Power-Optimal Network Beamforming in Single-Carrier Asynchronous Two-Way Relay Networks

by

Sahar Bastanirad

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Abstract

In this thesis, we consider a single-carrier *asynchronous* two-way relay network, where the relays employ amplify-and-forward (AF) signaling in a multiple access broadcast channel (MABC) protocol to enable a bi-directional communication link between two transceivers. The network we consider is asynchronous, meaning that each relaying path (which originates from one transceiver, goes through one of the relays, and ends at the other transceiver) causes a delay which is significantly different from the delays caused by other relaying paths. Such a two-way relay channel can be viewed as a multi-path link which can produce inter-symbol-interference at the two transceivers. Assuming a block transmission scheme, we resort to cyclic prefix insertion to eliminate inter-block-interference. We aim to obtain the relay complex beamforming weights and the transceivers' transmit powers such that the total power consumed in the whole network is minimized subject to two constraints on the transceivers' data rates. We rigorously prove that at the optimum, only a subset of the relays has to be turned on and the rest of the relays have to be switched off. More specifically, we prove that at the optimum, the end-to-end channel impulse response (CIR) will have only one non-zero tap, and hence, only those relays which contribute to that non-zero tap have to be turned on. We devise a simple search algorithm to optimally determine which tap of the end-to-end CIR has to be non-zero. Finally, we present a semi-closed-form solution for the optimal values of the design parameters, namely the relays' beamforming

weights and the transceivers' transmit powers. Our simulations results show that our proposed method significantly outperforms an equal power allocation scheme (i.e., when all the nodes in the network consume the same amount of power) which satisfies the same constraints on transceivers' data rates.

To my loving parents, Homa and Homayoun,

my amazing sister, kimia, and my wonderful brother, Sina.

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List of Acronyms

AF Amplify-and-Forward

AWGN Additive white Gaussian noise

BER Bit Error Rate

CIR Channel Impulse Response

CP Cyclic Prefix

CRLB Cramer-Rao Lower Band

CSI Channel State Information

DF Decode-and-Forward

EF Estimate-and-Forward

EPA Equal Power Allocation

FF Filter-and-Forward

FIR Finite Impulse Response

HDAF Hybrid Decode–Amplify–Forward

IBI Inter-Block-Interference

ICT Information and Communication Technology

ISI Inter-Symbol-Interference

LTI Linear Time Invariant

MABC Multiple Access Broadcast

MEPA Modified Equal Power Allocation

ML Maximum Likelihood

MIMO Multiple Input Multiple Output

MSE Mean Squared Error

OFDM Orthogonal Frequency Division Multiplexing

QoS Quality of Service

SISO Single-Input Single-Output

SNR Signal to Noise Ratio

TDBC Time Division Broadcast

Chapter 1

Introduction

1.1 A Brief Preview

Over the past three decades, wireless communication systems have drawn considerable attention from the research communities. There are numerous applications of wireless communication technologies due to their mobility, flexibility, easy installation, and cost saving compared to their wired counterparts. As such, cellular phones as a part of everyday life are now replacing traditional wired networks. Moreover, wireless local area networks are rapidly supplanting wireline systems in many business premises, campuses, and afcourse homes. The exponential growth of wireless communication industry is stimulating research interests in many new applications such as smart homes, wireless sensor networks and remote telemedicine devices. However, many technical limitations must be looked into make wireless networks more reliable.

1.2 Limitations in Wireless Communication

The high capacity and reliability of wired networks are still not achievable in wireless systems. Some features of wireless radio channel such as fading, shadowing, and path loss make a communication medium unpredictable. Path loss and shadowing are two main factors which cause variations in the received signal strength. The signal experiences random variations because of blockage from objects in the signal path [1]. The effects of wireless channel can be grouped into two categories: large-scale fading (including path loss and attenuation) and small-scale fading. Signal attenuation by large objects, such as buildings, causes the large-scale fading ,whereas the small-scale fading is due to constructive and destructive effects of the signals arrived over multiple propagation paths at the receiver. Signal fading due to multi-path propagation can be mitigated using diversity techniques. Diversity provides the destination node with several versions of the transmitted signals. Therefore, when one copy experiences fading, the destination may still have a chance to detect the received signal successfully from other received versions. Diversity in wireless systems can be obtained by frequency diversity, time diversity, spatial diversity, and many more. In multiple-input multiple-output (MIMO) systems, multiple antennas are used at the receiver and at the transmitter to exploit spatial diversity. MIMO systems provide many advantages over single-input single-output (SISO) schemes. Spatial diversity provided by multiple spatial paths can reduce sensitivity to fading [**?**]. Although transmit diversity introduces significant improvements to information rate and/or transmission reliability, a wireless node may not support more than one antenna at the transmitter due to size, cost, and hardware limitations.

On the other hand, explosive growth of high data rate applications with guaranteed quality of service (QoS) results in exceeding higher consumption. Meanwhile, limitations on battery lifetime and bandwidth shared among users are obstacles to develop efficient wireless applications. Considering these resource constraints, researchers aim to maximize the performance of the system. Several ideas which have been presented in the literature could not be effective enough due to the different channel qualities among users, unbalanced resource distributions and even worst the mobility of terminals. If users can cooperate in transmitting their messages as well as sharing their local resources, many of these issues can be resolved.

Recently, cooperative communications have become the promising idea which enables single-antenna nodes to share their antennas. In other words, cooperative communication help to realize a virtual multiple-antenna transmitter which allows the system to achieve transmit diversity. In addition, the relay-based communication, a specific kind of wireless cooperative communication, is an effective way to mitigate wireless fading through providing spatial diversity without the need of multi-antenna structure [2].

1.3 Cooperative Communications

The main idea behind cooperative communication can be traced back to the work of Cover and El Gamal [3] in 1980s. They studied the information theoretic properties of the relay channel. They considered a system model consisting of a source node, a relay node and a destination node, and showed that a discrete memoryless additive white Gaussian noise (AWGN) relay channel achieves higher capacity compared to the simple source-destination channel. They assumed three different random coding schemes for relay nodes to process information: Facilitation, Observation and Cooperation. Later in 1990s, Sendonaris *et al.* proposed a new form of spatial diversity which sparked the subsequent research on cooperative communications [4].

In cooperative wireless communication, users improve their quality of service by cooperation. It is assumed that each wireless user transmits its own information symbols as well as acts as a cooperative node for other users [5]. This cooperation scheme results in trade-offs in code rates and transmit power. In cooperative mode, the power consumption may increase since each user is supposed to transmit for both users. On the other hand, because of diversity, the baseline transmit power for both users can be decreased. Therefore, if the other factors in the network remain constant, the total transmit power of the network can be reduced. Also, for the rate of the system, one may argue that on one hand that since each user transmits its own data along with the other users' data, the transmission data rate can be decreased. However, because of cooperative diversity, the channel code rates may be increased. It is worth mentioning that cooperation of the users results in a more reliable communication link between the transceivers.

Designing the cooperative communication network requires considering important factors, such as the total interference in the network, hand-off and cooperation assignment and fairness of the communication link. Cooperative wireless communication has many advantages compared to a point-to-point communication model. Providing the destination node with proper combination of the received signals from different cooperative nodes is a differentiating factor between cooperative transmission scheme from traditional non cooperative systems [6]. Cooperative communication has been proposed to improve channel reliability, system throughput, and seamless service provisioning. Also, operation costs for both users and service providers may be decreased by cooperation in wireless communication [7].

Cooperative communication was introduced to enhance the overall point-to-point wireless network performance by the help of relay nodes with higher transmission data rates. In other words, a cooperative communication is selected based on the fact that sending data to a destination through a relay node with higher transmission rate is faster than directly sending information to the destination with low transmission rate. Also, since relays are power-efficient network components, they can increase the data rate and coverage with relatively low cost [8]. Relay-assisted cooperation improves throughput and efficiency of the wireless communication systems. However, the overall performance of the system can be enhanced by optimum resource allocation. Hence, different approaches such as selection of the best relays, allocation of optimum power, etc. have been proposed in the literature. The basic idea of relay communication is to use several relay terminals to participate in communication by retransmitting a signal traveling between a source and a destination thereby forming a distributed multi-antenna system [9]. The destination terminal receives several versions of the transmitted signals relayed by the relay nodes and combines them to decode the transmitted messages.

1.3.1 Challenges in Cooperative Communications

While cooperative communication opens new doors to the communication world, it also poses new challenges to researchers. Relays' performance at the cooperating node is one of the main key interests to realize cooperative communication. Another concern is sharing of potential relays among users and finding proper resource allocation schemes. The low transmission reliability because of channel characteristic of the transmission links is another challenge of cooperative communications. The trade-off between system reliability and efficiency in the cooperative systems with multiple relay nodes has received large attention of researchers. In order to achieve high capacity in cooperative networks, accurate resource allocation is needed due to the multi-hop transmission. Using traditional allocation scheme, which depends on the channel condition, may decrease the overall Quality of Service (QoS) of the relay network.

1.4 Motivation and Problem Statement

Information and communication technology (ICT) sector has an important role in worldwide warming and greenhouse gas emission. Even though ICT is responsible for small part of global greenhouse gas emissions (reported as 3 percent), this portion is expected to increase rapidly [10]. Wireless communication is an important part of ICT sector and should strive to save energy. Also, developing speed of battery technology is not as high as the increase of energy consumption rate of mobile terminals in wireless systems. On the other hand, new challenges arise by growing high data rate applications of wireless networks which results in more and more energy consumption. Supporting high data rate Internet services, and advanced multimedia such as mobile TV, gaming and smart phones increase demand for high data rates. We are motivated to save the energy for relatively high data rate transmission. Recently, power minimization has received much attention in both academia and industry. Technologies such as MIMO and cooperative communication are proposed to improve the efficiency of communication devices. Cooperative relaying can improve the performance of wireless system by several mechanisms, such as increasing spatial diversity or beamforming effects [4, 11]. One important issue is using a relay with minimal power consumption in the whole system. Two-way (bi-directional) relay networks have received significant attention from the research community [12]. Employing two-way relaying scheme enables a bi-directional communication where users (transceivers) can simultaneously exchange their information with the help of one or multiple relay nodes. Each relay provides a relaying path for transmitted signal to from one transceiver to transmit to the other transceiver. If all different relaying paths have the same delays, a bi-directional relay network is considered as synchronous. In a synchronous two-way relay network where the channel between each transceiver and each relay is frequency flat, the end-to-end channel can be modeled as frequency flat link as well.

In practice, due to the distributed nature of the nodes, the transceivers and the relaying nodes may receive transmitted signals at different times. In other words, each relay provides a relaying path which has different propagation delay from those of the other relaying paths. This different relaying propagation delay results in asynchronism in the network [1,13–16]. In this case, the end-to-end channel can be modeled as muti-path link which produces inter-symbol-interference (ISI) at the transceivers.

Our motivation which is saving energy for relatively high data rate applications in realistic scenario leads us to consider the power minimization problem for an asynchronous two-way relay network. In this thesis, *we are interested in designing a single-carrier asynchronous two-way AF relay network with a minimal power consumption in the whole network subject to two constraints on the transceivers' data rates.* Solving this problem is important as it allows system designers to obtain the smallest possible levels of power consumption in the whole network for any required pair of data rates at the transceivers. Assuming that the relays use AF-based multiple access broadcast channel (MABC) two-way relaying protocol, the end-to-end channel impulse response is determined by the relays' weights. The channel from one transceiver to the other one can be viewed as a multi-path end-to-end channel which causes ISI at the transceivers. In a block communication scheme, where the transmitted symbols are coded in blocks and the received signals are decoded in blocks at the transceiver, ISI results in intra-block-interference and inter-block-interference (IBI). To the best of our knowledge, the problem of minimization total power subject to data rates at the transceivers for asynchronous two-way AF relay network has not been studied in the literature . Hence, we are motivated to consider total power minimization problem thereby jointly obtaining optimal relay beamforming weights and transceivers' transmit power subject to constraints on transceivers' data rates.

The problem of minimization total consumed power subject to the transceivers' data rates for two-way asynchronous relay network has not been studied yet. In this work, we consider the problem of minimization the total consumed power in a asynchronous two-way relay network while maintaining the transceiver s's data-rate above the certain levels. We prove that this minimization leads to a certain relay selection scheme, which the relays contributing to the optimal tap of the end-to-end channel are active and the other reaming relays do not participate in relaying and are switched off.

1.5 Objective

The main objective is to solve the power minimization problem for an asynchronous two-way relay network subject to constraints on the transceivers' data rates. These constraints guarantee that the data rate achieved at the first and second transceivers are larger than the minimum required data rates at the both transceivers, respectively. Hence, we aim to design an asynchronous two-way relay where relays use simply AF relaying protocol to state the minimization problem for such a network.

1.6 Methodology

We first introduce the system model for an asynchronous two-way relay method which consists of two single-antenna transceivers and several single-antenna relays. In such a network, we consider that the relays use simple amplify-and-forward relaying protocol to enable a two-way communication link between the two transceivers. In this system, it is assumed that the transceivers receive the signals transmitted by different relays, at different times. In other words, the cooperative nodes are not perfectly time-synchronized and each relaying path has a propagation delay which is different from those of the other paths. This assumption leads us to view the end-to-end link as a frequency selective multi-path channel (i.e., with multiple taps), which produces ISI at the two transceivers. We model the transmission scheme, channel, received noise, received signal and total transmit power for an asynchronous two-way relay network. Then, we present an optimization problem which aims to minimize the total consumed power in the network under constraint on the transceivers' data rates. To model our data model as MIMO channel, we express data rate achieved at each transceiver.Using optimization methods, we simplify the problem to obtain the optimal beamforming weight vector and the optimal transceivers' transmit powers which are our optimization parameters. we show that the power minimization problem can be achieved by a semi-close form solution. We also prove that our approach leads to a relay selection scheme where only the relays contributing to the optimal tap of the end-to-end channel impulse response (CIR), are active and the other relays are turned off. We propose a simple algorithm to find the optimal tap of the end-to-end CIR. We evaluate the performance of our algorithm to compare the simulation results of it with other power allocation algorithms.

1.7 Contributions

In this subsection, we summarize our contributions in this thesis,

- We consider a single-carrier *asynchronous* two-way relay network, where the relays employ AF signaling in an MABC protocol to enable a bi-directional communication between two transceivers.
- We obtain the relay complex beamforming weights and the transceivers' transmit powers such that the total power consumed in the whole network is minimized subject to two constraints on the transceivers' data rates.
- We rigorously prove that at the optimum, the end-to-end CIR will have only one non-zero tap, and hence, only those relays which contribute to that non-zero tap have to be turned on.
- We devise a simple search algorithm to optimally determine which tap of the end-to-end CIR has to be non-zero
- We present a semi-closed-form solution for the optimal values of the design parameters, namely the relays' beamforming weights and the transceivers' transmit powers.

The full version of this thesis has been submitted to *IEEE Trans in Signal Processing* as

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  Also, part of this research will be presented at the Forthy-Ninth IEEE Asilomar conference on Signals, Systems, and Computers on Nov. 2015, as:

- S. Bastanirad, S. Shahbazpanahi, and A. Grami, " Jointly optimal distributed beamforming and power control in asynchronous two-way relay networks," will be presented in *IEEE Asilomar Conference on Signals, Systems, and Computers*, Nov. 2015.

1.8 Outline of Thesis

In this thesis, we focus on a single-carrier asynchronous two-way relay network. The reminder of the thesis is organized as follows: In Chapter 2, we first review the recent research results on cooperative communication including one- and two-way relay networks. We then summarize the recent studies on asynchronous two-relay networks. In Chapter 3, we study power-optimal network beamforming in single-carrier asynchronous two-way relay networks. For such a scheme, we propose a semi-closed-form algorithm to find the optimal transmit powers at the transceivers as well as the optimal relay beamforming weights. Simulation results are provided to show that our proposed scheme outperform equal power allocation method and modified equal power allocation scheme. In Chapter 4, we present conclusion as well as the potential future work in this area of research.

1.9 Notation

Matrices and vectors are denoted by bold upper and lower-case letters, respectively. $E\{\cdot\}$ and $tr\,(\cdot)$ stand for statistical expectation operator and trace of a matrix. Transpose, complex conjugate and Hermitian transpose are represented as $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$, respectively. We represent the l_2 norms as $\|\cdot\|$. The $N \times N$ identity matrix and the $N \times M$ all-zero matrix are denoted as $\mathbf{I}_N$ and $\mathbf{0}_{N\times M}$. $\mathrm{diag}(\mathbf{v})$ yields a diagonal matrix whose diagonal entries are the elements of the vector $\mathbf{v}$. We use $(\cdot)^{-1}$ represent the inverse of a matrix. The (i,j)-th element of matrix $\mathbf{B}$ is denoted as $\mathbf{B}(i,j)$. The determinant of a matrix is denoted as $\det(\cdot)$. We use $|\cdot|$ to represent the amplitude of a complex number.

Chapter 2

Literature Review

In this chapter, we review recent studies in areas related to this thesis. First, we provide a brief survey on the cooperative relay networks. Then, different relaying strategies used in literature are presented. We then review the related results published regarding onone-way and two-way relay networks. We finally review the literature on synchronous and asynchronous relay networks.

2.1 Cooperative Relay Networks

Cooperative communication is proposed to improve the capacity, transmission reliability and the coverage of the wireless communication systems [17]. Traditional point-to-point communication systems implement a direct transmission scheme, where the destination node processes only the data transmitted by the source node and signals from other nodes are considered as interference. In cooperative communication, nodes in the network help another node by relaying that node's transmitted signals while transmitting their own information [4]. Moreover, when the direct link between transceiver and receiver is not strong and/or reliable enough or even when there is no direct link between them, the relays help to enable communication link between transceivers [18]. In the simplest cooperative relay network, a source and a destination node use one relay node to exchange their information [3]. However, a general relay network can consist of multiple relays [5]. Cooperative communication performance depends on the relaying strategies including relaying protocols and relay types. Exchanging information between two terminals can be categorized into two main categories, namely uni-directional transmission and bi-directional transmission [2,19]

2.2 Relaying Strategies

Based on the relay operation on its received signal, different relaying protocols have been studied in the literature. Cooperative communication schemes can be categorized into four well known major types: 1) amplify-and-forward (AF), 2) decode-and-forward (DF), 3) estimate-and-forward (EF), 4) filter-and-forward (FF).

In the AF technique, the relays simply capture the signal received from the source, amplify it, and then retransmit it to the receiver [20]. There are two main types of amplifying gain used in AF relaying design which are variable gain and fixed gain [21, 22]. In the variable gain based approach, the relays estimate the channel thereby choosing the amplification gain which depends on instantaneous channel fading of each path. However, in fixed gain relaying method, the relaying gain is constant and can be calculated using the fading channel statistics of channel information (CSI). Though variable gain relaying systems outperform the fixed gain relaying schemes, the requirement of the availability of instantaneous CSI increases the complexity as well as the cost of the relays. While CSI is not needed for fixed gain relaying and the relay does not require to measure the channel from the source [23]. When the system has a strict requirement of transmission delay, fixed gain relaying is preferred due to its lower complexity. The AF technique is simple as the detection of the transmitted signals is not required at the relays, but this approach is more proper when the noise power at the relays is significantly lower than the signal power [24]. It is worth mentioning that AF method requires less processing power at the relays compared to other relaying strategies.

In the DF scheme, the relays decode the signal received from transmitter and then forward it to the destination node. The noise contaminates the signal can be surpassed at the relays and would not be passed to the receiver [25]. In other words, the DF technique generates a relatively clean version of retransmitted signal. It is worth mentioning that this technique needs high computing power at the relays [26], [19]. In [27], the authors investigate the performance of Hybrid decode–amplify–forward protocol (HDAF). They showed that this protocol has better performance than decode-forward (DF) one , and performs very close to the AF scheme.

Another relaying strategy is EF relaying, which is also known as compress-and-forward technique [3]. In this method, the relays transmit an estimated version of the transmitted signals to the destination without decoding the message [28]. Hence, an error containing version of signal is forwarded to the destination. In order to realize cooperative diversity, the destination node combines the error containing signal which is relayed with the signal from the source node. The first combination scheme was introduced in [**?**], which was based on the maximum likelihood (ML) detection theory. The authors improve the performance of EF relaying scheme by providing a destination node with a clipper function in the diversity combiner. It is shown in [29] that clipping the signal received at the destination can waste a large portion of power allocated to relays.

The FF strategy is an extended version of the conventional AF protocol which was first proposed in [30] to compensate the transmitter-to-relay and relay-to-destination channels. In the FF technique, the relay nodes are equipped with finite impulse response filters (FIR). This technique is proposed to use in two-way relay networks with frequency selectivity channels thereby employing the channel equalizer in a distributed manner [31, 32].

2.3 One-Way Relay Networks

One-way relay scheme is one of the earliest relaying methods which was proposed by E. van der Meulen, T.M. Cover and A. El Gamal in 1970. In [3], the authors considered a basic relay channel consisting of a source node, a relay node, and a destination node. In this pioneer work, the capacity of a three-terminal network is analyzed. The authors of [33] generalized relay network by using more than one relay to cooperate and showed that for large networks, multi-relay schemes outperform the corresponding single relay approaches. Employing one-way relay scheme provide uni-directional transmission between two nodes. Uni-directional transmission takes place in two time slots. In the first phase, the source terminal transmits the signal to the rely. In the second time slot, the relay node forwards the massage to the destination terminal. In early studies on one-way relaying scheme, it was assumed that the relays have perfect CSI [34–39].

2.4 Two-Way Relay Networks

In a two-way relaying scheme, relays are used to help two or more transceivers to exchange their information when there is no direct link between them. Moreover, two-way relay channel (TWRC) has potential to double the data rate transmission of a one-way relay channel. The simplest model of a two-way relay channel is a three-node relay channel system. In [40], the authors considered a relay node between two transceivers and studied the capacity region of this scheme. Also in [41], an upper bound of the capacity using the cut-set theorem is proposed and in [42] the achievable rate for this scheme is studied.

Based on the transmission scheme, there are different two-way relaying methods. In the simplest form of TWRC, two one-way relaying schemes are used. The end-to-end transmission takes 4 time slots. This straightforward method helps to avoid interference at the relays and at the Transceivers, but it is not bandwith efficient. In another scheme, which called time division broadcast (TDBC), the number of time slots is decreased to three using network coding based method. This approached is introduced in [43]. In multiple access broadcast (MABC) scheme two transceivers simultaneously transmit their information to a relay node which takes place in one time slot. In the second time slot which is the final step of transmission, a relay node retransmits its received signal to the transceivers [44]. It is obvious that in the absence of direct link between transceivers, MABC provides the highest bandwidth efficiency among other protocols used for TWRC. The MABC approach has been considerd in [38, 45–58].

The authors of [59] compared two bdirectional network schemes, MABC and TDBC. They showed that TDBC-based bidirectional network, outperforms MABC approach when there is a direct link between two transceivers due to providing additional degrees of freedom.

2.4.1 Synchronous versus Asynchronous Relay Networks

Due to the distributed nature of the rely nodes within cooperative systems, synchronization between all nodes may be difficult or even impossible to achieve. Considering this imperfection, each relaying path originating from one transceiver to the other transceiver, has a propagation delay which is different from those of other relaying paths. In most studies on cooperative relay network, it is assumed that cooperative nodes are perfectly time synchronized and propagation delays are identical between all relaying paths. In some publications space-time codes [60,61] and space-frequency codes [62] were used to mitigate the imperfect synchronization. In synchronous TWRC where the relay-transceiver channels are frequency flat, and the end-to-end channel between two transceivers can be modeled as frequency flat link. In recently published results, the asynchronous two-way relay network is considered where each node receives the signals transmitted by different nodes, at different times. This consideration leads to view the end-to-end link as a frequency selective multi-path channel which produces inter symbol interference (ISI) at both transceivers. In the following sections, we review results published on both synchronous and asynchronous models.

2.4.2 Synchronous Two-Way Relay Networks

In this subsection, we review several important studies on synchronous two-way relay network. The study in [12] is one of the pioneer studies on two-way AF relay networks which the nodes are assumed to be synchronized. The authors of [12] consider a MABC AF two-way relaying scheme consisting of two single antenna transceivers and multiple relays. Assuming that exchanging two symbols between the two transceivers in this two-way relaying channel needs two time slots, they minimize the total transmit power under the constraints on transceivers' received signal-to-noise ratios (SNRs). They obtain a simple iterative algorithm to find the optimal weight vector. They also maximize the smaller transceivers' SNRs subject to the total power budget. Their proposed scheme results in a unique solution where half of the maximum power budget is allocated to the two transceivers and the remaining half will be shared among all the relay nodes.

When only one relay is employed to make a communication link between transceivers, phase adjustment and synchronization at the relays are not needed. Meanwhile, this scheme is easier to implement and also the power allocation between relay and transceivers is simpler to achieve compared to using multiple relay nodes [63]. The authors of [63] propose an optimal joint relay selection and power allocation scheme for such a network. They maximize the smaller of the received signal-to-noise-ratios (SNRs) of the two transceivers while total transmit power budget is constrained.

Assuming the perfect synchronization, for a two-way relay network with two end-users and multiple relays, the autors of [64] present the problem of designing distributed beamforming at the relays and power control scheme. They maximize the smaller SNR at the two transceivers for both single relay and multiple relay networks. In case of single-relay network, they solve the maximization problem analytically while for the corresponding multiple-relay case, they present an iterative numerical algorithm. Their simulation results show that the sum-rate maximization approach and the max-min fair design approach have the same solution.

Another example of considering perfectly time synchronization between all nodes in AF bi-directional relay networks is presented in [59]. In this study, the problem of minimization the total power consumed in the network subject to quality of service (QoS) is considered for two different relaying strategies(MABC and TDBC).

In [65], the authors devise space-time coding for multi-antenna relay network with Rayleigh fading channels. Their results based on the assumption that the wireless network is synchronized at the symbol level.

2.4.3 Asynchronous Two-Way Relay Networks

In almost literature on two-way relay network, the authors assume that cooperative nodes are all perfectly time-synchronized. However, in practice, the relays and the transceivers may not be time-synchronized due to the different propagation delay of each relaying path. In this subsection we review recently published results on asynchronous two-way relay networks.

The authors of [66] extended the study of distributed space-time coding for synchronous wireless network [65] to asynchronous network model. They presented distributed space-time code design which allows full spatial diversity for an asynchronous relay network.

In [67], the authors considered an asynchronous AF two-way relay network. Due to the different propagation delays, this relay channel can be viewed as an artificial multi-path channel which can produce ISI at the transceivers. They deploy orthogonal frequency division multiplexing (OFDM) at the two transceivers to combat the ISI and obtain jointly optimal subcarrier power loading at the transceivers and beamforming weights at the relays by two max-min design approaches. Another approach to combat the ISI produced by relay asynchronism is employing multi-carrier technique where each subcarrier enables a bidirectional communication between two transceivers.

In [68], the authors of [67], consider an asynchronous bi-directional relay network when the communication between two single-antenna transceivers is performed using multiple single-antenna relays when the transceiver-relay paths have different propagation delays. They modeled this communication scheme as a frequency selective multi-path channel with high data rates. The transceivers utilize an OFDM scheme to combat the ISI. The subcarrier power loading at the transceivers along the relay beamforming weights are obtained employing the max-min fair design approach. Their simulation results proved that their schemes have better performance compared to m equal or maximum power allocation schemes.

The authors of [69] considering the sum-rate as the optimality criterion on asynchronous multi-carrier two-way relay network of [68], showed that for identical system model, the results of the max-min SNR fair design technique in [68] and the sum-rate maximization approach [69] are identical. Their Sum-rate maximization approach subject to total consumed power results in a relay selection scheme where only relays contributing to the one tap of the end-to-end CIR are active and all the remaining relays are deactivated.

The achievable SNR and rate regions under a total power constraint for an asynchronous bidirectional multi-carrier relay network is developed in [70]. In this research, the authors assume that the end-to-end CIR is frequency selective while relay-transceiver links are frequency flat. The authors employ OFDM technique at the two transceivers to equalize the frequency selective channel.

A single-carrier communication scheme is also considered in [71], for an asynchronous bi-directional AF relay network. The end-to-end link is a frequency selective channel which have multiple taps. Like other works on asynchronous relay network, ISI is inevitable at the two single-antenna transceivers. Using block communication scheme where the symbols are transmitted and are decoded in blocks, ISI results in IBI. The authors propose CP insertion and block post-channel equalization methods to combat the IBI. They obtain the block post-channel equalizers at the two transceivers, the transceivers transmit powers, and the relay beamforming weight vector through minimizing the total mean squared error (MSE) of the estimated received signals at both transceivers where total transmit power budget is limited.

2.5 Power Minimization

Resource allocation during relay transmission may result in additional time and power consumption. Bidirectional relaying technique can provide energy saving opportunity [72]. Hence, significant volume of studies in the literature investigate the problem of minimization the total consumed power in the network under different constraints [63,73–77]. In [12], the authors minimize the total transmit power under transceivers' received SNRs and find a unique optimal weight vector by an iterative algorithm. They also show that when the two constraints on the transceivers' SNRs are the same (symmetric relaying schemes), the two transceivers use the half of the total transmit power and relay nodes use the other half power.

In [49], in order to obtain the optimal rate region in a two-way relay channel, the authors introduced an iterative algorithm which solves a power minimization problem at every step subject to signal-to-interference-and-noise ratios.

The authors of [78] present a semi-closed form solution to find the relay beamforming weight vectors and the transceiver transmit powers when the total power of the network is minimized. They show that minimization of the total transmit power, subject to SNR constraints at the two transceivers, is equivalent to the minimization of the total power for a one-way relay network when the SNR of the receiving transceiver is equal to the sum of the SNRs of the two transceivers.

Another power minimization approach in the two-way relay networks is presented in [74]. They obtained a closed-form solution for relay selection approach on finding the optimal relay or relays to exchange the information between two transceivers when the consumed power is minimized subject to the transceivers' quality of services (QoS) be greater than specific thresholds.

The power minimization approach presented in this thesis is different form those in the literature. We consider a single-carrier asynchronous AF two-way relay wireless network and to the best of our knowledge the problem of minimization total power for such a network under the constraints on transceivers data rates has not been addressed in the literature.

Chapter 3

Total Power Minimization Method

3.1 System Model

We consider an asynchronous two-way relay network consisting of two single-antenna transceivers T_1 and T_2 and L single-antenna relay nodes. The relays use a simple amplify-and-forward relaying protocol to materialize a two-way communication link between the two transceivers. In this network, we assume that transceivers can communicate only through the relays, implying that there is no direct link between the two transceivers. Our focus in this thesis is on the two time-slot AF-based multiple access broadcast channel (MABC) two-way relaying protocol. In the first time-slot of this relaying protocol, the two transceivers simultaneously transmit their signals to the relays. Each relay receives a noisy mixture of the attenuated versions of the signals transmitted by the two transceivers. In the second time-slot, each relay transmits an amplified and phase-adjusted version of its received signal to the two transceivers. Having the global channel state information, each transceiver subtracts the self-interference signal (i.e., its own signal which is relayed back to this transceiver) from its received signal and uses the residual signal to decode the signal transmitted by the other transceiver. We assume that, due to the distributed nature of the nodes, each relaying path has a propagation delay which is different from those of the other paths. Thus, the end-to-end link can be viewed as a multi-path channel consisting of multiple taps which can produce inter symbol interference (ISI) at the two transceivers at sufficiently high data rates. In a block communication scheme, where the symbols are transmitted in blocks and are decoded in blocks at the receiver, ISI results in intra-block-interference and inter-block-interference (IBI). In order to avoid IBI, cyclic prefix (CP) can be inserted between successive transmitted blocks, as shown in Fig. 3.1. The effect of intra-block-interference however has to be somehow accounted for in the system performance optimization.

3.1.1 Transmission Scheme

Fig. 3.1 shows the transmission scheme for the single-carrier two-way asynchronous AF relay network we herein considered. At each transceiver, the information symbols to be transmitted are converted by a serial-to-parallel conversion block, denoted as "S/P", into blocks of N_s symbols. The i^{th} block of the symbols transmitted by Transceiver q is represented as an $N_s \times 1$ vector $\mathbf{s}_q(i)$:

$$\mathbf{s}_q(i) = \Big[s_q[iN_s] \quad s_q[iN_s+1] \quad \cdots s_q[iN_s+N_s-1]\Big]^T \tag{3.1}$$

where $s_q[k]$ is the k^{th} symbol transmitted by Transceiver q. In Fig. 3.1, the CP insertion is performed by multiplying $\mathbf{s}_q(i)$ with the matrix $\mathbf{T}_{\text{cp}} \triangleq [\tilde{\mathbf{I}}_{\text{cp}}^T \ \ \mathbf{I}_{N_s}^T]^T$, where $\tilde{\mathbf{I}}_{\text{cp}}$ contains the last N rows of the $N_s \times N_s$ identity matrix $\mathbf{I}_{N_s}$ and N is the length of the vector of the taps of the end-to-end channel impulse response (CIR)[1]. Thus, the $N_t \times 1$ vector of the output of the CP insertion block, denoted as $\overline{\mathbf{s}}_q(i)$, can be written as

$$\begin{aligned}\overline{\mathbf{s}}_q(i) &\triangleq \left[\begin{array}{cccc} \overline{s}_q[iN_t] & \overline{s}_q[iN_t+1] & \cdots & \overline{s}_q[iN_t+N_t-1] \end{array} \right]^T \\ &\triangleq \mathbf{T}_{\text{cp}}\mathbf{s}_q(i) \\ &= [s_q[(i+1)N_s-N] \; \cdots \; s_q[(i+1)N_s-1] \; s_q[iN_s] \; \cdots \; s_q[(i+1)N_s-1]]^T \end{aligned} \tag{3.2}$$

where $N_t \triangleq N + N_s$ is the length of the transmitted blocks and $\overline{s}_q[iN_t + k]$ is the k^{th} entry of $\overline{\mathbf{s}}_q(i)$. The data block $\overline{\mathbf{s}}_q(i)$ is then converted back to serial by the parallel-to-serial conversion block, denoted as "P/S" . The output serial signal is multiplied by $\sqrt{P_q}$, where P_q is the transmit power of Transceiver q.
At the receiver, the received signal goes through a parallel-to-serial conversion block and is thus converted into blocks of N_t symbols. Then, the vectors of the received signals is passed through self-interference cancellation block, denoted as "SIC". As each transceiver knows its own transmitted signal along with its channel gains to the relays, it can cancel its own transmitted signal which was relayed back to this transceiver from its received signal. The CP removal block is then employed to discard the CP which was added before transmission. To do so, the vectors of received signals are multiplied by the matrix $\mathbf{R}_{\text{cp}} \triangleq [\mathbf{0}_{N_s \times N} \;\; \mathbf{I}_{N_s}]$.

In this subsection, the discrete-time channel model for our system is presented. The link between each transceiver and each relay is assumed to be frequency flat and reciprocal. We define g_{lq} as the complex frequency flat coefficient of the channel between Transceiver q and the l^{th} relay, for $l \in \{1, 2, \cdots, L\}$. The signal traveling through the l^{th} relay undergoes

[1] We present our channel model in the next subsection.

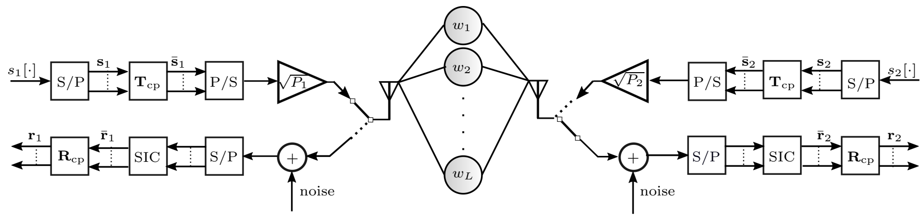

Figure 3.1: System Block Diagram

a total amplification/attenuation factor, denoted as b_l, which is given by

$$b_l \triangleq w_l g_{l1} g_{l2} \tag{3.3}$$

where w_l is the complex beamforming weight of the l^{th} relay. We denote the propagation delay of the signal traveling between the two transceivers via the l^{th} relay as τ_l which is the summation of three time delays: i) propagation delay from one transceiver to the l^{th} relay; ii) the time delay for the relaying process; and iii) the propagation delay from that relay to the other transceiver. We also define $\breve{n}_l$ as the discrete-time propagation delay of the l^{th} relaying path which must satisfy the following condition:

$$(\breve{n}_l - 1)T_s < \tau_l \leq \breve{n}_l T_s \tag{3.4}$$

where T_s is the symbol period. Note that only the relay or relays which can satisfy (3.4) can contribute to the tap $\breve{n}_l$ of the end-to-end CIR. We can now represent the n^{th} tap of the discrete-time impulse response of the linear time-invariant (LTI) end-to-end channel between Transceivers 1 and 2 as

$$h[n] = \sum_{l=1}^{L} b_l \delta[n - \breve{n}_l], \quad \text{for } \ 0 \leq n \leq N-1 \tag{3.5}$$

where N is the maximum number discrete-time delay spread of $h[\cdot]$ and it is defined as

$$N \triangleq 1 + \max_{1 \leq l \leq L} \lceil \frac{\tau_l}{T_s} \rceil. \tag{3.6}$$

Nothng that $h[\cdot]$ depends on the relay weight vector $\mathbf{w} \triangleq [w_1 \ \ w_2 \ \ \dots \ \ w_L]^T$, we define the channel coefficient vector $\mathbf{h}(\mathbf{w})$ as

$$\mathbf{h}(\mathbf{w}) \triangleq \left[\begin{array}{cccc} h[0] & h[1] & \cdots & h[N-1] \end{array} \right]^T. \tag{3.7}$$

To express the contribution of the l^{th} relay to the n^{th} tap of $h[\cdot]$, we define the $(n+1, l)^{\text{th}}$ element of the $N \times L$ matrix $\mathbf{B}$ as

$$\mathbf{B}(n+1, l) \triangleq \begin{cases} g_{l1} g_{l2}, & \frac{\tau_l}{T_s} \leq n < \frac{\tau_l}{T_s} + 1 \\ 0, & \text{otherwise} \end{cases} \tag{3.8}$$

where $n = 0, 1, \cdots, N-1$ and $l = 0, 1, \cdots, L$. We now use (3.3), (3.5), and (3.8) to write the vector of the taps of the end-to-end CIR as

$$\mathbf{h}(\mathbf{w}) = \mathbf{B}\mathbf{w} \tag{3.9}$$

where $\mathbf{h}(\mathbf{w})$ is given as in (3.7).

3.1.2 Received Noise Model

The signal passing through each relaying path is contaminated with noise. In order to model the noise added to the received signal, let us define τ'_{lq} as the propagation delay between Transceiver q and the l^{th} relay and n'_{lq} as an integer value which satisfies the condition $\frac{\tau'_{l_q}}{T_s} \leq n'_{lq} < \frac{\tau'_{l_q}}{Ts} + 1$. We also define $\upsilon_l[n]$ as the spatially and temporally white noise at the l^{th} relay which is zero-mean with variance σ^2. Since the relays use AF relaying protocol, first, at the l^{th} relay, the received noise is amplified by w_l, and then, it arrives at Transceiver q with delay n'_{lq}. We can now represent the expression for the superposition of the relay noises received at Transceiver q, denoted by $\xi_q[n]$, as

$$\xi_q[n] \triangleq \sum_{l=1}^{L} w_l g_{lq} \upsilon_l[n - n'_{lq}] = \mathbf{v}_{n,q}^T \mathbf{G}_q \mathbf{w} \tag{3.10}$$

where the following definitions are used: $\mathbf{v}_{n,q} \triangleq [\upsilon_1[n - n'_{1q}] \; \upsilon_2[n - n'_{2q}] \; \cdots \; \upsilon_L[n - n'_{Lq}]]^T$ and $\mathbf{G}_q \triangleq \text{diag}\{g_{1q}, g_{2q}, \cdots, g_{Lq}\}$. The total noise received at Transceiver q can be written as $\psi_q[n] = \xi_q[n] + \psi'_q[n]$, where $\psi'_q[n]$ is the measurement noise at Transceiver q. Using the following definitions: $\boldsymbol{\psi}_q(i) \triangleq [\psi_q[iN_t] \; \psi_q[iN_t + 1] \; \cdots \; \psi_q[iN_t + N_t - 1]]^T$, $\boldsymbol{\xi}_q(i) \triangleq [\xi_q[iN_t] \; \xi_q[iN_t + 1] \; \cdots \; \xi_q[iN_t + N_t - 1]]^T$, and $\boldsymbol{\psi}'_q(i) \triangleq [\psi'_q[iN_t] \; \psi'_q[iN_t + 1] \; \cdots \; \psi'_q[iN_t + N_t - 1]]^T$, we can write the vector of the total noise received at Transceiver q, corresponding to the i^{th} transmitted block, as

$$\boldsymbol{\psi}_q(i) = \boldsymbol{\Upsilon}_q(i) \mathbf{G}_q \mathbf{w} + \boldsymbol{\psi}'_q(i) \tag{3.11}$$

where $\boldsymbol{\Upsilon}_q(i) \triangleq \begin{bmatrix} \mathbf{v}_{iN_t,q} & \mathbf{v}_{iN_t+1,q} & \cdots & \mathbf{v}_{(iN_t+N_t-1),q} \end{bmatrix}^T$ is an $N_t \times L$ matrix whose l^{th} column is the l^{th} relay noise.

3.1.3 Received Signal Model

The i^{th} signal block received at the output of the self-interference cancellation block at Transceiver q can be expressed as [79]

$$\bar{\mathbf{r}}_q(i) = \sqrt{P_{\bar{q}}}\mathbf{H}_0(\mathbf{w})\bar{\mathbf{s}}_{\bar{q}}(i) + \sqrt{P_{\bar{q}}}\mathbf{H}_1(\mathbf{w})\bar{\mathbf{s}}_{\bar{q}}(i-1) + \boldsymbol{\psi}_q(i) \tag{3.12}$$

where we assume that for $q \in \{1, 2\}$ and for any k and q, $\mathrm{E}\{|s_q[k]|^2\} = 1$ and $\mathrm{E}\{s_q[k]\} = 0$ hold true. In (3.12), we define $\mathbf{H}_0(\mathbf{w})$ and $\mathbf{H}_1(\mathbf{w})$ as

$$\mathbf{H}_0(\mathbf{w}) \triangleq \begin{bmatrix} h[0] & 0 & 0 & \cdots & 0 \\ \vdots & h[0] & 0 & \cdots & 0 \\ h[N-1] & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & h[N-1] & \cdots & h[0] \end{bmatrix}$$

$$\mathbf{H}_1(\mathbf{w}) \triangleq \begin{bmatrix} 0 & \cdots & h[N-1] & \cdots & h[1] \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & h[N-1] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}. \tag{3.13}$$

To combat IBI, the received signal vector $\bar{\mathbf{r}}_q(i)$ is passed through the CP removal block, and thus, its first N entries are removed. Noting that $\mathbf{R}_{\mathrm{cp}}\mathbf{H}_1(\mathbf{w}) = \mathbf{0}_{N_s \times N_t}$ holds true, the output of the CP removal block is written as $\mathbf{r}_q(i)$ as

$$\begin{aligned} \mathbf{r}_q(i) \triangleq \mathbf{R}_{\mathrm{cp}}\bar{\mathbf{r}}_q(i) &= \sqrt{P_{\bar{q}}}\mathbf{R}_{\mathrm{cp}}\mathbf{H}_0(\mathbf{w})\mathbf{T}_{\mathrm{cp}}\mathbf{s}_{\bar{q}}(i) + \mathbf{R}_{\mathrm{cp}}\boldsymbol{\psi}_q(i) \\ &= \sqrt{P_{\bar{q}}}\widetilde{\mathbf{H}}(\mathbf{w})\mathbf{s}_{\bar{q}}(i) + \widetilde{\boldsymbol{\psi}}_q(i). \end{aligned} \tag{3.14}$$

Here, $\widetilde{\boldsymbol{\psi}}_q(i) \triangleq \mathbf{R}_{\mathrm{cp}}\boldsymbol{\psi}_q(i)$ is an $N_s \times 1$ noise vector and $\widetilde{\mathbf{H}}(\mathbf{w}) \triangleq \mathbf{R}_{\mathrm{cp}}\mathbf{H}_0(\mathbf{w})\mathbf{T}_{\mathrm{cp}}$ is an $N_s \times N_s$ circulant matrix whose $(k,l)^{\mathrm{th}}$ entry is given by $[\tilde{h}[(k-l) \mod N_s]$, where $\tilde{h}[n]$ is defined as

$$\tilde{h}[n] \triangleq \begin{cases} h[n], & \text{for} \quad 0 \leq n < N-1 \\ 0, & \text{for} \quad N \leq n < N_s - 1. \end{cases} \tag{3.15}$$

Since $\tilde{\mathbf{H}}(\mathbf{w})$ may be non-diagonal in general, it can produce intra-block-interference in the received block.

3.1.4 Total Transmit Power Expression

For our purpose, we need to express the total power consumed in the whole network in terms of transceiver transmit powers, P_1 and P_2, and the relay weight vector $\mathbf{w}$. Based on our system block diagram shown in Fig. 3.1, the $N_t \times 1$ vector $\overline{\mathbf{x}}_l(i)$ of the i^{th} signal block relayed by the l^{th} relay can be written as

$$\begin{aligned}\overline{\mathbf{x}}_l(i) &\triangleq \begin{bmatrix} \overline{x}_l[iN_t] & \overline{x}_l[iN_t+1] & \cdots & \overline{x}_l[iN_t+N_t-1] \end{bmatrix}^T \\ &= w_l\left[\sqrt{P_1}g_{l1}\overline{\mathbf{s}}_1(i) + \sqrt{P_2}g_{l2}\overline{\mathbf{s}}_2(i) + \boldsymbol{\upsilon}_l(i)\right] \end{aligned} \tag{3.16}$$

where the vector $\boldsymbol{\upsilon}_l(i) \triangleq [\upsilon_l[iN_t] \;\; \upsilon_l[iN_t+1] \;\; \cdots \;\; \upsilon_l[iN_t+N_t-1]]^T$ is the i^{th} block of measurement noise at the l^{th} relay and $\overline{x}_l[t]$ is the signal transmitted by the l^{th} relay at time t. Assuming that $\boldsymbol{\upsilon}_l(\cdot)$ is a stationary zero-mean stochastic vector process whose entries are uncorrelated and have a variance of σ^2, we can write the total average transmit power of the l^{th} relay as

$$\begin{aligned}\widetilde{P}_l &\triangleq \frac{1}{N_t}\mathrm{E}\left\{\overline{\mathbf{x}}_l^H(i)\overline{\mathbf{x}}_l(i)\right\} \\ &= \frac{|w_l|^2}{N_t}\mathrm{E}\left\{\left[\sqrt{P_1}g_{l1}^*\overline{\mathbf{s}}_1^H(i) + \sqrt{P_2}g_{l2}^*\overline{\mathbf{s}}_2^H(i) + \boldsymbol{\upsilon}_l^H(i)\right] \times \left[\sqrt{P_1}g_{l1}\overline{\mathbf{s}}_1(i) + \sqrt{P_2}g_{l2}\overline{\mathbf{s}}_2(i) + \boldsymbol{\upsilon}_l(i)\right]\right\} \\ &= \frac{P_1|g_{l1}|^2|w_l|^2}{N_t}\mathrm{E}\left\{\overline{\mathbf{s}}_1^H(i)\overline{\mathbf{s}}_1(i)\right\} + \frac{P_2|g_{l2}|^2|w_l|^2}{N_t}\mathrm{E}\left\{\overline{\mathbf{s}}_2^H(i)\overline{\mathbf{s}}_2(i)\right\} + \frac{|w_l|^2}{N_t}\mathrm{E}\left\{\boldsymbol{\upsilon}_l^H(i)\boldsymbol{\upsilon}_l(i)\right\} \\ &= |w_l|^2\left(|g_{l1}|^2P_1 + |g_{l2}|^2P_2 + \sigma^2\right) \end{aligned} \tag{3.17}$$

where it is assumed that $\bar{s}_1(\cdot)$, $\bar{s}_2(\cdot)$ and $\mathbf{v}_l(\cdot)$ are zero-mean mutually independent stationary random vector processes. The last equality follows from the assumption that the largest time differences between arrivals of the transmitted signals at the relay are negligible compared to the communication time frame [68]. Using (3.17), we can represent the total power consumed in the whole network as

$$\begin{aligned} P_{total} &\triangleq P_1 + P_2 + \sum_{l=1}^{L} \widetilde{P}_l = P_1 + P_2 + \sum_{l=1}^{L} |w_l|^2 \left(|g_{l1}|^2 P_1 + |g_{l2}|^2 P_2 + \sigma^2\right) \\ &= P_1\left(1 + \|\mathbf{G}_1\mathbf{w}\|^2\right) + P_2\left(1 + \|\mathbf{G}_2\mathbf{w}\|^2\right) + \sigma^2 \mathbf{w}^H\mathbf{w}. \end{aligned} \tag{3.18}$$

Note that $\sum_{l=1}^{L} \sigma^2 |w_l|^2 |g_{lq}|^2 = \mathbf{w}^H \mathbf{D}_q \mathbf{w}$, where we define $\mathbf{D}_q \triangleq \sigma^2 \text{diag}\{|g_{lq}|^2\}$, for $q \in \{1, 2\}$. The total power can be rewritten as

$$P_{total} = \frac{1}{\sigma^2}\left(\sum_{q=1}^{2} P_q \left(\mathbf{w}^H \mathbf{D}_q \mathbf{w} + \sigma^2\right) + \sigma^4 \mathbf{w}^H \mathbf{w}\right). \tag{3.19}$$

In our design, the goal is to minimize P_{total} subject to two constraints on the transceivers' data rate.

3.2 Power Minimization

3.2.1 Problem Definition

In this section, we aim to find the relay beamforming weight vector $\mathbf{w}$ and the transceivers' transmit powers P_1 and P_2 such that the total consumed power is minimized subject to two constraints on the transceivers' data rates. Mathematically, we solve the following optimization problem:

$$\begin{aligned} \min_{\mathbf{w},P_1,P_2} \quad & P_{total} \\ \text{subject to} \quad & R_1(P_2,\mathbf{w}) \geq r_1, \quad R_2(P_1,\mathbf{w}) \geq r_2 \\ & P_1 \geq 0, \quad P_2 \geq 0 \end{aligned} \tag{3.20}$$

where $R_1(P_2,\mathbf{w})$ and $R_2(P_1,\mathbf{w})$ are the data rates achieved at Transceiver 1 and 2, respectively and r_1 and r_2 are the corresponding minimum required data rates at Transceivers 1 and 2, respectively. We now view the data model in (3.14) as a multiple input multiple output (MIMO) channel, and hence, write its corresponding data rate as [79]

$$R_q(P_{\bar{q}},\mathbf{w}) = \frac{1}{2}\log_2\left(\det\left(\mathbf{I} + P_{\bar{q}}\mathbf{C}_q(\mathbf{w})^{-1/2}\widetilde{\mathbf{H}}(\mathbf{w})\widetilde{\mathbf{H}}^H(\mathbf{w})\mathbf{C}_q(\mathbf{w})^{-1/2}\right)\right), \text{ for } q = 1,2. \tag{3.21}$$

Here, the factor 1/2 signifies that the communication occurs in two time slots, whereas $\mathbf{C}_q(\mathbf{w})$, for $q = 1,2$, is the correlation matrix of the noise $\widetilde{\boldsymbol{\psi}}_q(k)$ in (3.14) and it is given as [71]

$$\begin{aligned} \mathbf{C}_q(\mathbf{w}) &\triangleq \mathrm{E}\{\widetilde{\boldsymbol{\psi}}_q(k)\widetilde{\boldsymbol{\psi}}_q^H(k)\} \\ &= \sigma^2\left(\mathbf{w}^H\mathbf{G}_q^H\mathbf{G}_q\mathbf{w} + 1\right)\mathbf{I}_{N_s} \\ &= \sigma^2\left(\|\mathbf{G}_q\mathbf{w}\|^2 + 1\right)\mathbf{I}_{N_s} \\ &= \left(\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2\right)\mathbf{I}_{N_s} \end{aligned} \tag{3.22}$$

where in the last equality, we used the fact that $\mathbf{D}_q = \sigma^2\mathbf{G}_q^H\mathbf{G}_q$ holds true for $q = 1,2$.

3.2.2 Problem Simplification

In order to simplify (3.21), we note that matrix $\tilde{\mathbf{H}}(\mathbf{w})$ is an $N_s \times N_s$ circulant matrix, and thus, it can be decomposed as

$$\tilde{\mathbf{H}}(\mathbf{w}) = \mathbf{F}^H \mathbf{D}(\mathbf{w}) \mathbf{F}, \tag{3.23}$$

where $\mathbf{D}(\mathbf{w})$, defined as

$$\mathbf{D}(\mathbf{w}) \triangleq \mathrm{diag}\{H(e^{j0}), H(e^{j\frac{2\pi}{N_s}}), \cdots, H(e^{j\frac{2\pi(N_s-1)}{N_s}})\}, \tag{3.24}$$

is an $N_s \times N_s$ diagonal matrix of the values of frequency response of the end-to-end channel at integer multiples of $\frac{1}{N_s}$, whereas $H(e^{j2\pi f}) \triangleq \sum_{n=0}^{N-1} h[n] e^{-j2\pi f n}$ is the frequency response of the end-to-end channel at the normalized frequency f, and $\mathbf{F}$ is the $N_s \times N_s$ DFT matrix whose $(k, k')^{\text{th}}$ element is defined as $\mathbf{F}(k, k') = N_s^{-\frac{1}{2}} e^{-j2\pi(k-1)(k'-1)/N_s}$, for $k, k' \in \{1, 2, \cdots, N_s\}$. Using (3.22) and (3.23) in (3.21), $R_q(P_{\bar{q}}, \mathbf{w})$ can be written as

$$\begin{aligned} R_q(P_{\bar{q}}, \mathbf{w}) &= \frac{1}{2}\log_2 \left(\det \left(\mathbf{I} + P_{\bar{q}} \mathbf{C}_q^{-1/2}(\mathbf{w}) \tilde{\mathbf{H}}(\mathbf{w}) \tilde{\mathbf{H}}^H(\mathbf{w}) \mathbf{C}_q^{-1/2}(\mathbf{w}) \right) \right) \\ &= \frac{1}{2}\log_2 \left(\det \left(\mathbf{I} + \frac{P_{\bar{q}}}{\mathbf{w}^H \mathbf{D}_q \mathbf{w} + \sigma^2} \mathbf{F}^H \mathbf{D}(\mathbf{w}) \mathbf{F} \mathbf{F}^H \mathbf{D}^H(\mathbf{w}) \mathbf{F} \right) \right). \end{aligned} \tag{3.25}$$

Using the fact that $\mathbf{F}\mathbf{F}^H = \mathbf{I}$ holds true, we can write (3.25) as

$$\begin{aligned} R_q(P_{\bar{q}}, \mathbf{w}) &= \frac{1}{2}\log_2\left(\det\left(\mathbf{I} + \frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2}\mathbf{F}^H\mathbf{D}(\mathbf{w})\mathbf{D}^H(\mathbf{w})\mathbf{F}\right)\right) \\ &= \frac{1}{2}\log_2\left(\det\left(\mathbf{F}^H\left(\mathbf{I} + \frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2}\mathbf{D}(\mathbf{w})\mathbf{D}^H(\mathbf{w})\right)\mathbf{F}\right)\right) \\ &= \frac{1}{2}\log_2\left(\det\left(\mathbf{F}^H\mathbf{F}\left(\mathbf{I} + \frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2}\mathbf{D}(\mathbf{w})\mathbf{D}^H(\mathbf{w})\right)\right)\right) \\ &= \frac{1}{2}\log_2\left(\det\left(\mathbf{I} + \frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2}\mathbf{D}(\mathbf{w})\mathbf{D}^H(\mathbf{w})\right)\right) \end{aligned} \tag{3.26}$$

where we use the fact that for any two square matrices $\mathbf{X}$ and $\mathbf{Y}$, the identity $\det(\mathbf{XY}) = \det(\mathbf{YX})$ holds true. To simplify (3.26), let us write matrix $\mathbf{D}(\mathbf{w})$ in (3.24) as

$$\mathbf{D}(\mathbf{w}) = \sqrt{N_s}\text{diag}\{\mathbf{f}_1^H\tilde{\mathbf{h}}(\mathbf{w}), \mathbf{f}_2^H\tilde{\mathbf{h}}(\mathbf{w}), \cdots, \mathbf{f}_{N_s}^H\tilde{\mathbf{h}}(\mathbf{w})\} \tag{3.27}$$

where we define the $N_s \times 1$ vector $\mathbf{f}_k$ as

$$\mathbf{f}_k \triangleq \frac{1}{\sqrt{N_s}}\begin{bmatrix} 1 & e^{j\frac{2\pi(k-1)}{N_s}} & \dots & e^{j\frac{2(N_s-1)(k-1)\pi}{N_s}} \end{bmatrix}^T, \quad \text{for} \quad k = 1, 2, \dots, N_s \tag{3.28}$$

and $\tilde{\mathbf{h}}(\mathbf{w})$ is the zero-padded version of the channel vector $\mathbf{h}(\mathbf{w})$, that is, using (3.15), we can write

$$\begin{aligned} \tilde{\mathbf{h}}(\mathbf{w}) &\triangleq [\mathbf{h}^T(\mathbf{w}) \;\; \mathbf{0}_{1\times(N_s-N)}]^T \\ &= [h[0] \; h[1] \; \cdots \; h[N_s-1] \;\; \mathbf{0}_{1\times(N_s-N)}]^T \\ &= \tilde{\mathbf{B}}\mathbf{w} \end{aligned}$$

where we define $\tilde{\mathbf{B}} \triangleq [\mathbf{B}^T \;\; \mathbf{0}^T_{(N_s-N)\times L}]^T$. Using (3.27), we can rewrite (3.26) as

$$R_q(P_{\bar{q}}, \mathbf{w}) = \frac{1}{2}\log_2\left(\prod_{k=1}^{N_s}\left(1+\frac{P_{\bar{q}}}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}|D_{kk}(\mathbf{w})|^2\right)\right) \tag{3.29}$$

where $D_{kk}(\mathbf{w})$ is the k^{th} diagonal entry of $\mathbf{D}(\mathbf{w})$. Now, using (3.19) and (**??**), the optimization problem (3.20) can be rewritten as

$$\begin{aligned}
\min_{\mathbf{w},P_1,P_2} \quad & \frac{1}{\sigma^2}\left(\sum_{q=1}^{2} P_q\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right)\\
\text{subject to} \quad & \frac{1}{2}\log_2\left(\prod_{k=1}^{N_s}\left(1+\frac{P_{\bar{q}}|D_{kk}(\mathbf{w})|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}\right)\right) \geq r_q, \text{ for } q=1,2.\\
& P_1 \geq 0 \quad, P_2 \geq 0.
\end{aligned} \tag{3.30}$$

At the optimum, the constraints in (3.30) will be satisfied with equality, otherwise, if at the optimum, any of these constraints is satisfied with inequality, one can scale down the corresponding transceiver's transmit power to satisfy the constraint with equality while furthermore reducing the objective function, thereby contradicting the optimality. Hence, we rewrite the optimization problem (3.30) as

$$\begin{aligned}
\min_{\mathbf{w},P_1,P_2} \quad & \frac{1}{\sigma^2}\left(\sum_{q=1}^{2} P_q\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right)\\
\text{subject to} \quad & \log_2\left(\prod_{k=1}^{N_s}\left(1+\frac{P_{\bar{q}}|D_{kk}(\mathbf{w})|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2}\right)\right) = 2r_q, \text{ for } q=1,2.\\
& P_1 \geq 0 \quad, P_2 \geq 0.
\end{aligned} \tag{3.31}$$

We define a set of new optimization variables γ_{qk} , for $k = 1, \cdots, N_s$ and $q = 1, 2$ as

$$\gamma_{qk} \triangleq \frac{P_{\bar{q}}|D_{kk}(\mathbf{w})|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2} = \frac{P_{\bar{q}}|\mathbf{f}_k^H\tilde{\mathbf{h}}(w)|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2} = \frac{P_{\bar{q}}|\mathbf{f}_k^H\tilde{\mathbf{B}}\mathbf{w}|^2}{\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2}, \tag{3.32}$$

where the first and second equalities follow from the expression of $\mathbf{D}(\mathbf{w})$ in (3.27) and that of $\mathbf{h}(\mathbf{w})$ in (3.9), respectively. Then using (3.32), we rewrite the first two constraints in (3.31) as

$$\sum_{k=1}^{N_s} \log_2(1 + \gamma_{qk}) = 2r_q, \qquad \text{for } q = 1, 2. \tag{3.33}$$

Using (3.32), we can write $P_{\bar{q}}$ as

$$P_{\bar{q}} = \frac{\gamma_{qk}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w} + \sigma^2\right)}{|\mathbf{f}_k^H\tilde{\mathbf{B}}\mathbf{w}|^2}. \tag{3.34}$$

Since (3.34) holds for any $k \in \{1, 2, \cdots, N_s\}$, we have the following constraints on $\{\gamma_{qk}\}_{k=1}^{N_s}$:

$$\frac{\gamma_{qk}}{|\mathbf{a}_k^H\mathbf{w}|^2} = \frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H\mathbf{w}|^2} \quad \text{for } q \in \{1, 2\} \tag{3.35}$$

where, we define $\mathbf{a}_k$ as

$$\mathbf{a}_k \triangleq \tilde{\mathbf{B}}^H\mathbf{f}_k, \quad \text{for } \ k \in \{1, 2, \cdots, N_s\}. \tag{3.36}$$

Also, in order to ensure that $P_q \geq 0$ holds true, we must ensure that $\gamma_{qk} \geq 0$, for $k \in \{1, 2, \cdots, N_s\}$ and $q \in \{1, 2\}$. Using (3.35), we can rewrite (3.34) as

$$P_{\bar{q}} = \frac{1}{N_s}\left(\mathbf{w}^H \mathbf{D}_q \mathbf{w} + \sigma^2\right) \sum_{k=1}^{N_s} \frac{\gamma_{qk}}{|\mathbf{a}_k^H \mathbf{w}|^2}. \tag{3.37}$$

Using (3.37), we now rewrite the objective function in (3.31) as

$$P_{total} = \frac{1}{\sigma^2}\left(\frac{1}{N_s}\sum_{k=1}^{N_s} \frac{\gamma_{1k} + \gamma_{2k}}{|\mathbf{a}_k^H \mathbf{w}|^2} \prod_{q=1}^{2}\left(\mathbf{w}^H \mathbf{D}_q \mathbf{w} + \sigma^2\right) + \sigma^4 \mathbf{w}^H \mathbf{w}\right). \tag{3.38}$$

We now define two vectors $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ as

$$\boldsymbol{\gamma}_1 \triangleq [\gamma_{11}\ \gamma_{21} \cdots\ \gamma_{N_s 1}]^T \tag{3.39}$$

$$\boldsymbol{\gamma}_2 \triangleq [\gamma_{12}\ \gamma_{22} \cdots\ \gamma_{N_s 2}]^T. \tag{3.40}$$

Using (3.38), (3.39), and (3.40) the optimization problem (3.31) can be rewritten as

$$\begin{aligned}
\min_{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \mathbf{w}} \quad & \frac{1}{\sigma^2}\left(\frac{1}{N_s}\sum_{k=1}^{N_s} \frac{\gamma_{1k} + \gamma_{2k}}{|\mathbf{a}_k^H \mathbf{w}|^2} \prod_{q=1}^{2}\left(\mathbf{w}^H \mathbf{D}_q \mathbf{w} + \sigma^2\right) + \sigma^4 \mathbf{w}^H \mathbf{w}\right) \\
\text{subject to} \quad & \frac{1}{2}\sum_{k=1}^{N_s} \log_2\left(1 + \gamma_{1k}\right) = r_1 \\
& \frac{1}{2}\sum_{k=1}^{N_s} \log_2\left(1 + \gamma_{2k}\right) = r_2 \\
& \frac{\gamma_{qk}}{|\mathbf{a}_k^H \mathbf{w}|^2} = \frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H \mathbf{w}|^2} \qquad \text{for } k, k' \in \{1, 2, \cdots, N_s\} \text{ and } q \in \{1, 2\} \\
& \gamma_{qk} \geq 0.
\end{aligned} \tag{3.41}$$

3.2.3 Relaxing the Problem and Solving the Relaxed Problem

To solve (3.41), let us consider the following optimization problem:

$$\begin{aligned}\min_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\mathbf{w}} \quad & \frac{1}{\sigma^2}\left(\frac{1}{N_s}\sum_{k=1}^{N_s}\frac{\gamma_{1k}+\gamma_{2k}}{|\mathbf{a}_k^H\mathbf{w}|^2}\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right)\\ \text{subject to} \quad & \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)=r_1\\ & \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=r_2, \end{aligned} \tag{3.42}$$

where we have considered only the first two constraints in (3.41). Let P_T^{min} denote the minimum value of P_{total} obtained by solving (3.42) and let the corresponding optimal value of the optimization variables be given by $(\boldsymbol{\gamma}_1^\circ, \boldsymbol{\gamma}_2^\circ, \mathbf{w}^\circ)$. We now consider the following optimization problem:

$$\begin{aligned}\max_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2,\mathbf{w}} \quad & \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)\\ \text{subject to} \quad & \frac{1}{2}\sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=r_2\\ & \frac{1}{\sigma^2}\left(\frac{1}{N_s}\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\sum_{k=1}^{N_s}\frac{\gamma_{1k}+\gamma_{2k}}{|\mathbf{a}_k^H\mathbf{w}|^2}+\sigma^4\mathbf{w}^H\mathbf{w}\right)\le P_T^{min}. \end{aligned} \tag{3.43}$$

Let $(\hat{\boldsymbol{\gamma}}_1^\circ, \hat{\boldsymbol{\gamma}}_1^\circ, \hat{\mathbf{w}}_1^\circ)$ be the solution to the optimization problem (3.43) and ${R_1}^{max}$ denote the maximum achievable value for the rate of Transceiver 1 obtained by solving (3.43) for the given budget P_T^{min}. We now argue that ${R_1}^{max} = r_1$ holds true. To show this, if ${R_1}^{max} < r_1$, then $(\boldsymbol{\gamma}_1^\circ, \boldsymbol{\gamma}_2^\circ, \mathbf{w}^\circ)$ results in a higher value for the objective function of (3.43). Indeed,

$(\boldsymbol{\gamma}_1^\circ, \boldsymbol{\gamma}_2^\circ, \mathbf{w}^\circ)$ being a solution to (3.42), results in a higher value for $\frac{1}{2}\sum_{k=1}^{N_s}\log_2(1+\gamma_{1k})$, while $\frac{1}{2}\sum_{k=1}^{N_s}\log_2(1+\gamma_{2k}) = r_2$, and at the same time, $P_{total} = P_T^{min}$ and this contradicts the optimality of $(\hat{\boldsymbol{\gamma}}_1^\circ, \hat{\boldsymbol{\gamma}}_1^\circ, \hat{\mathbf{w}}_1^\circ)$ for (3.43). On the other hand, it is easy to show that ${R_1}^{max}$ cannot be greater than r_1 either. Otherwise, if ${R_1}^{max} > r_1$, then one can scale down the optimal value $\hat{\boldsymbol{\gamma}}_1^\circ$ and such that $\frac{1}{2}\sum_{k=1}^{N_s}\log_2(1+\gamma_{1k}) = r_1$ holds true while $P_{total} < P_T^{min}$. In other words, the $(\hat{\boldsymbol{\gamma}}_1^\circ, \hat{\boldsymbol{\gamma}}_1^\circ, \hat{\mathbf{w}}_1^\circ)$ results in a lower P_{total} while satisfying the two constraints in (3.42). This contradicts the optimality of $(\boldsymbol{\gamma}_1^\circ, \boldsymbol{\gamma}_2^\circ, \mathbf{w}^\circ)$ for (3.42). Therefore, we can conclude that ${R_1}^{max} = r_1$ holds true meaning that any solution to the optimization problem (3.43) is indeed a solution to the optimization problem (3.42) and any solution to the optimization problem (3.42) inherits all the properties of the solutions to (3.43).

Optimization problem (3.43) has been studied in [69] where it has been shown that the optimal $\mathbf{w}$ belongs to the set $\mathcal{W}$ defined as

$$\mathcal{W} = \left\{\mathbf{w}\middle| |\mathbf{a}_k^H\mathbf{w}| = |\mathbf{a}_{k'}^H\mathbf{w}|, \forall k, k' \in \{1, 2, \cdots, N_s\}\right\}. \tag{3.44}$$

Since the solution to (3.42) inherits all the properties of the solutions to (3.43) , the optimal $\mathbf{w}$ of (3.42) also belongs to the set $\mathcal{W}$, i.e,

$$\mathbf{w} \in \mathcal{W}. \tag{3.45}$$

Hence, without any loss of optimality, we can restrict $\mathbf{w}$ such that it belongs to set $\mathcal{W}$ and rewrite the optimization problem (3.42) as

$$\begin{aligned}
\min_{\mathbf{w}} \min_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2} \quad & \frac{1}{\sigma^2}\left(\frac{1}{N_s}\prod_{q=1}^{2}\left(\mathbf{w}^H\mathbf{D}_q\mathbf{w}+\sigma^2\right)\frac{1}{|\mathbf{a}_k^H\mathbf{w}|^2}\left(\sum_{k=1}^{N_s}\gamma_{1k}+\sum_{k=1}^{N_s}\gamma_{2k}\right)+\sigma^4\mathbf{w}^H\mathbf{w}\right) \\
\text{subject to} \quad & \sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)=2r_1 \\
& \sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=2r_2 \\
& \mathbf{w}\in\mathcal{W}.
\end{aligned} \tag{3.46}$$

3.2.4 Solution to the Original Optimization Problem

We now show that any solution to the optimization problem (3.46) satisfies the constraints $\frac{\gamma_{qk}}{|\mathbf{a}_k^H\mathbf{w}|^2}=\frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H\mathbf{w}|^2}$ and $\gamma_{qk}\geq 0$ for $q\in\{1,2\}$, and thus, it is a solution to the optimization problem (3.41). To show this, we note that the inner minimization problem in (3.46) is equivalent to two separate minimization problems which are given as

$$\min_{\boldsymbol{\gamma}_1} \quad \sum_{k=1}^{N_s}\gamma_{1k}, \quad \text{subject to} \sum_{k=1}^{N_s}\log_2\left(1+\gamma_{1k}\right)=2r_1 \tag{3.47}$$

and

$$\min_{\boldsymbol{\gamma}_2} \quad \sum_{k=1}^{N_s}\gamma_{2k}, \quad \text{subject to} \sum_{k=1}^{N_s}\log_2\left(1+\gamma_{2k}\right)=2r_2. \tag{3.48}$$

In order to further simplify (3.47) and (3.48), we use the following lemma.

Lemma 3.2.1 *Consider the following optimization problem*

$$\begin{aligned} \min_{\mathbf{x}} \quad & \sum_{i=1}^{M} x_i \\ \textit{subject to} \quad & \sum_{i=1}^{M} log_2\left(1+x_i\right) = cte \end{aligned} \tag{3.49}$$

where $\mathbf{x} = [x_1\ x_2\ \cdots x_M]$ *is an* $1 \times M$ *vector. At the optimum of* (3.49)*, all* x_i*'s are equal for all* i *(i.e.,* $x_i = x_j$ *, for* $i, j \in \{1, 2, \cdots, M\}$ *).*

Proof: See the appendix.

Using Lemma 1, we conclude that for the optimization problems (3.47) and (3.48), at the optimum, $\gamma_{1k} = \gamma_{1k'} \triangleq \beta_1$ and $\gamma_{2k} = \gamma_{2k'} \triangleq \beta_2$ for any k and k'. Hence, we can rewrite the constraints in (3.46), as $\beta_1 = 2^{\frac{2r_1}{N_s}} - 1$ and $\beta_2 = 2^{\frac{2r_2}{N_s}} - 1$, and this, $\beta_1 = \gamma_{1k} \geq 0$ and $\beta_2 = \gamma_{2k} \geq 0$ hold true. Now, due to the fact that any optimal $\mathbf{w}$ belongs to $\mathcal{W}$ (i.e., $|\mathbf{a}_k^H \mathbf{w}| = |\mathbf{a}_{k'}^H \mathbf{w}|$ holds true for any k and k') we observe that $\frac{\gamma_{qk}}{|\mathbf{a}_k^H \mathbf{w}|^2} = \frac{\gamma_{qk'}}{|\mathbf{a}_{k'}^H \mathbf{w}|^2}$ holds true for any k and k'. Hence, any solution to (3.46) is also a solution to (3.41). As a result, we can solve (3.46) thereby providing a solution to (3.41). In order to solve (3.46), we can write the set $\mathcal{W}$ in (3.44) as [71]

$$\mathcal{W} = \bigcup_{n=0}^{N-1} \mathcal{U}_n \tag{3.50}$$

where $\mathcal{U}_n$ is the set of the relay weight vectors such that only the n^{th} tap of the end-to-end CIR is non-zero and the remainder of the taps are zero[2]. Since each relay contributes only

[2]Note that if no relay contributes to the n^{th} tap of the end-to-end CIR, $\mathcal{U}_n$ will be empty.

to one tap of the end-to-end CIR, we have the following condition on $\mathcal{U}_n$:

$$\mathcal{U}_n \bigcap \mathcal{U}_{n'} = \emptyset, \text{ for } n \neq n'. \tag{3.51}$$

Now, for any $\mathbf{w} \in \mathcal{W}$, we can write

$$|\mathbf{a}_k^H \mathbf{w}|^2 = \frac{1}{N_s} \sum_{k=1}^{N_s} |\mathbf{a}_k^H \mathbf{w}|^2 = \frac{1}{N_s} \sum_{k=1}^{N_s} |\mathbf{f}_k^H \mathbf{B}\mathbf{w}|^2, \tag{3.52}$$

where the first equality follows from the fact that for $\mathbf{w} \in \mathcal{U}_n$, $|\mathbf{a}_k^H \mathbf{w}|^2 = |\mathbf{a}_{k'}^H \mathbf{w}|^2$ holds true, for any k and k' and the second equality follows from the definition of $\mathbf{a}_k$ in (3.36).

Using Parseval's theorem, we can write

$$\sum_{k=1}^{N_s} |\mathbf{f}_k^H \tilde{\mathbf{B}}\mathbf{w}|^2 = \|\tilde{\mathbf{B}}\mathbf{w}\|^2 = \mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w} \tag{3.53}$$

where the second equality follows from the fact that the last $N_s - N$ rows of $\tilde{\mathbf{B}}\mathbf{w}$ are zero.

Using (3.52) and (3.53), the optimization problem (3.42) can be rewritten as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{1}{\sigma^2}\left(N_s \left(\mathbf{w}^H \mathbf{D}_1 \mathbf{w} + \sigma^2\right)\left(\mathbf{w}^H \mathbf{D}_2 \mathbf{w} + \sigma^2\right) \frac{\beta_1 + \beta_2}{\mathbf{w}^H \mathbf{B}^H \mathbf{B} \mathbf{w}} + \sigma^4 \mathbf{w}^H \mathbf{w}\right) \\ \text{subject to} \quad & \mathbf{w} \in \bigcup_{n=0}^{N-1} \mathcal{U}_n. \end{aligned} \tag{3.54}$$

Noting that as the sets $\{\mathcal{U}_n\}_{n=0}^{N-1}$ are mutually exclusive, we can decompose the optimization problem (3.54) into a set of N subproblems, and then, solve each of these subproblems separately. Therefore, we will have N candidate values for the optimal $\mathbf{w}$ and the one which leads to the smallest value for the total transmit power will be our solution.

In order to solve the optimization problem in (3.54), let us introduce $\mathbf{w}_n$ which is the vector of the weights of those relays which contribute to the n^{th} tap of the end-to-end CIR. For any $\mathbf{w} \in \mathcal{W}$, we can write

$$\tilde{\mathbf{h}}^H(\mathbf{w})\tilde{\mathbf{h}}(\mathbf{w}) = \mathbf{h}^H(\mathbf{w})\mathbf{h}(\mathbf{w}) = \mathbf{w}^H\mathbf{B}^H\mathbf{B}\mathbf{w} = \mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n, \tag{3.55}$$

where $\mathbf{b}_n$ is an the $L_n \times 1$ vector whose i^{th} entry is equal to the i^{th} non-zero entry of the $(n+1)^{\text{th}}$ column of matrix $\mathbf{B}^H$ and L_n is the number of the non-zero entries of the $(n+1)^{\text{th}}$ column of matrix $\mathbf{B}^H$. Now, we can solve N separate optimization problems, then find the n which leads to the minimum value for the objective function. Indeed, the optimal value of n is obtained such that the total power of the network is minimized. Using (3.55), the optimization problem (3.54) can be rewritten as

$$\min_{0\leq n\leq N-1} \min_{\mathbf{w}_n} \frac{1}{\sigma^2}\left(\frac{(\beta_1' + \beta_2')\left(\mathbf{w}^H\mathbf{D}_1^{(n)}\mathbf{w} + \sigma^2\right)\left(\mathbf{w}^H\mathbf{D}_2^{(n)}\mathbf{w} + \sigma^2\right)}{\mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n} + \sigma^4\mathbf{w}_n^H\mathbf{w}_n\right) \tag{3.56}$$

where $\beta_1' \triangleq N_s\beta_1$ and $\beta_2' \triangleq N_s\beta_2$ and $\mathbf{D}_q^{(n)}$, for $q = 1, 2$, is a diagonal matrix whose diagonal entries are a subset of those diagonal entries of $\mathbf{D}_q$ which correspond to the relays which contribute to the n^{th} tap of the end-to-end CIR. Defining $\mathbf{Q}_q^{(n)} \triangleq \frac{1}{\sigma^2}\mathbf{D}_q^{(n)}$, for $q = 1, 2$, we can further simplify (3.56) as

$$\min_{0\leq n\leq N-1} \min_{\mathbf{w}_n} \sigma^2\left(\frac{(\beta_1' + \beta_2')\left(1 + \mathbf{w}_n^H\mathbf{Q}_1^{(n)}\mathbf{w}_n\right)\left(1 + \mathbf{w}_n^H\mathbf{Q}_2^{(n)}\mathbf{w}_n\right)}{\mathbf{w}_n^H\mathbf{b}_n\mathbf{b}_n^H\mathbf{w}_n} + \mathbf{w}_n^H\mathbf{w}_n\right). \tag{3.57}$$

It is now well-known that the inner optimization problem in (3.57) is amenable to a semi-closed-form solution for the optimal $\mathbf{w}_n$, denoted $\mathbf{w}_n^{\mathrm{o}}$, which is given by [78]

$$\mathbf{w}_n^{\mathrm{o}} = \kappa_n \sqrt{\frac{z_n(\beta_1' + \beta_2')}{\lambda_n}} ((\beta_1' + \beta_2')\mathbf{Q}_2^{(n)} + \lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-1}\mathbf{b}_n \tag{3.58}$$

where

$$\kappa_n = \frac{1}{\sqrt{z_n\mathbf{b}_n^H(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n})((\beta_1' + \beta_2')\mathbf{Q}_2^{(n)} + \lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}\mathbf{b}_n}}\,. \tag{3.59}$$

Here, z_n is an intermediate parameter which can be obtained by solving the following non-linear equation:

$$\hbar_n(z_n) \triangleq \frac{1/z_n^2 + \lambda_n\mathbf{b}_n^H((\beta_1' + \beta_2')\,\mathbf{Q}_2^{(n)} - \lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}\mathbf{Q}_1^{(n)}\mathbf{b}_n}{\lambda_n^2\mathbf{b}_n^H((\beta_1' + \beta_2')\,\mathbf{Q}_2^{(n)} + \lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n})\mathbf{b}_n} - \frac{1}{\beta_1' + \beta_2'} = 0$$

and λ_n is the unique positive root of the following non-linear equation:

$$\sum_{i=1}^{L_n} \frac{z_n(z_n|g_{i1}^{(n)}|^2 + 1)^{-1}|g_{i1}^{(n)}|^2|g_{i2}^{(n)}|^2}{(\beta_1' + \beta_2')|g_{i2}^{(n)}|^2(z_n|g_{i1}^{(n)}|^2 + 1)^{-1} + \lambda_n} = 1. \tag{3.60}$$

Here, $g_{iq}^{(n)}$ represents the channel coefficient between Transceiver q and the i^{th} relay which contributes to the n^{th} tap. Now, in (3.57), we can find the optimal tap n, for $n = 0, 1, \ldots, N-1$, by determining which $\{\mathbf{w}_n^{\mathrm{o}}\}_{n=0}^{N-1}$ yields the smallest value for the objective function in (3.57). Hence, the optimal value of n, denoted as n^{o} is obtained as

$$n^{\mathrm{o}} = \arg \min_{0 \leq n \leq N-1} \quad \sigma^2 \left(\frac{(\beta_1' + \beta_2') \left(\mathbf{w}_n^{\mathrm{o},H} \mathbf{Q}_1^{(n)} \mathbf{w}_n^{\mathrm{o}} + 1 \right) \left(\mathbf{w}_n^{\mathrm{o},H} \mathbf{Q}_2^{(n)} \mathbf{w}_n^{\mathrm{o}} + 1 \right)}{\mathbf{w}_n^{\mathrm{o},H} \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n} + \mathbf{w}_n^{\mathrm{o},H} \mathbf{w}_n^{\mathrm{o}} \right) \tag{3.61}$$

which means that the set of the relays which contribute to the tap n^{o} of the end-to-end CIR leads to the smallest value for the total transmit power among other relay sets. The search for the optimal value of n is restricted only to the non-zero taps of $h[\cdot]$. If for a certain value of n, no relay contributes to $h[n]$, then $h[n] = 0$, which means that the $(n+1)^{\mathrm{th}}$ row of matrix B is zero. Having n^{o}, we can obtain the optimal values of P_1, and P_2, denoted as P_1^{o} and P_2^{o}, respectively as

$$P_1^{\mathrm{o}} = \frac{\beta_2' \left(1 + \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{Q}_2^{(n^{\mathrm{o}})} \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}} \right)}{\mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{b}_{n^{\mathrm{o}}} \mathbf{b}_{n^{\mathrm{o}}}^H \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}}} \tag{3.62}$$

$$P_2^{\mathrm{o}} = \frac{\beta_1' \left(1 + \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{Q}_1^{(n^{\mathrm{o}})} \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}} \right)}{\mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o},H} \mathbf{b}_{n^{\mathrm{o}}} \mathbf{b}_{n^{\mathrm{o}}}^H \mathbf{w}_{n^{\mathrm{o}}}^{\mathrm{o}}}. \tag{3.63}$$

Our proposed algorithm is summarized as Algorithm 1.

Algorithm 1 : Proposed power-optimal network and beamforming

1: Set $n = 0$.

2: If no relays contributes to the n^{th} tap of $h[\cdot]$ (i.e., if the $(n+1)^{\text{th}}$ row of the matrix $\mathbf{B}$ is zero), go to Step 12.

3: Let $\mathbf{b}_n$ be an the $L_n \times 1$ vector whose i^{th} entry is equal to the i^{th} non-zero entry of the $(n+1)^{\text{th}}$ column of matrix $\mathbf{B}^H$ and L_n be the number of the non-zero entries of the $(n+1)^{\text{th}}$ column of matrix $\mathbf{B}^H$. Define $\mathbf{Q}_q^{(n)} \triangleq \frac{1}{\sigma^2}\mathbf{G}_q^{(n)}$, for $q = 1, 2$, where $\mathbf{G}_q^{(n)}$ is an $L_n \times L_n$ diagonal matrix whose diagonal entries are a subset of the diagonal entries of $\mathbf{G}_q$ which correspond to those relays which contribute to the n^{th} tap of the end-to-end CIR. Define $g_{iq}^{(n)}$ as the channel coefficient between Transceiver q and the i^{th} relay which contributes to the n^{th} tap of the end-to-end CIR. Let $\mathbf{g}_1^{(n)}$ be the channel vector between the Transceiver 1 and the relays which contribute to the n^{th} tap. Assume N_s as the number of symbols and define $\beta_1' \triangleq N_s\left(2^{\frac{2r_1}{N_s}} - 1\right)$ and $\beta_2' \triangleq N_s\left(2^{\frac{2r_2}{N_s}} - 1\right)$. Also, define $\hbar_n(z)$ as

$$\hbar_n(z) \triangleq \frac{1/z^2 + \lambda_n \mathbf{b}_n^H((\beta_1'+\beta_2')\mathbf{Q}_2^{(n)} - \lambda_n(z\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}\mathbf{Q}_1^{(n)}\mathbf{b}_n}{\lambda_n^2 \mathbf{b}_n^H((\beta_1'+\beta_2')\mathbf{Q}_2^{(n)} + \lambda_n(z\mathbf{Q}_1^{(n)} + \mathbf{I}))^{-2}(z\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n})\mathbf{b}_n} - \frac{1}{\beta_1'+\beta_2'}, \quad \text{for } n = 0, 1, \ldots, N-1$$

where for any given value of z, λ_n is the unique positive root of the following non-linear equation:

$$\sum_{i=1}^{L_n} \frac{z(z|g_{i1}^{(n)}|^2+1)^{-1}|g_{i1}^{(n)}|^2|g_{i2}^{(n)}|^2}{(\beta_1'+\beta_2')|g_{i2}^{(n)}|^2(z|g_{i1}^{(n)}|^2+1)^{-1}+\lambda_n} = 1, \quad \text{for } n = 0, 1, \ldots, N-1.$$

4: Let $p_l \triangleq \dfrac{\beta_1'+\beta_2'}{\|\mathbf{g}_1^{(n)}\|^2}$, choose p_u to be a sufficiently large number, and let ϵ to an arbitrarily small real-valued scalar.

5: Set $k = 1$ and choose $z_n^{(k)} = (p_l + p_u)/2$

6: If for $\hbar_n(z_n^{(k)}) > 0$, then set $p_u = z_n^{(k)}$. If $\hbar_n(z_n^{(k)}) < 0$, then set $p_l = z_n^{(k)}$.

7: $z_n^{(k+1)} = (p_l + p_u)/2$

8: If $|z_n^{(k+1)} - z_n^{(k)}| > \epsilon$, then $k = k + 1$ and go to Step 6. Otherwise go to Step 9

9: Set $z_n = z_n^{(k)}$ and calculate κ_n using

$$\kappa_n = \frac{1}{\sqrt{z_n \mathbf{b}_n^H(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))\lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-2}\mathbf{b}_n}}$$

10: Obtain the optimal value of the weight vector $\mathbf{w}_n$, denoted as $\mathbf{w}_n^{\text{o}}$, as

$$\mathbf{w}_n^{\text{o}} = \kappa_n \sqrt{\frac{z_n(\beta_1'+\beta_2')}{\lambda_n}}((\beta_1'+\beta_2')\mathbf{Q}_2^{(n)} + \lambda_n(z_n\mathbf{Q}_1^{(n)} + \mathbf{I}_{L_n}))^{-1}\mathbf{b}_n$$

11: Calculate the cost function $f_n(\mathbf{w}_n^{\text{o}})$ as

$$f_n(\mathbf{w}_n^{o}) = \sigma^2 \left(\frac{(\beta_1' + \beta_2') \left(\mathbf{w}_n^{\text{o},H} \mathbf{Q}_1^{(n)} \mathbf{w}_n^{\text{o}} + 1\right) \left(\mathbf{w}_n^{\text{o},H} \mathbf{Q}_2^{(n)} \mathbf{w}_n^{\text{o}} + 1\right)}{\mathbf{w}_n^{\text{o},H} \mathbf{b}_n \mathbf{b}_n^H \mathbf{w}_n^{\text{o}}} + \mathbf{w}_n^{\text{o},H} \mathbf{w}_n^{\text{o}} \right)$$

12: Set $n = n + 1$, if $n \geq N$ go to the next step, otherwise go to Step 2.

13: Let $\mathbf{w}_{opt}$ denote the optimal relay weight vector. If the l^{th} relay contributes to tap n^{o} of the end-to-end CIR, then the l^{th} entry of $\mathbf{w}_{opt}$ is equal to the element of $\mathbf{w}_{n^{\text{o}}}^{\text{o}}$ which corresponds to the l^{th} relay and if the l^{th} relay does not contribute to tap n^{o} of the end-to-end CIR, then the l^{th} entry of $\mathbf{w}_{opt}$ is zero.

14: Calculate the transceiver transmit powers as

$$P_1^{\text{o}} = \frac{\beta_2' \left(1 + \mathbf{w}_{n^{\text{o}}}^{\text{o},H} \mathbf{Q}_2^{(n^{\text{o}})} \mathbf{w}_{n^{\text{o}}}^{\text{o}}\right)}{\mathbf{w}_{n^{\text{o}}}^{\text{o},H} \mathbf{b}_{n^{\text{o}}} \mathbf{b}_{n^{\text{o}}}^H \mathbf{w}_{n^{\text{o}}}^{\text{o}}} \tag{3.64}$$

$$P_2^{\text{o}} = \frac{\beta_1' \left(1 + \mathbf{w}_{n^{\text{o}}}^{\text{o},H} \mathbf{Q}_1^{(n^{\text{o}})} \mathbf{w}_{n^{\text{o}}}^{\text{o}}\right)}{\mathbf{w}_{n^{\text{o}}}^{\text{o},H} \mathbf{b}_{n^{\text{o}}} \mathbf{b}_{n^{\text{o}}}^H \mathbf{w}_{n^{\text{o}}}^{\text{o}}}. \tag{3.65}$$

3.3 Simulation Results

We model a single-carrier asynchronous two-way AF relay network where $L = 60$ single-antenna relays are used to exchange information between two transceivers T_1 and T_2. Using a block transmission scheme, the signals are transmitted in blocks of $N_s = 64$ symbols. In each simulation run, the frequency-flat channel coefficient between each relay and each transceiver is modeled as an independent zero-mean complex Gaussian random variable and has a variance which is inversely proportional to the path loss. The path loss corresponding to the propagation between each transceiver and relay is assumed to be proportional to the corresponding propagation delay to the power of 3. For each transceiver-relay link, the propagation delay is assumed to be a random variable which is uniformly distributed in the interval $[T_s,\ 4T_s]$.

Fig. 3.2 shows the average minimum total transmit power P_{total}, the average of the corresponding relay transmit powers P_r, and the average of the corresponding transmit transceiver powers P_1 and P_2 versus $r = r_1 = r_2$ in dB. In this figure, the performance of our proposed method scheme is compared with two approaches, namely, the equal power allocation (EPA) technique and the modified equal power allocation (MEPA) method. In the EPA algorithm, the total power of the system is uniformly distributed among all nodes including Transceivers 1 and 2 and all the relays. Hence, each node in the system has $1/(L + 2)$ fraction of the total power. In the MEPA approach, half of the total power is allocated to the two transceivers and the other half is distributed uniformly among all the relays. As shown in Fig. 3.2, our proposed method significantly outperforms the EPA algorithm and also has better performance compared to the MEPA scheme. Also, it is shown in Fig. 3.2 that for our algorithm, the average relay transmit power is always 3 dB

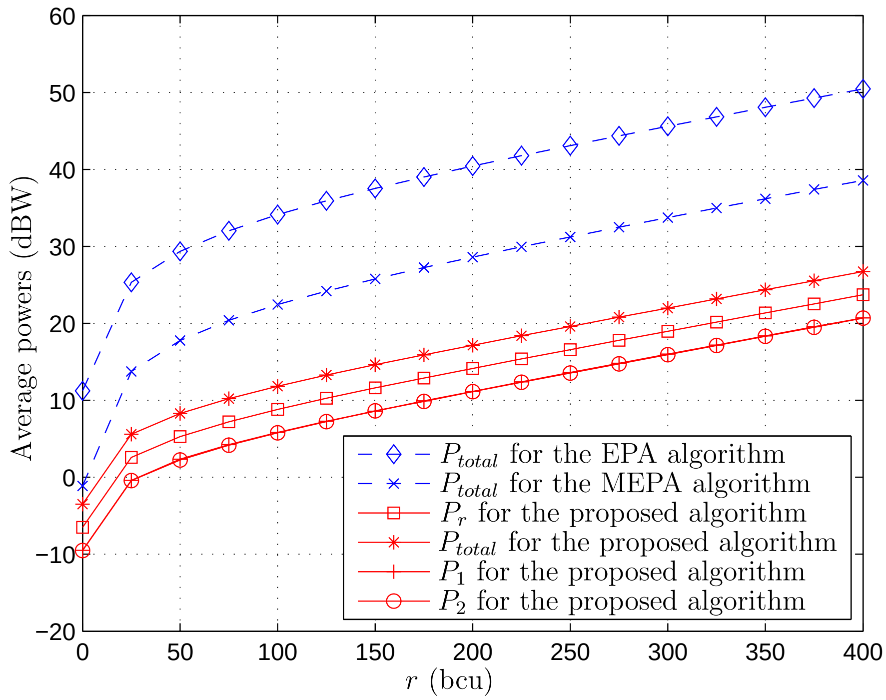

Figure 3.2: The average minimum total transmit power P_{total}, the corresponding average relay power P_r, the corresponding average transceiver powers P_1 and P_2, for the total power minimization method, the average total power P_{total} for the EPA technique and the average total power P_{total} for the MEPA method versus $r_1 = r_2 = r$.

below that average minimum total transmit power. Indeed, in our power allocation scheme, *for any channel realization*, half of the power budget is allocated to the two transceivers and the remaining half is shared among all the relay nodes. It can be seen from this figure that the *average* of transceiver powers P_1 and P_2 are identical. Note that P_1 and P_2 may not be identical for a given channel realization.

In Fig. 3.3, assuming QPSK modulation, the end-to-end average bit error rates (BERs)

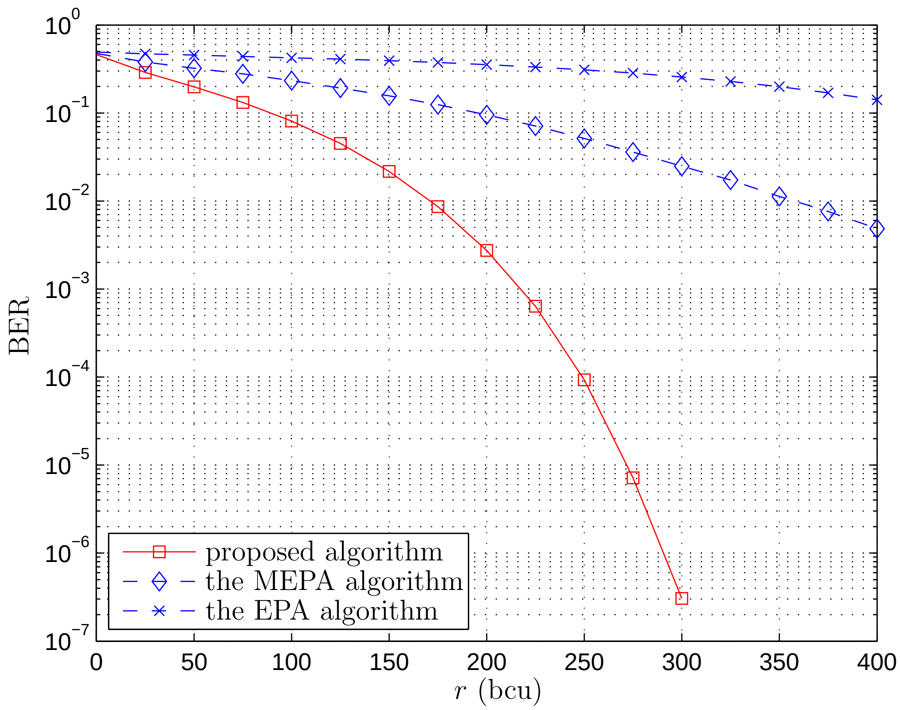

Figure 3.3: Bit error rate versus $r_1 = r_2 = r$, for different methods.

performance for our proposed algorithm is plotted for $r \triangleq r_1 = r_2$ and is compared with those of the EPA and the MEPA schemes. As can be seen from this figure, our power minimization approach outperforms the EPA and the MEPA methods.

Fig. 3.4 shows the minimum total power P_{total} achieved by our power minimization method for one random channel realization, versus $\beta'_1 = N_s(2^{\frac{2r_1}{N_s}} - 1)$ and for different values of $\beta'_1 + \beta'_2$. It can be seen from this figure that as long as the sum of β'_1 and β'_2 remains constant, changing β'_1 does not change the minimum total power of the network. This phenomenon is the direct result of the fact that the solution to the total power

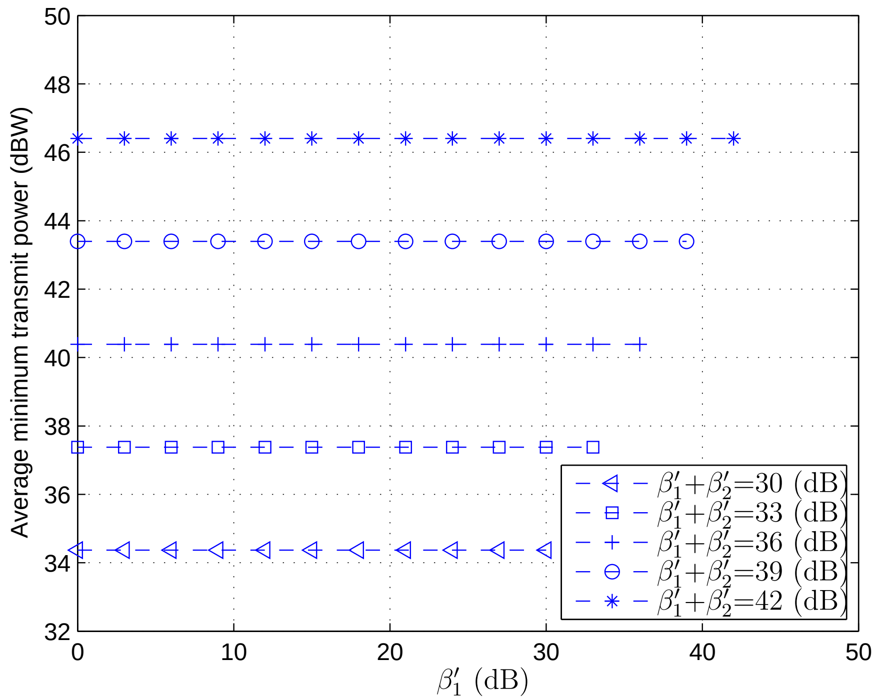

Figure 3.4: The minimum total transmit power versus β_1', for different values of β_2'

minimization approach depends on $\beta_1' + \beta_2'$.

Chapter 4

Conclusion and Future work

4.1 Conclusion

We considered a single-carrier *asynchronous* two-way relay network, where the relays employ amplify-and-forward (AF) signaling in an multiple access broadcast channel (MABC) protocol to enable a bi-directional communication between two transceivers. The network we considered is asynchronous, meaning that each relaying path (which originates from one transceiver, goes through one of the relays, and ends at the other transceiver) causes a delay which is significantly different form the delays caused by other relaying paths. Such a two-way relay channel can be viewed as a multi-path link which can produce inter-symbol-interference at the two transceivers. Assuming a block transmission scheme, we resorted to cyclic prefix insertion to eliminate inter-block-interference. We obtained the relay complex beamforming weights and the transceivers' transmit powers such that the total power consumed in the whole network is minimized subject to two constraints on the transceivers' data rates. We rigorously proved that at the optimum, only a subset of the relays have to be turned on and the rest of the relays have to be switched off. More specifically, we proved that at the optimum, the end-to-end channel impulse response (CIR) will have only one non-zero tap, and hence, only those relays which contribute to that non-zero tap have to be turned on. We devised a simple search algorithm to optimally determine which tap of the end-to-end CIR has to be non-zero. Finally, we presented a semi-closed-form solution for the optimal values of the design parameters, namely the relays' beamforming weights and the transceivers' transmit powers. Our simulations results showed that our proposed method significantly outperforms an equal power allocation scheme (i.e., when all the nodes in the network consume the same amount of power) with the same total transmit power.

4.2 Future work

Possible directions for future work can be listed as below:

- In this work, we considered that there is no direct link between two transceivers. Assuming direct link changes the communication scheme from MABC to TDBC, hence a change in the signal model, then it become a challenging direction to study.

- The transceivers and relays which are used in this work are equipped with a single antenna (SISO). Developing the communication scheme to employ multiple-antenna relays changes the SISO system model into MIMO model and is an interesting work which can be done in future.

- In this thesis, we assume a single-carrier communication scheme. Another research in this area could extend the proposed scheme from single-carrier model to multi-carrier system model.

- Another possibility for future research area would be to consider multi-user systems which allow asynchronous relay channel to be shared among all users.

Appendix A

Appendix

A.1 Proof of Lemma 3.2.1

The Lagrangian of the optimization problem (3.49) can be written as

$$\mathcal{L}(\mathbf{x}, \lambda) \triangleq \sum_{i=1}^{M} x_i - \lambda \left(\sum_{i=1}^{M} \log_2 (1 + x_i) - cte \right). \tag{A.1}$$

The derivative of Lagrange function with respect to x_i is

$$\frac{\partial}{\partial \mathbf{x}} \mathcal{L}(x_i) = 1 - \frac{\lambda}{1 + x_i} - cte. \tag{A.2}$$

Equating (A.2) to zero results in

$$x_i = \frac{\lambda}{1 - cte} - 1. \tag{A.3}$$

Equation (A.3) holds true for any value of $i = 1, 2, \cdots, M$. That means x_i is independent of i and all x_i's are the same. The proof is complete. ■

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