JOINT UPLINK-DOWNLINK BEAMFORMING IN MULTI-ANTENNA RELAYING SCHEMES

by

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A Thesis Submitted in Partial Fullfilment
of the Requirements for the Degree of

Master of Applied Science

in

The Faculty of Engineering and Applied Science

Program

Univeristy of Ontario Institute of Technology

November 2009

Certificate of Examination

Abstract

The thesis examines the problem of joint receive and transmit beamforming for a wireless network which consists of one relay node equipped with multiple antennas. The transmitter and the receiver are single antenna systems. The communication system consists of two phases. In the first phase the transmitter sends the information symbol to the relay while in the second phase, the relay re-transmits a linearly transformed version of the vector of its received signals. The concept of general-rank beamforming is applied to this communication scheme for the case of the uplink (transmitter-relay) and downlink (relay-receiver) channel vectors being statistically independent and statistically dependent. In the general-rank beamforming approach, the multi-antenna relay multiplies the received signal vector with a general-rank complex weight matrix and re-transmits each entry of the output vector on the corresponding antenna. The thesis presents a closed form solution to the general-rank beamforming power minimization problem with proof that for statistically independent uplink and downlink channels, the general-rank beamforming approach results in a rank-one solution for the beamforming matrix. The simulation results have shown that when the general-rank beamformer is applied to the case of statistically dependent uplink and downlink channels, the general-rank beamforming technique significantly outperforms the separable receive and transmit beamforming method.

Dedication

To my parents: with gratitude for their love and support

To my sister: thank you for always being there for me.

Acknowledgements

I want to express my gratitude to my supervisor Dr. Shahram Shahbazpanahi for his inspiration, advice, and guidance without which this thesis would not have been possible.

I would like to thank my supervisor and mentor Dr. Ali Grami, whose guidance, foresight and knowledge gave me a sense of direction and the chance to accomplish my dreams.

Lastly, I would like to thank all my colleagues, friends and classmates whose stimulating conversations, insights and company will always be remembered.

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Glossary

AP-AF	all participate - amplify-and-forward, 8
CSIT	channel state information at the transmitter, 8
dB	decibels, 36
DL	downlink, 8
GRB	General-Rank Beamformer, 33
i.i.d	independent identically distributed, 37
ISM	Industrial, Scientific and Medical, 2
Mbps	mega bytes per second, 2
MISO	multiple input single output, 20
MRT	maximal ratio transmission, 20
OFDMA	OFDM with multiple access, 8
PSD	positive semi-definite, 32

QPSK	quadrature phase shift keying, 36
S-AF	selective-amplify-and-forward, 8
SDP	semi-definite programming, 8
SER	symbol error rate, 8
SINR	signal to interference plus noise ratio, 8
SISO	single input single output, 20
SNR	Signal to Noise Ratio, 3, 8
SOCP	second order cone programming, 8
TDMA	time division multiple access, 8
UL	uplink, 8
WLAN	wireless local area networks, 2

Nomenclature

$(\cdot)^*$ denotes the complex conjugate transpose of a matrix

$(\cdot)^H$ denotes the hermitian conjugate transpose of a matrix

$\mathbf{A}$ boldface uppercase letters represent matrices

$\mathbf{a}$ boldface lowercase letters are used to represent vectors

$\mathbf{A}^{-1}$ inverse of matrix $\mathbf{A}$

$\mathbf{A}^T$ transpose of matrix $\mathbf{A}$

$\mathcal{P}\{\cdot\}$ represents the principal eigenvector of a matrix

$\lambda_{max}(\cdot)$ represents the largest eigenvalue of a matrix

$\otimes$ represents the Kronecker product

$\mathrm{tr}\{\cdot\}$ represents the trace of a matrix

$\mathrm{vec}(\mathbf{A})$ stacks all columns of the matrix $\mathbf{A}$ one below the other

$E\{\cdot\}$ denotes the statistical expectation

Chapter 1

Introduction

A communication network may be simply defined as a set of equipment that facilitates the transfer of information between two or more users. Some of the most familiar examples of a communication network are the telephone network, cellular networks and the Internet. Communication networks have become an essential infrastructure in today's society [1]. This development has led to the devices that transmit information at very high speeds and provide users with the ability to perform certain actions from remote locations.

Wireless communication has become a fast growing segment of the communications industry. Cellular and wireless systems have experienced an explosive growth and have become a critical necessity in everyday life. Today wireless networks are replacing existing wired networks in most homes and businesses. The shift towards portable and hand-held devices has further increased the need for an improved and more secure wireless communication system. Even with the rapid development of wireless technologies, there still remain many technical challenges in the design and implementation of wireless systems.

1.1 A History of Wireless Technologies

Some of the first traces of wireless communications were seen in the pre-industrial age. These early systems were restricted by the line of sight and were only able to transfer information within a certain distance. As time progressed, the distance over which information could be transferred was also increased. New methods such as smoke signals, flashing mirrors, signal flares and telescopes were invented. In 1838 Samuel Morse invented the telegraph network which led to the replacement of these systems. The telegraph network was then replaced by the telephone network in 1875 by Alexander Graham Bell [2]. Radio communications was first discovered in 1895 when Marconi demonstrated the use of radio transmission between the Isle of Wright and a tugboat 18 miles away. After the discovery of radio technology, major advances were rapidly made to enable transmissions over larger distances while trying to reduce the size, cost and power utilization of the devices. The period between 1960 and 1980 saw a shift from analog signals to digital signals. *Packet radio* [3] was a term coined for radio communications using digital signals. During the late 1980's radio communication services were only available to the military but in the early 1990s, radio communication became available for commercial use as well. The low data rates, high cost and few applications prevented a good market for this system from materializing. Ethernet technologies were developed in the early 1970's and steered many major commercial companies away from the use of radio communications. The 10 Megabits per second (Mbps) data rate was far superior to anything available using radio communications.

The early wireless local area networks (WLAN) used the Industrial, Scientific and Medical (ISM) frequency bands for wireless data transfer. Since the public WLANs were secondary users in this system,it had very poor performance in terms of the data rate and the coverage. The result of this poor performance, security concerns, lack of standardization and high cost of the devices led to the slow development of wireless technologies during this period.

One of the most successful and well-known wireless applications today is the cellular network. Its conception came about in 1915, with the wireless voice transmission system between New York and San Francisco [4]. The inefficient use of the radio spectrum and the state of technology in the 1990's lead to a limit in the capacity of the cellular system. AT&T Bell Laboratories developed a solution by the reuse of the radio spectrum. It was known at that time that, the range of a cellular system was limited by its transmission power as after a certain range the transmitted signal was severely attenuated. Therefore, users separated by a sufficient distance could reuse the same frequency with very low interference between the users. This solution allowed the effective use of the cellular system and the frequency band.

1.2 Technical Challenges Faced by Wireless Networks

In order to increase the robustness of wireless networks for the future, many technical issues must be addressed. Current issues with wireless devices include the addition of the ability to support multiple modes of operations that also support different applications and features. Innovative design breakthroughs in circuit design are needed to implement this multi-mode system on small handheld devices. The power requirement for such a design would be large as the device would have to perform signal processing and data transmission. Keeping this requirement in mind, the total power of the device would have to be minimized. Since most wireless systems require a significant amount of signal process and computation, wireless networks generally place much of their processing burden at fixed locations with large power resources. This leads to the creation of critical points and bottle necks which is not preferred in a wireless system. In some networks, wireless nodes cannot recharge their batteries. These nodes are termed as critical nodes. Therefore, the conservation of energy plays an important role, as the lifetime and robustness of the entire wireless network depends on these nodes. Thus special attention must be paid to the power and energy requirements of the system during the development phase of the network. There is high uncertainty in wireless networks as the transmission medium is unpredictable and difficult to anticipate. In the cellular network example, the transmitter, the receiver and the surrounding objects are moving randomly in time. This makes the channel experience random fluctuations in time. Thus, the design of a reliable wireless system with a guaranteed performance is difficult. The limited bandwidth of the system and random variations and fluctuations of channels would require the creation of a robust application that degrades slowly as the performance of the network degrades [5].

Another major problem faced by wireless networks is the allocation of the limited radio spectrum for different applications. The buying of this scare bandwidth is expensive therefore this spectrum must be used as efficiently as possible. Consequently, the spectrum allocation is governed by regulatory bodies, both nationally and internationally. Most wireless systems operate at frequencies of about several giga-hertz. This shortage of bandwidth could be greatly reduced with the development of new systems with the same cost and performance to enable the use of higher frequencies. Security is an important design consideration when designing wireless networks. Analog systems have no security and one can easily listen in after scanning the frequency band. Most digital systems incorporate some level of encryption but this encryption can be broken with a sufficient amount of time. These are some of the most well know technical challenges faced by wireless systems today. Ongoing research and development in wireless technologies provides for more robust and secure wireless systems for tomorrow.

The thesis shows new and innovative methods that can be used to further extend the range for the transmission of data. As the transmission distance increase, the power required to maintain a strong signal increases exponentially. The signal to noise ratio (SNR) must be maintained above a certain threshold to allow decoding of the transmitted signal at the receiver. Many new methods have been introduced to increase the range of the system. One such method is the use of multiple input multiple output (MIMO) systems. MIMO systems have the ability to increase the throughput of a system as well as the transmission range without increasing the systems overheads like bandwidth or power consumption. MIMO systems achieve this by using higher spectral efficiency (more bits per second per hertz of bandwidth) and link reliability or diversity (reduced fading). MIMO systems aim to separate data streams occupying the same bandwidth by relying on the de-correlation of the multiple received signals in the presence of multi-path. Multi-path is a phenomenon that occurs when a radio transmission starts at a point A and then reflects or passes through multiple surfaces before arriving, via multiple paths, at point B. MIMO systems technology uses its multiple antennas to collect and organize signals arriving via these paths.

Another method used to increase the transmission distance is the use of relays present between the transmitter and the receiver. There are two types of scenarios in which relays could be used to extend the transmission distance. In the first scenario a set of relays, each with a single antenna, could be used to receive the transmitted signal and then forward it towards the receiver. A second scenario would be to have one relay. However this relay would be equipped with multiple antennas to implement the gain achieved by beamforming to receive and re-transmit the data. The basic concept in the use of relay schemes however remains the same. During the first phase, the signal is transmitted from the transmitter to the relays. During the next phase, the relays amplitude- and phase-adjust the received signal and re-transmit it to the receiver.

This thesis shows the benefits of using a relaying scheme where a single relay with multiple antennas is used. From a geographical stand point all antennas are located at one fixed location. Thus the antennas can communicate and make use of spatial diversity to further reduce the power consumption. Spatial diversity is another means employed to improve the reliability and the quality of the transmission link. Spatial diversity is especially effective at mitigating multi-path situations. Multiple antennas give the receiver several observations of the same signal. In the event that one antenna is experiencing a deep fade, it is highly likely that another has a sufficient signal. Beamforming is a spatial technique that typically satisfies two main criteria, namely it passes undistorted the signals with a given direction of arrival θ and it attenuates all other direction of arrivals from different θ. Therefore beamforming can be viewed as forming a (reception) beam in the direction of θ. Beamformers use information such as the location of the antennas and the direction of wave of interest to form the beam in that direction. The beamformer attenuates the signals from all other directions. There are two types of beamformers conventional beamformers and adaptive beamformers. The main difference between the two being that a conventional beamformer has a fixed weighting matrix, whereas an adaptive beamformer continuously tries to improve the direction of interest and attenuate signals from all other directions. Thus, the use of beamforming can provide a robust link for the system. Also in this thesis, we consider the problem of the joint uplink-downlink beamforming in multi-antenna relaying schemes. We aim to minimize the total relay transmit power, subject to maintaining the SNR above a certain threshold. Reference [6] has considered the problem of the maximization of the receiver SNR subject to a total transmit power constraint. We have considered the duality between these two optimization problems. This duality allows us to show that the result obtained by the minimization of the total transmit power subject to a constraint on the SNR is related to the results obtained by the maximization of SNR subject to the total transmit power constraint presented in [6]. We prove that the beamforming weight matrix obtained in the minimization of power subject to a constraint on the receiver SNR is rank one. Reference [6] has proved that the resultant beamforming weight matrix is of rank-one.

The rest of this thesis is organized as follows. In the next chapter we discuss a diversified approach to joint uplink-downlink beamforming. We analyze, discuss and compare these approaches against our own approach. In the third chapter, we discuss the concepts of the general-rank beamformer. Initially the system model used for our work is elaborated upon and then we proceed to formulate the problem. We then solve the general-rank beamformer based on the power minimization problem. Having solved the general- rank beamforming problem, we prove that the resultant beamforming weight matrix obtained is of rank-one. In the fourth chapter, we present our simulation results where we first discuss how channel variations affect the total transmit power for statistically independent channels. We then discuss the performance of the general-rank beamformer versus the rank-one beamformer for statistically dependent channels and show that the general-rank beamformer outperforms the rank-one beamformer by a significant margin. Following this, the dependency of the uplink and downlink channels is quantified to show how the level of dependence between the two channels affects the performance of both the rank-one beamformer and the general-rank beamformer. Chapter five reinstates the results while concluding the thesis.

Chapter 2

Literature Review

In this chapter, a diversified approach to the joint uplink-downlink beamforming is discussed. We describe in detail the various system models that resemble our own system model. We mention some of the different methods proposed in the literature to maximize the total system capacity, minimize the total relay transmit power or to improve the quality of service. We will then perform a detailed analysis on a few system models that very closely resemble our system model, while providing the variations between them.

The demand for higher capacity and higher performance wireless networks has lead to the use of beamforming. Beamforming is a technique used in signal processing where the sensor arrays are used for directional signal transmission or reception. Fixed or adaptive receive and transmit beam patterns are used to achieve spatial selectivity. A beamformer during the transmission phase controls the phase and the amplitude of the signal, in doing so it creates constructive and destructive interference patterns. When receiving the information from various sensors, the signals are combined to form a predefined pattern that is expected at the receiver. In the case where receive beamforming is used, the signal from each antenna could be amplified by a

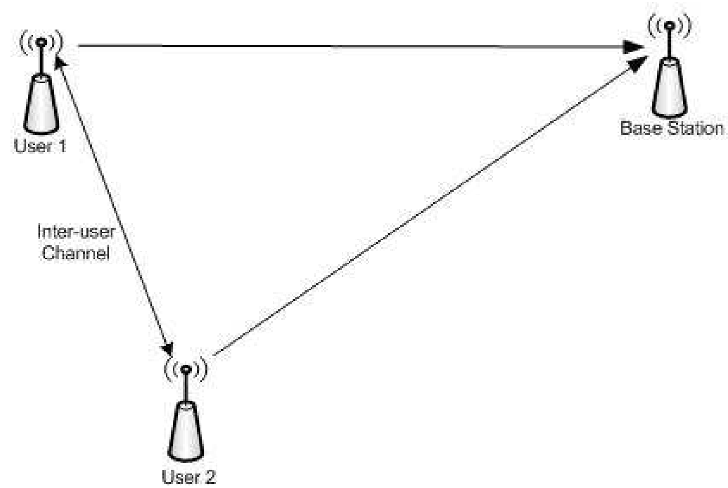

Fig. 2.1: A Two user partnering scheme.

"weight". By multiplying the signal by different weights different weighting patterns can be obtained to achieve the most desirable sensitivity patterns. In simpler terms a beamformer is a filter that satisfies two main criteria, namely it passes undistorted the signals with a given direction of arrival θ and it attenuates all other direction of arrivals from different θ. Therefore the spatial filter can be viewed as forming a beam in the direction of θ. The reception beam can be termed as a main lobe and all other direction of arrivals have small side lobes. The beamformer can control the width of the main lobe as well as the widths of the side lobes . The beamformer can also control the position of the null. Conventional beamformers use weights that are fixed to combine the signals from the sensors in the array. Adaptive beamformers use properties of the signals that are received by the array to typically improve the direction of interest signal while attenuating all other signals. One of the major drawback of beamforming is that it can be computationally intensive.

In all wireless communication systems the channel is always unpredictable. Due to the randomness of their channel, users may experience severe variations in signal attenuation. In most wireless systems, the data rate, the available power and the capacity of a system are limited, it therefore becomes vital to conserve these limited recourses. The authors in reference [7] and [8] have proposed a new form of spatial diversity. This new technique involves the use of cooperation between two mobile users in a system (see Fig. 2.1). Each mobile user is responsible for transmitting, not only their own information but also the information of the mobile user it has partnered with. Reference [7] proposes that when one of the mobile users uses the antenna of the other user (i.e., partner) to transmit its data, spatial diversity is achieved. It has been assumed here that the channel between the two mobile users is noisy and that each of the mobile users have their own information to transmit. Therefore, each mobile user receives an attenuated and a noisy version of its partners transmitted signal and then combines that received signal with its own data to construct its transmit signal. The common base station then receives a noisy version of the sum of the two transmitted signal from both the mobile users.

The authors of [7] first formulate the signal received at each mobile user as the sum of the signal transmitted by its partner multiplied by the fading coefficients between the two mobile users and the additive channel noise. The signal received by the base station is the sum total of the transmitted signal from both mobile users multiplied by the fading coefficients between itself and the base station respectively and the additive channel noise. The authors have assumed that there is no contribution between the two received signals at the base station. In most scenarios that use cooperation there is always some contribution between two cooperating users. Thus in the above mentioned cooperative scenario the problem lies in finding the best strategy for both users to construct their transmit signals, given that each user transmits its own data and the received partners data. Also, the base station must employ a reception scheme such that the mobile users are able to maximize their data rates between themselves and the base station. If the transmitters know the amplitude and phase of the fading they should be able to allocate their power depending on the different fading states. This can be done, while still maintaining their average power constraint. It is know that the mobile unit has very limited power capabilities, which renders the power allocation for different fading states infeasible. Reference [7] aimed to introduce the concept of user co-operation schemes. In addition to the possibility of user cooperative schemes, the authors have also analyzed the throughput and the overall performances of this scheme.

The authors of reference [7] perform a capacity analysis of user cooperation and have concluded that if the channels from the users to the base station have similar quality, and the channel between the two mobile users is of good quality, the cooperation scheme greatly improves the maximum achievable data rate. As the channel between the two mobile users degrades, the fading between the two mobile user's increases, until performance of the overall system approaches a point of no achieved gain from the use of user cooperation. The authors have further investigated the probability of outage, their results show that the probability of outage for a cooperation scheme is smaller than the probability of outage under no cooperation. These investigations have demonstrated that user cooperation increases robustness of a system against channel variations. Another study was performed on the increase in overall capacity of the system to an equivalent increase in the cell coverage area for a cooperative scheme. The authors numerically demonstrated that the percentage of increase in cell area coverage is roughly equal to the percentage increase in total capacity of the system. The decrease in the sensitivity of the data rate to channel variations adds a significant gain to the system.

Reference [8] further expands on the network designs presented in [7], it investigates the design and implementation of a cooperative system. The authors have further performed an analysis on optimal and suboptimal receiver designs for the conventional code division multiple access (CDMA) implementation. CDMA is a channel access method utilized by various radio communication technologies. The CDMA system uses “spread spectrum”technology, and spreads the information contained in a particular signal over a much greater bandwidth than the original signal. One of the major advantages of using a CDMA system is that it has no hard limit for the number of users who may share one base station. Additional users can connect to the system until the base station determines that call quality of service would suffer with the increase of users. The authors described a cooperation strategy with a conventional CDMA system and have calculated various probabilities of bit errors associated with the given scheme. Using the CDMA implemented system the authors have performed an in-depth analysis on the throughput and outage. The results showed that even though the increase in throughput due to cooperation is moderate, the resulting system is significantly more robust against channel variations.

If the channel quality between the users and the base station is fixed, user cooperation gains results in an increase in the area coverage. The overall increase in throughput is a linear function of the gains seen in the increase of the area of coverage. In reference [8] the authors have assumed that the fading phase is known at the transmitters. However, in most practical applications this assumption is trivial and hence the authors have presented a cooperative scheme that does not require the transmitters to know the phase of the fading coefficient between them and the base station. This scheme increases the data rate of the system. This increase could also be translated into reduced power usage for the mobile users. User cooperation then proves to be beneficial as the mobile users need to use less power to achieve a certain data rate. Since the mobile users have limited power capabilities, user cooperation maybe used to prolong the battery life of the devices which would in turn extend the life of the entire network.

Some of the mainly used relaying schemes are amplify and forward (AF) and decode and forward (DF). As the name suggests a relay using amplify and forward relaying scheme amplifies and retransmits the received signal. A relay using decode and forward relaying scheme decodes the received signal and then retransmits the signal. Both systems have their advantages and disadvantages; however the AF relaying scheme is preferred due to its simplistic approach. The authors of reference [9] have performed a comparative analysis of an AF system versus a DF system. They consider the case where the two user channels are slow fading with similar channel qualities and the channel gain between the users is larger than 10 dB. From their simulations they have seen that the performance of the AF and the DF system depends upon the channel conditions. Their findings show that when the inter-user channel is statistically worse than the two user channels, the AF scheme offers a higher channel capacity. In the event that the user channels are statistically worse than the inter-user channel then the DF scheme offers a higher channel capacity. The results show that the average performance of the AF scheme versus the DF scheme is comparatively the same.

Reference [10] investigates power-constrained networks with large bandwidth resources and a large number of nodes. The authors have shown that the maximum achieved rate increases linearly with the network power. In their study the authors have presented an optimum power allocation scheme which can be viewed as a form of maximum ratio combining by indicating the favorable relay positions in the network. The authors have then considered a network with large bandwidth resources, this network uses orthogonal transmissions at the relays in order to maximize the SNR. The authors have concluded that the optimum power allocation among the AF relays can be viewed as a form of maximum ratio combining where the powers are proportional to the channel gains that depend on the uplink and downlink channel gains. Reference [11] investigates the capacity of OFDM and OFDMA networks consisting of one source/destination pair and multiple AF relays. Orthogonal frequency-division multiplexing (OFDM) is a frequency-division multiplexing scheme which is utilized as a digital multi-carrier modulation method. Data is carried by a large number of closely-spaced orthogonal sub-carriers. The data stream which is in serial mode is divided into several parallel data streams, one stream for each sub-carrier. Each subcarrier is then modulated with a conventional modulation scheme at a low symbol rate. Thus this system maintains the same data rate as the original conventional single-carrier modulation scheme while having the same bandwidth. An OFDMA network is defined as a network where there is only one node transmitting on each subcarrier at any time. An important feature of the OFDMA scheme is its ability to exploit frequency selectivity and enable multiuser diversity by intelligent resource allocation. As mentioned before code division multiple access uses spread-spectrum technology. Spread-spectrum technology spreads the electromagnetic energy that is generated by a certain bandwidth to other frequencies in the frequency domain. This results in a larger bandwidth and more resistance to interference. CDMA also uses a special coding scheme to allow multiple users to use the same channel at the same time. This results in higher data rate.

The authors of [11] have performed a comparative analysis on two multiple access schemes namely joint coding/decoding multiple access and OFDMA combined with two relaying schemes namely AF and DF. Joint coding/decoding multiple access is defined as a network where on each subcarrier the source nodes are jointly coded. The

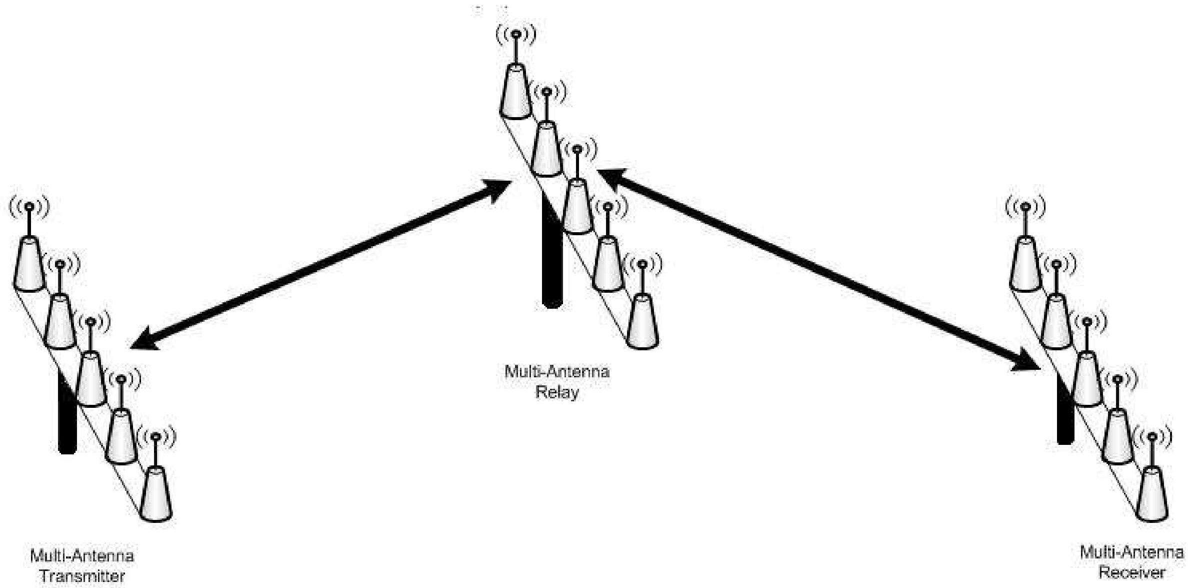

Fig. 2.2: A two hop multi-antenna transmitter, relay and receiver.

authors make a note that using a joint coding/decoding system is highly impractical in a power constrained network. The results observed show that if the DF relaying scheme is used the joint coding/decoding multiple access scheme, it provides a much higher capacity than the OFDMA scheme. However, if the AF relaying scheme is used instead of the DF scheme then the OFDMA multiple access scheme provides a higher capacity.

Reference [12] has considered a system model with a transmitter, AF relay and a receiver (see Fig. 2.2). The transmitter, receiver and relay are all multi-antenna systems. The system operates in a half duplex mode. In other words, the data can be transmitted in both directions on a signal carrier, but not at the same time. The receiver is not in direct communication range of the transmitter and therefore the relay is used to transmit the data. The authors have assumed that the channel state information at the transmitter (CSIT) is known and that all three nodes have perfect CSI. The authors have then examined the power allocation over the sub-channels in the frequency and space domains. This analysis has lead to the maximization of the instantaneous rate of the link. The authors of [12] optimize the power allocation by the separate optimization of the transmit power of the source and transmit power of the relay, with an individual per node transmit power constraint. The authors have presented the optimal power allocations for the source and the relay. Furthermore they showed that alternately repeating the optimization of the source and the relay power allocation improves the achievable rate. The authors have further investigated the optimization of the source and the relay power allocation, with a joint transmit power constraint and have observed that the power allocation is now jointly optimized over the sub-channels at the source and the relay. Thus the authors have combined the individual power allocation scheme into one combined power allocation scheme. In most practical systems the source and the relay have independent power supplies. However, the joint power constraint gives a deeper insight into the behavior of the required transmit power per communication link. A high SNR approximation per sub-channel is used. Since the obtained problem was not concave the authors of [12] have obtained an approximated solution and find that the rates achieved by this power allocation scheme are quite near the optimal power allocations scheme.

The amplify and forward relaying scheme is the simplest relaying scheme as the relay only amplifies the received signal and retransmits the amplified signal to receiver. Reference [13] provides a new alternative design for the capacity maximization for a non-regenerative cooperative transmission. The authors assume that the relay the has knowledge of the transmitter-relay and relay-receiver channel state information and that the relays are not simple non-regenerative relays, but have added intelligence and are able to carry out some signal processing. However, the relay is not required to demodulate or re-modulate the symbols it receives, as in the case of regenerative relaying schemes. The capacity of the system has been maximized under different levels of the channel state information available at the relays. There are three levels of the channel state information that can be considered at the relay station. The first level is when the CSI at the relay only contains information about the uplink (transmitter-relay) channel. The second level is when the CSI at the relay contains information of the uplink and downlink (relay-receiver) channel. The third level of CSI is when the relay contains information of all links in the transmission, i.e. uplink, downlink and the direct link. The results are then compared against the performance of the conventional AF approach. They are also compared against the performance of regenerative relays and direct non-cooperative transmission. Results show that an increase in capacity can be obtained when the uplink and downlink CSI are known at the relay. Despite the relay knowing this information, this solution is not the capacity-maximizing solution. The capacity-maximizing solution is only available when the CSI of the direct link between the transmitter and the receiver is known. However the loss due to no knowledge of the CSI between the transmitter and the receiver is not significant. The authors have noted that in most practical applications the knowledge of the CSI of the direct link is difficult to obtain. Other interesting results obtained by the authors in [13] show that when the number of antenna on the relay is the same as the number of antennas at the destination, the improvement obtained from the use of the channel state information reduces. However, when the number of antennas at the relay station is greater than the number of antennas at the destination, a significant capacity improvement can be obtained.

The authors of [14] consider an ad hoc network consisting of n source-destination pairs and R relaying nodes. Each transmitting source has to transmit its data through the relay network to its corresponding destination. Each relay in the network transmits a phase and amplitude adjusted version of its received signal. In doing so, it cooperates with other relays in the network to deliver each source's data to its corresponding destination. The authors have made the assumption that there is minimal cooperation between the relaying nodes. The authors then design a distributed beamformer such that the total relay transmit power dissipated by all relays is minimized while the quality of service at all destinations are guaranteed to be above certain thresholds. The authors of [14] use a semi-definite relaxation approach [15], and therefore the power minimization problem can be turned into a semi-definite programming (SDP) problem, and therefore, it can be solved efficiently using interior point methods. Due to the semi-definite relaxation, the matrix obtained by solving the optimization problem may be of rank one. The authors have used randomization techniques in their simulations and have stated that in a the majority of trials for $n \geq 3$ they obtained a rank one solution for the optimization problem. The results show that the distributed relay multiplexing is possible and may be beneficial depending on the channel conditions. Through their simulation the authors have shown that as the level of uncertainty in the uplink and downlink channel coefficients is increased, the QoS constraints become harder to satisfy. The authors have also shown that with the increase in the number of source-destination pairs more power is required to satisfy the QoS constraints.

It is known that in order to achieve maximum bandwidth in a system it becomes necessary to use spectral multiplexing. In budget constrained networks, the only other alternative is to use a distributed antenna systems. Reference [16] has proposed three schemes that could achieve high bandwidth effectiveness. One scheme analyzed is the distributed antenna system with decode and forward in the access point. The second scheme analyzed is the distributed antenna scheme with linear processing in the access point and the third scheme discussed is linear relaying without any information exchange between the relay nodes. It is assumed that the channels between the sources, destination and relay node is known. The authors of [16] have derived the gain allocation strategies for linear relaying systems. Their strategies are based on the analysis of a large system. Also, the relays are not required to have global channel knowledge. The beamforming matrix is obtained so as to minimize the noise received at the destination node as well as the sources not targeting the node. Additional constraints are imposed that preserve the received signal from each source at its targeted destination node. The resulting optimization problem has been shown to be a convex optimization problem and can be formulated as a second order cone program (SOCP). It is then solved using the interior points method [17], [18]. With the help of simulations the authors have compared the SINR and SER of their beamforming technique to the zero forcing beamforming technique [19]. The results show their beamforming technique to have superior performance when compared to the zero forcing beamforming technique. Another result shown here is that the overall performance of the system increased with the use of multi-antenna relays as opposed to single antenna relays.

Reference [20] considers a system which includes a single antenna transmitter, receiver and multiple relay nodes. In this system one could assume that all the relay nodes participate in the transmission and that the available power and bandwidth resources would be equally distributed among all the nodes. The authors of [20] have considered this approach and have proven it to be suboptimal. The authors have established the basis of their system model from reference [21]. In reference [21] the system shares its resources and the nodes transmit data in orthogonal channels. They have then presented a new optimal power allocation scheme to maximize the system throughput for a conventional AF system. This new power allocation scheme also minimizes the system error rate. The authors have then presented a new approach where only the best relay is chosen to participate in the transmission. The destination is assumed to have full knowledge of the channel gains as well as the knowledge of the channel between the source and the relays. Therefore the destination node is best suited for the implementation of this scheme. One of the objectives of this scheme is to minimize the system outage probability based on complete channel state information and the channel statistics. The selection scheme maintains a full diversity order and at reasonable power levels it has shown better outage behavior and throughput than a conventional method scheme. The authors assume that the channel state information is known as they obtain the optimal power allocation for a conventional AF system. They have imposed constraints on the total available power as well as the individual power constraints on each participating node. Since the destination has maximum knowledge of the channel variations, it notifies each relay of its power usage. In the event that a relay node is assigned zero power, it will remain idle during the transmission. The instantaneous throughput is maximized for each set of channel realizations, hence the minimum outage probability is achieved. It is noted here that in many practical application the feedback channel from the destination to the relays has a very low capacity. This makes the optimal power allocation scheme sub-optimal.

In time division multiple access (TDMA) each node transmits within its time slot and uses the entire frequency band during its transmission slot.. In orthogonal transmission, every node can only transmit in a time slot with duration $1/(m+1)$ of the entire block where m is the maximum length of the block. Even though orthogonal transmission achieves a full diversity order, the factor of $1/(m+1)$ has a large adverse effect on the throughput when m is large. The authors have used a selection scheme previously presented in [22]. This scheme used the relay delay process as the basis of its selection scheme. This scheme has the potential to fail when packet collision is taken into consideration. Therefore, the authors of [20] have derived a new scheme called the selection amplify and forward (S-AF), where the transmission is divided into two slots. The first slot implements a data sharing phase of all the participating

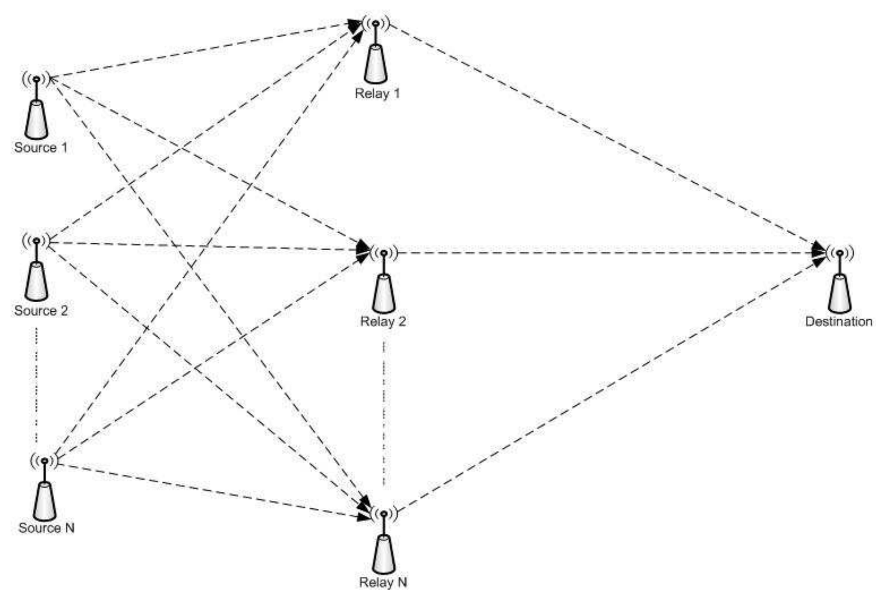

Fig. 2.3: A Two Hop Multi-Source Parallel Relay Network.

amplify and forward (AP-AF) relays. However the relaying phase of S-AF contains one slot, in which a relay node is selected by the destination, this relay then amplifies and forwards its received signal from the source towards the destination. Through simulations, the authors have shown that the S-AF method out performs the conventional or (AP-AF) scheme and the optimal power allocation scheme as the number of relay nodes m is increased. In this S-AF scheme the destination node need only to select and notify the best relay, instead of computing and feeding back the power allocated to every relay node. This reduces the computational time taken at the destination. It also reduces the complexity and the outage probability of the S-AF scheme. S-AF has a higher throughput as it only repeats the information once.

In beamforming, the amplitude and phase of each antenna is controlled. A beamformer applies a complex weight to the transmit signal thus shifting its phase and setting the amplitude for each element of the antenna array. In most wireless relay networks, relay noise is correlated. This occurs due to a variety of reasons namely common interference or the noise propagation from preceding hops. Fig. 2.3 shows a parallel relay network, where L nodes wish to communicate with a common destination through a set of N relays. The transmission of data occurs in two time slots. In the first slot the L sources transmit their data to the relays and in the second slot, all the relays simultaneously forward their received signal to the destination. The authors assume that there is no direct link between the source and the destination and that all nodes are equipped with a single antenna. An optimal relaying scheme is designed in order to increase the signal power and to reduce the noise power. The authors of [23] have specifically studied the performance of the relay networks under the effects of noise. Attention is paid to cases where the relays have full knowledge of the correlation and where the relays have no knowledge of the correlation structure. The two main approaches in studying the effects of noise correlation are to use a very high relay transmit power and to use a very high transmit power. It is found that for very high relay transmit powers, depending on the noise signals being either positively correlated or negatively correlated, then the correlation helps as part of the noise gets canceled out. However, if both the signal and the noise components are correlated in the same direction, then correlation has an adverse effect as it increases the noise power. In the scenario where the noise is correlated and the relays are aware of the correlation, the relays can achieve a much higher maximum achievable SNR than the scenario where the noise is correlated and the relays are unaware of it. In the approach using a high transmit power, the system acts like a multiple input single output system (MISO). This scheme represents a maximal ratio transmission (MRT), where the relays act as a multiple antenna transmitter. Here the noise at the relays is negligible when compared to the source power and therefore the effect of the noise covariance matrix is relatively small. The authors of [23] have obtained a closed-form solution for the optimal relay amplification vector and the maximum sum rate. Through numerical simulations the maximum sum rate is compared against the relays having correlation knowledge and the relays having no correlation knowledge and it has been found that correlation is beneficial even if the relays do not have knowledge of the correlation.

Following [24] which states that space time coding can achieve full diversity if there is no channel state information present at the receiver. The authors of reference [25] have considered a single antenna transmitter, receiver and relay nodes. It is assumed that the relays and receiver have perfect knowledge of channel state information. Every node in the network has its own individual power constraint. The relays use the amplify-and-forward protocol to retransmit the information symbol to the receiver. The relays use channel direction information and channel strength information to form a beam towards the receiver and to adaptively adjust their transmit powers respectively. The results of [25] show that the optimal power is not a binary value between zero and the maximum power. In some cases relay must use a partial amount of its power. The value of this partial amount of power is obtained by configuring each relay based on its own channel and also other channels. This is because every relay has two effects on the transmission. On one hand it helps the transmission by forwarding the information, while on the other hand it has an adverse effect on the transmission by also forwarding noise. The relay power has a non-linear effect on the power of both the signal and noise, which make the optimization solution a non-binary (i.e., on or off) solution. Since all relay are allowed to cooperate in the transmission, it is hard to measure how much each relay contributes. The authors have noted that in a given network beamforming, there can be more than one relay in the network using its maximum power.

Reference [26] considers the problem of distributed beamforming when the second order statistics of the CSI are known. The system model Fig. 2.4 consists of a trans-

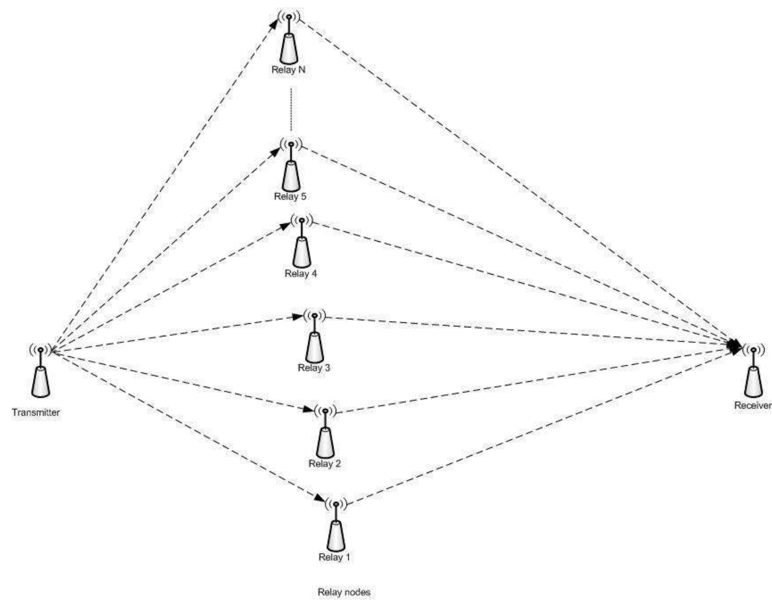

Fig. 2.4: Single antenna transmitter and receiver with N relay nodes.

mitter, a receiver and N relay nodes. The transmitter, receiver and relay nodes are all single antenna systems. The authors consider two approaches in the design of the beamformer. For the first approach the authors design a beamformer by minimizing the total relay transmit power subject to a quality of service constraint at the receiver. In the second approach, the beamformer weights are obtained through the maximization of the receiver SNR subject to two different power constraints namely, the total relay transmit power constraint and the individual relay transmit power constraint. The signal received at the destination has two components namely the desired signal component and the unwanted noise component. The uncertainty in the channel is modeled through the introduction of the covariance matrices of the channel coefficients. The authors then obtain the optimization problem as the minimization of the total relay transmit power subject to the SNR being great than a certain threshold. The authors obtained a closed form solution for the optimum beamforming weight matrix. In the next optimization problem the authors maximize the receiver SNR subject to a constraint on the total relay transmit power being less than a predefined maximum. The authors obtain a closed form solution for this problem as well. Their simulation results show that the maximum achievable SNR is decreased as the uncertainty in the channel is increased. The authors have then considered a variation of the maximization presented earlier problem where they have considered maximizing the receiver SNR subject to an individual power constraint on the relay nodes. The authors have used semi-definite programming and have show that the solution is attained using the interior points method. Due to the computational complexity of the semi-definite programming approach the authors have also derived a simplified technique which results in a closed-form beamforming weight vector. The simulation results show that in both methods the maximum achievable SNR is decreased as the uncertainty in the channel is increased. Through further numerical simulations it is found that the maximum achievable SNR with a total power constraint and the maximum achievable SNR with the individual power constraint are very close.

Reference [27] considers a system that consists of a multi-antenna transmitter, receiver and relay node which is similar to the system model considered in [12]. An optimal non-regenerative MIMO relay matrix is designed so as to increase the capacity of the system between the transmitter and the receiver. If the weighting matrix is applied at the MIMO relay then it follows an optimal set of canonical coordinates. These coordinates are governed by the channel matrices. The canonical coordinates then decompose the MIMO relay channel into several single input single output (SISO) channels. The authors have then presented an upper bound and two lower bounds on the optimal capacity of the MIMO relay channel when a direct link between the source and the destination is considered. Reference [27] develops a non-regenerative multi-antenna relaying strategy through the maximization of the capacity between the source and the destination. The resulting beamforming weight matrix turns out to be full rank. This concept has been further studied in [28] where an SNR maximization approach is used to obtain the beamforming matrix. Reference [28] assumes that the receiver and the relay nodes have perfect knowledge of the transmitter-relay and relay-receiver CSI.

Reference [28] considers a system that consists of a two hop MIMO scheme with a transmitter, an AF relay and a receiver. The transmitter, receiver and relay are multi-antenna systems that are operating in half duplex mode. The authors have assumed that the relay has perfect knowledge of the CSI between the source and the relay and the receiver has perfect knowledge of the CSI between the relay and the receiver. Thus the source has no knowledge of the CSI for any links. They study the optimal power allocation scheme between the source and the destination by not taking into consideration the direct link. The authors of [28] have assumed this problem to be a one dimensional beamforming problem where the source beamforming vector, the destination combining vector and the relay weighting matrix have been optimized so as to maximize the received SNR subject to a power constraint. In the second half of the paper the authors study the same system model however they now consider the direct link between the source and the destination. They assume that the relay knows the source-relay and relay-destination channels and thus derive the optimal source beamforming vector. The authors are unable to show the proof of this solution analytically, however they show that for i.i.d. Rayleigh channels the solution to this problem is uniformly distributed on the sphere of unit radius.

The authors of reference [6] have investigated the problem of joint receive and transmit beamforming for a wireless network consisting of a transmitter, a receiver and a relay node. The relay node is equipped with multiple antennas where, are the transmitter and the receiver are single antenna systems. There are two main phases in the communication cycle, in the first phase the transmitter transmits its information to the relays. In the second phase the relays retransmit an amplitude- and phase-adjusted version of the received signal to the destination. The authors have maximized the receiver SNR subject to a constraint on the total relay transmit power being less than a certain threshold. They have shown that this approach leads to the separation of the two beamformers for the case when the uplink and downlink channel coefficients are statistically independent of each other. The uncertainty in the channel is introduced through the channel covariance matrices. In the next section of the paper the authors have introduced a novel concept called the general-rank beamformer. This general-rank beamformer has superior performance when compared to a rank-one beamformer. The general-rank beamformer is implemented at the relay, the relay multiplies the received signal with a complex beamforming weight matrix and then transmits the output vectors on their corresponding antenna. The authors have solved the general-rank beamforming problem and have arrived at a closed form solution. The authors have proved that for statistically independent transmitter-relay and relay-receiver channels the general-rank beamforming problem results in a beamforming weight matrix of rank-one. Simulations have compared the performance of the general-rank beamformer with the performance of the rank-one beamformer. Results show that for statistically dependent transmitter-relay and relay-receiver channels the general-rank beamformer is far superior to the separable receive and transmit beamformer.

In this thesis we minimize the power consumed by the general-rank beamformer while imposing a constraint on the SNR. We wish to obtain the minimum power required in order to satisfy a certain threshold of SNR. We have obtained a closed-form solution and have shown that this solution obtained is the same as the optimization problem presented in [6].

The difference between our results and the results of [27] is that we use the receiver SNR as the underlying criterion while the approach in [27] is based on the maximization of the relay channel capacity. Another difference is that our work is based on the availability of the second order statistics of the channel coefficients while [27] assumes instantaneous channel state information. One of the main differences between our work and the work of [28] is that the authors of [28] assume that the instantaneous channel state information is available. In addition to this, the network set up in [28] is different in the sense of the receiver and transmitter deploy multiple antennas, where as our work considers a single antenna transmitter and a single antenna receiver. Since we assume single antennas at the receiver and transmitter, the complexity at the transmitter and receiver is reduced as there is no spatial requirement. However [28] requires spatial processing. It also considers a quantization method for the two beamforming coefficients that are to be transmitted at the source and the relays. This quantization method would introduce errors and further reduce the receiver SNR.

Chapter 3

The General-Rank Beamformer

In this chapter, we discuss the general rank beamforming approach for the minimization of the total relay power subject to a constraint on the receiver SNR. In the first section we discuss our approach to formulate the general-rank beamforming problem. In the second section, we solve the power minimization problem and find the minimum total relay transmit power associated with a certain threshold of SNR. In the third section, we prove that the resultant beamforming weight matrix is of rank one. We compare this resultant optimal beamforming weight matrix with that the beamforming weight matrix in [6].

3.1 Problem Formulation

We consider a three node communication systems consisting of a single antenna transmitter, a single antenna receiver and a relaying node which is equipped with n_r antennas. We assume a poor quality of the transmitter to receiver channel and therefore there is no direct link between the transmitter and the receiver. Let s denote the information symbol broadcasted by the transmitter to the relays and let $\mathrm{E}\{|s|^2\} = 1$. We model the flat fading channel between the transmitter and the ith antenna of the relay as a complex random variable f_i (see Fig. 3.1). The channel between the ith

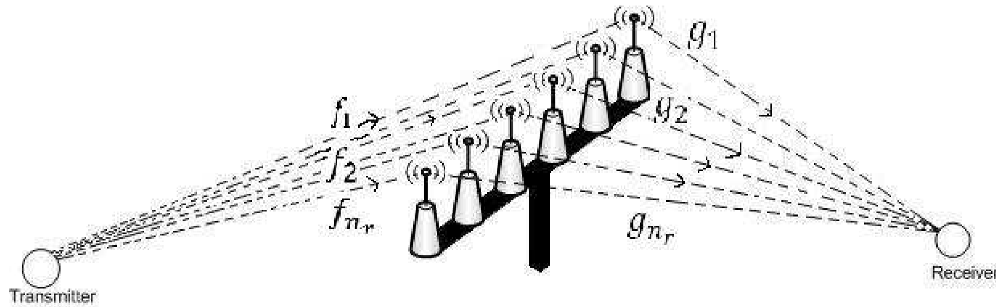

Fig. 3.1: System Model

antenna of the relay and the receiver is modeled as a complex random variable g_i. We also assume the correlation matrices of the channel vectors $\mathbf{f} \triangleq [f_1 \;\; f_2 \;\; f_3 \;\; \ldots f_{n_r}]^T$ and $\mathbf{g} \triangleq [g_1^* \;\; g_2^* \;\; g_3^* \;\; \ldots g_{n_r}^*]^T$ are known and they are, respectively, given by,

$$\begin{aligned} \mathbf{R}_f &\triangleq E\{\mathbf{f}\,\mathbf{f}^H\} \\ \mathbf{R}_g &\triangleq E\{\mathbf{g}\mathbf{g}^H\}\,. \end{aligned}$$

In the vector notation, the $n_r \times 1$ complex vector r of the relay received signals can be expressed as,

$$\mathbf{r} = \mathbf{f}s + \mathbf{n}, \tag{3.1}$$

where $\mathbf{n}$ is the $n_r \times 1$ vector of noise or interference with the known covariance matrix $\mathbf{R}_n$. Using a $n_r \times n_r$ beamforming weight matrix, the relay transmit vector is given by $\mathbf{t} = \mathbf{W}\mathbf{r}$ where $\mathbf{W}$ can have any arbitrary rank. We can therefore express the signal obtained at the receiver as,

$$y = \mathbf{g}^H\mathbf{W}\mathbf{r} + \eta, \tag{3.2}$$

here η is measurement of noise at the receiver. We can further expand (3.2) as,

$$\begin{aligned} y &= g^H\mathbf{W}(\mathbf{f}s + n) + \eta \\ y &= \underbrace{\mathbf{g}^H\mathbf{W}\mathbf{f}s}_{\text{desired signal component}} + \underbrace{(\mathbf{g}^H\mathbf{W}\mathbf{n} + \eta)}_{\text{total noise}}\,. \end{aligned} \tag{3.3}$$

If $\mathbf{h} \triangleq \text{vec}\{\mathbf{f}g^H\}$ and $\mathbf{w} \triangleq \text{vec}\{\mathbf{W}^H\}$, we can write the equation for the signal power P_s as,

$$\begin{aligned} P_s &= E\{|\mathbf{g}^H\mathbf{W}\mathbf{f}s|^2\} \\ &= E\{|\mathbf{g}^H\mathbf{W}\mathbf{f}|^2\}\underbrace{E\{|s|^2\}}_{1} \\ &= E\{|\text{tr}(\mathbf{W}\mathbf{f}\mathbf{g}^H)|^2\} \\ &= E\{|\mathbf{w}^H\mathbf{h}|^2\} \\ &= E\{\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}\} \\ &= \mathbf{w}^H\mathbf{R}_h\mathbf{w}. \end{aligned} \tag{3.4}$$

Similarly the noise power can be obtained as,

$$\begin{aligned} P_n &= E\{|\mathbf{g}^H\mathbf{W}\mathbf{n}|^2\} + \sigma_\eta^2 \\ &= E\{|\text{tr}(\mathbf{W}\mathbf{n}\mathbf{g}^H)|^2\} + \sigma_\eta^2 \\ &= E\{|\mathbf{w}^H\text{vec}(\mathbf{n}\mathbf{g}^H)|^2\} + \sigma_\eta^2 \\ &= \mathbf{w}^H E\{\text{vec}(\mathbf{n}\mathbf{g}^H)\text{vec}(\mathbf{n}\mathbf{g}^H)^H\}\mathbf{w} + \sigma_\eta^2 \\ &= \mathbf{w}^H E\{(\mathbf{g}^* \otimes \mathbf{n})(\mathbf{g}^T \otimes \mathbf{n})^H\}\mathbf{w} + \sigma_\eta^2 \\ &= \mathbf{w}^H E\{(\mathbf{g}^*\mathbf{g}^T \otimes \mathbf{n}\mathbf{n}^H)\}\mathbf{w} + \sigma_\eta^2 \\ &= \mathbf{w}^H(\mathbf{R}_g^* \otimes \mathbf{R}_n)\mathbf{w} + \sigma_\eta^2, \end{aligned} \tag{3.5}$$

where $\sigma_\eta^2 \triangleq E\{|\eta|^2\}$. We have used the fact that $\text{vec}(\mathbf{a}\mathbf{b}^T) = \mathbf{b}\otimes\mathbf{a}$ in the fifth equality and $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$ in the sixth equality. We now obtain the total relay transmit power P_T as,

$$\begin{aligned} P_T &= E\{\mathbf{r}^H\mathbf{W}^H\mathbf{W}\mathbf{r}\} \\ &= \text{tr}(\mathbf{W}E\{\mathbf{r}\mathbf{r}^H\}\mathbf{W}^H) \\ &= \text{tr}(\mathbf{W}(\mathbf{R}_f + \mathbf{R}_n)\mathbf{W}^H) \qquad (3.6) \\ &= \mathbf{w}^H(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n))\mathbf{w}\,. \qquad (3.7) \end{aligned}$$

Thus the optimization problem for the minimization of the total relay transmit power subject to a constraint on the receiver SNR can be written as,

$$\begin{aligned} \min \quad & P_T \\ \text{subject to} \quad & SNR \geq \gamma. \end{aligned} \tag{3.8}$$

Using (3.4), (3.5) and (3.7) the total relay transmit power minimization subject to a constraint on the receiver SNR is written as,

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n)\right) \mathbf{w} \\ \text{subject to} \quad & \frac{\mathbf{w}^H \mathbf{R}_h \mathbf{w}}{\mathbf{w}^H \left(\mathbf{R}_g^* \otimes \mathbf{R}_n\right) \mathbf{w} + \sigma_\eta^2} \geq \gamma. \end{aligned} \tag{3.9}$$

3.2 General-Rank Beamformer Power Minimization Approach

We now solve the optimization problem presented in (3.9).

We introduce a new vector $\tilde{\mathbf{w}}$ such that,

$$\tilde{\mathbf{w}} = \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n)\right)^{1/2} \mathbf{w}. \tag{3.10}$$

Therefore, we re-write the optimization problem in (3.9) as,

$$\begin{aligned} \min_{\tilde{\mathbf{w}}} \quad & ||\tilde{\mathbf{w}}||^2 \qquad (3.11) \\ \text{subject to} \quad & \frac{\tilde{\mathbf{w}}^H \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n)\right)^{-1/2} \mathbf{R}_h \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n)\right)^{-1/2} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n)\right)^{-1/2} \left(\mathbf{R}_g^* \otimes \mathbf{R}_n\right) \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n)\right)^{-1/2} \tilde{\mathbf{w}} + \sigma_\eta^2} \geq \gamma. \end{aligned}$$

If we let $\mathbf{A} = \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n)\right)^{-1/2}$, then the optimization problem in (3.11) may be re-written as

$$\begin{aligned} \min_{\tilde{\mathbf{w}}} \quad & ||\tilde{\mathbf{w}}||^2 \\ \text{subject to} \quad & \frac{\tilde{\mathbf{w}}^H \mathbf{A} \mathbf{R}_h \mathbf{A}^H \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{A} \left(\mathbf{R}_g^* \otimes \mathbf{R}_n\right) \mathbf{A}^H \tilde{\mathbf{w}} + \sigma_\eta^2} \geq \gamma. \end{aligned} \tag{3.12}$$

We further manipulate the variables in (3.12) to obtain,

$$\begin{aligned} \min_{\tilde{\mathbf{w}}} \quad & ||\tilde{\mathbf{w}}||^2 \\ \text{subject to} \quad & \tilde{\mathbf{w}}^H \mathbf{A} \left(\mathbf{R}_h - \left(\mathbf{R}_g^* \otimes \gamma \mathbf{R}_n \right) \right) \mathbf{A}^H \tilde{\mathbf{w}} \geq \gamma \sigma_\eta^2. \end{aligned} \tag{3.13}$$

We let $\mathbf{R}_h = \mathbf{R}_g^* \otimes \mathbf{R}_f$, therefore we re-write the optimization problem in (3.13) as,

$$\begin{aligned} \min_{\tilde{\mathbf{w}}} \quad & ||\tilde{\mathbf{w}}||^2 \\ \text{subject to} \quad & \tilde{\mathbf{w}}^H \mathbf{A} \left[\left(\mathbf{R}_g^* \otimes \mathbf{R}_f \right) - \left(\mathbf{R}_g^* \otimes \gamma \mathbf{R}_n \right) \right] \mathbf{A}^H \tilde{\mathbf{w}} \geq \gamma \sigma_\eta^2. \end{aligned} \tag{3.14}$$

As a result, the minimization of the total relay power with a constraint on the receiver SNR is written as,

$$\begin{aligned} \min_{\tilde{\mathbf{w}}} \quad & ||\tilde{\mathbf{w}}||^2 \\ \text{subject to} \quad & \tilde{\mathbf{w}}^H \mathbf{Q} \tilde{\mathbf{w}} \geq \gamma \sigma_\eta^2. \end{aligned} \tag{3.15}$$

where $\mathbf{Q} = \mathbf{A} \left[\mathbf{R}_g^* \otimes (\mathbf{R}_f - \gamma \mathbf{R}_n) \right] \mathbf{A}^H$. It is worth mentioning here that $\mathbf{A}$ is positive semi-definite (PSD), since $\mathbf{R}_f$ and $\mathbf{R}_n$ are the covariance matrices and are positive semi-definite. If $(\mathbf{R}_f - \gamma \mathbf{R}_n) \leq 0$ it would in turn make $\mathbf{Q} \leq 0$ (i.e., negative definite) and thus make the problem infeasible.

We assume that $\tilde{\mathbf{w}}_{opt}$ is the optimal beamforming weight matrix obtained by solving 3.15. The constraint in (3.15) must satisfy the equality because if the optimal value of $\tilde{\mathbf{w}}_{opt}$ does not satisfy the equality in the constraint, then one can further scale down $\tilde{\mathbf{w}}_{opt}$ so that the constraint is satisfied with an equality. This decreases the objective function and thus contradicts the optimality of $\tilde{\mathbf{w}}_{opt}$. The objective function in (3.15) can be minimized if $\tilde{\mathbf{w}}$ is chosen to be the principal eigenvector of $\mathbf{Q}$. Thus, the solution to the optimization problem in (3.15) is written as,

$$\mathbf{w}_{opt} = \sqrt{\frac{\gamma \sigma_\eta^2}{\lambda_{max}(\mathbf{Q})}} \left(\mathbf{I} \otimes (\mathbf{R}_f + \mathbf{R}_n) \right)^{-\frac{1}{2}} \mathcal{P}\{\mathbf{Q}\}. \tag{3.16}$$

Also, the minimum achievable relay transmit power is given by,

$$P_T{}^{min} = \frac{\gamma \sigma_\eta^2}{\lambda_{max}(\mathbf{Q})}. \tag{3.17}$$

3.3 Proof: The General-Rank Beamformer has a Rank One Solution

In this section for the scenario where the uplink and downlink channel vectors $\mathbf{f}$ and $\mathbf{g}$ are statistically independent, we show that the general-rank beamformer through power minimization under the receiver SNR constraint results in the optimal beamforming weight matrix $\mathbf{W}_{\text{opt}}$ whose vectorized version is given in (3.16) being rank one.

Lemma: *The optimal beamforming matrix* $\mathbf{W}_{\text{opt}}$ *whose vectorized version is given in (3.16) is rank one.*

Proof: Let $\mathbf{W}_{\text{opt}}$ be the solution to the following optimization problem:

$$\begin{aligned} \min_{\mathbf{W}} \quad & P_T(\mathbf{W}) \\ \text{subject to} \quad & \text{SNR}(\mathbf{W}) \geq \gamma. \end{aligned} \tag{3.18}$$

where $P_T^{min}(\mathbf{W}_{opt})$ is the minimum power obtained by solving (3.18). Note in (3.18), we have replaced P_T and SNR with $P_T(\mathbf{W})$ and SNR($\mathbf{W}$) respectively, to emphasize the fact that P_T and SNR are functions of ($\mathbf{W}$). Now consider the following SNR maximization problem:

$$\begin{aligned} \max_{\mathbf{W}} \quad & \text{SNR}(\mathbf{W}) \\ \text{subject to} \quad & P_T(\mathbf{W}) \ \leq P_T^{min}(\mathbf{W}_{opt}) \end{aligned} \tag{3.19}$$

Let the maximum SNR obtained by solving (3.19) be $\text{SNR}^{max}(\tilde{\mathbf{W}}_{opt})$, where $\tilde{\mathbf{W}}_{opt}$ is any solution to (3.19). Note that the inequality constraint in (3.19) will be satisfied with an equality at the optimum, otherwise $\tilde{\mathbf{W}}_{opt}$ can be scaled up and this will further increase the SNR. This in turn contradicts the optimality of $\tilde{\mathbf{W}}_{opt}$, therefore $P_T(\tilde{\mathbf{W}}_{opt})$ is therefore equal to $P_T^{min}(\mathbf{W}_{opt})$. If the SNR($\tilde{\mathbf{W}}_{opt}$) is less than γ which is in turn less than equal to SNR($\mathbf{W}_{opt}$), this would contradict the optimality of $\tilde{\mathbf{W}}_{opt}$ for (3.19), simply because $\mathbf{W}_{opt}$ would give us a higher SNR and it also belongs to the feasible set of (3.19). Since SNR($\tilde{\mathbf{W}}_{opt}$) is greater than or equal to γ and $P_T(\tilde{\mathbf{W}}_{opt})$ is equal to $P_T^{min}(\mathbf{W}_{opt})$, we conclude that $\tilde{\mathbf{W}}_{opt}$ is a solution to (3.18). But we know from [6] that (3.19) has a rank one solution. If we choose $\tilde{\mathbf{W}}_{opt}$ to be a rank one solution to (3.19) then this rank one solution $\tilde{\mathbf{W}}_{opt}$ will also be the solution to (3.18), thereby completing the proof.

Let Fig. 3.3 be the plot of the minimum achieveable total transmit power P_T^{min} versus SNR. We first consider the optimization problem presented in (3.18), where we minimize the total transmit power subject to a constraint on the SNR. Let us assume that in Fig. 3.3 the solution to this optimization problem or the minimum total relay transmit power is represented by the horizontal line (i.e., $P_T = P_T^{min}$). This horizontal line intercets the curve at point A. Thus we can assume that this point A is the optimal solution to (3.18). We see that the solution approaches point A from the left of the curve. Now, let us consider the optimization problem presented in (3.19). Here we are maximizing the total achieveable SNR subject to a constraint on the total relay transmit power. Let us assume that in Fig. 3.3 the solution to (3.19) is represented by the vertical line (i.e., SNR $= \gamma$). This vertical line intersects the given curve at point A. Thus we can assume that point A is the optimal solution to (3.19). However, here we see that the solution approaches point A from the right. Thus point A satisfies the solutions for both optimization problems.The rank one property of the resultant beamforming weight matrix, allows us to reduce the comptational complexity when using the general-rank beamformer.

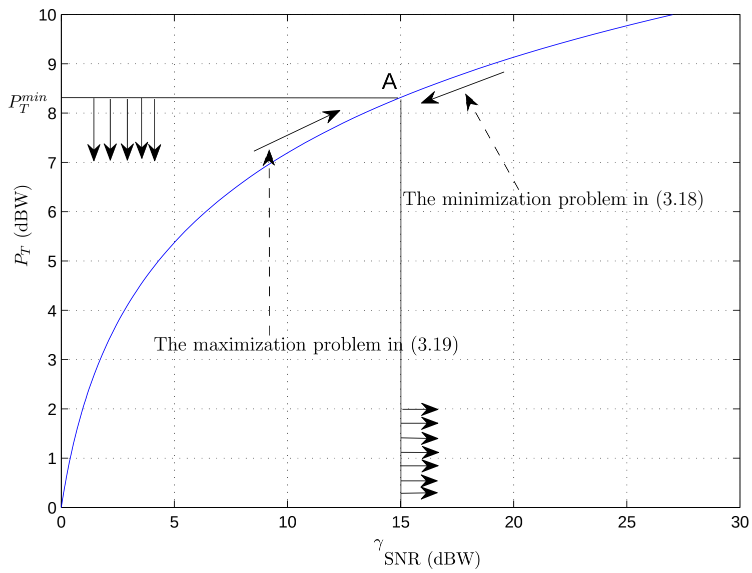

Fig. 3.2: Total Transmit Power versus SNR.

Chapter 4

Simulations

In this chapter, we discuss the simulations that have been performed on different variations of the system and under different channel conditions. In the first section we assume the uplink and downlink channels vectors to be statistically independent of each other and discuss the effects of different levels of channel uncertainty on the total relay transmit power. The simulation results show that for higher levels of uncertainty in the channel, the total relay transmit power required to maintain a certain threshold of the receiver SNR is also higher. In the next section we perform a comparison between a rank-one beamformer and a general-rank beamformer. The obtained simulations results show the performance gain of the general-rank beamformer over the rank-one beamformer in terms of the total relay transmit power. In the last section we consider the case where the uplink and downlink channel vectors are statistically dependent. We perform an analysis on the effects of channel dependency on the total relay transmit power. Our results show that with an increase in the channel dependency there is an increase in the performance of the general-rank beamformer.

4.1 Channel Coefficient Variations for Statistically Independent Channels

In our simulations, we have assumed that $\mathbf{R}_n = \sigma_\eta^2 \mathbf{I}$ and that $\sigma_\eta^2 = \sigma_n^2 = 0$ dBW. The information symbols are considered to be drawn from the QPSK constellation. In the first example, we consider a scenario where the two channel vectors $\mathbf{f}$ and $\mathbf{g}$ are statistically independent. We assume that the relay node has 20 antennas, therefore the channel vectors are modeled as,

$$\mathbf{f} = \bar{\mathbf{f}} + \tilde{\mathbf{f}} \tag{4.1}$$

$$\mathbf{g} = \bar{\mathbf{g}} + \tilde{\mathbf{g}}, \tag{4.2}$$

where we know that $\bar{\mathbf{f}}$ and $\bar{\mathbf{g}}$ are the mean of the channel vectors $\mathbf{f}$ and $\mathbf{g}$, respectively. The vectors $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$ are the zero-mean circularly symmetric Gaussian random vectors. $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$ have been modeled as the variations of the channel vectors around their nominal values. The mean vectors $\bar{\mathbf{f}}$ and $\bar{\mathbf{g}}$ are known and their ith entries are respectively generated as,

$$\bar{f}_i = \frac{e^{j\theta_i}}{\sqrt{1+\alpha_f}} \tag{4.3}$$

$$\bar{g}_i = \frac{e^{j\phi_i}}{\sqrt{1+\alpha_g}}, \tag{4.4}$$

where $\bar{f}_i$ and $\bar{g}_i$ are the ith entry of $\bar{\mathbf{f}}$ and $\bar{\mathbf{g}}$ respectively. We have defined two random variables θ_i and ϕ_i which are independently chosen from the interval $[0 \;\; 2\pi]$. The ith entries of the vectors $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$ are i.i.d. zero-mean Gaussian random variables whose variances are given by,

$$E\{|\tilde{f}_i|^2\} = \frac{\alpha_f}{1+\alpha_f} \tag{4.5}$$

$$E\{|\tilde{g}_i|^2\} = \frac{\alpha_g}{1+\alpha_g}, \tag{4.6}$$

where $\tilde{f}_i$ and $\tilde{g}_i$ are the ith entry of $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$, respectively. Based on this channel model we have defined two parameters α_f and α_g that are used to quantify the level of uncertainty in our knowledge about the channel vectors. The *true* channel covariance matrices are given by,

$$\mathbf{R}_f = \bar{\mathbf{f}}\bar{\mathbf{f}}^H + \frac{\alpha_f}{1+\alpha_f} \quad (4.7)$$

$$\mathbf{R}_g = \bar{\mathbf{g}}\bar{\mathbf{g}}^H + \frac{\alpha_g}{1+\alpha_g}. \quad (4.8)$$

Large values of α_f and α_g correspond to the case where our knowledge of the uplink channel vector $\mathbf{f}$ and the downlink channel vector $\mathbf{g}$ is highly uncertain, therefore increasing the values of α_f and α_g results in the decrease in the norm of the mean vectors $\bar{\mathbf{f}}$ and $\bar{\mathbf{g}}$ and increases the variance of the elements present in $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$.

In Fig. 4.1 to Fig. 4.8, we have shown and analyzed the simulation plots of total relay transmit power P_T^{min} versus the receiver SNR for different levels of uncertainty in our knowledge of the uplink channel vector α_f and the downlink channel vector α_g. It is worth mentioning here that a limit exists on the minimum value of total relay transmit power. A point is reached where the problem becomes negative definite and hence the minimum total relay transmit power tends to infinity. Here any further increase in the total transmit power does not result in any maximum feasible SNR.

Fig. 4.1 shows the P_T^{min} versus SNR for $\alpha_f = -15$ dB and for different values of α_g. In Fig. 4.1 we first analyze the plot where our knowledge about the channel is most certain (i.e. $\alpha_f = \alpha_g = -15$ dB). Here we see that in order to maintain an SNR of 13 dB, the minimum total relay transmit power required (P_T^{min}) is 16.83 dB. We then observe the gradual increase in the level of uncertainty in our knowledge of the downlink channel vector. We see that as the values of α_g increase, the P_T^{min} required to maintain the SNR at 13 dB also increases. We then analyze the case where the uncertainty in the downlink channel is at its maximum (i.e. $\alpha_f = -15$ dB and $\alpha_g = 10$ dB) and observe the increase in P_T^{min} to 24.5 dB. Thus there is a 7.67 dB increase in the total relay transmit power.

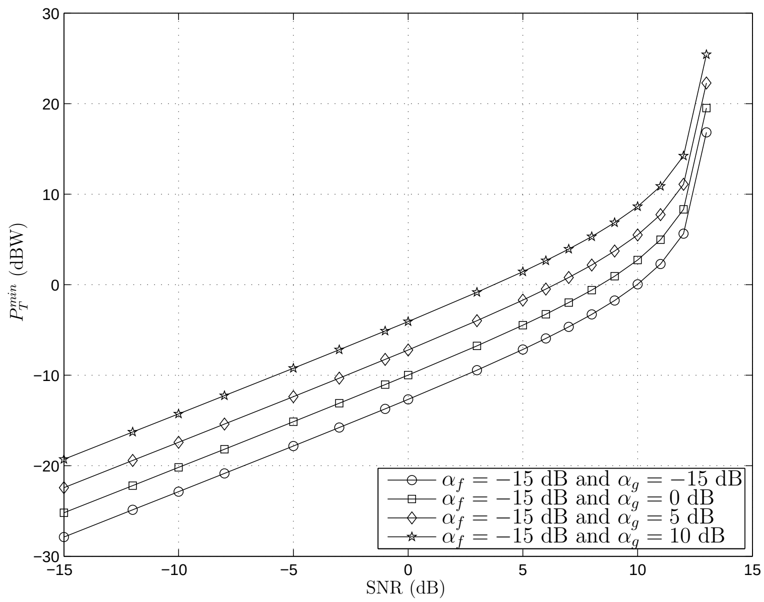

Fig. 4.1: The maximum total relay transmit power versus SNR for $\alpha_f = -15$ dB and for different values of α_g

In Fig. 4.2 the uncertainty in our knowledge of the uplink channel has increased to 0 dB, here we see that in order to maintain an SNR of 10 dB, the minimum total relay transmit power required P_T^{min} is 7.5 dB for $\alpha_f = 0$ dB and $\alpha_g = -15$ dB. We then observe the gradual increase in the level of uncertainty in our knowledge about the downlink channel vector. Again, we see that the increase in the level of uncertainty in our knowledge of the channels leads to an increase in the total relay transmit power required to maintain the 10 dB SNR threshold. When the level of uncertainty is at its maximum (i.e. $\alpha_f = 0$ dB and $\alpha_g = -10$ dB) a P_T^{min} of 16.2 dB is required. Thus there is a 8.7 dB increase in the total relay transmit power.

Fig. 4.3 and Fig. 4.4 show the effects of the increase in the level of uncertainty in our knowledge of the channel vectors on the minimum total relay transmit power and that the increase in the level of uncertainty in the channel vectors corresponded to a considerable decrease in the maximum feasible values of SNR.

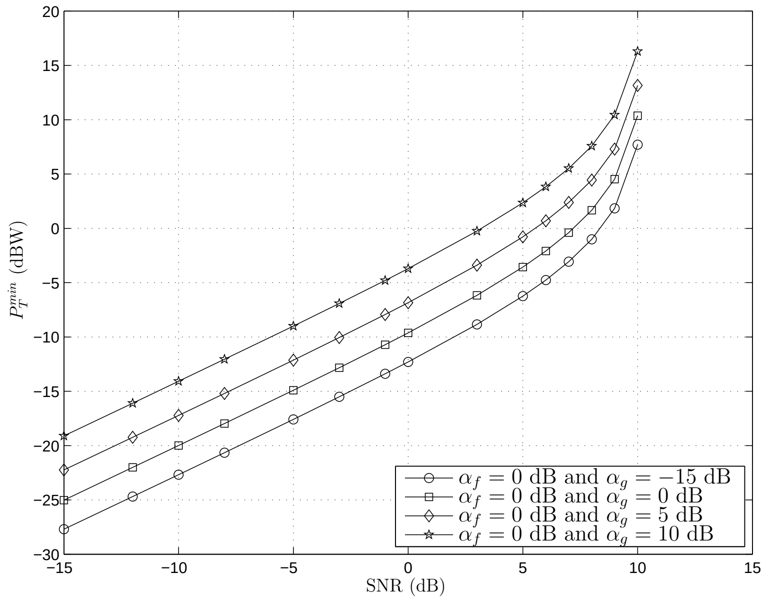

Fig. 4.2: The maximum total relay transmit power versus SNR for $\alpha_f = 0$ dB and for different values of α_g

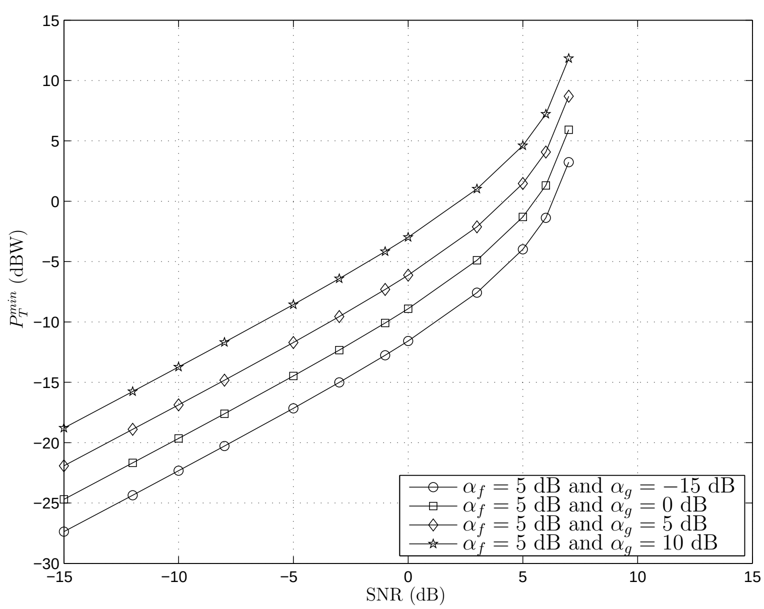

Fig. 4.3: The maximum total relay transmit power versus SNR for $\alpha_f = 5$ dB and for different values of α_g

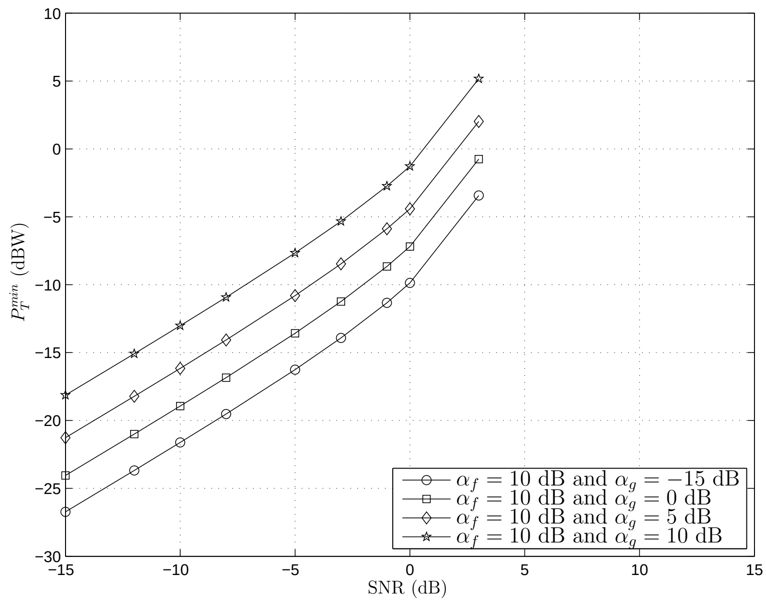

Fig. 4.4: The maximum total relay transmit power versus SNR for $\alpha_f = 10$ dB and for different values of α_g

In Fig. 4.5 to Fig. 4.8 we have kept the level of uncertainty in our knowledge of the downlink channel vector α_g constant for each plot. We have then varied the level of uncertainty in our knowledge of the uplink channel vector. Fig. 4.5 illustrates the P_T^{min} versus SNR for $\alpha_g = -15$ dB and for different values of α_f.

We observe that in order to satisfy an SNR of 10 dB, the minimum total relay transmit power required is 0 dB when the level of uncertainty in our knowledge of both the uplink and downlink channel vectors is at the lowest (i.e. $\alpha_f = \alpha_g = -15$ dB). However if we were to increase the value of α_f to 0 dB. The minimum P_T^{min} required is 7 dB. Considering higher levels of uncertainty in the channel, we observe that satisfying even lower values of SNR approximately 3 dB becomes harder. It is here that an infeasibility problem exists and P_T^{min} tends to infinity.

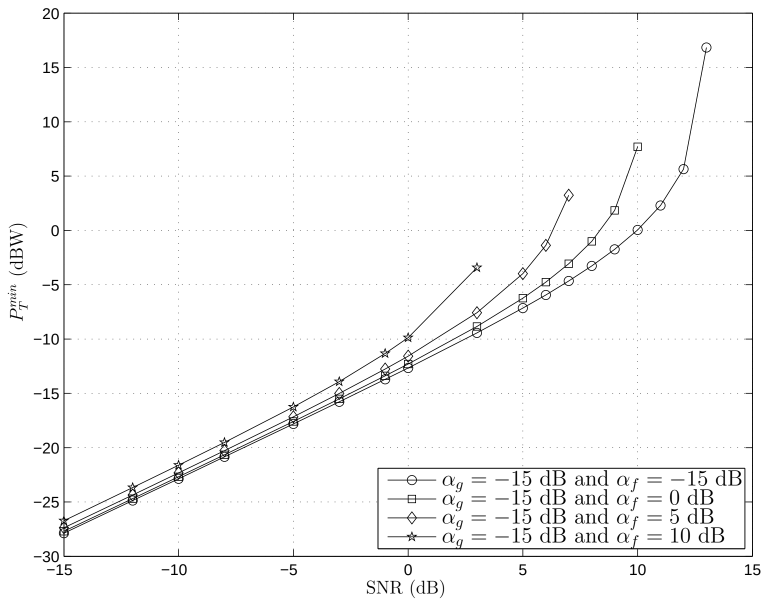

Fig. 4.5: The maximum total relay transmit power versus SNR for $\alpha_g = -15$ dB and for different values of α_f

In Fig. 4.6 to Fig. 4.8 our knowledge of the level of uncertainty in the downlink channel is increased. The effects of this increase on the total relay transmit power P_T^{min} is analyzed for different values of α_f. Our results show that the increase in the level of uncertainty in our knowledge of the channel vectors leads to an increase in the total relay transmit power required to satisfy the a given SNR threshold.

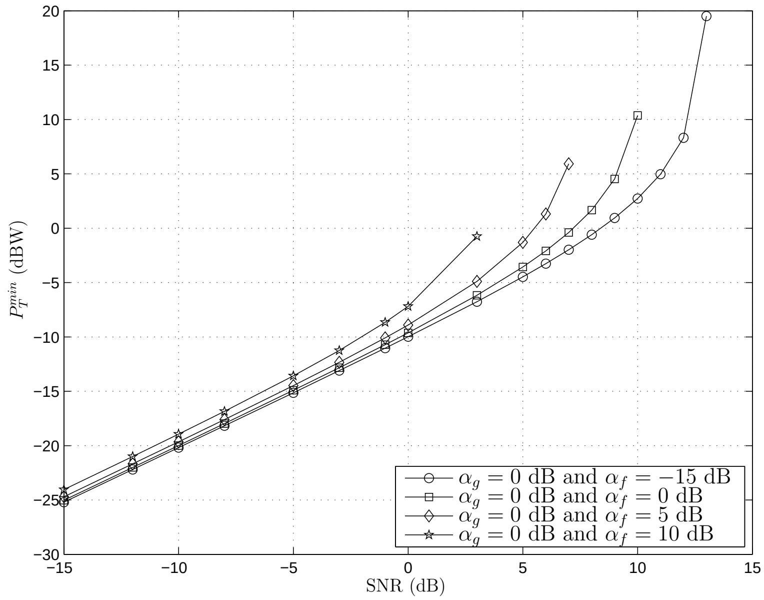

Fig. 4.6: The maximum total relay transmit power versus SNR for $\alpha_g = 0$ dB and for different values of α_f

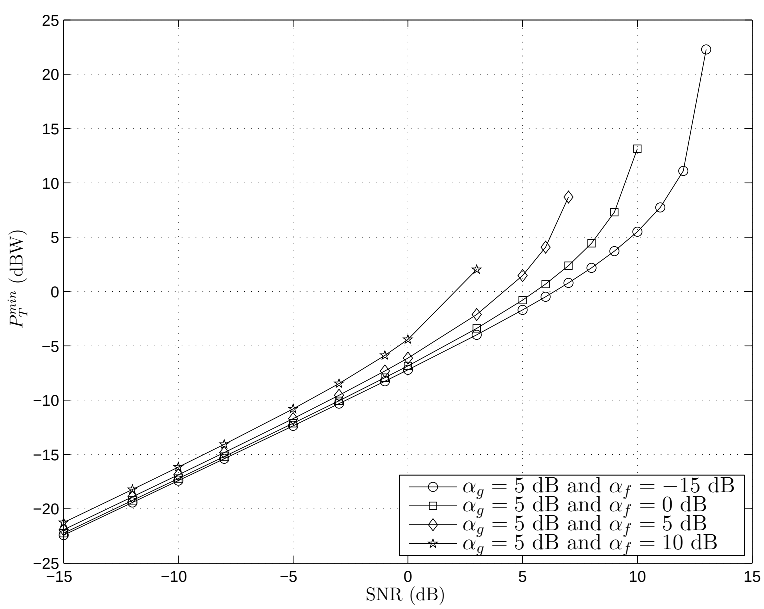

Fig. 4.7: The maximum total relay transmit power versus SNR for $\alpha_g = 5$ dB and for different values of α_f

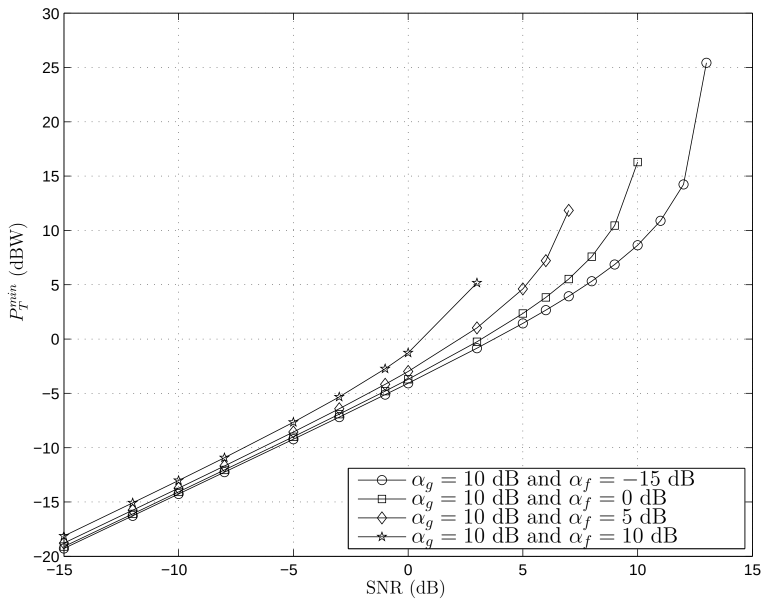

Fig. 4.8: The maximum total relay transmit power versus SNR for $\alpha_g = 10$ dB and for different values of α_f

4.2 Comparison between a General-Rank Beamformer and a Rank-One Beamformer

In the second example, we consider a scenario where the channel vectors **f** and **g** are statically dependent. We assume that the channel vectors comply with the model presented in the first example, however the vectors of channel variations are generated as,

$$\tilde{\mathbf{f}} = \sqrt{\frac{\alpha_f}{1+\alpha_f}}\mathbf{z}$$
$$\tilde{\mathbf{g}} = \sqrt{\frac{\alpha_g}{1+\alpha_g}}\mathbf{z},$$

where **z** is a zero-mean complex Gaussian random vector with covariance matrix equal to **I**. For this second example, the variation vectors in the transmitter - relay and relay - receiver channels are different scalings of the random vector **z**, and are therefore statistically dependent. This model still leads to the channel vectors **f** and **g**

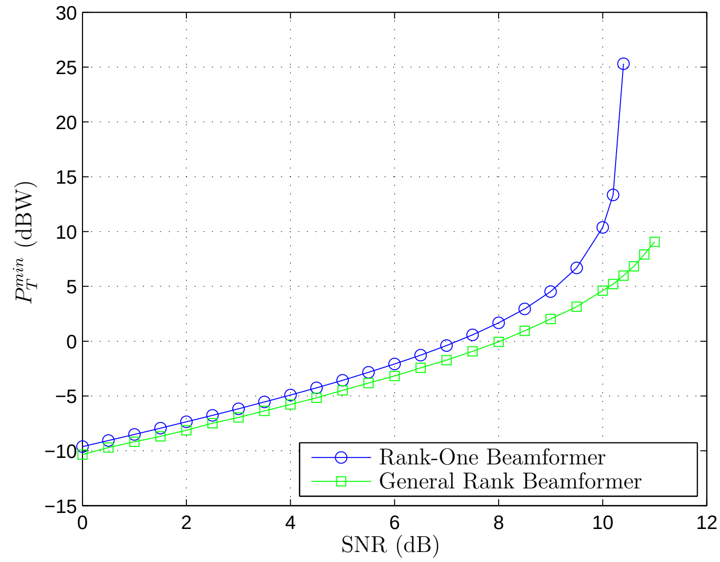

Fig. 4.9: The minimum total relay transmit power versus SNRs for the General Rank beamformer versus a Rank One beamformer with $\alpha_f = \alpha_g = 0$ dB

with the same correlation matrices as given in (4.7) and (4.8), respectively. However for the case of statistically dependent channels,

$$\mathbf{R}_h = \mathrm{E}\{\mathbf{h}\mathbf{h}^H\} \neq \mathbf{R}_g^* \otimes \mathbf{R}_f. \tag{4.9}$$

The last inequality is true because $\mathbf{f}$ and $\mathbf{g}$ are statistically dependent.

In Fig. 4.9 and Fig. 4.10 we have shown the plot of P_T^{min} versus SNR obtained by using two methods, the rank-one beamformer method and the general-rank beamformer method. The rank-one beamformer method uses a combination of receive beamforming followed by transmit beamforming. The receive beamformer is used to obtain a linear estimate of the transmitted symbol at the relay by maximizing the relay SNR, while the transmit beamformer is used to maximize the receiver SNR under the total relay transmit power constraint. The output of these two beamformers is properly scaled to satisfy the SNR constraint. In Fig. 4.9, we have kept our knowledge of the channel uncertainty fixed at $\alpha_f = \alpha_g = 0$. Therefore, in order to satisfy an SNR of 9 dB, the rank-one beamformer requires a total relay transmit power P_T^{min} of 10.5

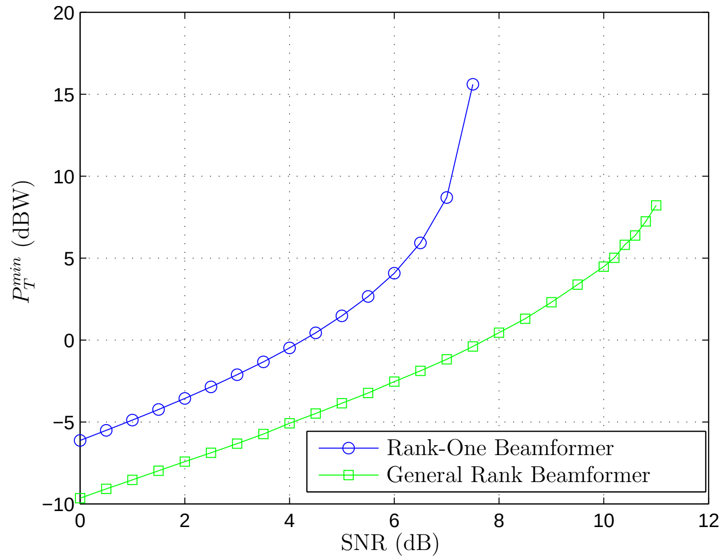

Fig. 4.10: The minimum total relay transmit power versus SNRs for the General Rank beamformer versus a Rank One beamformer with $\alpha_f = \alpha_g = 5$ dB

dBW. However, the general rank beamformer requires a total relay transmit power of only 5 dBW. Thus, we see a performance gain of 3 dBW.

In Fig. 4.10, we have increased the level of our knowledge of the uncertainty in the channel vectors. Thus, we have considered $\alpha_f = \alpha_g = 5$. In order to satisfy an SNR of 7.5 dB, we see that the rank-one beamformer requires a minimum power P_T^{min} of at 16 dBW. The general-rank beamformer requires a P_T^{min} of 0 dB. Thus we observe a 16 dB performance gain with the general-rank beamformer. Such a significant performance gain is the result of considering the statistical dependence of the transmitter-relay and relay-receiver channel vectors and using a rank-one restriction on the beamforming weight matrix. Thus the general-rank beamformer outperforms the rank-one beamformer in terms of the total relay transmit power.

4.3 The Effect of Channel Dependency on the Total Relay Power

In the third example, we study the effect of the correlation between the channel coefficients on the SNR, $\tilde{f}$ and $\tilde{g}$ can be modeled respectively, as

$$\begin{aligned} \tilde{f} &= \sqrt{\frac{\alpha_f}{1+\alpha_f}}(\beta \mathbf{z} + \sqrt{1-\beta^2}\mathbf{z}_1) \\ \tilde{g} &= \sqrt{\frac{\alpha_g}{1+\alpha_g}}(\beta \mathbf{z} + \sqrt{1-\beta^2}\mathbf{z}_2), \end{aligned}$$

where $\mathbf{z}$ represents the correlated components of $\tilde{f}$ and $\tilde{g}$, $\mathbf{z}_1$ and $\mathbf{z}_2$ are the uncorrelated components of $\tilde{f}$ and $\tilde{g}$ respectively. The parameter β quantifies the correlation between $\tilde{f}$ and $\tilde{g}$. In Fig. 4.11, we assume a relay with 15 antennas. We see the total relay transmit power P_T^{min} of the rank-one beamformer and the general-rank beamformer versus the correlation factor β to satisfy an SNR of 6 dB and for $\alpha_f = \alpha_g = 5$ dB. Thus performance gap between the two beamformers increases as β increases and when the two channels are totally correlated (i.e., $\beta = 1$), we see a reduction of P_T^{min} by 12 dB. In Fig. 4.12, we have assumed the system to have a relay with 20 antennas. Again, we see a performance gain of 2.5 dB when the channels are totally correlated. As seen from Fig. 4.11 and Fig. 4.12 the total relay transmit power for the general-rank beamformer is significantly reduced as β is increased.

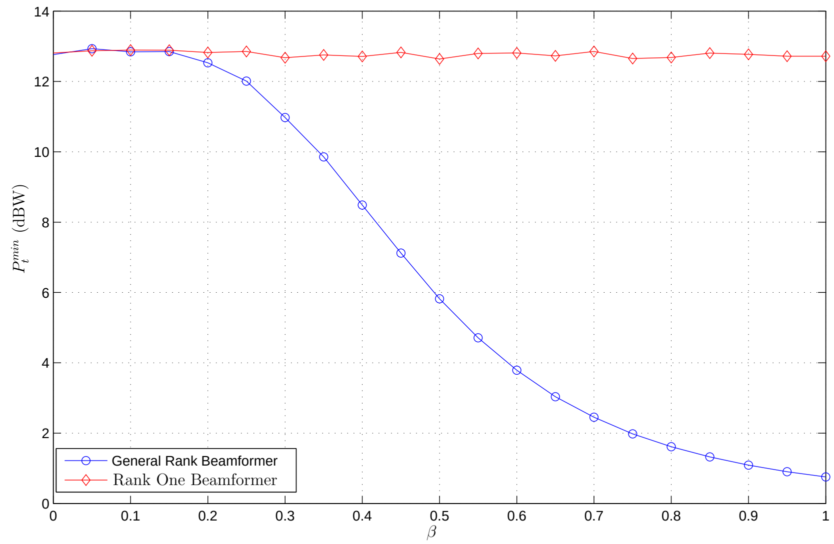

Fig. 4.11: The general-rank beamformer versus the rank-one beamformer for 15 antennas on the relay.

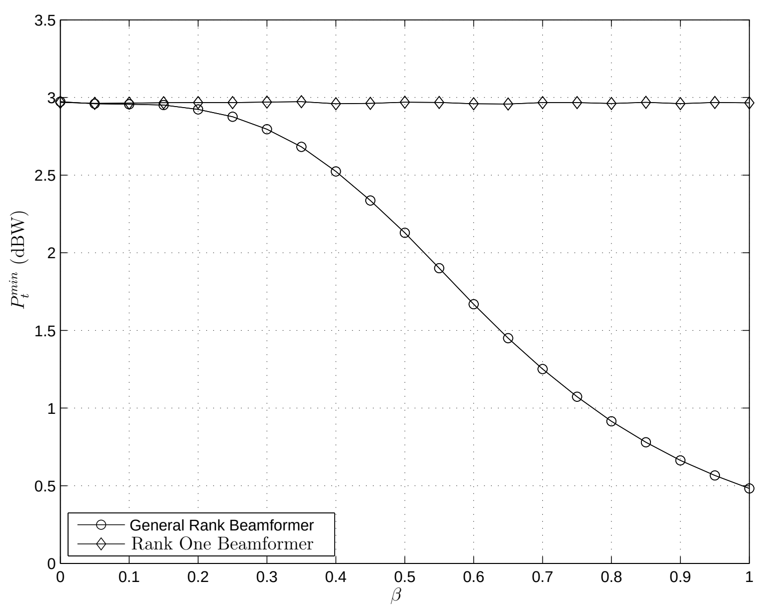

Fig. 4.12: The general-rank beamformer versus the rank-one beamformer for 20 antenna on the relays

Chapter 5

Conclusion

In this thesis, we studied the problem of joint receive and transmit beamforming for a wireless network. Our system was made up of a transmitter, a receiver and a relay node. The relay node is equipped with multiple antennas while the transmitter and the receiver are single antenna systems. Our communication scheme consisted of two phases. In the first phase, the transmitter sends the information symbols to the relays. In the second phase, the relay re-transmits an amplitude- and-phase adjusted version of its received signal. We have used the concept of general-rank beamforming and have applied it to our communication scheme for cases where the uplink and downlink channel vectors are statistically independent and dependent. In the general-rank beamforming approach, the multi-antenna relay multiplies the vector of its received signals with a general-rank complex beamforming weight matrix and re-transmits each entry of the output vector on the corresponding antenna. Through our studies, we have obtained a closed-form solution to the general-rank beamforming power minimization problem. We have also proved that for statistically independent uplink and downlink channels, the general-rank beamforming approach results in a rank-one solution for the beamforming weight matrix. Through our simulation results, we have shown that as the level of uncertainty in our knowledge of the channel vectors is increased, the power required to satisfy a given SNR increases. However, it is noted that there is a limit on the minimum value of total relay transmit power. A point is reached where the problem becomes negative definite and hence the minimum total relay transmit power tends to infinity. In this case, any further increase in the total transmit power does not result in any maximum feasible SNR. We compared the performance of the general-rank beamformer and a rank-one beamformer versus the total relay transmit power. Our simulation results showed us that the general-rank beamformer is able to satisfy the given threshold of SNR for a much lower total relay transmit power. In some cases, the general-rank beamformer outperforms the rank-one beamformer by 16 dB in terms of the total relay transmit power. In a study to see how the statistical dependency between the uplink and downlink channel vectors affected the performance of the two beamformers, we saw that the general rank beamformer utilizes this channel dependency to reduce the total relay transmit power by 12 dB when compared to the rank-one beamformer.

The main aim of our work was to minimize the power consumed by the system, thus prolonging the life of the entire system. Therefore, the future work in this field would be to develop and test different system models that are variations from our system model by including certain aspects that are most common in today's commercial applications. Some of the future work in this field would be the utilization of the general-rank beamformer with a per antenna power constraint so that, not all antennas have to transmit data using the maximum transmit power allotted to them. This scheme not only has the potential to increase the life of the entire network by saving on the power usage, but also the potential to reduce the overall transmit power of the system. Our current system model is set up for the unidirectional flow of data from the transmitter to the receiver via the relay. However, most commercial applications today require a bidirectional flow of data by the use of transceivers. Therefore our communication scheme could be expanded to use transceivers instead of a transmitter and a receiver. Hence flow of data would then be bidirectional between the two transceivers. In this new bidirectional data flow scheme, we can impose power constraints on each of the antenna present on the relay and compare the power saving with that of the total relay transmit power constraint approach. Again, in our current system since we have used a single antenna transmitter and receiver, we have not been able to make use of spatial diversity at the transmitter or the receiver. However, if we were to use a multi-antenna transmitter or receiver we could make use of spatial diversity gains to further increase the SNR of the system or further minimize the total relay transmit power of the system. This could be another research direction.

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