SUM-RATE MAXIMIZATION FOR TWO-WAY ACTIVE CHANNEL WITH UNEQUAL SUBCHANNEL NOISE POWERS

by

Ashkan Kiani

A thesis submitted in conformity with the requirements
for the degree of Master of Applied Sciences
Graduate Department of Electrical and Computer Engineering
University of Ontario Institute of Technology
June 2016

Abstract

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Wireless parallel channels refer to certain wireless links, where the information is conveyed from a source to a destination through a set of parallel orthogonal subchannels. In such channels, the transmitted signals undergo path loss, phase shift, and can also be affected by multi-path propagation effects. In conventional wireless channel models, there is no control over the gain of each individual subchannel. We herein refer to these conventional channels as passive channels. In this thesis, we study the problem of joint power allocation and design of a parallel channel with orthogonal subchannels where not only the source transmit power(s) over subchannels can be adjusted, but also the powers of each subchannel can be carefully chosen for optimal performance. We herein refer to such wireless links as active channels. We study the problem of sum-rate maximization of a two-way active channel with unequal noise powers over the subchannels, conveying information between two sources (two transceivers that can transmit and receive signals), subject to three constraints. The first two constraints are on the sources' total transmit powers and the last constraint is on the total active channel's power. Although this maximization problem is not convex, we develop an analytical method with efficient computations for optimal sources and channel power allocation, by utilizing Karush-Kuhn-Tucker (KKT) conditions. To do so, we first show that not all subchannels, but only a subset may receive transmit power from the sources and the active channel. We then use KKT conditions to determine the necessary conditions for optimality. By searching through the solutions obtained by KKT conditions, we find the number of subchannels which should be active in order for the power constraints to be feasible. We subsequently obtain the optimal channel power allocation for any feasible number of active subchannels. Hence, the optimal solution can be obtained by comparing a finite number of points in the feasible set and introducing the point which yields the best sum-rate performance, as the optimal point that represents the maximum sum-rate.

Dedication

To my lovely family

Acknowledgments

I would like to express my sincere gratitude to many people who supported me during the course of my studies. First and foremost, I would like to thank my supervisor, Professor Shahram ShahbazPanahi for his technical support and guidance. I would like to thank my friends all over the world for their friendship and support in every aspect of my graduate life. Most importantly, I would like to thank my father, Bijan Kiani, for his continuing support and words of encouragement, my mother, Zahra Radhoush, for her unconditional love, support, and thoughtful advice through different stages of my life, and also my one and only sister, Niloufar Kiani, for all her ongoing love, support, and mentorship in welcoming me to the engineering world.

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Nomenclature

BER	Bit Error Rate
CSI	Channel State Information
CSIT	Channel State Information at the Transmitter
ECG	Electrocardiogram
ISI	Inter-Symbol Interference
KKT	Karush-Kuhn-Tucker
LHS	Left Hand Side
MIMO	Multiple-Input Multiple-Output
MMSE	Minimum Mean-Squared Error
MSE	Mean Square Erorr
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
QoS	Quality of Service
RHS	Right Hand Side
SDV	Singular Value Decomposition
SIMO	Single-Input Multi-Output
SISO	Single-Input Single-Output
SNR	Signal to Noise Ratio
TDMA	Time Devision Multiple Access

Chapter 1

Introduction

1.1 Overview

Communication industry has witnessed an enormous demand for fast, high-quality, and reliable communication at a low cost over the last two decades. This demand has resulted in the replacement of traditional means of communication with wireless communication. Wireless communication services are mostly designed and manipulated to meet the requirements of high quality video and audio streaming and fast internet services. In order to satisfy these requirements, the system should be capable of supporting high data-rates, as well as providing a reliable transmission in a resource-limited world. These limitations can be either imposed by the system itself (e.g., energy constraints), or by the environment (e.g., shadowing or fading effects). Hence, in order to achieve robust communication, it is necessary to make the most out of available resources such as energy, frequency bandwidth, and spectrum. For instance, cognitive radios have been utilized, in order to reuse the frequency spectrum. In another instance to overcome the fading effects of the environment, different diversity techniques in different domains and approaches (such as time domain, frequency domain, multi-antenna systems, and coding techniques) have been utilized.

Wireless channels can be categorized depending on their features. However, the focus here is on the wireless parallel channels, such as fading channels, multi-relay systems, and multi-antenna systems. In traditional parallel channels, subchannels' gains are not adjustable and such channels will be referred to as passive channels in this thesis. Here, we study another type of parallel channels, where the energy (or the gain) of each subchannel can be adjusted judiciously. Such channels are referred to as active channels, since they differ from the passive channels in sense of having control over each subchannel's power (i.e., subchannels' gains adjustability). The active channels optimally adjust the gain of each subchannels in order to optimize a certain performance criterion (e.g., maximize sum-rate). One instance of the active channels is the relay networks (channels), where the end-to-end subchannel gains can be adjusted at the relays [1–8]. Another example of the active channels is the Raman amplifier, where the raman gain is caused by the transfer of power from one optical beam to another which is downshifted in frequency by the energy of an optical phonon [9]. We herein bring our attention to the two-way active channels which carry information in both directions. In this thesis, the objective is to maximize the sum-rate of two-way active channels under both transmit power and channel energy constraints, in a practical environment where each subchannel has a different noise power.

1.2 Wireless Parallel Channel

Parallel channels refer to certain wireless links, where the information is conveyed from a source to a destination through a set of subchannels with independent noise, gain and power (subchannels are linked, only through the total energy constraint). Some examples of parallel channels are the fading channels, inter-symbol interference (ISI) channels, multiple input multiple output (MIMO) channels and multi-antenna systems [10–16]. Time varying fading channels, orthogonal frequency division multiplexing (OFDM), and multi-antenna communication are some examples of parallel channels. For instance, consider a frequency flat-fading channel with a time-varying gain. The channel at each instance of time can be assumed as a subchannel with a different gain at that specific instance. In other words, each fading state is interpreted as a subchannel [14,16]. As other example, through ISI channels where orthogonal non-overlapping frequency bands are utilized (as in OFDM), the signals are transmitted over these different frequency bands (i.e., multi-tone transmission) [10]. Interpreting each frequency band as a parallel subchannel, this channel can be considered to be parallel. In the multi-antenna communication, a MIMO system refers to a system with multiple antennas at the transmitter and receiver. These antennas are used to improve the performance in terms of bit error rate (BER) and capacity, and also to increase the data rate through multiplexing. Goldsmith [17] explains that the initial work on MIMO systems have been conducted by several pioneers, including Winter [18], Foschini [19], Foschini and Gans [20], and Telatar [21]. They all have predicted spectral efficiencies for wireless systems with multiple transmit and receive antennas. A MIMO channel can be decomposed into a number of parallel independent channels, as delicately studied by Telatar in [21]. Telatar shows that the singular value decomposition (SVD) of the channel gain matrix represents a set of parallel subchannels whose gains are given by the singular values of the MIMO channel. By multiplexing independent data onto these independent subchannels, more efficiency is achieved in comparison to a system with only one antenna at the transmitter and the receiver. It is also shown in [11, 12] that the number of these independent parallel subchannels is equal to the rank of the channel response matrix.

1.3 Cooperative Communications

As discussed in previous sections, MIMO system are widely known to be beneficial. For example some transmit diversity methods, such as Alamouti signaling, have appeared in wireless standards. However, MIMO system's applications may not be practically widespread due to certain issues such as size, cost, or the hardware complexity restrictions. Hence, cooperative communication has been exploited in a multiuser environment with single-antenna mobiles. In cooperative communication systems, users are able to share their antenna with other users within the network, and create a virtual multi-antenna network [22, 23]. In other words, all users in a cooperative network, not only transmit their own data, but also play a relaying role for other users in the network, in terms of receiving and transmitting their data to/from the base station or the next user which plays the role of another relay. Hence, different relaying strategies, schemes and topologies might be used. In such networks, considering each user as a relay and the network as a multi-relay network, we can consider each different path from one node to another as a subchannel. Hence, we can utilize the enhancements of the parallel channels in cooperative networks as well.

One of the major concerns in cooperative diversity networks is a decrease in transmission rates; since each node (i.e., user) transmits its own data in addition to some other users' data. It is noteworthy that, many aspects should be taken into account in designing a cooperative communication network; Aspects such as, cooperative assignments, the total interference of the network, fairness, reliability, and the transceivers' requirements [24]. Compared to point-to-point communication, cooperative communication has many advantages, such as a wider coverage area, larger capacity region, and enhanced communication reliability [25]. By utilizing the terminals distributed in space, a significant enhancement in wireless network performances is achievable [26, 27]. For instance, cooperation between two adjacent nodes with channel state information (CSI) in beamforming a signal toward a destination, leads to a higher total throughput/capacity of the network as compared to the case in which this pair of nodes do not cooperate. In [28–30], extensive studies on results of the cooperation among users have been conducted.

1.4 Water-filling Power Allocation

The water-filling power allocation technique is a well-known optimal solution for the sum-rate maximization over a parallel channel. A parallel channel contains a set of parallel subchannels works as a communication link that carries information from a transmitter to a receiver [17, 31]. We herein, consider a wireless parallel channel with N orthogonal parallel subchannels, while the noise over each subchannel is independent. In [32], it has been shown that the sum-rate of such channel can be written as:

$$\sum_{i=1}^{N} \log(1 + p_i c_i), \tag{1.1}$$

where p_i is the assigned power of the ith subchannel. Note that c_i is defined as $c_i \triangleq \alpha_i |h_i|^2$, where α_i and $|h_i|^2$ denote inverse of the noise power and the power of the ith subchannel. When the CSI is available (c_i's are known), the problem will be to find the optimal power allocation scheme, at which the maximum sum-rate has been achieved while the power constraint, $\sum_{i=1}^{N} p_i = P_T$, is satisfied. Hence, the optimization problem can be rewritten as:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{N} \log(1 + p_i c_i) \\ \text{subject to} \quad & \sum_{i=1}^{N} p_i = P_T, \quad p_i \geq 0 \quad \text{for} \;\; i = 1, 2, \ldots, N \end{aligned} \tag{1.2}$$

where P_T denotes the total available power at the transmitter and the vector $\mathbf{p} \triangleq [p_1, p_2, \ldots, p_N]$ is the vector of the powers of the different subchannels.

Introducing Lagrangian multipliers $\lambda^* \in \mathbb{R}^N$ for the inequality constraints $p_i \geq 0$, a multiplier $\nu^* \in \mathbb{R}$ for the equality constraint, and the p_i^* as the optimal value of the p_i, the KKT conditions can be obtained as:

$$\begin{aligned} p_i^* \succeq 0, \quad \mathbf{1}^T p_i^* = \mathbf{P}_T, \quad \lambda^* \succeq 0, \quad \lambda_i^* p_i^* = 0, & \qquad \text{for} \;\; i = 1, 2, \ldots, N, \\ -\frac{c_i}{(1 + p_i^* c_i)} + \nu^* - \lambda^* = 0, & \qquad \text{for} \;\; i = 1, 2, \ldots, N. \end{aligned} \tag{1.3}$$

where $\mathbf{x} \succeq \mathbf{0}$ means that all of the entries of the vector $\mathbf{x}$ are positive and $\mathbf{1}$ is an all-one vector. Considering λ^* as a slack variable, (1.3) can be rewritten as

$$\begin{aligned} p_i^* \succeq 0, \quad \mathbf{1}^T p_i^* = \mathbf{P}_T, \quad \lambda^* \succeq 0, \quad p_i^* \left(-\frac{c_i}{(1+p_i^* c_i)} + \nu^* \right) = 0, \quad \text{for} \ \ i = 1, 2, \ldots, N, \\ \nu^* \geq \frac{c_i}{(1+p_i^* c_i)}, \quad \text{for} \ \ i = 1, 2, \ldots, N. \end{aligned} \tag{1.4}$$

Assuming $p_i^* > 0$, we can write

$$\nu^* = \frac{c_i}{(1+p_i^* c_i)}. \tag{1.5}$$

Therefore

$$p_i^* = \frac{1}{\nu^*} - \frac{1}{c_i} \qquad \text{for} \ \ \nu^* < c_i. \tag{1.6}$$

It can be seen that if $\nu^* > c_i$, then our assumption about $p_i^* > 0$ does not hold true and consequently $\nu^* > \frac{c_i}{(1+p_i^* c_i)}$ that contradicts the complementary slackness condition. Thus we obtain

$$p_i^* = \begin{cases} \dfrac{1}{\nu^*} - \dfrac{1}{c_i} & \nu^* < c_i, \\ 0 & \nu^* \geq c_i. \end{cases} \tag{1.7}$$

We can write the optimal solution to (1.2), as

$$p_i^* = \max\{(\frac{1}{\nu^*} - \frac{1}{c_i}), 0\} \qquad \text{for} \ \ i = 1, 2, \ldots, N. \tag{1.8}$$

Assuming the total power as an amount of water poured over N connected containers where the bottom level of the ith container is $\frac{1}{c_i}$, the water settles with a depth of $\frac{1}{\nu^*}$, where ν^* is chosen to satisfy the equality constraint in (1.2). Hence, this method of power allocation is called *Water-Filling* or *Water-Pouring* [31–35].

1.5 Motivation

As mentioned in Section 1.4, the water-filling power allocation scheme in conventional wireless parallel channels, allots more power to the subchannels that have better quality (i.e., higher SNR) within the channels. These channels were referred to as passive channels, where we have no control over the gain of each subchannel. We obviously witnessed that in such channels, the maximum achievable sum-rate depends not only on the source transmit power, but also the quality of the individual subchannels. Hence, this problem has motivated us to define a similar scenario with a difference in its parallel channel. This difference is in the sense that not only the transmit power is adjustable, but also the parallel channel itself can be properly designed or adjusted in a way to achieve a higher sum-rate, compared to the traditional passive channels. In other words, we are defining a channel that can allocate a power (or gain) to its subchannels, in order to achieve the highest sum-rate. The sum-rate maximization problem for one-way active channels has been studied in [36–38], and for a two-way active channel with equal subchannel noise in [39, 40], where all have shown that the achievable sum-rate over the active channel can be larger than that of a passive channel. This improvement motivated us to define a more general and practical scenario of the active channel and solve the problem of sum-rate maximization for a two-way active channel with unequal subchannel noise.

1.6 Objective

We consider the sum-rate maximization problem for an active channel with unequal subchannel noise, which carries information between two sources, where each of them can act as a transmitter or a receiver. In such a scenario, we can inject the power, not only at the transceivers, but also at each of the unequally noise corrupted subchannels. By this means, we assume a scenario of maximization of the sum-rate, while not only the two transceiver powers are constrained, but also the power of the active channel is limited. In other words, we have three power constraints for our optimization problem: the first and second ones limit the total power consumed at the first and second transceivers, while the third limits the total power used by the active channel (injected into subchannels). Specifically, we define the subchannel powers as the square of each subchannel gains, and the total power constraint as the summation of these subchannel powers. Hence, the main objective is to jointly optimize the injected power into each subchannel by the active channel and the allocated power to each subchannel by the transceivers, in order to achieve the maximum sum-rate.

1.7 Methodology

For solving the optimization problem of maximizing the sum-rate over a two-way active channel, which we defined in a fully practical scenario where each subchannel was corrupted by an unequal independent noise, we first formulate the maximization problem under all power constraints (over the two transceivers and the channel). This mathematical representation consists of two maximizations. The inner maximization is to obtain the optimal power allocation and the outer is to obtain the optimal number of activated subchannels. We then see that this is a non-convex optimization problem. Nevertheless, the KKT conditions can be used to develop the necessary conditions for optimality. These necessary conditions limit the feasible set of possible solutions. We then explore all points of the feasible set that satisfy the necessary conditions, whereby we acquire a computationally efficient algorithm for obtaining the optimal power allocation scheme for any feasible number of activated subchannels. Once the optimal sources and channel power allocation is obtained, we calculate the sum-rate for any feasible number of activated subchannels and compare the sum-rates to determine the maximum sum-rate. The largest value yields the maximum sum-rate for the active channel with unequal subchannel noise powers.

1.8 Outline of Thesis

In this thesis, we consider a two-way active channel, whose subchannels experience an unequal noise powers, and aim to maximize the sum-rate for this channel, while both transceivers and the channel are energy-limited. The remainder of this thesis is organized as follows:

Chapter 2 reviews the passive parallel channels in Section 2.1, and then investigates the research results on power allocation schemes in different applications of the passive parallel channels in Section 2.2. In Section 2.3, we consider the sum-rate maximization over active channels. Section 2.4 scrutinizes the sum-rate maximization over the one-way active channels while Section 2.5 reviews the sum-rate maximization for a two-way active channel. In Chapter 3, we rigorously study the sum-rate maximization problem for a two-way active channel, where different subchannel of the active channel experience different noise powers and explain our proposed method in details. Chapter 4 presents our simulation analysis and results for our proposed method. In Chapter 5, we finally present the conclusion for our work and also point out several potential directions for further research that can be pursued.

Chapter 2

Literature Review

As discussed in Chapter 1, parallel channels are those used in modern communication techniques, such as MIMO systems, ISI channels, time division multiple access (TDMA), and orthogonal frequency division multiple access (OFDMA). They also play a major role in cooperative communication schemes to help create enhanced communication reliability. In this chapter, we first review the parallel passive channels and the techniques for maximizing the sum-rate of the parallel channels. Furthermore, we explore the idea behind the active channels and investigate several types and scenarios, such as one way active channels with equal and unequal subchannel noise power, as well as two way active channels with equal subchannel noise power.

2.1 Parallel Passive Channels

As discussed earlier, in Section 1.5, we refer to the parallel passive channels as those which convey information from a transmitter (i.e., source) to a receiver (i.e., destination), through a set of parallel subchannels, while there is no control over the gain of the subchannels. This noise has clearly been one of the limits for an efficient performance in the sense of the sum-rate. Such channels are beneficial to many different applications. As an instance of these applications, using parallel channels in biomedical signal processing, not only provides the possibility of forming different classes of informative parameters (i.e., amplitude, time, frequency, or another resulting from conversion of biomedical signal characteristics) due to separation of individual subchannels, but also the possibility to eliminate interference and minimize the relative error in determination of informative parameters [41]. For example, in [41], the use of parallel channels in preprocessing of the electrocardiogram signal (ECG) gives an increase in sensitivity of the electrocardiographic examination, by allowing a more accurate estimation of ECG element forms, under the impact of interference.

Other aforementioned channels, such as those with inter-symbol interference, fading channels, and multi-antenna systems, can be analyzed as a family of parallel Gaussian channels. As an example, in the case of ISI channels, each of the parallel channels corresponds to a carrier frequency; in the case of fading channels, each corresponds to a fading state [16]. There are other studies of wireless parallel channels, such as: [10, 11, 11–15].

When a transmitter can measure and track a channel, it is well known that capacity can be achieved by an optimal power allocation over the parallel channels [16]. Furthermore, the optimal power allocation can be computed via a simple water-filling construction [34]. As shown in Section 1.4, as well as in [32], this water-filling power allocation can be obtained by defining an optimization problem such as in (1.2). In the following section, we will provide a more detailed review of power allocation techniques in parallel passive channels.

2.2 Power Allocation in Parallel Passive Channels

As mentioned in the previous section, the use of parallel passive channels, and their corresponding power allocation techniques, have recently been a trend. Hence, we will review some of them in this section.

OFDM:

OFDM systems divide an entire channel into orthogonal narrowband subcarriers [42], and each set of these orthogonal subcarriers are modulated during each transmission. It is the orthogonality between the subcarriers that ensures the prefect reconstruction of the symbol stream at the receiver [43]. If we assume these subcarriers as orthogonal subchannels, the OFDM-based communication systems can be considered as a parallel channel. In [12, 44, 45], the water-filling power allocation scheme has been considered as an efficient method when the channel state information at the transmitter (CSIT) is available. This approach has been exploited to minimize the bit error rate in [46, 47], to minimize the transmit power in [48], and to maximize the throughput of the system and the spectral efficiency in [49–52]. Meanwhile, in a scenario in which CSIT is not entirely available, a modified form of the water-filling is proposed in order to maximize the sum-rate [53]. Under the circumstance of partially available CSIT, the authors in [54] develop this optimal power allocation in order to maximize the spectral efficiency, and the authors in [55] aim to minimize the bit error rate in a parallel channel. In a similar scenario of partially available CSIT, in [42], the authors apply the water-filling power allocation to the average of the channel gains (instead of the instantaneous gains), which leads to maximization of the upper bound for the sum-rate. They refer to their method as statistical water-filling. Their analysis was developed by the authors in [56] for parallel channels consisting of single-input multiple output (SIMO) with Nakagami-m fading over each subchannel. Consequently, by using statistical water-filling power allocation, the sum-rate is maximized (rather than the upper bound on the ergodic capacity in [42]).

OFDMA schemes also rely on the basis of the parallel channels, where each subset of subcarriers, assigned to each user, can be considered as a subchannel of the parallel channel. Power allocations in OFDMA systems, as well as subcarrier allocations, are studied in the literature [57–61]. In [58], the authors formulate the cross-layer optimization problem of maximizing the sum of the utilities over all active users in OFDMA, subject to a feasible rate region which is determined by an adaptive resource allocation scheme, and also subject to the current channel conditions. They present the conditions for optimal subcarrier assignment and power allocation, based on the utility. In the optimization problem of dynamic resource allocation on the downlink of the OFDM networks, they then focus on the fundamentals of maximizing the aggregate network utility. Consequently, they derive a criterion for utility-based subcarrier assignment and the water-filling method. The authors in [59] develop a transmit power adaptation method that maximizes the sum-rate of OFDM systems in a downlink transmission, which is based on assigning each subcarrier to only one user with the best channel gain within that subcarrier and allocating the transmit power over the subcarriers by the water-filling power allocation scheme.

MIMO Channels:

The studies on the single input single output (SISO) channels have been recently replaced by research studies and investigations in MIMO channels. Hence, using multiple antennas, both at the transmitter and the receiver, produces diversity in the communication system and/or leads to a significant improvement in the capacity of the channels (i.e., by creation of a set of independent parallel subchannels) [12, 21, 62, 63]. In other words, exploiting propagation diversity through multiple-element transmitter and receiver antenna arrays offers an increase in the data rate [63]. In early works, the information capacity of certain MIMO channels with memory was derived by Brandenburg and Wyner [64]. Studies by Cheng and Verdu [65] later derived the capacity region for more general multi-access MIMO channels.

It is shown in [19] and [21] that in Rayleigh flat-fading environments, a link comprising multiple-element antennas has a theoretical capacity which increases linearly with the least number of transmitter and receiver antennas. In [21], the authors research the use of multiple transmitting and/or receiving antennas for a single user communication over the additive Gaussian channel, and consequently show that the use of multiple antennas significantly increases the achievable rates on the fading channels if the channel parameters can be estimated at the receiver and if the path gains between different antenna pairs behave independently.

Optimal resource allocation in MIMO communications has become a research trend, recently. Optimal resource (e.g., power) allocation has been conducted in [12, 31, 44, 66–89] to improve the capacity of the channel, and in [45, 90–96] to minimize the BER and/or the MSE. In most aforementioned studies, multiple-antennas at the transmitter and receiver with their connecting links are considered as a parallel channel with a set of orthogonal subchannels. In [21, 62], it is shown that the optimum capacity of such systems can be obtained by summing the water-filled singular subchannels, which originate from the orthogonality of the MIMO channel matrix (i.e., which is often denoted as $\mathbf{H}$).

For frequency selective MIMO channels, one common capacity achieving method is a multi-tone transmission, where each subcarrier experiences a flat fading MIMO channel [12, 62, 97]. It is shown that, in the case where the transmitter and the receiver are designed such that the channel matrices at subcarriers are diagonalized and the CSI is available at the transmitter and receiver, the capacity of the frequency selective MIMO channel is achievable. The water-filling power allocation scheme over the subchannels should be then utilized [12, 31, 98].

When the problem of joint optimization of multiple transmitter-receiver pairs to minimize the MMSE gained attention, one of the earliest contributions to this problem was made by Salz [94], who developed the optimum linear MMSE vector transmission and reception filters for $M \times M$ channels with no excess bandwidth. Some of the later works on MIMO equalizers includes the linear equalizer with excess bandwidth and the decision feedback equalizer [93, 99, 100]. The authors in [62] consider a MIMO channel and point to design a pre-coder and a decoder for this transmission system using a weighted MMSE criterion, subject to the total transmit power constraint in order to minimize the MSE. The authors in [92, 101] generalize the joint optimization of the pre-coder and decoder of the MIMO channel, not only to minimize the MSE, but also to achieve the maximum sum-rate and satisfy a certain threshold for Quality of Service (QoS) over each subchannel, under a total power constraint. Similar structures and their corresponding water-filling power allocation schemes are extensively studied in [45, 90, 91, 93–95, 99, 102, 103].

In most of the mentioned studies, the water-filling scheme is utilized, where two parameters are required: a single water level, and a total transmit power constraint. For obtaining the water level, some methods are proposed (e.g., bisection). We can classify these methods as either *iterative algorithms*, in which the water level is obtained by iterative procedures [93, 104–106], or *exact algorithms* which lead to an exact value for the water level by a finite number of iterations [45, 92, 107].

2.3 Sum-Rate Maximization for Active Channels

Maximizing the sum-rate over the MIMO Gaussian broadcast channels under both transmit power and channel energy constraint (e.g., Frobenius norm constraint in [108]) is studied in [108–111]. In this section, we review several types of active channels, as well as the problem of maximization of the sum-rate over them.

The problems of optimal transmit power allocation and channel design, and/or channel power allocation under transmit power and/or channel energy constraints, are defined to find an upper bound for the MIMO channel capacity [85] and also the characteristics of channels with a maximum sum-rate [36–40]. These characteristics of the channel can be then utilized in designing the adaptive antenna arrays [108, 109, 111].

Single User MIMO Active Channels:

The authors in [108, 109] mostly focus on the study of single-user MIMO active channels with equal subchannel noise power.

In [109], the authors' objective is to maximize the sum-rate of the MIMO channel under both transmit power and channel energy constraints, while the locations of the antennas for the both transmitter and receiver can be optimally adjusted. In order to transform the MIMO channel into a parallel channel with a set of subchannels, the eigenvalue decomposition is used. In such a scenario, each eigenmode represents a subchannel and the power of each subchannel is defined by the square of the corresponding eigenvalue of the channel matrix. Moreover, the degree of freedom in choosing the locations of the antennas makes them capable of modifying a new eigenvalue of the channel matrix. Hence, maximizing the sum-rate is equivalent to optimal positioning of the antennas in order to achieve the optimal eigenvalues. For SNRs larger than a criteria, it is shown that the maximum sum-rate can be achieved by obtaining equal eigenvalues or, in other words, equally splitting the power between the subchannels, thereby satisfying the channel energy constraint.

The capacity of a point-to-point MIMO channel under the transmit power constraint and also the channel Frobenius norm constraint (i.e., Euclidean norm of a matrix is defined as the square root of the sum of the absolute squares of its elements [112]), is investigated by the authors in [108]. This study shows that the maximum sum-rate can be achieved when the channel has equal singular values for all non-zero eigenmodes. The authors then conduct a global search in order to find the number of these non-zero eigenmodes, while equally distributing the power between them. Consequently the maximum sum-rate is achieved.

Multiuser MIMO Active Channel:

In [111], the authors consider a multi-user MIMO active channel with the assumption of equal noise power over different subchannels, since their motivation was to find the class of channels which provides the best sum capacity under both total transmit power and total channel energy constraints. This study shows that in a case that $N_t \geq KN_r$, where N_t, N_r, and K denote the number of transmitting antennas, the number of receiving antennas, and the number of the users in the system, respectively, the best sum-rate is achieved when the user channels are mutually orthogonal to each other. Furthermore, the authors prove that, for each individual user, equal energy is distributed between all non-zero spatial eigenmodes. Finally, they optimize the number of non-zero eigenmodes for all users and the optimal power distribution among users.

2.4 One-Way Active Channel

In [36–38], the authors investigate the problem of joint power allocation and channel design for an active link, which conveys information from a transmitter to a receiver. They explain that the power can be injected through the subchannels, not only at the source, but also at each subchannel. Their objective is to maximize the sum-rate in such a scenario, under both total source power and channel power constraints. In [38], they assume that the noise power over all subchannels are equal (i.e., equal subchannel SNRs), while in [36, 37], they consider a more practical scenario where the subchannel noise powers are assumed to be unequal. The analysis and simulation results in [36–38] show that the active channel can offer a significantly higher sum-rate compared to their passive counterparts.

Equal Subchannel Noise Powers:

In [38], the authors assume that all of the subchannels of the active channel are corrupted by equal noise powers, and aim to find a closed form solution for the problem of maximizing the sum-rate for such a channel. Although their optimization problem is not convex, they present an efficient solution by showing that, at the optimum, only a subset of these subchannels should be active and the rest will remain inactive (i.e., the channel power should be distributed among a certain number of the subchannels), which is in contrast with passive parallel channels with equal subchannel SNRs, where the water-filling power allocation scheme, under the total transmit power constraint, yields the maximum sum-rate by equally allotting the power to all subchannels. Moreover, the authors not only prove that the number of activated subchannels depends on the product of the source and channel powers, but also prove that in a scenario where the total available power to the transmitter and the channel is limited, the optimal sum-rate is achieved when the total available power is equally divided between the transmitter and the channel [8].

Unequal Subchannel Noise Powers:

The authors in [36, 37] consider the same problem in [38] but with a different assumption regarding the subchannel noise powers. In [36,37], they investigate a more practical scenario where different subchannels are corrupted by noises with different powers. Despite the non-convexity of their optimization problem, they determine the number of subchannels that can be activated for the source power constraint to be feasible, by utilizing the KKT conditions. For any feasible number of the active subchannels, they then calculate the optimal power allocation. In [36, 37], they represent in detail that, for any feasible number of the activated subchannels, one or two solution(s) for the optimal power allocation can be obtained. Comparing sum-rates of this finite set of optimal power allocations, they obtain the point with the best sum-rate performance [36, 37].

2.5 Two-Way Active Channel

In common with [39, 40], we consider a two-way active channel, which exchanges information between two transceivers and power can be injected not only at each transceiver, but also at each subchannel and the objective is to maximize the sum-rate in such a sce-

nario, under three power constraints: 1) The power constraint of the first transceiver 2) The power constraint of the second transceiver 3) The total channel power constraint. In [39, 40], the authors assume that the noise power over all subchannels is equal (either in reciprocal or non-reciprocal channel scenarios), while we consider a more practical scenario in which the reciprocal subchannel noise powers are assumed to be unequal. The non-reciprocal two-way active channel case does not fit in the scope of this thesis.
The analysis and simulation results in both [39, 40] and this thesis, show that the active channel can offer a significantly higher sum-rate compared to the passive counterparts.

Equal Subchannel Noise Powers:

In [39, 40], the authors assume that all of the subchannels of the active channel are corrupted by equal noise. They explain that a two-way active channel can be a reciprocal channel, meaning that the gains of its subchannels in both communication directions are identical. They also investigate the case where the gains of the subchannels in the two communication directions are different. This case is referred to as a non-reciprocal subchannel. The authors' concentration is on the problem of maximizing the sum-rate over such active channels. Their optimization problem is not a convex one. Hence, they utilize the KKT conditions techniques. They prove that, at optimum, the channel power should be equally distributed among only a certain number of subchannels. Hence, the maximum sum-rate is achieved by comparing a finite number of optimal sum-rates for any feasible number of activated subchannels.

Unequal Subchannel Noise Powers:

In this thesis, we study the problem of sum-rate maximization over a two-way active channel in a practical scenario, where different subchannel undergoes noises with different powers. Although this maximization is a non-convex optimization problem, we develop an analytical method with efficient numerical computations for the optimal source and channel power allocation, by utilizing the KKT conditions. To do so, we show that, only a subset, rather than all of subchannels, may receive power from the sources and the channel. Using KKT conditions, we determine the number of the active subchannels, in order for the power constraints to be feasible. We then obtain the optimal sources and channel power distribution for any feasible number of the activated subchannels. We finally obtain the optimal solution, by comparing a finite number of points in the feasible set and identifying the point that yields the best sum-rate performance.

2.6 Research Contribution

In this thesis, we introduce the active channels as parallel channels, whose subchannel gains can be optimally adjusted within a bound, created by the channel power constraint. We consider the problem of sum-rate maximization over a two-way active channel, whose subchannels undergo noises with different powers, subject to three power constraints, the first two on the total transceiver powers, and the third on the total channel energy. We jointly optimize the injected power onto each subchannel by the channel, as well as the allocated power to each subchannel by each of the transceivers. After formulating our maximization problem, we show that this optimization problem is not a convex optimization problem. Hence, we utilize the KKT conditions and develop an analytical method with efficient computational complexity. We show that, unlike passive parallel channels, not all of the subchannels receive power and the considered transceivers and channel allocate power only to a subset of available subchannels. Consequently, for any feasible number of the activated subchannel, we analytically prove and obtain the optimal power allocation in order to achieve the maximum sum-rate in each case. Meanwhile, we show that these achieved maximum values are unique. Comparing a finite set of obtained optimal sum-rates in such feasible cases, we achieve the global maximum sum-rate for our problem. The analysis and simulation results are obvious indications of the advantage of our proposed two-way active channels over the passive counterparts which utilize the water-filling power allocation scheme.

Chapter 3

Sum-Rate Maximization

In this chapter, we model a system where an active channel conveys information between two transceivers. We formulate the problem of sum-rate maximization for such a two-way active channel. Using KKT conditions, we then develop a computationally efficient solution to such optimization problem.

3.1 System Model and Sum-Rate Maximization

We consider a communication system which consists of two transceivers transmitting and receiving information over a parallel channel containing N subchannels. The parallel channel is active, meaning that the power of each subchannel (i.e., the squared magnitude of the subchannel gain) can be judiciously obtained such that a certain performance metric, such as sum-rate, is optimized. We represent the transmit powers of the first and the second transceivers over the ith subchannel as $\tilde{p}_{1i}$ and $\tilde{p}_{2i}$, respectively, and use $\tilde{h}_i$ to denote the complex gain of the ith subchannel. The signal received at Transceiver 2 (which has been transmitted by Transceiver 1) over the ith subchannel, is modeled as $r_{2i} = \sqrt{\tilde{p}_{1i}}\tilde{h}_i s_{1i} + n_{1i}$, where s_{1i} and n_{1i} are the unit-power signal transmitted by Transceiver 1 and the received noise over the ith subchannel, respectively. The signal received at Transceiver 1 over the ith subchannel is modeled as $r_{1i} = \sqrt{\tilde{p}_{2i}}\tilde{h}_i s_{2i} + n_{2i}$, where s_{2i} and n_{2i} are the unit-power signal transmitted by Transceiver 2 and the received noise over the ith subchannel, respectively. In this thesis, the active channel is assumed to be reciprocal (i.e., the subchannel gains are identical in both communication directions). The non-reciprocal two-way active channel case does not fit in the scope of this thesis. Unlike [40], we assume unequal noise power over different subchannels, and use α_i to denote the inverse of the noise power over the ith subchannel. Without loss of generality, we assume that

$$\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_{N-1} \geq \alpha_N. \tag{3.1}$$

We consider three power constraints which limit the total transmit powers of the transceivers as well as the total power of the active channel. If we represent the maximum total transceivers' powers by $P_{\rm s1}$ and $P_{\rm s2}$, respectively, and that of the active channel by $P_{\rm c}$, we can then express these three power constraints as

$$\sum_{i=1}^{N} \tilde{p}_{1i} \leq P_{\rm s1}\ , \quad \sum_{i=1}^{N} \tilde{p}_{2i} \leq P_{\rm s2}\ , \quad \sum_{i=1}^{N} |\tilde{h}_i|^2 \leq P_{\rm c} \tag{3.2}$$

where $|\tilde{h}_i|^2$ represents the power of the ith subchannel. The objective of the current study is to maximize the sum-rate under these three power constraints. That is, we solve the following optimization problem:

$$\begin{aligned} \max_{\tilde{\mathbf{p}}_1 \succeq \mathbf{0}, \tilde{\mathbf{p}}_2 \succeq \mathbf{0}, \tilde{\mathbf{h}} \succeq \mathbf{0}} \quad & \sum_{i=1}^{N} \log(1 + \alpha_i \tilde{p}_{1i} |\tilde{h}_i|^2) + \log(1 + \alpha_i \tilde{p}_{2i} |\tilde{h}_i|^2) \\ \text{subject to} \quad & \mathbf{1}^T \tilde{\mathbf{p}}_1 \leq P_{\text{s}1} \\ & \mathbf{1}^T \tilde{\mathbf{p}}_2 \leq P_{\text{s}2} \\ & \mathbf{1}^T \tilde{\mathbf{h}} \leq P_{\text{c}} \end{aligned} \tag{3.3}$$

where $\log(\cdot)$ represents the natural logarithm, $\mathbf{x} \succeq 0$ means that all of the entries of the vector $\mathbf{x}$ are non-negative, $\mathbf{1}$ denotes an all-one vector, and the following definitions are used: $\tilde{\mathbf{p}}_1 \triangleq [\tilde{p}_{11} \;\; \tilde{p}_{12} \;\; \cdots \;\; \tilde{p}_{1N}]^T$, $\tilde{\mathbf{p}}_2 \triangleq [\tilde{p}_{21} \;\; \tilde{p}_{22} \;\; \cdots \;\; \tilde{p}_{2N}]^T$, and $\tilde{\mathbf{h}} \triangleq [|\tilde{h}_1|^2 \;\; |\tilde{h}_2|^2 \;\; \cdots \;\; |\tilde{h}_N|^2]^T$. In an active channel, the power allocation scheme is different from that in a conventional (i.e., passive) parallel channel. Indeed, for an active channel, the subchannel gains are adjustable within the set defined by a constraint which limits the total channel power, while the passive channel, the subchannel gains are fixed.

It can be shown that our optimization problem (3.3) is not convex (see Appendix A). Nevertheless, we show how this optimization problem can be solved efficiently. To do so, we first show that at the optimum, all three constraints are satisfied with equality. To show this, if we assume that at optimum, $\mathbf{1}^T \tilde{\mathbf{p}}_1 < P_{\text{s}1}$ and/or $\mathbf{1}^T \tilde{\mathbf{p}}_2 < P_{\text{s}2}$ and/or $\underline{1}^T \tilde{\mathbf{h}} < P_{\text{c}}$ hold true, we can scale up the entries of the optimal value of $\tilde{\mathbf{p}}_1$ and/or those of $\tilde{\mathbf{p}}_2$ and/or those of the $\tilde{\mathbf{h}}$, such that $\mathbf{1}^T \tilde{\mathbf{p}}_1 = P_{\text{s}1}$, $\mathbf{1}^T \tilde{\mathbf{p}}_2 = P_{\text{s}2}$, and $\mathbf{1}^T \tilde{\mathbf{h}} = P_{\text{c}}$ hold true. Scaling up the entries of any of the three vectors, $\tilde{\mathbf{p}}_1$, $\tilde{\mathbf{p}}_2$, and $\tilde{\mathbf{h}}$, will increase the objective function, thereby contradicting the optimality. Hence, the optimization problem (3.3), can be rewritten as

$$\begin{aligned}
\max_{\tilde{\mathbf{p}}_1 \succeq \mathbf{0}, \tilde{\mathbf{p}}_2 \succeq \mathbf{0}, \tilde{\mathbf{h}} \succeq \mathbf{0}} \quad & \sum_{i=1}^{N} \log(1+\alpha_i \tilde{p}_{1i} h_i) + \log(1+\alpha_i \tilde{p}_{2i} h_i) \\
\text{subject to} \quad & \mathbf{1}^T \tilde{\mathbf{p}}_1 = P_{\mathrm{s1}} \\
& \mathbf{1}^T \tilde{\mathbf{p}}_2 = P_{\mathrm{s2}} \\
& \mathbf{1}^T \tilde{\mathbf{h}} = P_{\mathrm{c}}
\end{aligned} \tag{3.4}$$

where we define $h_i \triangleq |\tilde{h}_i|^2$. Note that at the optimum, if the ith entry of any of the vectors, $\tilde{\mathbf{p}}_1$, $\tilde{\mathbf{p}}_2$, or $\tilde{\mathbf{h}}$ is zero, the corresponding entries in the other two vectors will be zero. Hence, in the optimal power allocation scheme, each subchannel may or may not receive power at the transceivers and from the active channel. Let n represent the number of activated subchannels (i.e., subchannels which receive power). The optimization problem (3.4) can then be rewritten as

$$\begin{aligned}
\max_{n} \ \max_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{h}} \quad & \sum_{i=1}^{n} \log(1+\alpha_i p_{1i} h_i) + \log(1+\alpha_i p_{2i} h_i) \\
\text{subject to} \quad & \mathbf{1}^T \mathbf{p}_1 = P_{\mathrm{s1}} \ , \ \mathbf{1}^T \mathbf{p}_2 = P_{\mathrm{s2}} \ , \ \mathbf{1}^T \mathbf{h} = P_{\mathrm{c}} \\
& \mathbf{p}_1 \succ \mathbf{0} \ , \ \mathbf{p}_2 \succ \mathbf{0} \ , \ \mathbf{h} \succ \mathbf{0}
\end{aligned} \tag{3.5}$$

where we use, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{h}$ to represent the vectors containing the non-zero entries of $\tilde{\mathbf{p}}_1$, $\tilde{\mathbf{p}}_2$, and $\tilde{\mathbf{h}}$, respectively. Moreover, p_{1i}, p_{2i} and h_i are the ith entries of the vectors, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{h}$, respectively.

3.2 Karush-Kuhn-Tucker (KKT) Conditions

For the optimization problem (3.5), the KKT conditions can be used to develop necessary conditions for the optimality, provided that the duality gap is zero. As the constraints of our optimization problem are linear, these constraints satisfy the linearity constraint qualification [113]. Hence, the duality gap is zero, and thus, the KKT conditions provide the necessary conditions for optimality. Such necessary conditions limit the feasible set to the set of possible solutions. We can then search among these possible solutions, which satisfy the necessary conditions and find the solution which results in the largest value for the sum-rate. More specifically for any fixed n, we aim to find all the possible solutions to inner maximization in (3.5) and the corresponding maximum sum-rates. For that value of n, we choose the solution which yields the highest sum-rate. We repeat this process for $n = 1, 2, \ldots, N$, the value of n which yields the highest sum-rate introduce the solution to the problem. For any n, we now consider the inner maximization problem in (3.5) and construct the Lagrangian function as

$$\begin{aligned}\mathcal{L}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{h}) \;=\; & -\sum_{i=1}^{n} \log(1 + \alpha_i p_{1i} h_i) - \sum_{i=1}^{n} \log(1 + \alpha_i p_{2i} h_i) \\ & + \lambda_1(\mathbf{1}^T \mathbf{p}_1 - P_{\mathrm{s1}}) + \lambda_2(\mathbf{1}^T \mathbf{p}_2 - P_{\mathrm{s2}}) + \lambda_3(\mathbf{1}^T \mathbf{h} - P_{\mathrm{c}}) - \boldsymbol{\mu}_1^T \mathbf{p}_1 - \boldsymbol{\mu}_2^T \mathbf{p}_2 - \boldsymbol{\mu}_3^T \mathbf{h}\end{aligned} \tag{3.6}$$

where the scalars λ_1, λ_2, and λ_3, as well as the $n \times 1$ vectors $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_2$, and $\boldsymbol{\mu}_3$, represent the Lagrange multipliers. The KKT conditions for optimality are listed below.

- Primal feasibility

$$\mathbf{1}^T\mathbf{p}_1 = P_{\mathrm{s1}} \tag{3.7}$$

$$\mathbf{1}^T\mathbf{p}_2 = P_{\mathrm{s2}} \tag{3.8}$$

$$\mathbf{1}^T\mathbf{h} = P_{\mathrm{c}} \tag{3.9}$$

$$-\mathbf{p}_1 \prec \mathbf{0} \tag{3.10}$$

$$-\mathbf{p}_2 \prec \mathbf{0} \tag{3.11}$$

$$-\mathbf{h} \prec \mathbf{0}. \tag{3.12}$$

- Dual feasibility

$$\boldsymbol{\mu}_1 \succeq 0, \ \ \boldsymbol{\mu}_2 \succeq 0, \ \ \boldsymbol{\mu}_3 \succeq 0. \tag{3.13}$$

- Complementary slackness

$$\lambda_1(\mathbf{1}^T\mathbf{p}_1 - P_{\mathrm{s1}}) = 0 \tag{3.14}$$

$$\lambda_2(\mathbf{1}^T\mathbf{p}_2 - P_{\mathrm{s2}}) = 0 \tag{3.15}$$

$$\lambda_3(\mathbf{1}^T\mathbf{h} - P_{\mathrm{c}}) = 0 \tag{3.16}$$

$$\boldsymbol{\mu}_1 \odot \mathbf{p}_1 = \mathbf{0} \tag{3.17}$$

$$\boldsymbol{\mu}_2 \odot \mathbf{p}_2 = \mathbf{0} \tag{3.18}$$

$$\boldsymbol{\mu}_3 \odot \mathbf{h} = \mathbf{0}. \tag{3.19}$$

- Stationary condition

$$\frac{\partial \mathcal{L}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{h})}{\partial p_{1i}} = \frac{-\alpha_i h_i}{(1+\alpha_i p_{1i} h_i)} + \lambda_1 - \mu_{1,i} = 0 \tag{3.20}$$

$$\frac{\partial \mathcal{L}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{h})}{\partial p_{2i}} = \frac{-\alpha_i h_i}{(1+\alpha_i p_{2i} h_i)} + \lambda_2 - \mu_{2,i} = 0 \tag{3.21}$$

$$\frac{\partial \mathcal{L}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{h})}{\partial h_i} = \frac{-\alpha_i p_{1i}}{(1+\alpha_i p_{1i} h_i)} + \frac{-\alpha_i p_{2i}}{(1+\alpha_i p_{2i} h_i)} + \lambda_3 - \mu_{3,i} = 0 \tag{3.22}$$

where $\mu_{1,i}$, $\mu_{2,i}$, and $\mu_{3,i}$ are the ith entries of $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_2$, and $\boldsymbol{\mu}_3$, respectively. Furthermore, $\mathbf{x} \prec 0$ means that all of the entries of the vector $\mathbf{x}$ are negative and $\odot$ stands for element-wise (Schur-Hadamard) product and $\mathbf{0}$ denotes an all-zero vector. It follows from (3.10)-(3.12) and from (3.17)-(3.19) that $\mu_{1,i} = \mu_{2,i} = \mu_{3,i} = 0$ holds true, for $i = 1, 2, ..., n$.

3.3 Solving KKT Conditions

Using the KKT conditions presented in the previous section, we now aim to develop a computationally efficient solution to the sum-rate maximization problem. Using (3.20), we can obtain p_{1i} as

$$p_{1i} = \frac{\alpha_i h_i - \lambda_1}{\alpha_i h_i \lambda_1} = \frac{1}{\lambda_1} - \frac{1}{\alpha_i h_i}. \tag{3.23}$$

Substituting (3.23) in (3.7), we can write

$$P_{\mathrm{s}1} = \sum_{i=1}^{n} \left(\frac{1}{\lambda_1} - \frac{1}{\alpha_i h_i} \right) = \frac{n}{\lambda_1} - \sum_{i=1}^{n} \frac{1}{\alpha_i h_i}. \tag{3.24}$$

It follows from (3.24) that

$$\frac{1}{\lambda_1} = \frac{P_{\mathrm{s}1}}{n} + \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\alpha_i h_i}. \tag{3.25}$$

Substituting (3.25) in (3.23), we can rewrite p_{1i} in terms of $\{h_i\}_{i=1}^{n}$ as

$$p_{1i} = \frac{P_{\mathrm{s}1}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j} - \frac{1}{\alpha_i h_i}. \tag{3.26}$$

Similarly, we can express p_{2i} as

$$p_{2i} = \frac{P_{\mathrm{s}2}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j} - \frac{1}{\alpha_i h_i}. \tag{3.27}$$

Using (3.26) and (3.27), we can rewrite (3.22) as

$$\frac{-\alpha_i\left(\frac{P_{\mathrm{s}1}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j} - \frac{1}{\alpha_i h_i}\right)}{\left(1+\alpha_i h_i\left(\frac{P_{\mathrm{s}1}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j} - \frac{1}{\alpha_i h_i}\right)\right)} + \frac{-\alpha_i\left(\frac{P_{\mathrm{s}2}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j} - \frac{1}{\alpha_i h_i}\right)}{\left(1+\alpha_i h_i\left(\frac{P_{\mathrm{s}2}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j} - \frac{1}{\alpha_i h_i}\right)\right)} + \lambda_3 = 0 \tag{3.28}$$

or, equivalently, as

$$\frac{-\alpha_i\left(\frac{P_{\mathrm{s}1}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j}\right) + \frac{1}{h_i}}{\alpha_i h_i\left(\frac{P_{\mathrm{s}1}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j}\right)} + \frac{-\alpha_i\left(\frac{P_{\mathrm{s}2}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j}\right) + \frac{1}{h_i}}{\alpha_i h_i\left(\frac{P_{\mathrm{s}2}}{n} + \frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j}\right)} + \lambda_3 = 0. \tag{3.29}$$

Simplifying (3.29) yields

$$-\frac{1}{h_i}-\frac{\frac{-1}{\alpha_i h_i^2}}{\left(\frac{P_{s1}}{n}+\frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j}\right)}-\frac{1}{h_i}-\frac{\frac{-1}{\alpha_i h_i^2}}{\left(\frac{P_{s2}}{n}+\frac{1}{n}\sum_{j=1}^{n}\frac{1}{\alpha_j h_j}\right)}+\lambda_3=0. \tag{3.30}$$

Using the following definitions:

$$A \triangleq \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\alpha_i h_i} \tag{3.31}$$

$$Q_n(A) \triangleq \left(\frac{1}{\frac{P_{s1}}{n}+A}+\frac{1}{\frac{P_{s2}}{n}+A}\right) \tag{3.32}$$

we can rewrite (3.30) as

$$Q_n(A)-2\alpha_i h_i+\lambda_3\alpha_i h_i^2=0. \tag{3.33}$$

For a feasible value[1] of A, h_i can take one of the two solutions to (3.33), as given

$$h_i^+ \triangleq \frac{\alpha_i+\sqrt{\alpha_i^2-\lambda_3 Q_n(A)\alpha_i}}{\lambda_3\alpha_i}=\frac{1+\sqrt{1-\lambda_3\frac{Q_n(A)}{\alpha_i}}}{\lambda_3} \tag{3.34}$$

$$h_i^- \triangleq \frac{\alpha_i-\sqrt{\alpha_i^2-\lambda_3 Q_n(A)\alpha_i}}{\lambda_3\alpha_i}=\frac{1-\sqrt{1-\lambda_3\frac{Q_n(A)}{\alpha_i}}}{\lambda_3}. \tag{3.35}$$

[1]For any value of n, not every value of $A \in (0,+\infty)$ is feasible. In Section 3.4, we discuss how the feasible values of A, can be determined for a given value of n

We now show that, if $\exists i$ such that $\alpha_i > \alpha_{i+1} > 0$, only the solution in (3.34), h_i^+, is acceptable for h_i. To do so, we consider (3.33) for two subchannels indices i and $i+1$:

$$Q_n(A) - 2\alpha_i h_i + \lambda_3 \alpha_i h_i^2 = 0 \quad (3.36)$$

$$Q_n(A) - 2\alpha_{i+1} h_{i+1} + \lambda_3 \alpha_{i+1} h_{i+1}^2 = 0. \quad (3.37)$$

Using (3.36) and (3.37), we can write

$$-2\alpha_i h_i + \lambda_3 \alpha_i h_i^2 = -2\alpha_{i+1} h_{i+1} + \lambda_3 \alpha_{i+1} h_{i+1}^2 \quad (3.38)$$

or, equivalently,

$$\alpha_i (2h_i - \lambda_3 h_i^2) = \alpha_{i+1} (2h_{i+1} - \lambda_3 h_{i+1}^2). \quad (3.39)$$

Knowing that $Q_n(A)$ is a positive value, from (3.36) and (3.37), it follows that both sides of (3.39) are positive values. Hence, referring to our initial assumption that $\alpha_i > \alpha_{i+1} > 0$ and using (3.39), we arrive at the following inequality:

$$2h_i - \lambda_3 h_i^2 < 2h_{i+1} - \lambda_3 h_{i+1}^2 \quad (3.40)$$

or, equivalently, at

$$2(h_i - h_{i+1}) < \lambda_3 (h_i^2 - h_{i+1}^2). \quad (3.41)$$

Note that $\alpha_i > \alpha_{i+1}$ implies that at the optimum $h_i > h_{i+1}$ holds true, otherwise we can swap h_i and h_{i+1} and achieve a higher sum-rate, thereby contradicting optimality. Using the fact that $h_i > h_{i+1}$, along with (3.41), we arrive at

$$2 \;<\; \lambda_3 h_i + \lambda_3 h_{i+1}. \tag{3.42}$$

Now replacing h_i and h_{i+1}, with their possible solutions in (3.34) and (3.35), we will have

$$2 \;<\; \left(1 \pm \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}}\right) + \left(1 \pm \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_{i+1}}}\right) \tag{3.43}$$

or, equivalently,

$$0 \;<\; \pm\sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}} \;\pm \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_{i+1}}}. \tag{3.44}$$

Based on whether we choose the positive or negative signs in (3.44), the following four cases are possible:

$$\text{Case I}: \quad 0 < -\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_i}} \;-\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_{i+1}}} \tag{3.45}$$

$$\text{Case II}: \quad 0 < -\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_i}} \;+\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_{i+1}}} \tag{3.46}$$

$$\text{Case III}: \quad 0 < +\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_i}} \;-\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_{i+1}}} \tag{3.47}$$

$$\text{Case IV}: \quad 0 < +\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_i}} \;+\sqrt{1 - \lambda_3 \tfrac{Q_n(A)}{\alpha_{i+1}}} \quad . \tag{3.48}$$

It is now clear that Case I with both negative signs is not acceptable. From Case II, we can write

$$\sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}} \;<\; \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_{i+1}}} \tag{3.49}$$

or, equivalently,

$$\alpha_i \ < \ \alpha_{i+1}. \tag{3.50}$$

Note however that (3.50) contradicts our initial assumption that $\alpha_i > \alpha_{i+1}$. Hence, Case II can never occur either. Considering Cases III and IV as the possible cases, both cases have a positive sign for the first term on their right hand sides (RHSs) of the inequality that corresponds to the ith subchannel, which assures us that h_i always chooses h_i^+. Similarly if $\alpha_{i+1} > \alpha_{i+2}$, we can show that h_{i+1} will choose h_i^+, as well. Note that for any given value of n, we cannot assume that h_n in (3.33), will be equal to h_i^+ in (3.34). In general, if k out of the n smallest values of $\{\alpha_i\}_{i=1}^n$, (i.e., $\{\alpha_i\}_{i=n-k+1}^n$) are equal, for $n > k \geq 1$, then $\{h_i\}_{i=n-k+1}^n$ may accept both solutions in (3.33) (i.e., both h_i^+ and h_i^+). If the first $n-k$ values of α_i's, are distinct from the last k equal α_i's, $\{h_i\}_{i=1}^{n-k}$ will follow the solution in (3.33) with positive sign (i.e., h_i^+). Hence, we define two possible scenarios:

Scenario 1:

$$h_i = h_i^+ = \frac{1 + \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}}}{\lambda_3}, \qquad \text{for} \ \ i = 1, 2, \ldots, n. \tag{3.51}$$

Scenario 2:

$$\begin{aligned} h_i = h_i^+ &= \frac{1 + \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}}}{\lambda_3}, \qquad \text{for} \quad i = 1, 2, \ldots, n-k, \\ h_i = h_i^- &= \frac{1 - \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}}}{\lambda_3}, \qquad \text{for} \quad i = n-k+1 \ , \ n-k+2 \ , \ldots, \ n. \end{aligned} \tag{3.52}$$

where k is the number of smallest identical α_i's. As mentioned earlier, those subchannels with the equal subchannel noise powers, are assigned the same subchannel power [40]. Hence, in a case that the last k subchannel noise powers are identical, it is clear that $h_i = h_j, \quad \text{for } i, j \in \{n-k+1, n-k+2, \ldots, n\}$. Each of these two sets of solutions in Scenarios 1 and 2 represents a local maximum for the objective function. Therefore, calculating the sum-rates corresponding to these two scenarios and comparing them, we can obtain the maximum sum-rate for that value of n. We explore both scenarios in next two subsections in order to determine their corresponding values for A and consequently their corresponding sum-rates, but before doing so, we first aim to express λ_3 in terms of A, since both A and λ_3 exist in (3.51) and (3.52). To do this, we divide both sides of (3.33) with $\alpha_i h_i$, sum up all equations, for $i = 1, 2, \ldots, n$, and arrive at

$$Q_n(A) \sum_{i=1}^{n} \frac{1}{\alpha_i h_i} - \sum_{i=1}^{n} 2 + \lambda_3 \sum_{i=1}^{n} h_i = 0. \tag{3.53}$$

Using (3.9) and (3.31), we can rewrite (3.53) as follows

$$Q_n(A)\ n\ A - 2n + \lambda_3\ P_{\mathrm{c}} = 0. \tag{3.54}$$

Therefore, λ_3 can be written as

$$\lambda_3 = \frac{n(2 - AQ_n(A))}{P_{\mathrm{c}}}. \tag{3.55}$$

3.3.1 Scenario 1

In this scenario, h_i's are obtained as in (3.51). Using (3.51) in (3.9), we can obtain

$$\sum_{i=1}^{n} \left(\frac{1 + \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}}}{\lambda_3} \right) = P_{\mathrm{c}} \tag{3.56}$$

or, equivalently,

$$\sum_{i=1}^{n} \left(1 + \sqrt{1 - \lambda_3 \frac{Q_n(A)}{\alpha_i}} \right) = \lambda_3 P_{\mathrm{c}}. \tag{3.57}$$

In other words, the parameter A has to satisfy (3.57) at the optimum. Substituting (3.55), we write (3.57) as

$$\sum_{i=1}^{n} \left(1 + \sqrt{1 - \frac{n(2 - AQ_n(A))Q_n(A)}{P_{\mathrm{c}}\alpha_i}} \right) = n(2 - AQ_n(A)). \tag{3.58}$$

We now define $F_{1n}(A)$, as

$$F_{1n}(A) \triangleq \sum_{i=1}^{n} \left(1 + \sqrt{1 - \frac{n(2 - AQ_n(A))Q_n(A)}{P_{\mathrm{c}}\alpha_i}} \right) - n(2 - AQ_n(A)). \tag{3.59}$$

Note that solving (3.58) means that we are looking for the solution to $F_{1n}(A) = 0$. We now prove that (3.58), or equivalently $F_{1n}(A) = 0$, has at most one solution. To do so, we differentiate (3.59) with respect to A (i.e., $\frac{\partial F_{1n}(A)}{\partial A}$), over the feasible values for A[2] and write the derivative of (3.59) as

$$\frac{\partial F_{1n}(A)}{\partial A} = \sum_{i=1}^{n}\left[\frac{(\frac{nQ_n(A)}{P_{\mathrm{c}}\alpha_i})(\frac{\frac{P_{\mathrm{s1}}}{n}}{(\frac{P_{\mathrm{s1}}}{n}+A)^2}+\frac{\frac{P_{\mathrm{s2}}}{n}}{(\frac{P_{\mathrm{s2}}}{n}+A)^2})+(\frac{n}{P_{\mathrm{c}}\alpha_i})(2-AQ_n(A))(\frac{1}{(\frac{P_{\mathrm{s1}}}{n}+A)^2}+\frac{1}{(\frac{P_{\mathrm{s2}}}{n}+A)^2})}{2\sqrt{1-\frac{n(2-AQ_n(A))Q_n(A)}{P_{\mathrm{c}}\alpha_i}}}\right]$$
$$+n(\frac{\frac{P_{\mathrm{s1}}}{n}}{(\frac{P_{\mathrm{s1}}}{n}+A)^2}+\frac{\frac{P_{\mathrm{s2}}}{n}}{(\frac{P_{\mathrm{s2}}}{n}+A)^2}). \tag{3.60}$$

We can clearly see that $\frac{\partial F_{1n}(A)}{\partial A}$ is a positive value over all feasible values of A:

$$\frac{\partial F_{1n}(A)}{\partial A} > 0. \tag{3.61}$$

Hence, the function $F_{1n}(A)$ is a monotonically increasing function of A and we can conclude that $F_{1n}(A)$ or equivalently (3.58) has at most one solution for A within its feasible values. In the next section, we show that the set of feasible values for A is either $(0, +\infty)$ or a subset of this interval. Hence, we can use a bisection method over this feasible set to obtain the solution to $F_{1n}(A) = 0$ or equivalently, the solution to (3.58) (if there exists any)[3].

[2] A is a positive value, however, in Section 3.4, we obtain the feasible values of A for this problem, in order for the subchannels powers (i.e., h_i's) and consequently $F_{1n}(A)$ to be real.

[3] Method of finding the possible solution by a bisection method is explained in details in Section 3.4

3.3.2 Scenario 2

In this scenario, h_i's are given as in (3.52). Knowing $h_i = h_j$ for $i, j \in \{n-k+1, n-k+2, \ldots, n\}$ and using (3.52) in (3.9), we can write

$$\sum_{i=1}^{n-k}\left(1+\sqrt{1-\frac{n(2-AQ_n(A))Q_n(A)}{P_\mathrm{c}\alpha_i}}\right)+k\left(1-\sqrt{1-\frac{n(2-AQ_n(A))Q_n(A)}{P_\mathrm{c}\alpha_n}}\right)= n(2-AQ_n(A)). \tag{3.62}$$

We now define the $F_{2n}(A)$ as

$$F_{2n}(A) \triangleq \sum_{i=1}^{n-k}\left(1+\sqrt{1-\frac{n(2-AQ_n(A))Q_n(A)}{P_\mathrm{c}\alpha_i}}\right)+k\left(1-\sqrt{1-\frac{n(2-AQ_n(A))Q_n(A)}{P_\mathrm{c}\alpha_n}}\right) -n(2-AQ_n(A)). \tag{3.63}$$

Solving (3.62) means that we are looking for the solution to $F_{2n}(A)) = 0$. We conjecture that $F_{2n}(A)$ has at most one root and consequently conclude that (3.62) has at most one solution in its feasible set. Proving or disproving this conjecture does not appear to be feasible at this time. However, we observed through our simulations that (3.62) has at most one solution for A in its feasible set of values and this solution (if exists) can be obtained using a bisection method.

3.4 Feasibility and Solution

In both aforementioned scenarios, all of the subchannel powers (i.e., h_i's) can be obtained, if the optimal value of A is obtained. Knowing h_i's are real valued, we can obtain a condition for the feasible values of A based on (3.34) and (3.35). To this end, we can write

$$1 - \lambda_3 \frac{Q_n(A)}{\alpha_i} \geq 0, \qquad \text{for} \quad i = \{1, 2, \ldots, n\} \tag{3.64}$$

or, equivalently,

$$\alpha_i \geq \lambda_3 Q_n(A), \qquad \text{for} \quad i = \{1, 2, \ldots, n\}. \tag{3.65}$$

We now show that (3.65) might enable us to narrow down the acceptable range for the optimum values for A, when solving (3.58) and (3.62), thereby decreasing the computational complexity of our proposed method.

Using (3.55), we rewrite the feasibility constraint (3.65) as

$$\alpha_i \geq \frac{n(2 - AQ_n(A))Q_n(A)}{P_\mathrm{c}}, \qquad \text{for} \quad i = \{1, 2, \ldots, n\}. \tag{3.66}$$

Based on our initial assumption in (3.1) (i.e., where we sorted α_i's in a non-increasing order), α_n is the smallest among $\{\alpha_i\}_{i=1}^{n}$ and consequently, if the feasibility constraint

(3.66) holds true for $i = n$, that will hold true for $\{\alpha_i\}_{i=1}^{n}$. Hence, (3.66) is equivalent to

$$\alpha_n \geq \frac{n(2 - AQ_n(A))Q_n(A)}{P_{\mathrm{c}}}. \tag{3.67}$$

We now define

$$G_n(A) \triangleq 1 - \frac{n(2 - AQ_n(A))Q_n(A)}{P_{\mathrm{c}}\alpha_n}. \tag{3.68}$$

Using (3.31), we can rewrite (3.67) as

$$G_n(A) = 1 - \frac{2rA^2 + (4s + r^2)A + 2rs}{t(A^2 + rA + s)^2} \geq 0, \tag{3.69}$$

where $r \triangleq \frac{P_{\mathrm{s1}}+P_{\mathrm{s2}}}{n}$, $s \triangleq \frac{P_{\mathrm{s1}}P_{\mathrm{s2}}}{n^2}$, $t \triangleq \frac{P_{\mathrm{c}}\alpha_n}{n}$. Therefore, we can rewrite (3.69) as

$$G_n(A) = \frac{tA^4 + 2rtA^3 + (r^2t + 2st - 2r)A^2 + (2rst - r^2 - 4s)A + (s^2t - 2rs)}{t(A^2 + rA + s)^2} \geq 0. \tag{3.70}$$

Note that for any n, the feasible values of A are such that (3.70) holds (i.e. $G_n(A) \geq 0$). Note that root(s) of $G_n(A)$ originate from the 4^{th} order polynomial (i.e., which is referred to as, quartic equation) in the numerator of $G_n(A)$, which we write as

$$tA^4 + 2rtA^3 + (r^2t + 2st - 2r)A^2 + (2rst - r^2 - 4s)A + (s^2t - 2rs) = 0. \tag{3.71}$$

Generally a quartic equation such as (3.71), has at most 4 roots, however, here we first show that $G_n(A)$ is a monotonically increasing function of A over the interval $(0, +\infty)$ and at most has one positive root. To do this, using (3.68), we differentiate $G_n(A)$ with respect to A as

$$\frac{\partial G_n(A)}{\partial A} = (\frac{n}{P_{\mathrm{c}}\alpha_n})\Bigg[Q_n(A)(\frac{\frac{P_{\mathrm{s1}}}{n}}{(\frac{P_{\mathrm{s1}}}{n}+A)^2}+\frac{\frac{P_{\mathrm{s2}}}{n}}{(\frac{P_{\mathrm{s2}}}{n}+A)^2})+(2-AQ_n(A))(\frac{1}{(\frac{P_{\mathrm{s1}}}{n}+A)^2}+\frac{1}{(\frac{P_{\mathrm{s2}}}{n}+A)^2})\Bigg]. \quad (3.72)$$

From (3.72), we can conclude that $\frac{\partial G_n(A)}{\partial A} > 0$ for $A \in (0, +\infty)$ and consequently only the following two cases can occur:

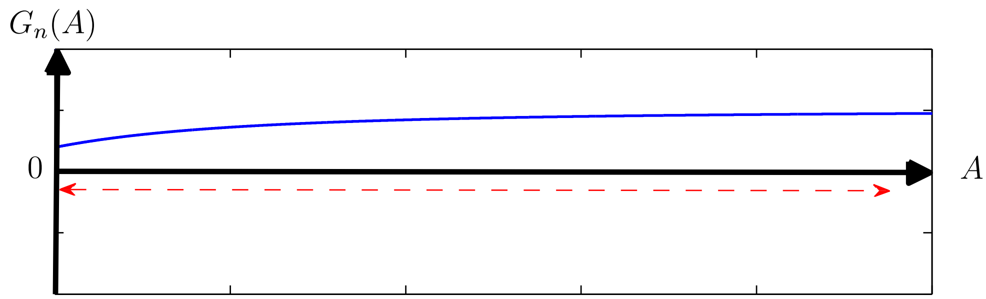
- Case 1: $G_n(A)$ has no positive root (Figure 3.1.a). In this case, we can conclude that the acceptable range for the optimum A is $(0, +\infty)$ and we conduct the bisection search for both scenarios over this interval.

- Case 2: $G_n(A)$ has one positive root (Figure 3.1.b) denoted as a_1. In this case, we can conclude that the acceptable range for the optimum A is $(a_1, +\infty)$ and we conduct the bisection search for both scenarios over this interval.

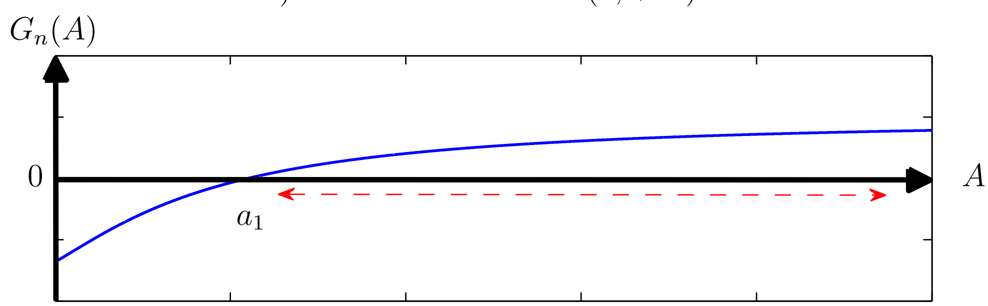
Hence, we first solve (3.71) using an algebraic method developed by Ferrari [114], and obtain all roots of the quartic equation (see Appendix B). We denote the largest root as a_1. In order to determine a general form for the feasible set of the values for A, as $(A_0, +\infty)$, considering both possible cases in Figure 3.1, we define

$$A_0 \triangleq \max\{0, a_1\} \quad (3.73)$$

Thus, the feasible set of values for A would be the interval $(A_0, +\infty)$ and to find the optimum A in both Scenarios 1 and 2 (as described in Subsections 3.3.1 and 3.3.2, respectively) we conduct a bisection method over this interval. To do so, we evaluate the values of $F_{1n}(A)$ and $F_{2n}(A)$ at the beginning and at the end of the interval $(A_0, +\infty)$. Consid-



a) Case 1: Feasible set $(0, +\infty)$



b) Case 2: Feasible set $(a_1, +\infty)$

Figure 3.1: Possible cases for feasibility function $G_n(A)$.

ering $\lim_{A\to+\infty} F_{1n}(A) = 2 > 0$ and $\lim_{A\to+\infty} F_{2n}(A) = n - k > 0$ and knowing (or presuming) that each of $F_{1n}(A)$ and $F_{2n}(A)$ has at most one root in the interval $A_0, +\infty)$, we can evaluate the values of these two functions at the beginning of the interval of feasible values of A (i.e., $F_{1n}(A_0)$ and $F_{2n}(A_0)$), to check whether an optimal A exists for each, and if so, obtain the solution(s). Hence, if $F_{1n}(A_0) \leq 0$ and/or $F_{2n}(A_0) \leq 0$, we conclude that $F_{1n}(A)$ and/or $F_{2n}(A)$ has a solution for A and we conduct a bisection method over the interval $(A_0, +\infty)$ to find these/this root(s). Once the optimal values of A's are obtained, we use (3.31) and (3.55) to obtain $Q_n(A)$ and λ_3, respectively, for each scenario. We then use (3.51) and (3.52) to calculate h_i's for Scenarios 1 and 2, respectively. Knowing the h_i's, we consequently obtain p_{1i}'s and p_{2i}'s using (3.26) and (3.27), respectively, for each scenario. We finally, use (3.5) to derive the sum-rates of the Scenarios 1 and 2 (for that value of n) and the larger value of these two sum-rates yields the maximum sum-rate of the active channel for that value of n. If $F_{1n}(A_0) \leq 0$ and $F_{2n}(A_0) > 0$ ($F_{1n}(A_0) > 0$ and $F_{2n}(A_0) \leq 0$), this means that only Scenario 1 (Scenario 2) does have a solution and we conduct the bisection method and obtain the optimal A for that Scenario which has a solution. If $F_{1n}(A_0) > 0$ and $F_{2n}(A_0) > 0$, this means that neither of scenarios has a real solution and a maximum sum-rate cannot be obtained for that value of n. Furthermore, in Appendix C, we show that, if $F_{1n}(A)$ and $F_{2n}(A)$ do not have a root in the interval $(A_0, +\infty)$ for a given value of n, they do not have a root for any larger value of n.

3.5 Algorithm for Finding the Maximum Sum-Rate

In this chapter, we proposed a computationally efficient solution to obtain the maximum sum-rate over a two-way active channel with unequal subchannel noise powers. In this section, we summarize our proposed method as an algorithm. For any given source powers (i.e. P_{s1} and P_{s2}), active channel power (i.e., P_{c}), unequal independent subchannel noise powers (i.e., α_i's) and the number of available subchannels (i.e., N), we can obtain the maximum sum-rate of the active channel (denoted as SR_{max}). To do so, we first find the sum-rate of the channel for different number of activated subchannels (denoted as $R(n)$ for $n = 1, 2, \ldots, N$), and by comparing them and finding the largest value among them, we determine the optimal number of activated subchannels which corresponds to the maximum sum-rate of the channel. To do so, we first obtain the sum-rate corresponding to the initial case of having only one activated subchannel (denoted as $R(1)$). In this case, all transmit powers and active channel power are assigned to that single subchannel (i.e., subchannel with the largest inverse noise power α_1). We then form the 4^{th} order polynomial numerator of $G_n(A)$ as in (3.71), for different number of activated subchannels (i.e., for $n = 2, 3, \ldots, N$). After solving (3.71) using Ferrari's solution, we obtain the interval $(A_0, +\infty)$ which represents the feasible values for A, for each value of n. We then evaluate $F_{1n}(A_0)$ and $F_{2n}(A_0)$ in order to check the existence of an optimal A as the solutions of (3.58) and (3.62), respectively. If $F_{1n}(A_0)$ and $F_{2n}(A_0)$ were both non-positive, we conduct a bisection method over the interval $(A_0, +\infty)$ to find the roots of $F_{1n}(A)$ and $F_{2n}(A)$ in this interval. Once the optimal values of A's are obtained for Scenarios 1 and 2, we use (3.31) and (3.55) to obtain $Q_n(A)$ and λ_3, respectively, for each scenario. We then use (3.51) and (3.52) to calculate h_i's for Scenarios 1 and 2, respectively. Knowing the h_i's, we consequently obtain p_{1i}'s and p_{2i}'s using (3.26) and (3.27), respectively, for each scenario. We subsequently obtain their corresponding sum-rates (denoted as R_1 and R_2)using (3.5). After comparing the sum-rates, the larger value yields the maximum sum-rate for that value of n (i.e., number of activated subchannels), that is $R(n) = \max\{R_1, R_2\}$. If only one of the functions $F_{1n}(A_0)$ and $F_{2n}(A_0)$ were non-positive, or equivalently, only one of the scenarios has a solution, we obtain its optimal A and consequently its corresponding sum-rate and for the other scenario which does not have a solution and a sum-rate cannot be obtained, we set its corresponding sum-rate equal to zero. Hence, the sum-rate corresponding the scenario with a solution is the maximum sum-rate for that value of n. If for a certain value of n, neither of the scenarios has a solution in the interval $(A_0, +\infty)$, we conclude that the problem is infeasible for that specific value of n and consider the previous number of activated subchannels as the last feasible number of subchannels and denote it as $n_{max} \triangleq n - 1$. In Appendix C, we prove that for $n = n_{max} + 1, n_{max} + 2, \ldots, N$, the proposed method is not feasible. This means that, the maximum does not exists for the sum-rate of the channel for any larger value of n (more than n_{max}), and we set its corresponding maximum sum-rate equal to zero (i.e., $R(n) = 0, \quad \text{for } n = n_{max} + 1, n_{max} + 2, \ldots, N$).

We can obtain the maximum sum-rate of the active channel by comparing these finite number of maximum sum-rates (i.e., N) for different number of activated subchannels ($SR_{max} = \max\{R(n)\}_{n=1}^{N}$).

Algorithm 1 Finding the maximum sum-rate

Input: P_{s1}, P_{s2}, and P_{c} are given positive real values as sources and channel powers, N is a given integer value as number of available subchannels, and $\alpha_1, \alpha_2, \ldots, \alpha_N$ are given unequal positive real values sorted in a non-ascending order.

Output: R_1, R_2, and $R(n)$ represent the obtained sum-rates in Scenario 1, Scenario 2, and the maximum sum-rate of the active channel for the corresponding number of activated subchannels (i.e., n), respectively. SR_{max} is a positive real value, representing the maximum sum-rate of active channel and hence the output of the algorithm.

- **Step 1:** For $n = 1$, choose $p_{11} = P_{\mathrm{s1}}$, $p_{21} = P_{\mathrm{s2}}$, and $h_1 = P_{\mathrm{c}}$ and calculate the sum-rate as $R(1) = \log(1 + \alpha_1 p_{11} h_1) + \log(1 + \alpha_1 p_{21} h_1)$.
- **Step 2:** If $n < N$, choose $n = n + 1$, otherwise go to **Step 8**.
- **Step 3:** Calculate $r = \frac{P_{\mathrm{s1}}+P_{\mathrm{s2}}}{n}$, $s = \frac{P_{\mathrm{s1}}P_{\mathrm{s2}}}{n^2}$, and $t = \frac{P_{\mathrm{c}}\alpha_n}{n}$, and find all roots of (3.71) using the Ferrari method and denote the largest root as a_1. Determine $A_0 = \max\{0, a_1\}$ and indicate the feasible values of A as $(A_0, +\infty)$.
- **Step 4:** If $F_{1n}(A_0) \leq 0$:

  Step 5.1: Using a bisection method, find the root of $F_{1n}(A)$ in the interval $(A_0, +\infty)$ and introduce that as the optimal value of A for Scenario 1.

  Step 5.2: Obtain the corresponding value of $Q_n(A)$ for Scenario 1, using (3.31).

  Step 5.3: Calculate the corresponding value of λ_3 for Scenario 1, using (3.55).

  Step 5.4: Use (3.51) to calculate the values of h_i's for Scenarios 1.

  Step 5.5: Obtain p_{1i}'s and p_{2i}'s using (3.26) and (3.27), respectively.

  Step 5.6: Using (3.5) calculate the sum-rate for Scenario 1 and denote it as R_1.

  Otherwise, $R_1 = 0$.
- **Step 5:** If $F_{2n}(A_0) \leq 0$:

  The optimal A only for Scenario 2 exists over the interval $(A_0, +\infty)$.

  Step 6.1: Using a bisection method, find the root of $F_{2n}(A)$ in the interval $(A_0, +\infty)$ and introduce that as the optimal value of A for Scenario 2.

  Step 6.2: Obtain the corresponding value of $Q_n(A)$ for Scenario 2, using (3.31).

  Step 6.3: Calculate the corresponding value of λ_3 for Scenario 1, using (3.55).

  Step 6.4: Use (3.52) to calculate the values of h_i's for Scenarios 2.

  Step 6.5: Obtain p_{1i}'s and p_{2i}'s using (3.26) and (3.27), respectively.

  Step 6.6: Using (3.5) calculate the sum-rate for Scenario 2 and denote it as R_2.

  Otherwise, $R_2 = 0$.

Algorithm 2 (Continuation of Algorithm 1) Finding the maximum sum-rate

- **Step 6:** If $R_1 = 0$ and $R_2 = 0$:
  The maximum sum-rate does not exists over the interval $(A_0, +\infty)$ for either of the scenarios. Set $n_{max} = n - 1$ and then set $R(n) = 0$ for $n = n_{max} + 1, \ldots, N$ and go to **Step 8**.
- **Step 7:** $R(n) = \max\{R_1, R_2\}$. Go to **Steps 2**.
- **Step 8:** SR_{max} can be obtained as $SR_{max} = \max\{R(n)\}_{n=1}^{N}$.

Chapter 4

Simulation Results

In this chapter, we compare the performance of the active channel with its passive counterpart. To do so, we consider three different resource allocation cases for the active channel and compare their maximum sum-rates (i.e., performances) with that of their passive counterpart. We first compare them for a fixed number of available subchannels for different total powers and then for a fixed total power for different number of available subchannels. We define the total consumed power as $P_{\mathrm{T}} \triangleq P_{\mathrm{s1}} + P_{\mathrm{s2}} + P_{\mathrm{c}}$ for the active channel and consider three different cases of power distribution between the transceivers and the active channel as: 1) $P_{\mathrm{s1}} = P_{\mathrm{s2}} = \frac{P_{\mathrm{c}}}{2} = \frac{P_{\mathrm{T}}}{4}$, 2) $P_{\mathrm{s1}} = P_{\mathrm{s2}} = P_{\mathrm{c}} = \frac{P_{\mathrm{T}}}{3}$, 3) $P_{\mathrm{s1}} = P_{\mathrm{s2}} = \frac{P_{\mathrm{c}}}{2} = \frac{P_{\mathrm{T}} + N}{4}$, where N represents the number of available subchannels for the certain channel. We model the unequal subchannel noise powers (i.e., inverse of α_i's) by Rayleigh distributed random variables and sort the α_i's in a non-ascending order. For the passive channel, as explained within the thesis, the subchannels' powers are not adjustable. Hence, the total available (i.e., consumed) power will be allocated to the transceivers (i.e.,

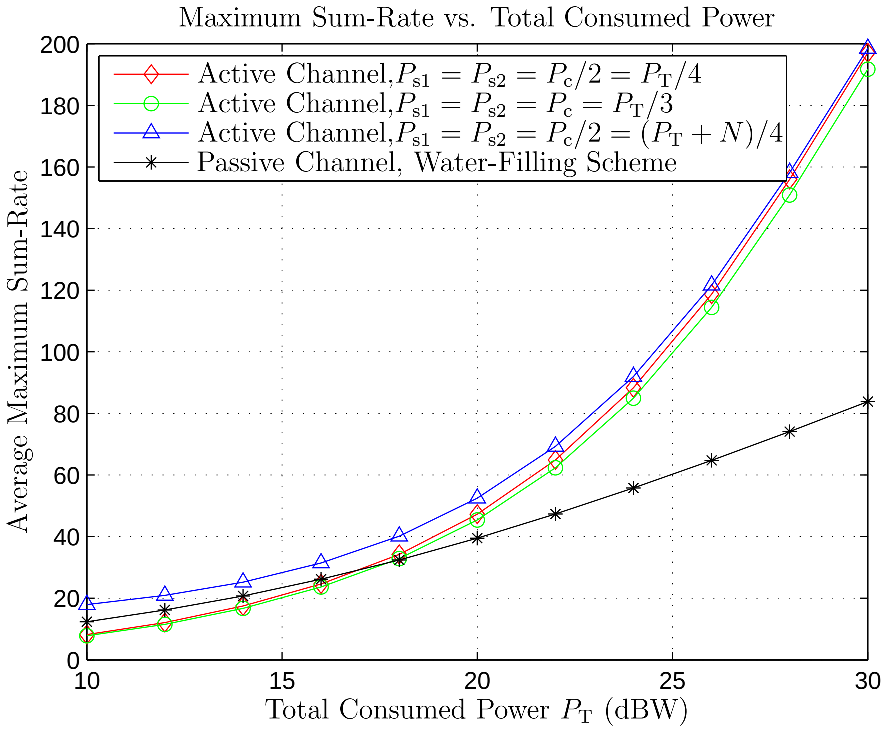


Figure 4.1: Maximum sum-rate versus the total consumed power, for both active and passive channels with $N = 16$ subchannel.

$P_{s1} = P_{s2} = \frac{P_T}{2}$) and we model the subchannel gains by a complex Gaussian distributed random variables with unit variance. In such passive channel, the norm of the channel is on average equal to N. Hence, the passive channel's power can be considered as $P_c = N$. In order to maximize the sum-rate over the passive channel water-filling power allocation scheme has been utilized.

In Figure 4.1, we plot maximum sum-rate of the active channel in three aforementioned cases of power distribution for the active channel, as well as that of the passive channel, versus the total consumed power for a given number of subchannels $N = 16$. It can obviously be seen that the active channel outperforms its passive counterpart, specifically at high values of the total consumed power. The reason behind this superior performance of the active channel, is the additional degrees of the freedom offered by them when compared to its passive counterpart. However, for small values of total consumed power, the passive channel slightly outperforms the active channel in the first two cases of power distribution. The reason is that the consumed power by the first two active channels (P_T) is less than the average power assigned to the passive channel (N) for low power assignments. Consequently this difference in the assigned power, exceeds the advantages of the additional degrees of the freedom offered by the active channel, which results in a higher sum-rate (i.e., better performance) by the passive channel rather than the active channel in lower consumed power.

Secondly, we compare the performance of the active channel, with that of the passive channel for different number of available subchannels (i.e., N) for a given power, considering the same three cases of power distribution for the active channel and the same modeling for the active channel's subchannels noise powers and the gains of the subchannels for the passive channel.

In Figure 4.2, we plot the maximum sum-rate of the active channel in three different cases of resource allocation and a passive channel, versus the total number of available subchannels (i.e., N), for $P = 25$ (dBW). In all four curves, we witness an increase in the maximum sum-rate, as the number of available subchannels increases, until a point where the sum-rate growth becomes saturated in active channel and the maximum sum-rate of the active channel stays constant afterward. The reason is that, while increasing the num-

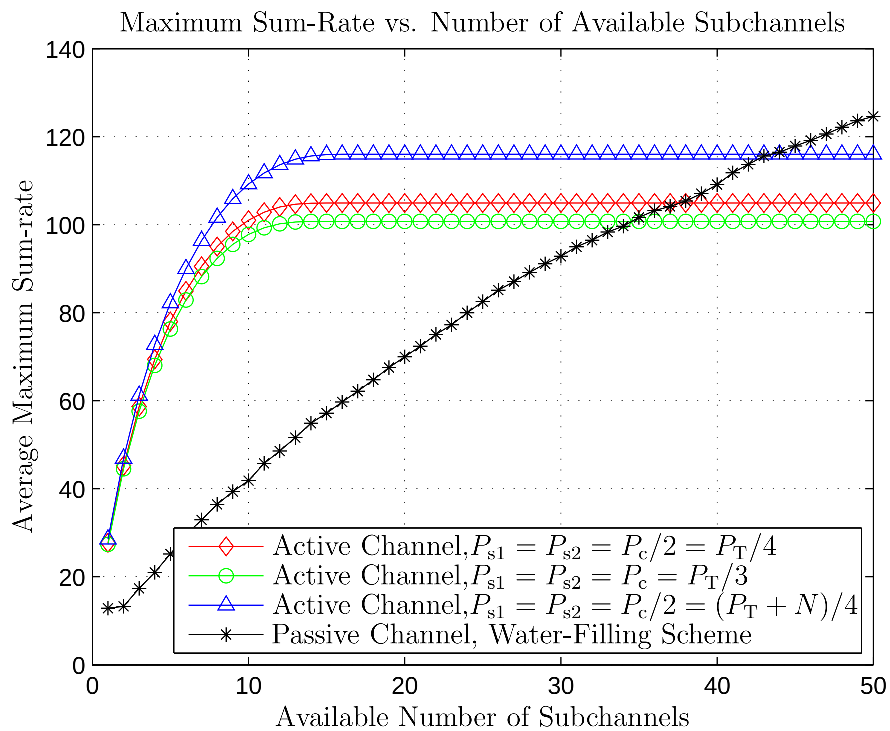


Figure 4.2: Maximum sum-rate versus the number of available subchannels for both active and passive channels for $P = 25$(dBW).

ber of activated subchannels, for a value of n and all larger values of that specific value, the maximum sum-rate does not exist within the feasible set. The corresponding number of activated subchannels in this saturation point, was referred to as n_{max} in Section 3.5 and the reason behind it was proved in Appendix C. In such a case, the active channel takes the previous number of the available subchannels into account. Hence, the maximum sum-rate for the active channel does not change anymore and stays constant afterwards.

In two previous simulation results, we plotted the outcome of our algorithm. Now we want to show some details of our proposed method, since it is interesting for us to probe

our algorithm which relies on our proposed solutions in Scenarios 1 and 2 (Subsections 3.3.1 and 3.3.2). To do this, we compare the obtained sum-rates of each of the scenarios discussed in Subsections 3.3.1 and 3.3.2 (which are denoted as R_1 and R_2 in Section 3.5). We again, define the consumed power in active channel as $P_{\mathrm{T}} \triangleq P_{\mathrm{s1}} + P_{\mathrm{s2}} + P_{\mathrm{c}}$ and explore two different cases of power distribution between the transceivers and the active channel as: 1) $P_{\mathrm{s1}} = P_{\mathrm{s2}} = \dfrac{P_{\mathrm{c}}}{2} = \dfrac{P_{\mathrm{T}}}{4}$ and 2) $P_{\mathrm{s1}} = P_{\mathrm{s2}} = P_{\mathrm{c}} = \dfrac{P_{\mathrm{T}}}{3}$.
The independent unequal subchannel noise powers (i.e., α_i's) have been modeled as Rayleigh distributed random variables, as well.

In both Figures 4.3 and 4.4, we plot the sum-rate obtained in both scenarios, versus the total consumed power for a given number of subchannels $N = 16$. It can be seen that in both cases of resource allocation, Scenario 1 yields a slightly higher sum-rate, which is chosen as the maximum sum-rate for the active channel within the algorithm.

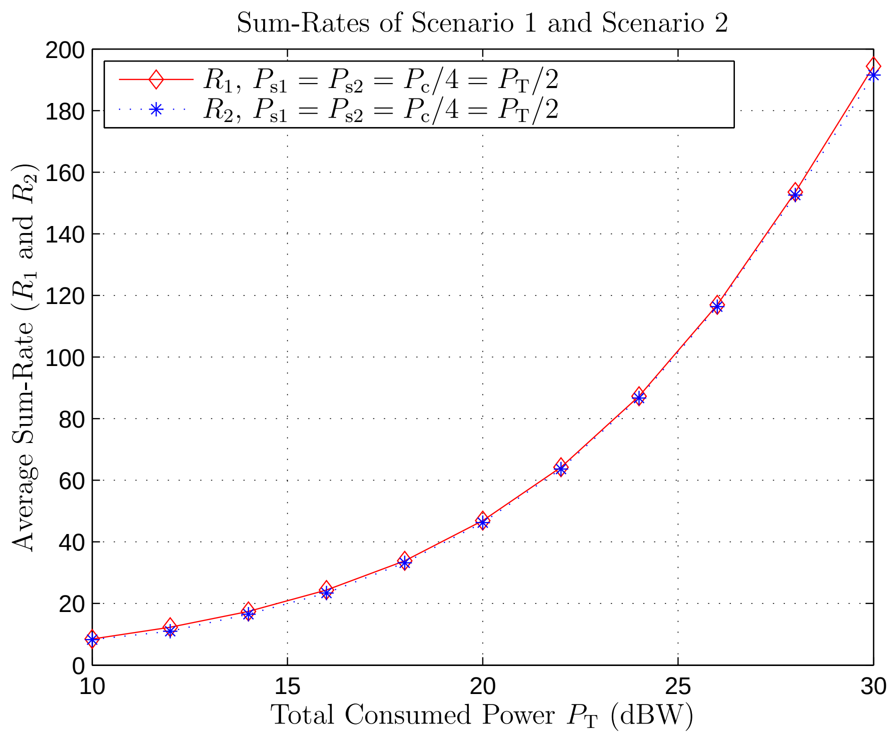


Figure 4.3: Sum-rate of both Scenarios 1 and 2 versus the total consumed power for first case of power distribution between the transceivers and the active channel with $N = 16$ subchannel.

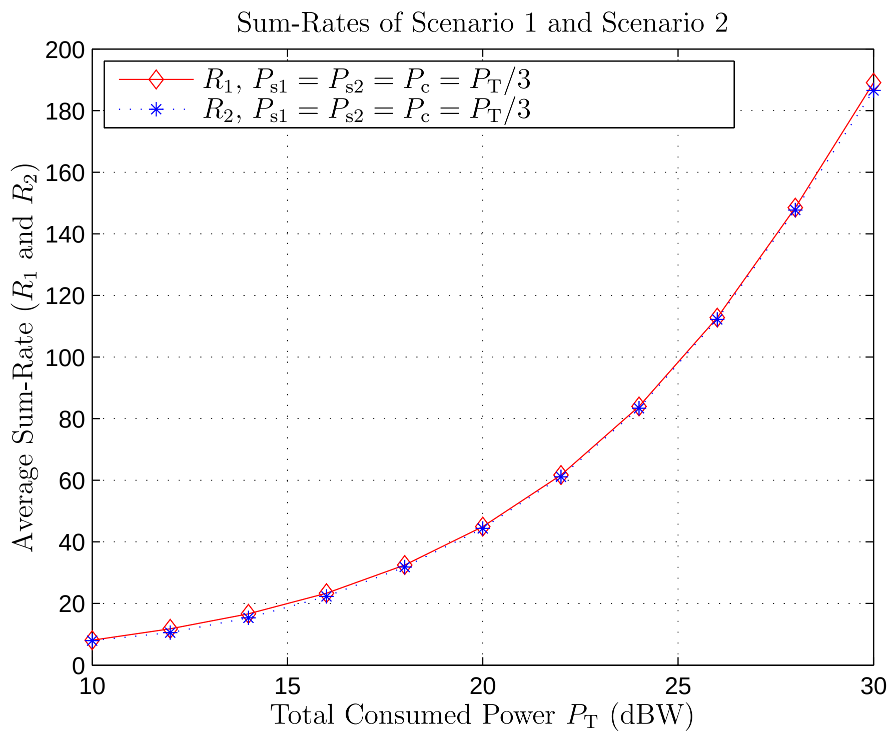


Figure 4.4: Sum-rate of both Scenarios 1 and 2 versus the total consumed power for the second case of power distribution between the transceivers and the active channel with $N = 16$ subchannel.

Chapter 5

Conclusion and Future Work

5.1 Conclusion

In this thesis, wireless parallel channels, with their various applications and realizations, were reviewed in recent studies. The maximum achievable sum-rate over such channels not only depends on the source transmit power(s), but also on the quality of individual subchannels (e.g., noise power over each subchannel). The conventional parallel channels were then referred to as passive channels, where there is no control over the gain of each subchannel. We herein investigate parallel channels, where not only the transmit power(s), but also the channel itself can be controlled (i.e., adjusted or designed), in order to achieve the maximum sum-rate over such channels. We referred to such wireless links as active channels. Maximization of the sum-rate over active channels has become a trend in recent studies. One-way active channels with equal/unequal noise powers over different subchannels and two-way active channels with equal subchannel noise powers are well investigated. We were inspired to develop a practical scenario in this thesis and to extensively study the sum-rate maximization over two-way active channels with unequal noise powers over different subchannels. We conducted a joint optimization of the transmit powers of two transceivers and the channel energy over a set of two-way active subchannels, under three power constraints: the first two on the transmit powers of the two transceivers, and the third constraint on the total energy of the channel. To solve such a problem, we utilized the KKT conditions and analytically developed a method with efficient computational complexity. We indicated that unlike passive parallel channels, not all of the subchannels, but only a subset of them receive power. We then obtained the sum-rate for any feasible number of activated subchannels and after comparing these finite number of sum-rates, introduced the largest value as the maximum sum-rate of the active channel.

5.2 Future Work

In this thesis, sum-rate maximization of two-way active channels with unequal noise powers over different subchannels is comprehensively discussed. This work, can be continued in several scenarios and applications as follows:

- Deriving the optimal power allocation for two-way non-reciprocal active channels with unequal subchannel noise powers over different subchannels.
- Deriving the achievable rate region for two-way active channels.
- Deriving the optimal power allocation for multi-way active channels.

- Deriving the optimal power allocation for active broadcast channels.

Appendices

Appendix A

Proof of Non-Convexity of (3.3)

In order to show optimization problem (3.3) is a non-convex one, we can show the Hessian matrix of the objective function in (3.4) is not always negative definite. The Hessian matrix will be written as

$$\text{blkdg}\left\{\left[\begin{array}{ccc} \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \tilde{p}_{1i}^2} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \tilde{p}_{1i}\partial \tilde{p}_{2i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \tilde{p}_{1i}\partial \bar{h}_i} \\ \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \tilde{p}_{2i}\partial \tilde{p}_{1i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \tilde{p}_{2i}^2} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \tilde{p}_{2i}\partial \bar{h}_i} \\ \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \bar{h}_i\partial \tilde{p}_{1i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \bar{h}_i\partial \tilde{p}_{2i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial \bar{h}_i^2} \end{array}\right]\right\}_{i=1}^{N}, \qquad \text{(A.1)}$$

where blkdg$\{\mathbf{A}\}_{\mathbf{i=1}}^{\mathbf{N}}$ represents a block diagonal matrix whose diagonal matrices are $\{\mathbf{A}\}_{\mathbf{i=1}}^{\mathbf{N}}$ and $f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})$ denotes the cost function (3.4). For any $i = 1, 2, ..., n$ we can write

$$\frac{\partial f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{1i}} = \frac{\tilde{\alpha}_i \bar{h}_i}{1 + \tilde{\alpha}_i \tilde{p}_{1i} \bar{h}_i} \tag{A.2}$$

$$\frac{\partial f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{2i}} = \frac{\tilde{\alpha}_i \bar{h}_i}{1 + \tilde{\alpha}_i \tilde{p}_{2i} \bar{h}_i} \tag{A.3}$$

$$\frac{\partial g(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \bar{h}_i} = \frac{\tilde{\alpha}_i \tilde{p}_{1i}}{1 + \tilde{\alpha}_i \tilde{p}_{1i} \bar{h}_i} + \frac{\tilde{\alpha}_i \tilde{p}_{2i}}{1 + \tilde{\alpha}_i \tilde{p}_{2i} \bar{h}_i}. \tag{A.4}$$

Then for any $i = 1, 2, ..., n$, we can also obtain

$$\frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{1i}^2} = \frac{-\tilde{\alpha}_i \bar{h}_i^2}{(1 + \tilde{\alpha}_i \tilde{p}_{1i} \bar{h}_i)^2} \tag{A.5}$$

$$\frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{2i}^2} = \frac{-\tilde{\alpha}_i \bar{h}_i^2}{(1 + \tilde{\alpha}_i \tilde{p}_{2i} \bar{h}_i)^2} \tag{A.6}$$

$$\frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{h}_i^2} = \frac{-\tilde{\alpha}_i \tilde{p}_{1i}^2}{(1 + \tilde{\alpha}_i \tilde{p}_{1i} \bar{h}_i)^2} + \frac{-\tilde{\alpha}_i \tilde{p}_{2i}^2}{(1 + \tilde{\alpha}_i \tilde{p}_{2i} \bar{h}_i)^2} \tag{A.7}$$

$$\frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{1i} \partial \tilde{p}_{2i}} = \frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{2i} \partial \tilde{p}_{1i}} = 0 \tag{A.8}$$

$$\frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{1i} \partial \bar{h}_i} = \frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \bar{h}_i \partial \tilde{p}_{1i}} = \frac{\tilde{\alpha}_i}{(1 + \tilde{\alpha}_i \tilde{p}_{1i} \bar{h}_i)^2} \tag{A.9}$$

$$\frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \tilde{p}_{2i} \partial \bar{h}_i} = \frac{\partial^2 f(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{h}})}{\partial \bar{h}_i \partial \tilde{p}_{2i}} = \frac{\tilde{\alpha}_i}{(1 + \tilde{\alpha}_i \tilde{p}_{2i} \bar{h}_i)^2}. \tag{A.10}$$

Therefore, for any block of (A.1), we can obtain the determinant as

$$\det\begin{bmatrix} \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\tilde{p}_{1i}^2} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\tilde{p}_{1i}\partial\tilde{p}_{2i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\tilde{p}_{1i}\partial\bar{h}_i} \\ \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\tilde{p}_{2i}\partial\tilde{p}_{1i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\tilde{p}_{2i}^2} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\tilde{p}_{2i}\partial\bar{h}_i} \\ \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\bar{h}_i\partial\tilde{p}_{1i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\bar{h}_i\partial\tilde{p}_{2i}} & \dfrac{\partial^2 f(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{h}})}{\partial\bar{h}_i^2} \end{bmatrix} =$$

$$\frac{\tilde{\alpha}_i^3\bar{h}_i^2}{(1+\tilde{\alpha}_i\tilde{p}_{1i}\bar{h}_i)^2(1+\tilde{\alpha}_i\tilde{p}_{2i}\bar{h}_i)^2}\left(\frac{1-\bar{h}_i^2\tilde{p}_{1i}^2}{(1+\tilde{\alpha}_i\tilde{p}_{1i}\bar{h}_i)^2}+\frac{1-\bar{h}_i^2\tilde{p}_{2i}^2}{(1+\tilde{\alpha}_i\tilde{p}_{2i}\bar{h}_i)^2}\right). \tag{A.11}$$

As it can be seen in (A.11), the ith block of the Hessian matrix is not always negative definite which means that the optimization problem (3.5) is non-convex.

Appendix B

Solving 4^{th} Order Polynomial (Quartic) Equation

The first algebraic technique for solving a general quartic was developed by Ferrari and then stolen and published in Cardano's book, Ars Magna [114]. We use the closed form solutions based on the same algebraic technique. In this technique, we first consider a general 4^{th} order polynomial as

$$a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0. \tag{B.1}$$

Dividing both sides of (B.1) by the a_4, we can rewrite it as a monic quartic function as

$$x^4 + ax^3 + bx^2 + cx + d = 0, \tag{B.2}$$

where $a \triangleq \frac{a_3}{a_4}$, $b \triangleq \frac{a_2}{a_4}$, $d \triangleq \frac{a_1}{a_4}$, and $e \triangleq \frac{a_0}{a_4}$. Substituting x with $y - \frac{1}{4}a$, we arrive at a quartic function in standard form (i.e., without any 3rd order term) in terms of the substituted variable y, as

$$y^4 + fy^2 + gy + h = 0, \tag{B.3}$$

where

$$\begin{aligned} f &\triangleq b - \frac{3}{8}a^2 \\ g &\triangleq c - \frac{1}{2}ab + \frac{1}{8}a^3 \\ h &\triangleq e - \frac{1}{4}ad + \frac{1}{16}a^2b - \frac{3}{256}a^4 \end{aligned} \tag{B.4}$$

In order to be able to write (B.3) in form of difference of two square terms, we add and subtract the term $y^2\nu + \nu^2/4$ to and from (B.3), where ν is an arbitrary quantity. Hence we can write

$$y^4 + y^2\nu + \frac{\nu^2}{4} - y^2\nu + \frac{\nu^2}{4} + fx^2 + gx + h = 0, \tag{B.5}$$

or, equivalently,

$$(y^2 + \frac{\nu}{2})^2 - \left((\nu - f)y^2 - gy + (\frac{\nu^2}{4} - h)\right) = 0, \tag{B.6}$$

The first term in (B.6) is square and the second term can be square by adjusting the arbitrary quantity (i.e., ν). To find the appropriate value of ν, we assume that the second term of (B.6) is square and name it S^2

$$S^2 \triangleq (\nu - f)\left(y^2 - (\frac{g}{\nu - f})y + \frac{\frac{\nu^2}{4} - h}{\nu - f}\right). \tag{B.7}$$

For the S^2 to be a perfect square, one of the many possible representations is

$$S^2 = (\nu - f)\left(y - \sqrt{\frac{\frac{\nu^2}{4} - h}{\nu - f}}\right)^2. \tag{B.8}$$

For (B.7) and (B.8) to be equivalents, (B.9) must hold

$$\sqrt{\frac{\frac{\nu^2}{4} - h}{\nu - f}} = \frac{g}{\nu - f}, \tag{B.9}$$

or, equivalently,

$$\nu^3 - f\nu^2 - (4h)\nu - (4fr + g^2) = 0, \tag{B.10}$$

which is a monic cubic function. The solution to 3^{rd} order equations is known [115]. Thus solving (B.10), we replace ν with one of three solutions of (B.10) (e.g., the real root called ν_1) in (B.8) and obtain S as

$$S = (\sqrt{\nu_1 - f})y - \frac{g}{2\sqrt{\nu_1 - f}}. \tag{B.11}$$

Therefore, (B.6) can be rewritten as

$$\left(y^2 + \frac{\nu_1}{2}\right)^2 - \left((\sqrt{\nu_1 - f})y - \frac{g}{2\sqrt{\nu_1 - f}}\right)^2 = 0, \tag{B.12}$$

or, equivalently,

$$\left(y^2+(\sqrt{\nu_1-f})y-\frac{g}{2\sqrt{\nu_1-f}}+\frac{\nu_1}{2}\right)\left(y^2-(\sqrt{\nu_1-f})y-\frac{g}{2\sqrt{\nu_1-f}}+\frac{\nu_1}{2}\right)=0. \tag{B.13}$$

Solving (B.13) is equivalent to solving two quadratic equations and we can easily find the solutions for y as

$$\begin{aligned} y_{1,2} &= \frac{1}{2}\left(-\sqrt{\nu_1-f}\pm\left(-\nu_1-f+\frac{2g}{\sqrt{\nu_1-f}}\right)\right) \\ y_{3,4} &= \frac{1}{2}\left(+\sqrt{\nu_1-f}\pm\left(-\nu_1-f+\frac{2g}{\sqrt{\nu_1-f}}\right)\right). \end{aligned} \tag{B.14}$$

Considering our first substitution (i.e., $x=y-\frac{1}{4}a$), we obtain all four roots of the quartic equation (B.1), as

$$\begin{aligned} x_1 &= -\frac{1}{4}a+\frac{1}{2}\left(-\sqrt{\nu_1-f}+\left(-\nu_1-f+\frac{2g}{\sqrt{\nu_1-f}}\right)\right) \\ x_2 &= -\frac{1}{4}a+\frac{1}{2}\left(-\sqrt{\nu_1-f}-\left(-\nu_1-f+\frac{2g}{\sqrt{\nu_1-f}}\right)\right) \\ x_3 &= -\frac{1}{4}a+\frac{1}{2}\left(+\sqrt{\nu_1-f}+\left(-\nu_1-f+\frac{2g}{\sqrt{\nu_1-f}}\right)\right) \\ x_4 &= -\frac{1}{4}a+\frac{1}{2}\left(+\sqrt{\nu_1-f}-\left(-\nu_1-f+\frac{2g}{\sqrt{\nu_1-f}}\right)\right). \end{aligned} \tag{B.15}$$

Appendix C

Proof of Reduced Computational Complexity

Within our proposed method, we form the function $G_n(A)$ for different number of activated subchannels (i.e., n), in order to obtain the interval of feasible values of A (i.e., $(A_0, +\infty)$) and then search for the optimal A within this interval. Once the optimal A is obtained (if exists), we calculated the corresponding sum-rate for that number of activat subchannel (i.e., n). We then compar all obtained sum-rates for different number of activated subchannels to find the maximum sum-rate of the active channel. In this appendix, we prove that if an optimal real A does not exists for both Scenarios 1 and 2 for a number of activated subchannels (i.e., n), it does no exist for any larger value of n. This fact limits our search for different number of activated subchannels (i.e, n's) to a largest feasible n denoted as n_{max}. Thus, feasibility check prevents us from further sum-rate calculations for larger values of n's once the first infeasible number of activated subchannel is determined, and this results in an enhancement in computational efficiency. To show this, we prove that $G_n(A)$ is a decreasing function of the number of activated subchannel (i.e., n) and also an increasing function of α_n. This fact guarantees that if for a value of n an optimal real A does exist over $(A_0, +\infty)$ for both Scenarios 1 and 2 (i.e., or equivalently to say that (3.70) does not hold for the optimal A), an optimal real A does exist for any larger value of n since the new $G_n(A)$ would be a smaller and (3.70) does not hold for this value of n. To do so, we first rewrite (3.68) as

$$G_n(A) = 1 - \frac{W_n(A)}{P_{\rm c}\alpha_n}, \tag{C.1}$$

where $W_n(A) \triangleq n(2 - AQ_n(A))Q_n(A)$. If we show that $W_n(A)$ is an increasing function of n (i.e., $W_{n+1}(A) > W_n(A)$), we then can show that $G_n(A)$ is a decreasing function of n. Hence, ignoring momentarily that n is an integer, we differentiate $W_n(A)$ with respect to n as

$$\frac{\partial W_n(A)}{\partial n} = (2 - AQ_n(A))Q_n(A) + n(2 - AQ_n(A))\left[\frac{\partial Q_n(A)}{\partial n}\right] - nAQ\left[\frac{\partial Q_n(A)}{\partial n}\right]. \tag{C.2}$$

Using (3.31), we can rewrite (C.2) as

$$\frac{\partial W_n(A)}{\partial n} = \left(\frac{\frac{P_{s1}}{n}}{(\frac{P_{s1}}{n} + A)} + \frac{\frac{P_{s2}}{n}}{(\frac{P_{s2}}{n} + A)}\right)\left(\frac{2A + \frac{(P_{s1}+P_{s2})}{n}}{(\frac{P_{s1}}{n} + A)(\frac{P_{s2}}{n} + A)}\right) + n(1 - AQ_n(A))\left[\frac{\partial Q_n(A)}{\partial n}\right]. \tag{C.3}$$

Using (3.31), we obtain $\frac{\partial Q_n(A)}{\partial n}$ as

$$\frac{\partial Q_n(A)}{\partial n} = \left(\frac{\frac{P_{s1}}{n^2}}{(\frac{P_{s1}}{n}+A)^2}\right) + \left(\frac{\frac{P_{s2}}{n^2}}{(\frac{P_{s2}}{n}+A)^2}\right) = \frac{\frac{P_{s1}}{n^2}(\frac{P_{s2}}{n}+A)^2 + \frac{P_{s2}}{n^2}(\frac{P_{s1}}{n}+A)^2}{(\frac{P_{s1}}{n}+A)^2(\frac{P_{s2}}{n}+A)^2}. \tag{C.4}$$

Using (C.4), we can rewrite (C.3) as

$$\begin{aligned}
\frac{\partial W_n(A)}{\partial n} &= \left(\frac{\{2\frac{P_{s1}P_{s2}}{n^2} + A(\frac{P_{s1}+P_{s2}}{n})\}\{\frac{P_{s1}+P_{s2}}{n}+2A\}}{(\frac{P_{s1}}{n}+A)^2(\frac{P_{s2}}{n}+A)^2}\right) \\
&\quad + n(1 - AQ_n(A))\left[\frac{\frac{P_{s1}}{n^2}(\frac{P_{s2}}{n}+A)^2 + \frac{P_{s2}}{n^2}(\frac{P_{s1}}{n}+A)^2}{(\frac{P_{s1}}{n}+A)^2(\frac{P_{s2}}{n}+A)^2}\right] \\
&= \left(\frac{\{2\frac{P_{s1}P_{s2}}{n^2} + A(\frac{P_{s1}+P_{s2}}{n})\}\{\frac{P_{s1}+P_{s2}}{n}+2A\}}{(\frac{P_{s1}}{n}+A)^2(\frac{P_{s2}}{n}+A)^2}\right) \\
&\quad + n(-1 + 2 - AQ_n(A))\left[\frac{\frac{P_{s1}}{n^2}(\frac{P_{s2}}{n}+A)^2 + \frac{P_{s2}}{n^2}(\frac{P_{s1}}{n}+A)^2}{(\frac{P_{s1}}{n}+A)^2(\frac{P_{s2}}{n}+A)^2}\right] \\
&= \left[\frac{\left(\{2\frac{P_{s1}P_{s2}}{n^2} + A(\frac{P_{s1}+P_{s2}}{n})\}\{\frac{P_{s1}+P_{s2}}{n}+2A\}\right) - \left[\frac{P_{s1}}{n}(\frac{P_{s2}}{n}+A)^2 + \frac{P_{s2}}{n}(\frac{P_{s1}}{n}+A)^2\right]}{(\frac{P_{s1}}{n}+A)^2(\frac{P_{s2}}{n}+A)^2}\right] \\
&\quad + (2 - AQ_n(A))\left[\frac{\frac{P_{s1}}{n}(\frac{P_{s2}}{n}+A)^2 + \frac{P_{s2}}{n}(\frac{P_{s1}}{n}+A)^2}{(\frac{P_{s1}}{n}+A)^2(\frac{P_{s2}}{n}+A)^2}\right].
\end{aligned} \tag{C.5}$$

Knowing that $[2 - AQ_n(A)] > 0$, in order to prove that $\frac{\partial W_n(A)}{\partial n} > 0$, we only have to show

$$J_n(A) \triangleq \left(\{2\frac{P_{s1}P_{s2}}{n^2} + A(\frac{P_{s1}+P_{s2}}{n})\}\{\frac{P_{s1}+P_{s2}}{n}+2A\}\right) - \left[\frac{P_{s1}}{n}(\frac{P_{s2}}{n}+A)^2 + \frac{P_{s2}}{n}(\frac{P_{s1}}{n}+A)^2\right] \geq 0. \tag{C.6}$$

We can write (C.6) as

$$\begin{aligned} J_n(A) &= 2\frac{P_{s1}P_{s2}(P_{s1}+P_{s2})}{n^3} + 4A\frac{P_{s1}P_{s2}}{n^2} + 2A^2\frac{P_{s1}+P_{s2}}{n} + A\left(\frac{P_{s1}+P_{s2}}{n}\right)^2 \\ &\quad -2\frac{P_{s1}P_{s2}(P_{s1}+P_{s2})}{n^3} - 8A\frac{P_{s1}P_{s2}}{n^2} - 2A^2\frac{P_{s1}+P_{s2}}{n} \\ &= A\left(\frac{P_{s1}+P_{s2}}{n}\right)^2 - 4A\frac{P_{s1}P_{s2}}{n^2}. \end{aligned} \tag{C.7}$$

Thus, we can rewrite $J_n(A)$ as a perfect square as

$$J_n(A) = A\left(\frac{P_{s1}-P_{s2}}{n}\right)^2. \tag{C.8}$$

From (C.8) we clearly can see that $J_n(A) \geq 0$ and consequently $\frac{\partial W_n(A)}{\partial n} > 0$. Hence, from (C.1) we can conclude that

$$W_{n+1}(A) > W_n(A). \tag{C.9}$$

Considering our initial assumption that $\alpha_n \geq \alpha_{n+1}$ along with (C.9), we can write

$$G_{n+1}(A) = 1 - \frac{W_{n+1}(A)}{P_c\alpha_{n+1}} \;<\; 1 - \frac{W_n(A)}{P_c\alpha_n} = G_n(A) < 0. \tag{C.10}$$

Assuming that the optimal A for $n = n_{max}+1$ does not exists within its feasible set, or equivalently $G_{n_{max}+1}(A) < 0$, by increasing the number of activated subchannels to $n_{max}+2$, $G_n(A)$ descends.

$$G_{n_{max}+2}(A) \;<\; G_{n_{max}+1}(A) \;<\; 0. \tag{C.11}$$

Thus, it is guaranteed that the problem is not feasible for $n = n_{max} + 1, n_{max} + 2, \ldots, N$.

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