_2 \triangleq \mathbf{G}_2 \mathbf{G}_2^H$ , and  $\mathbf{h} \triangleq \mathbf{G}_1 \mathbf{g}_2 = \mathbf{G}_2 \mathbf{g}_1 = \mathbf{g}_1 \odot \mathbf{g}_2$ , the received SINRs in (4.8) and (4.10) can be respectively rewritten as

$$\text{SINR}_1 = \frac{p_2 \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w}}{\boldsymbol{\alpha}^H \mathbf{D}_1 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 2\sigma^2}, \quad (4.11)$$

$$\text{SINR}_2 = \frac{p_1 \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w}}{\boldsymbol{\alpha}^H \mathbf{D}_2 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 2\sigma^2}. \quad (4.12)$$

In light of (4.11), the data rate achieved at  $S_1$  to decode  $x_1$  is given by

$$R_1 = \frac{1}{3} \log_2(1 + \text{SINR}_1), \quad (4.13)$$

where the factor  $\frac{1}{3}$  is used because three time slots are utilized in the overall transmission.

Similarly, in light of (4.12), the data rate achieved at  $S_2$  to decode  $x_2$  can be mathematically expressed as

$$R_2 = \frac{1}{3} \log_2(1 + \text{SINR}_2). \quad (4.14)$$

The signal  $y_3$  received at  $R_2$  in phase 2 can be written as

$$\begin{aligned}
y_3 &= f x_{R1} + n_{R2} \\
&= f(w_1^* y_{R1} + \alpha_1^* x_3) + n_{R2} \\
&= f w_1^* (\sqrt{p_1} g_{11} x_2 + \sqrt{p_2} g_{12} x_1 + n_{R1}) + f \alpha_1^* x_3 + n_{R2} \\
&= \underbrace{f \alpha_1^* x_3}_{\text{desired signal}} + \underbrace{\sqrt{p_2} f w_1^* g_{12} x_1 + \sqrt{p_1} f w_1^* g_{11} x_2}_{\text{interference}} + \underbrace{f w_1^* n_{R1} + n_{R2}}_{\text{noise}}. \tag{4.15}
\end{aligned}$$

Unlike the two transceivers  $S_1$  and  $S_2$ ,  $R_2$  has no knowledge about  $x_1$  or  $x_2$ ; therefore, the SINR achieved at  $R_2$  to decode its desired signal  $x_3$ , under the interference from the two primary signals  $x_1$  and  $x_2$ , is given by

$$\text{SINR}_3 = \frac{|f|^2 |\alpha_1|^2}{p_2 |f|^2 |w_1|^2 |g_{12}|^2 + p_1 |f|^2 |w_1|^2 |g_{11}|^2 + \sigma^2 |f|^2 |w_1|^2 + \sigma^2}, \tag{4.16}$$

where we have used the assumptions that  $E[|x_3|^2] = 1$ .

Hence, in light of (4.16), the data rate achieved at  $R_2$  to decode  $x_3$  can be written as

$$R_3 = \frac{1}{3} \log_2(1 + \text{SINR}_3), \tag{4.17}$$

where the factor  $\frac{1}{3}$  is used because the secondary transmission occurs only in one out of three time slots.

The signal  $y_4$  received  $R_1$  in phase 3 can be written as

$$\begin{aligned}
y_4 &= f x_{R2} + n_{R1} \\
&= f(w_2^* y_{R2} + \alpha_2^* x_4) + n_{R1} \\
&= f w_2^* (\sqrt{p_1} g_{21} x_2 + \sqrt{p_2} g_{22} x_1 + n_{R2}) + f \alpha_2^* x_4 + n_{R1} \\
&= \underbrace{f \alpha_2^* x_4}_{\text{desired signal}} + \underbrace{\sqrt{p_2} f w_2^* g_{22} x_1 + \sqrt{p_1} f w_2^* g_{21} x_2}_{\text{interference}} + \underbrace{f w_2^* n_{R2} + n_{R1}}_{\text{noise}}. \tag{4.18}
\end{aligned}$$

Unlike the two transceivers  $S_1$  and  $S_2$ ,  $R_1$  has no knowledge about  $x_1$  or  $x_2$ ; therefore, the SINR achieved at  $R_1$  to decode its desired signal  $x_4$ , under the interference from the two primary signals  $x_1$  and  $x_2$ , is given by

$$\text{SINR}_4 = \frac{|f|^2 |\alpha_2|^2}{p_2 |f|^2 |w_2|^2 |g_{22}|^2 + p_1 |f|^2 |w_2|^2 |g_{21}|^2 + \sigma^2 |f|^2 |w_2|^2 + \sigma^2}, \tag{4.19}$$

where we have used the assumptions that  $E[|x_4|^2] = 1$ .

Therefore, the data rate achieved at  $R_1$  to decode  $x_4$  can be expressed, by using (4.19), as

$$R_4 = \frac{1}{3} \log_2(1 + \text{SINR}_4). \tag{4.20}$$

## 4.2 Relay Data Rate Maximization

In this chapter, we aim to optimally obtain the beamforming weights  $\{w_i\}_{i=1}^2$  and the coefficients  $\{\alpha_i\}_{i=1}^2$  such that the smaller of  $R_3$  and  $R_4$  is maximized when the transmit powers  $p_1$  and  $p_2$  of primary users  $S_1$  and  $S_2$  and the relay powers  $p_3$  and  $p_4$  are given under the constraints that  $R_1$  and  $R_2$  are not smaller than two given thresholds  $\eta_1$  and  $\eta_2$ , respectively. Mathematically, we solve the following optimization problem:

$$\begin{aligned}
 & \max_{\alpha, \mathbf{w}} && \min (R_3, R_4) \\
 \text{subject to} & && R_1 \geq \eta_1 \\
 & && R_2 \geq \eta_2 \\
 & && P_{R1} \leq p_3 \\
 & && P_{R2} \leq p_4
 \end{aligned} \tag{4.21}$$

And at the same time, we assume that the information symbols  $x_1, x_2, x_3$ , and  $x_4$  and the relay noises  $n_{R1}$  and  $n_{R2}$  are all zero-mean mutually independent random variables. Using (4.1)



and (4.2), the transmit power  $P_{R1}$  of relay  $R_1$  can be expressed as

$$\begin{aligned}
P_{R1} &\triangleq E\{|x_{R1}|^2\} \\
&= E\{|w_1|^2|y_{R1}|^2 + 2w_1y_{R1}\alpha_1x_3 + |\alpha_1|^2|x_3|^2\} \\
&= |w_1|^2E\{|y_{R1}|^2\} + 2w_1\alpha_1E\{y_{R1}x_3\} + |\alpha_1|^2 \\
&= |w_1|^2E\{p_1|g_{11}|^2|x_2|^2 + p_2|g_{12}|^2|x_1|^2 + |n_{R1}|^2\} + \\
&\quad 2w_1\alpha_1E\{(\sqrt{p_1}g_{11}x_2 + \sqrt{p_2}g_{12}x_1 + n_{R1})x_3\} + |\alpha_1|^2 \\
&= |w_1|^2(p_1|g_{11}|^2 + p_2|g_{12}|^2 + \sigma^2) + \\
&\quad 2w_1\alpha_1E\{\sqrt{p_1}g_{11}x_2 + \sqrt{p_2}g_{12}x_1 + n_{R1}\}E\{x_3\} + |\alpha_1|^2
\end{aligned}$$

where in the fifth equality, we have used the assumptions that the information symbol  $x_3$  is statically independent for the information symbols  $x_1$  and  $x_2$  and the relay noise  $n_{R1}$ , and at the same time, we have  $E\{x_3\} = 0$ . Therefore, we can write

$$P_{R1} = |w_1|^2(p_1|g_{11}|^2 + p_2|g_{12}|^2 + \sigma^2) + |\alpha_1|^2. \quad (4.22)$$

Similarly, using (4.1) and (4.2), the transmit power  $P_{R2}$  of relay  $R_2$  can be written as

$$P_{R2} \triangleq E\{|x_{R2}|^2\} = |w_2|^2(p_1|g_{21}|^2 + p_2|g_{22}|^2 + \sigma^2) + |\alpha_2|^2. \quad (4.23)$$

Substituting (4.13), (4.14), (4.17), (4.20), (4.22) and (4.23) into (4.21), the optimization

problem in (4.21) can be equivalently written as

$$\begin{aligned}
& \max_{\boldsymbol{\alpha}, \mathbf{w}} && \min (\text{SINR}_3, \text{SINR}_4) \\
\text{subject to} &&& \text{SINR}_1 \geq 2^{3\eta_1} - 1 \\
&&& \text{SINR}_2 \geq 2^{3\eta_2} - 1 \\
&&& |w_1|^2(p_1|g_{11}|^2 + p_2|g_{12}|^2 + \sigma^2) + |\alpha_1|^2 \leq p_3 \\
&&& |w_2|^2(p_1|g_{21}|^2 + p_2|g_{22}|^2 + \sigma^2) + |\alpha_2|^2 \leq p_4
\end{aligned} \tag{4.24}$$

Using (4.11) and (4.12), the optimization in (4.24) can be expressed as

$$\begin{aligned}
& \max_{\boldsymbol{\alpha}, \mathbf{w}} && \min (\text{SINR}_3, \text{SINR}_4) \\
\text{subject to} &&& \frac{p_2 \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w}}{\boldsymbol{\alpha}^H \mathbf{D}_1 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 2\sigma^2} \geq 2^{3\eta_1} - 1 \\
&&& \frac{p_1 \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w}}{\boldsymbol{\alpha}^H \mathbf{D}_2 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 2\sigma^2} \geq 2^{3\eta_2} - 1 \\
&&& |w_1|^2(p_1|g_{11}|^2 + p_2|g_{12}|^2 + \sigma^2) + |\alpha_1|^2 \leq p_3 \\
&&& |w_2|^2(p_1|g_{21}|^2 + p_2|g_{22}|^2 + \sigma^2) + |\alpha_2|^2 \leq p_4
\end{aligned} \tag{4.25}$$

or, equivalently, as

$$\begin{aligned}
& \max_{\boldsymbol{\alpha}, \mathbf{w}} && \min (\text{SINR}_3, \text{SINR}_4) \\
\text{subject to} &&& \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_1} - 1)(\boldsymbol{\alpha}^H \mathbf{D}_1 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 2\sigma^2)}{p_2} \\
&&& \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_2} - 1)(\boldsymbol{\alpha}^H \mathbf{D}_2 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 2\sigma^2)}{p_1} \\
&&& |w_1|^2(p_1 |g_{11}|^2 + p_2 |g_{12}|^2 + \sigma^2) + |\alpha_1|^2 \leq p_3 \\
&&& |w_2|^2(p_1 |g_{21}|^2 + p_2 |g_{22}|^2 + \sigma^2) + |\alpha_2|^2 \leq p_4
\end{aligned} \tag{4.26}$$

Now, we define

$$t \triangleq \min (\text{SINR}_3, \text{SINR}_4) \text{ with } t > 0.$$

Note that without loss of optimality, we can assume that

$$\text{SINR}_3 = t,$$

$$\text{SINR}_4 = t.$$

Otherwise, if one of them is satisfied with inequality, let us say

$$\text{SINR}_4 > t \tag{4.27}$$

holds true at the optimum, then substituting (4.19) into (4.27), hence (4.27) can be equivalently

expressed as

$$\frac{|f|^2|\alpha_2|^2}{p_2|f|^2|w_2|^2|g_{22}|^2 + p_1|f|^2|w_2|^2|g_{21}|^2 + \sigma^2|f|^2|w_2|^2 + \sigma^2} > t. \quad (4.28)$$

In light of (4.28), we can obtain the following conditions on the optimal value of  $|\alpha_2|^2$ , namely

$$|\alpha_2|^2 > \frac{tp_2|f|^2|w_2|^2|g_{22}|^2 + tp_1|f|^2|w_2|^2|g_{21}|^2 + t\sigma^2|f|^2|w_2|^2 + t\sigma^2}{|f|^2}. \quad (4.29)$$

And at the same time, we can obtain the following conditions by using the constraints in (4.26), which are expressed as

$$\mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_1} - 1)(\boldsymbol{\alpha}^H \mathbf{D}_1 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 2\sigma^2)}{p_2}, \quad (4.30)$$

$$\mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_2} - 1)(\boldsymbol{\alpha}^H \mathbf{D}_2 \boldsymbol{\alpha} + \sigma^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 2\sigma^2)}{p_1}, \quad (4.31)$$

$$|w_2|^2(p_1|g_{21}|^2 + p_2|g_{22}|^2 + \sigma^2) + |\alpha_2|^2 \leq p_4. \quad (4.32)$$

If the optimal value of  $|\alpha_2|^2$  is reduced such that it becomes equal to the right-hand side of (4.29), the right-hand sides of (4.30) and (4.31) are also reduced, and hence, the new value of  $|\alpha_2|^2$  being equal to the right-hand side of (4.29) is feasible, because it does not violate the con-

straints in (4.30) and (4.31). Additionally, the left-hand side of (4.32) is an increasing function of  $|\alpha_2|^2$ . In other words, with  $|\alpha_2|^2$  decreasing, the left-hand side of (4.32) reduces such that the new value of the left-hand side of (4.32) still remains smaller than  $p_4$ ; hence, the new value of  $|\alpha_2|^2$  being equal to the right-hand side of (4.29) does not violate the constraint in (4.32). For this reason, this new value of  $|\alpha_2|^2$  relaxes these three constraints in (4.30), (4.31), and (4.32). Hence, without loss of optimality, we can say that

$$\text{SINR}_4 = t. \quad (4.33)$$

Therefore, based on the fact that at the optimum,  $\text{SINR}_3 = \text{SINR}_4 = t$  holds true, the optimization problem in (4.26) can be mathematically expressed in the following form:

$$\begin{aligned} & \max_{\alpha, \mathbf{w}, t} && t \\ & \text{subject to} && \text{SINR}_3 = t \\ & && \text{SINR}_4 = t \\ & && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_1} - 1)(\alpha^H \mathbf{D}_1 \alpha + \sigma^2 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 2\sigma^2)}{p_2} \\ & && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_2} - 1)(\alpha^H \mathbf{D}_2 \alpha + \sigma^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 2\sigma^2)}{p_1} \\ & && |w_1|^2(p_1 |g_{11}|^2 + p_2 |g_{12}|^2 + \sigma^2) + |\alpha_1|^2 \leq p_3 \\ & && |w_2|^2(p_1 |g_{21}|^2 + p_2 |g_{22}|^2 + \sigma^2) + |\alpha_2|^2 \leq p_4 \\ & && t > 0 \end{aligned} \quad (4.34)$$

Substituting (4.16) and (4.19) into (4.34), then the optimization problem in (4.34) can be

equivalently written as

$$\begin{aligned}
& \max_{\alpha, \mathbf{w}, t} && t \\
\text{subject to} &&& \frac{|f|^2 |\alpha_1|^2}{p_2 |f|^2 |w_1|^2 |g_{12}|^2 + p_1 |f|^2 |w_1|^2 |g_{11}|^2 + \sigma^2 |f|^2 |w_1|^2 + \sigma^2} = t \\
&&& \frac{|f|^2 |\alpha_2|^2}{p_2 |f|^2 |w_2|^2 |g_{22}|^2 + p_1 |f|^2 |w_2|^2 |g_{21}|^2 + \sigma^2 |f|^2 |w_2|^2 + \sigma^2} = t \\
&&& \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_1} - 1)(\alpha^H \mathbf{D}_1 \alpha + \sigma^2 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 2\sigma^2)}{p_2} \\
&&& \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \frac{(2^{3\eta_2} - 1)(\alpha^H \mathbf{D}_2 \alpha + \sigma^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 2\sigma^2)}{p_1} \\
&&& |w_1|^2 (p_1 |g_{11}|^2 + p_2 |g_{12}|^2 + \sigma^2) + |\alpha_1|^2 \leq p_3 \\
&&& |w_2|^2 (p_1 |g_{21}|^2 + p_2 |g_{22}|^2 + \sigma^2) + |\alpha_2|^2 \leq p_4 \\
&&& t > 0
\end{aligned} \tag{4.35}$$

Using the constraints in (4.35), we can obtain the following conditions on  $|\alpha_1|^2$  and  $|\alpha_2|^2$ :

$$\begin{aligned}
|\alpha_1|^2 &= \frac{tp_2 |f|^2 |w_1|^2 |g_{12}|^2 + tp_1 |f|^2 |w_1|^2 |g_{11}|^2 + t\sigma^2 |f|^2 |w_1|^2 + t\sigma^2}{|f|^2} \\
&= tp_2 |w_1|^2 |g_{12}|^2 + tp_1 |w_1|^2 |g_{11}|^2 + t\sigma^2 |w_1|^2 + \frac{t\sigma^2}{|f|^2} \\
&= t |w_1|^2 (p_2 |g_{12}|^2 + p_1 |g_{11}|^2 + \sigma^2) + \frac{t\sigma^2}{|f|^2},
\end{aligned} \tag{4.36}$$

$$\begin{aligned}
|\alpha_2|^2 &= \frac{tp_2|f|^2|w_2|^2|g_{22}|^2 + tp_1|f|^2|w_2|^2|g_{21}|^2 + t\sigma^2|f|^2|w_2|^2 + t\sigma^2}{|f|^2} \\
&= tp_2|w_2|^2|g_{22}|^2 + tp_1|w_2|^2|g_{21}|^2 + t\sigma^2|w_2|^2 + \frac{t\sigma^2}{|f|^2} \\
&= t|w_2|^2(p_2|g_{22}|^2 + p_1|g_{21}|^2 + \sigma^2) + \frac{t\sigma^2}{|f|^2}.
\end{aligned} \tag{4.37}$$

Using the following definitions:

$$\mathbf{M}_1 \triangleq \begin{bmatrix} p_2|g_{12}|^2 + p_1|g_{11}|^2 + \sigma^2 & 0 \\ 0 & 0 \end{bmatrix} \tag{4.38}$$

$$\mathbf{M}_2 \triangleq \begin{bmatrix} 0 & 0 \\ 0 & p_2|g_{22}|^2 + p_1|g_{21}|^2 + \sigma^2 \end{bmatrix} \tag{4.39}$$

therefore,  $|\alpha_1|^2$  and  $|\alpha_2|^2$  in (4.36) and (4.37) can be equivalently written as

$$|\alpha_1|^2 = t\mathbf{w}^H \mathbf{M}_1 \mathbf{w} + \frac{t\sigma^2}{|f|^2}, \tag{4.40}$$

$$|\alpha_2|^2 = t\mathbf{w}^H \mathbf{M}_2 \mathbf{w} + \frac{t\sigma^2}{|f|^2}. \tag{4.41}$$

In light of (4.40) and (4.41), we know that both  $|\alpha_1|^2$  and  $|\alpha_2|^2$  can be expressed in terms of

w. Based on this fact,  $\alpha^H \mathbf{D}_1 \alpha$  and  $\alpha^H \mathbf{D}_2 \alpha$  can also be expressed in terms of  $\mathbf{w}$ , that is

$$\begin{aligned}
\alpha^H \mathbf{D}_1 \alpha &= \alpha^H \mathbf{G}_1 \mathbf{G}_1^H \alpha \\
&= |g_{11}|^2 |\alpha_1|^2 + |g_{21}|^2 |\alpha_2|^2 \\
&= |g_{11}|^2 (t \mathbf{w}^H \mathbf{M}_1 \mathbf{w} + \frac{t \sigma^2}{|f|^2}) + |g_{21}|^2 (t \mathbf{w}^H \mathbf{M}_2 \mathbf{w} + \frac{t \sigma^2}{|f|^2}) \\
&= \mathbf{w}^H (t |g_{11}|^2 \mathbf{M}_1 + t |g_{21}|^2 \mathbf{M}_2) \mathbf{w} + (\frac{t |g_{11}|^2}{|f|^2} + \frac{t |g_{21}|^2}{|f|^2}) \sigma^2, \tag{4.42}
\end{aligned}$$

$$\begin{aligned}
\alpha^H \mathbf{D}_2 \alpha &= \alpha^H \mathbf{G}_2 \mathbf{G}_2^H \alpha \\
&= |g_{12}|^2 |\alpha_1|^2 + |g_{22}|^2 |\alpha_2|^2 \\
&= |g_{12}|^2 (t \mathbf{w}^H \mathbf{M}_1 \mathbf{w} + \frac{t \sigma^2}{|f|^2}) + |g_{22}|^2 (t \mathbf{w}^H \mathbf{M}_2 \mathbf{w} + \frac{t \sigma^2}{|f|^2}) \\
&= \mathbf{w}^H (t |g_{12}|^2 \mathbf{M}_1 + t |g_{22}|^2 \mathbf{M}_2) \mathbf{w} + (\frac{t |g_{12}|^2}{|f|^2} + \frac{t |g_{22}|^2}{|f|^2}) \sigma^2. \tag{4.43}
\end{aligned}$$

And at the same time, we can obtain

$$|w_1|^2 (p_1 |g_{11}|^2 + p_2 |g_{12}|^2 + \sigma^2) = \mathbf{w}^H \mathbf{M}_1 \mathbf{w} \tag{4.44}$$

$$|w_2|^2 (p_1 |g_{21}|^2 + p_2 |g_{22}|^2 + \sigma^2) = \mathbf{w}^H \mathbf{M}_2 \mathbf{w}. \tag{4.45}$$



Substituting (4.40), (4.41), (4.42), (4.43), (4.44), and (4.45) into (4.35), the optimization problem in (4.35) can be mathematically expressed as

$$\begin{aligned}
& \max_{\mathbf{w}, t} && t \\
\text{subject to} & && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \\
& && \frac{(2^{3\eta_1} - 1)[\mathbf{w}^H (t|g_{11}|^2 \mathbf{M}_1 + t|g_{21}|^2 \mathbf{M}_2) \mathbf{w} + (\frac{t|g_{11}|^2}{|f|^2} + \frac{t|g_{21}|^2}{|f|^2})\sigma^2 + \sigma^2 \mathbf{w}^H \mathbf{D}_1 \mathbf{w} + 2\sigma^2]}{p_2} \\
& && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \\
& && \frac{(2^{3\eta_2} - 1)[\mathbf{w}^H (t|g_{12}|^2 \mathbf{M}_1 + t|g_{22}|^2 \mathbf{M}_2) \mathbf{w} + (\frac{t|g_{12}|^2}{|f|^2} + \frac{t|g_{22}|^2}{|f|^2})\sigma^2 + \sigma^2 \mathbf{w}^H \mathbf{D}_2 \mathbf{w} + 2\sigma^2]}{p_1} \\
& && \mathbf{w}^H \mathbf{M}_1 \mathbf{w} + t \mathbf{w}^H \mathbf{M}_1 \mathbf{w} + \frac{t\sigma^2}{|f|^2} \leq p_3 \\
& && \mathbf{w}^H \mathbf{M}_2 \mathbf{w} + t \mathbf{w}^H \mathbf{M}_2 \mathbf{w} + \frac{t\sigma^2}{|f|^2} \leq p_4 \\
& && t > 0
\end{aligned} \tag{4.46}$$

or, equivalently, as

$$\begin{aligned}
& \max_{\mathbf{w}, t} && t \\
\text{subject to} & && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \\
& && \frac{(2^{3\eta_1} - 1)[\mathbf{w}^H (t|g_{11}|^2 \mathbf{M}_1 + t|g_{21}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_1) \mathbf{w} + (\frac{t|g_{11}|^2}{|f|^2} + \frac{t|g_{21}|^2}{|f|^2} + 2)\sigma^2]}{p_2} \\
& && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq \\
& && \frac{(2^{3\eta_2} - 1)[\mathbf{w}^H (t|g_{12}|^2 \mathbf{M}_1 + t|g_{22}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_2) \mathbf{w} + (\frac{t|g_{12}|^2}{|f|^2} + \frac{t|g_{22}|^2}{|f|^2} + 2)\sigma^2]}{p_1} \\
& && \mathbf{w}^H (\mathbf{M}_1 + t\mathbf{M}_1) \mathbf{w} \leq p_3 - \frac{t}{|f|^2} \sigma^2 \\
& && \mathbf{w}^H (\mathbf{M}_2 + t\mathbf{M}_2) \mathbf{w} \leq p_4 - \frac{t}{|f|^2} \sigma^2 \\
& && t > 0
\end{aligned} \tag{4.47}$$

Now, we define

$$\begin{aligned}
C_1(t) &= \frac{(2^{3\eta_1} - 1)(\frac{t|g_{11}|^2}{|f|^2} + \frac{t|g_{21}|^2}{|f|^2} + 2)\sigma^2}{p_2} > 0 \\
C_2(t) &= \frac{(2^{3\eta_2} - 1)(\frac{t|g_{12}|^2}{|f|^2} + \frac{t|g_{22}|^2}{|f|^2} + 2)\sigma^2}{p_1} > 0,
\end{aligned} \tag{4.48}$$

then, the optimization problem (4.47) can be equivalently expressed as

$$\begin{aligned}
& \max_{\mathbf{w}, t} && t \\
\text{subject to} & && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq C_1(t) + \frac{(2^{3\eta_1} - 1) \mathbf{w}^H (t|g_{11}|^2 \mathbf{M}_1 + t|g_{21}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_1) \mathbf{w}}{p_2} \\
& && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq C_2(t) + \frac{(2^{3\eta_2} - 1) \mathbf{w}^H (t|g_{12}|^2 \mathbf{M}_1 + t|g_{22}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_2) \mathbf{w}}{p_1} \\
& && \mathbf{w}^H (\mathbf{M}_1 + t\mathbf{M}_1) \mathbf{w} \leq p_3 - \frac{t}{|f|^2} \sigma^2 \\
& && \mathbf{w}^H (\mathbf{M}_2 + t\mathbf{M}_2) \mathbf{w} \leq p_4 - \frac{t}{|f|^2} \sigma^2 \\
& && t > 0
\end{aligned} \tag{4.49}$$

For fixed  $t$ , the optimization problem in (4.49) can be mathematically written as a second-order convex feasibility problem, given below

$$\begin{aligned}
& \text{Find} && \mathbf{w} \\
\text{such that} & && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq C_1(t) + \frac{(2^{3\eta_1} - 1) \mathbf{w}^H (t|g_{11}|^2 \mathbf{M}_1 + t|g_{21}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_1) \mathbf{w}}{p_2} \\
& && \mathbf{w}^H \mathbf{h} \mathbf{h}^H \mathbf{w} \geq C_2(t) + \frac{(2^{3\eta_2} - 1) \mathbf{w}^H (t|g_{12}|^2 \mathbf{M}_1 + t|g_{22}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_2) \mathbf{w}}{p_1} \\
& && \mathbf{w}^H (\mathbf{M}_1 + t\mathbf{M}_1) \mathbf{w} \leq p_3 - \frac{t}{|f|^2} \sigma^2 \\
& && \mathbf{w}^H (\mathbf{M}_2 + t\mathbf{M}_2) \mathbf{w} \leq p_4 - \frac{t}{|f|^2} \sigma^2
\end{aligned} \tag{4.50}$$

To obtain the optimal value of  $t$ , we use a bisection method where at each step the second-order convex feasibility problem in (4.50) is solved.

Note that in the second-order convex feasibility problem of (4.50), the objective function

or the constraints will not be affected by multiplying the optimal beamforming weight vector  $\mathbf{w}$  with  $e^{j\phi}$ . Therefore, without loss of generality, we assume that  $\mathbf{w}^H \mathbf{h}$  is a real number. Based on this fact and using the following definitions:

$$\tilde{\mathbf{w}} \triangleq [1 \ \mathbf{w}^T]^T$$

$$\tilde{\mathbf{h}} \triangleq [0 \ \mathbf{h}^T]^T$$

$$\mathbf{U}_1(t) \triangleq \begin{bmatrix} C_1(t) & \mathbf{0}^T \\ \mathbf{0} & \frac{(2^{3\eta_1} - 1)(t|g_{11}|^2 \mathbf{M}_1 + t|g_{21}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_1)}{p_2} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{U}_2(t) \triangleq \begin{bmatrix} C_2(t) & \mathbf{0}^T \\ \mathbf{0} & \frac{(2^{3\eta_2} - 1)(t|g_{12}|^2 \mathbf{M}_1 + t|g_{22}|^2 \mathbf{M}_2 + \sigma^2 \mathbf{D}_2)}{p_1} \end{bmatrix}^{\frac{1}{2}}$$

$$\mathbf{U}_3(t) \triangleq \begin{bmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & \mathbf{M}_1 + t\mathbf{M}_1 \end{bmatrix}$$

$$\mathbf{U}_4(t) \triangleq \begin{bmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & \mathbf{M}_2 + t\mathbf{M}_2 \end{bmatrix}$$

therefore, the second-order convex feasibility problem in (4.50) can be equivalently written as

$$\begin{aligned} & \text{Find} && \tilde{\mathbf{w}} \\ & \text{such that} && \Re\{\tilde{\mathbf{w}}^H \tilde{\mathbf{h}}\} \geq \|\mathbf{U}_1(t)\tilde{\mathbf{w}}\| \\ & && \Re\{\tilde{\mathbf{w}}^H \tilde{\mathbf{h}}\} \geq \|\mathbf{U}_2(t)\tilde{\mathbf{w}}\| \\ & && \Im\{\tilde{\mathbf{w}}^H \tilde{\mathbf{h}}\} = 0 \\ & && \tilde{\mathbf{w}}^H \mathbf{U}_3(t)\tilde{\mathbf{w}} \leq p_3 - \frac{t}{|f|^2}\sigma^2 \\ & && \tilde{\mathbf{w}}^H \mathbf{U}_4(t)\tilde{\mathbf{w}} \leq p_4 - \frac{t}{|f|^2}\sigma^2 \end{aligned} \quad (4.51)$$

where  $\Re\{\cdot\}$  and  $\Im\{\cdot\}$  denote, respectively, the real part and the imaginary part of a complex number and  $\|\cdot\|$  stands for the Euclidean norm of a vector.

The optimization problem in (4.51) is a second-order cone convex feasibility problem, which can be efficiently solved using interior-point methods. In order for the second-order cone convex feasibility problem in (4.51) to be feasible, the following conditions must hold true, that is

$$\begin{aligned} p_3 - \frac{t}{|f|^2}\sigma^2 &\geq 0 \\ p_4 - \frac{t}{|f|^2}\sigma^2 &\geq 0 \end{aligned}$$

or, equivalently, as

$$t \leq \frac{p_3 |f|^2}{\sigma^2} \quad (4.52)$$

$$t \leq \frac{p_4 |f|^2}{\sigma^2}. \quad (4.53)$$

Let us define

$$t_1 \triangleq \frac{p_3 |f|^2}{\sigma^2} \quad (4.54)$$

$$t_2 \triangleq \frac{p_4 |f|^2}{\sigma^2}, \quad (4.55)$$

then the search for optimal  $t$  will be limited to the interval  $(0, \min(t_1, t_2)]$ .

We summarize the bisection method as Algorithm II.

**Algorithm 2** : Algorithm-Bisection Scheme

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**Input:**  $p_1, p_2, p_3, p_4, \mathbf{g}_1, \mathbf{g}_2, f, \eta_1, \eta_2, \sigma^2$  and  $\epsilon$ .  
**Calculate:**  $t_1$  and  $t_2$ .

- 1: Initialize  $t_{low} = 0$  and  $t_{up} = \min(t_1, t_2)$ .
- 2: Set  $t_{mid} = \frac{t_{up} + t_{low}}{2}$ .
- 3: **while**  $|t_{up} - t_{low}| > \epsilon$  **do**
- 4:     Set  $t = t_{mid}$ .
- 5:     Calculate  $\mathbf{U}_1(t), \mathbf{U}_2(t), \mathbf{U}_3(t)$ , and  $\mathbf{U}_4(t)$ .
- 6:     **if** the second-order cone convex feasibility problem of (4.51) is feasible **then**
- 7:          $t_{low} = t_{mid}$
- 8:          $t_{mid} = \frac{t_{low} + t_{up}}{2}$
- 9:     **else**
- 10:          $t_{up} = t_{mid}$
- 11:          $t_{mid} = \frac{t_{low} + t_{up}}{2}$
- 12:     **end if**
- 13:     Repeat Step 3 to Step 12 until the optimal  $t_{opt} = t_{mid}$  is found.
- 14: **end while**

**Output:** the optimal  $t_{opt}$ .

---

### 4.3 Simulation Results

The complex channel coefficient between the transceiver  $S_i$  and relay  $R_j$  is denoted as  $g_{ji}$ , where  $i, j \in 1, 2$ , and we have  $g_{ji} \sim \mathcal{CN}(0, 1)$ . The complex channel coefficient between relays  $R_1$  and  $R_2$  is denoted as  $f$ . We assume that the channel coefficients  $g_{ji}$  and  $f$  are known and assume  $\sigma^2 = 1$ .

For Figure 4.2, we choose  $p_1 = p_2 = 15$  dBW and  $f \sim \mathcal{CN}(0, 10)$ . We assume that  $\eta_1 = \eta_2 = \eta$ , which changes from 0.1 b/cu to 0.6 b/cu, and that the relay powers  $p_3$  and  $p_4$  of  $R_1$  and  $R_2$  are equal, which change from 5 dBW to 20 dBW. Moreover, we set the number of channel realizations as 100.

Figure 4.2 displays the average data rate  $R_3$  ( $R_4$ ) versus  $\eta$  for  $p_1 = p_2 = 15$  dBW. Figure 4.2 clearly shows that with  $\eta$  increasing, the average data rate  $R_3$  ( $R_4$ ) decreases. This is because

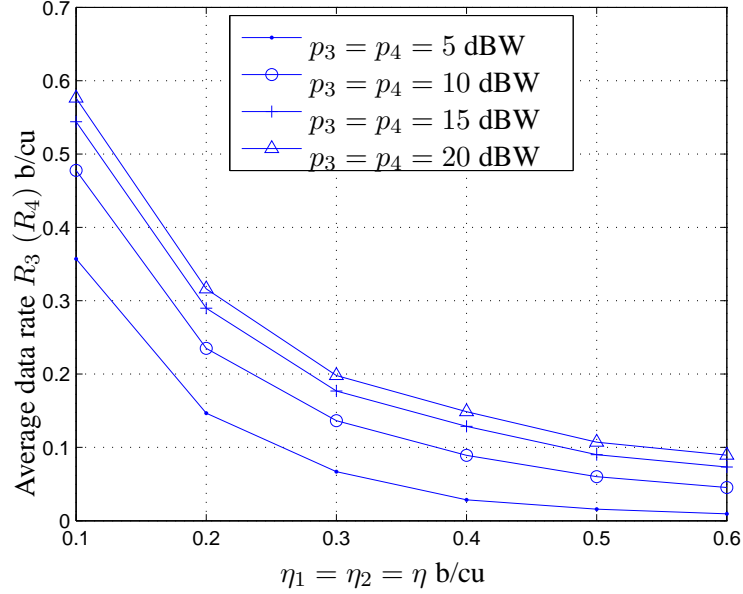


Figure 4.2: Average data rate  $R_3$  ( $R_4$ ) versus  $\eta$  when  $p_1 = p_2 = 15$  dBW.

$U_1(t)$  and  $U_2(t)$  increase while  $\eta$  increasing, which will violate the constraints in (4.51). In order for the second-order cone convex feasibility problem (4.51) to be feasible,  $t$  should decrease while  $\eta$  increasing. As mentioned earlier, without loss of optimality,  $\text{SINR}_3 = \text{SINR}_4 = t$ , therefore, the average data rate  $R_3$  ( $R_4$ ) is a decreasing function of  $\eta$ . From this figure, we observe that the average data rate  $R_3$  ( $R_4$ ) is an increasing function of  $p_3$  ( $p_4$ ). This is because the constraints in (4.51) will be relaxed while  $p_3$  ( $p_4$ ) increasing, as a result, more relay powers are allocated to provide secondary transmission with  $p_3$  ( $p_4$ ) increasing. As the curves shown in Figure 4.2, we see about 0.1 b/cu gap for the curves between  $p_3 = p_4 = 5$  dBW and  $p_3 = p_4 = 10$  dBW when  $\eta = 0.1$ . Although the performance gap for the curves between  $p_3 = p_4 = 5$  dBW and  $p_3 = p_4 = 10$  dBW gradually reduces with  $\eta$  increasing, it still remains larger than the performance gap for the curves between  $p_3 = p_4 = 15$  dBW and  $p_3 = p_4 = 20$  dBW. Therefore,



we know that the performance gap for large values of  $p_3$  ( $p_4$ ) seems not to change significantly over the range of  $\eta$ .

For Figure 4.3, we choose  $p_1 = p_2 = p_3 = p_4 = 15$  dBW and assume that  $\eta_1 = \eta_2 = \eta$ , which changes from 0.1 b/cu to 0.6 b/cu. Also, it is assumed that  $\sigma_f^2$  changes from 1 to 1000. Moreover, we set the number of channel realizations as 100.

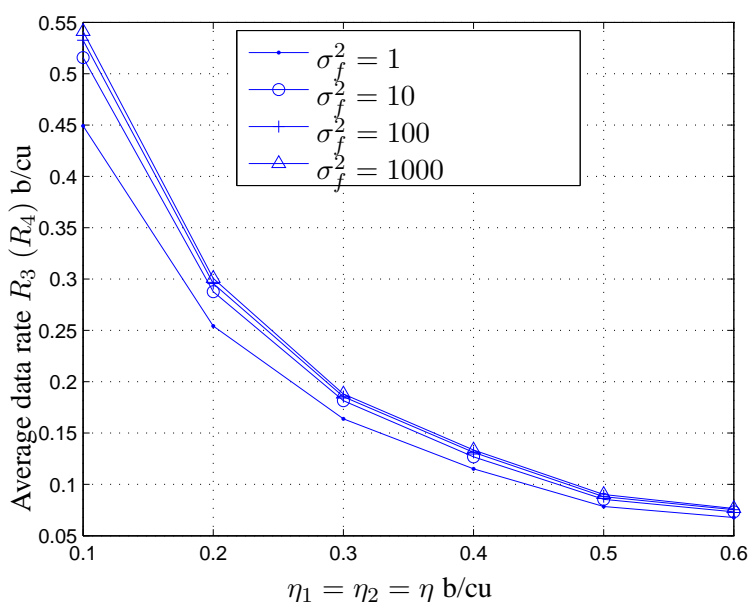


Figure 4.3: Average data rate  $R_3$  ( $R_4$ ) versus  $\eta$  when  $p_1 = p_2 = p_3 = p_4 = 15$  dBW.

Figure 4.3 displays the average data rate  $R_3$  ( $R_4$ ) versus  $\eta$  for  $p_1 = p_2 = p_3 = p_4 = 15$  dBW. From this figure, we can see that with  $\sigma_f^2$  increasing, the average data rate  $R_3$  ( $R_4$ ) increases; therefore, the average data rate  $R_3$  ( $R_4$ ) is an increasing function of  $\sigma_f^2$ . The performance gap for different values of  $\sigma_f^2$  changes significantly when  $\eta$  is smaller. However, with  $\eta$  increasing to large values, the performance gap for different values of  $\sigma_f^2$  changes only slightly. As the curves shown in Figure 4.3, we see about 0.07 b/cu gap for the curves between  $\sigma^2 = 1$  and

$\sigma^2 = 10$  when  $\eta = 0.1$ . Although the performance gap for the curves between  $\sigma_f^2 = 1$  and  $\sigma_f^2 = 10$  gradually reduces with  $\eta$  increasing, it still remains larger than the performance gap for the curves between  $\sigma_f^2 = 100$  and  $\sigma_f^2 = 1000$ . Therefore, we know that the performance gap for large values of  $\sigma_f^2$  seems not to change significantly over the range of  $\eta$ .

## 4.4 Summary

In this chapter, we consider a communication scheme consisting of two transceiver nodes and two relay nodes. The relays enable a bidirectional communication between the two transceivers, and at the same time, exchange their own information with each other. We aim to optimally obtain the beamforming weights and the coefficients to maximize the smaller of the data rates of two relays under the constraints that the data rate of each transceiver is not smaller than a given threshold. We prove that at the optimum, the data rates of two relays are equal. We show that the respective rate maximization problem can be written as a combination of a bisection technique and second-order cone convex feasibility problem. Analytical and simulation results show that with the two given thresholds increasing, the data rates of two relays present a downward trend.

## Chapter 5

# Conclusions and Future Work

In this thesis, a two-way relaying scheme is introduced due to its ability to improve the spectral efficiency. In a two-way relaying scheme, the relays can establish a bidirectional communication link between the two transceivers. We considered the data rate maximization problem in two-way relaying schemes and presented the solutions to two main problems.

For the first problem, we aim to optimally obtain the power allocation to the transceiver signal and the relay signal such that the data rate of link between the relay node and the receiving node is maximized under the constraints that the data rate of each transceiver is not smaller than a given threshold. We use the structure of the corresponding rate maximization problem to obtain the optimal power allocation scheme in a closed form. The results show that with the power allocation factor increasing, less and less relay power is allocated to provide opportunities for secondary transmission.

For the second problem, we aim to optimally obtain the beamforming weights and the value of the relay amplification factor for their own signals to maximize the smaller of the data rates

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of two relays under the constraints that the data rate of each transceiver is not smaller than a given threshold. We show that at the optimum, the data rates of two relays are equal. By using the beamforming weight vector as the design parameter, we show that the respective rate maximization problem can be written as a combination of a bisection technique and second-order cone convex feasibility problem.

## 5.1 Future Work

This work can be continued in several directions as listed below:

- Studying the problem of minimizing the total transmit power under the constraints that the data rate of each transceiver is not smaller than a given threshold.
- Studying the problem of minimizing the total relay power under the constraints that the data rate of each transceiver is not smaller than a given threshold.
- Studying the problem of maximizing the smaller of the data rates of two transceivers under the constraint that the total transmit power is not larger than a given threshold.
- Studying the problem of maximizing the sum-rate subject to an individual power constraint on each node.

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