

Cooperative Relay Beamforming Design
for Multi-Cluster Relay Interference Networks

by

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Abstract

In this work, we consider a multi-cluster amplified-and-forward (AF) relay interference network and design relay beam matrix for each cluster to maximize the minimum signal-to-interference-and-noise ratio (SINR) at destinations subject to the relay power budget within each cluster. Both single and multiple source-destination (S-D) pair scenarios are considered. We propose a beam matrix structured as a weighted sum of two types of beam matrices: zero forcing (ZF) beam matrix for inter-cluster interference suppression, and another beam matrix for beamforming gain maximization within a cluster. Maximum ratio combining (MRC) and minimal mean square error (MMSE) are chosen to design the second beam matrix in single and multiple S-D pair scenarios, respectively. The optimal solution for each type of beam matrix is obtained in closed form. We then obtain the optimal weights to each type of beam matrix by transforming the max-min SINR problem and solving it via the SDR approach.

Compared with applying the direct SDR approach to the original problem, our solution offers similar performance with significantly lower computational complexity. In addition, our proposed structured beam matrix clearly reveals the power shift between interference suppression among clusters and beamforming gain maximization within the cluster as the distance among clusters or the size of clusters changes.

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Chapter 1

Introduction

1.1 Overview

Thanks to the rapid proliferation of rich multimedia services like social media, e-business and smart phones, the past decade have witnessed a fast increasing demand for wireless data services. A report from the Cellular Telephone Industries Association (CTIA) [1] shows that the data traveled across U.S. wireless networks has increased explosively by some 20 times from approximate 191 billion megabytes (MB) in 2009 to more than 4 trillion MB in 2014. To cater this huge demand, numerous wireless technologies have been developed and put into practice to improve the speed, quality and reliability of a wireless system. In this thesis, we focus on two of such techniques: cooperative relay networks and beamforming.

Cooperative relaying is an important technique adopted in many practical wireless systems or applications, such as Long-Term Evolution (LTE) for the 4th-generation cellular networks [2], wireless local network (WLAN), etc. It can achieve spacial diversity without the need of multi-antennas, and has the potential of significantly improving the coverage, capacity and energy saving of a network. In cooperative relay beamforming, a set of relays work together and form a virtual multi-antenna array

to process and forward received signals. By combining channels coherently, transmit power is then focused towards the direction of each desired user to form both power gain and diversity gain at destination for higher received signal-to-noise ratio (SNR).

In a cooperative relay networks, relay interference is a common issue when there are multiple source-destination pairs communicating through relays. In a multi-user peer-to-peer (MUP2P) relay network, interference comes from other sources in the same cluster. In a relay interference network, multiple source-destination (S-D) pairs, each communicating through their own dedicated relay(s), form multiple relaying clusters. In this scenario, interference could also come from users and relays from other relay clusters, possibly from adjacent cell. Examples include relay-assisted transmission at neighboring cells in a cellular network forming multiple relaying clusters, where each S-D pair suffers from inter-cluster interference. Another example would be the wireless ad-hoc network where each participating nodes can serve as a relay by forwarding data for other nodes; the capacity of the network is limited by the mutual interference from concurrent transmissions among nodes [3]. An efficient design of cooperative relaying to reduce inter-cluster interference for simultaneous transmissions is important in such networks. In this work, we study the design of amplified-and-forward (AF) multi-antenna relaying in relay interference networks. We consider multi-antenna relay beamforming technique for signal processing and forwarding. The goal is to design relay beam matrix in such multi-cluster relay interference networks to reduce inter-cluster interference and maximize beamforming gain from source towards each destination.

1.2 Motivation and Objective

Most existing works consider single cluster relay networks. Very few literature works consider multi-cluster relay interference networks, either with single or multiple S-D pairs. However, such scenario often arises in practical systems such as cellular networks, where S-D pairs may be at neighboring cells and do not share the common relays. Also, studies show that it maybe beneficial to divide a large relay network into multiple clusters, either geographically or functionally, to improve the transmission efficiency [4]. Relay beamforming design in a multi-cluster network has to consider interference from other sources and relays in other clusters.

Furthermore, most of the existing works that consider multi-user beamforming, either in MUP2P or multi-cluster cases, choose relay power minimization as the design objective instead of signal-to-interference-and-noise ratio (SINR) maximization. Such formulated optimization problem has the feasibility issue¹. In addition, existing works often use the semidefinite relaxation (SDR) method as a typical approach to obtain a solution for many similar relay beamforming optimization problems, which can be computationally expensive when the problem size is large.

Considering all the above factors, in this thesis, we investigate into the relay beamforming design in a multi-cluster relay interference network, and aim at developing a low-complexity solution which maximizes the minimum SINR for all users subject to the relay power budget within each cluster.

¹For power minimization problem, there might be cases when there is no solution that satisfy the constraints, thus the problem is infeasible

1.3 Thesis Contribution

In this thesis, we consider relay beamforming design in a multi-cluster relay interference network for both single and multiple S-D pair cases. We aim to develop a low-complexity solution which maximizes the minimum SINR for all users subject to a per-cluster relay power budget.

For single S-D pair scenario, we propose a structured relay beam matrix as a weighted sum of two types of beam matrices: zero forcing (ZF) beam matrix for inter-cluster interference suppression, and the maximum ratio combining (MRC) beam matrix for maximizing the beamforming gain within the cluster. We obtain the optimal solution for each type of the beam matrix in closed form. The optimal weight to each type of beam matrix is then obtained by transforming the max-min SINR problem and solving it via the SDR approach. Comparing with applying the direct SDR approach to the original problem, our proposed solution offers both similar performance and significant computational complexity reduction. Furthermore, our proposed structured beam matrix clearly revealed the power shift between interference suppression among clusters and beamforming gain maximization within the cluster as the distance among clusters changes.

For multiple S-D pairs scenario, a similar structured relay beam matrix consisting of ZF and minimum mean squared error (MMSE) beam matrix is proposed for interference suppression and beamforming gain maximization, respectively. The optimal weight to each type of beam matrix is also obtained by transforming and solving the question through the SDR approach. Simulations show that our proposed solution

provides similar SINR performance with that of direct approach, and the performance gap decreases as the number of relays increases. Additionally, proposed approach also has significant lower computational complexity and the ability of revealing the power shift as the distance among clusters or size of cluster changes.

1.4 Thesis Organization

The rest of this thesis is organized as follows: In Chapter 2, a literature review is carried out on the recent studies of relay beamforming and commonly used optimization objectives and approaches in dealing with these problems. In Chapter 3, single S-D pair relay beamforming design in multi-cluster relay interference networks is developed. In Chapter 4, a multi-cluster relay beamforming problem with multiple S-D pairs in each cluster is then investigated. The conclusion is provided in Chapter 5.

1.5 Notations

Trace, Hermitian, transpose, and conjugate of \mathbf{A} are denoted by $\text{tr}[\mathbf{A}]$, \mathbf{A}^H , \mathbf{A}^T , and \mathbf{A}^* , respectively. The Kronecker product is denoted as \otimes . $\text{vec}(\mathbf{A})$ vectorizes $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_N]$ to $[\mathbf{a}_1^T, \dots, \mathbf{a}_N^T]^T$. A semi-positive definite matrix \mathbf{A} is denoted as $\mathbf{A} \succcurlyeq 0$, and \mathbf{I} denotes the identity matrix. Notation $\text{bldg}(\mathbf{A}_1, \dots, \mathbf{A}_N)$ denotes a block diagonal matrix with \mathbf{A}_i being the i th diagonal block. $[\mathbf{A}]_{i,j}$ denotes the (i, j) th entry of \mathbf{A} , and $[\mathbf{a}]_i$ means the i th element of \mathbf{a} .

Chapter 2

Literature Review

2.1 Relay Network

Due to fading effect, transmission through wireless channels suffers from severe attenuation in signal strength. As the effect of fading is related with the distance between source and destination, long transmission distance will result in weak receive signal and poor SNR performance.

To combat fading, relay techniques are adopted to help forward signal between two nodes. An example of a relay network is shown in Fig. 2.1, where a relay station (RS) receives signal from the source and then forwards the signal to the destination. Early studies of relay can be dated back to [5, 6]. Later on, different aspects of relay networks, such as capacity, diversity gain and performance improvement have been studied [7–21].

In a relay network, each relay not only forwards the desired signal, but also sends amplified noise at the relay to the destination. When there are multiple S-D pairs, the relay forwards interference originated from other sources to the destination, too. To mitigate the effect of interference, different relaying approaches can be adopted, such as the AF scheme [22–24] and the decode and forward scheme (DF) [25, 26]. The

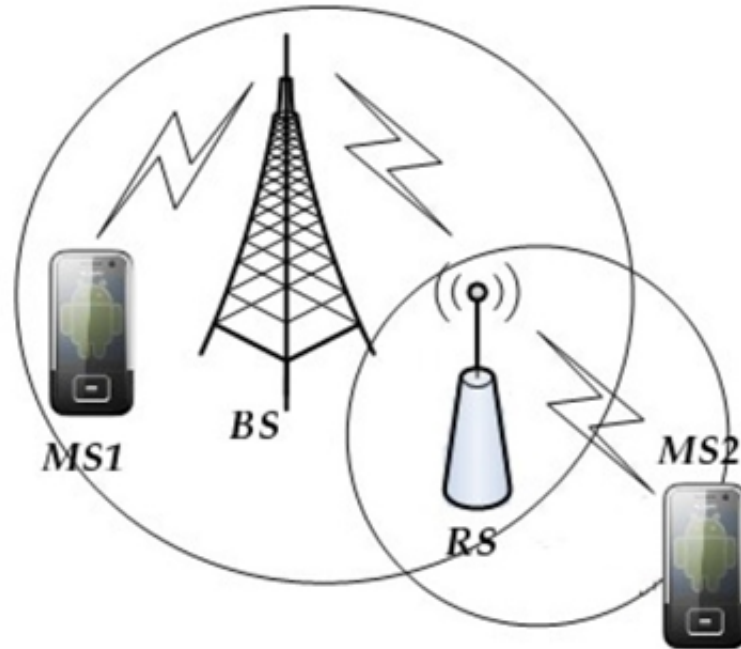


Figure 2.1: A relay network

AF scheme is easy to apply, where the relay simply amplifies and retransmits the signal received from the source. In a DF scheme, the relay first decodes the received signal and then retransmits the decoded and regenerated symbols to the destination. It produces better signals at the cost of more complex hardware, as a decoding block is needed at the relay.

2.2 Beamforming Technique

In a multi-antenna system, beamforming technique can be used to obtain diversity and power gains [27]. For beamforming, all the antennas transmit the same symbol and then combine signal over each antenna coherently at a destination, as shown in Fig. 2.2. By utilizing channel information (direction and strength) at transmitter side, beamforming can achieve certain benefits, including improved SNR, power usage reduction and extended transmission range. Depending on the capability of

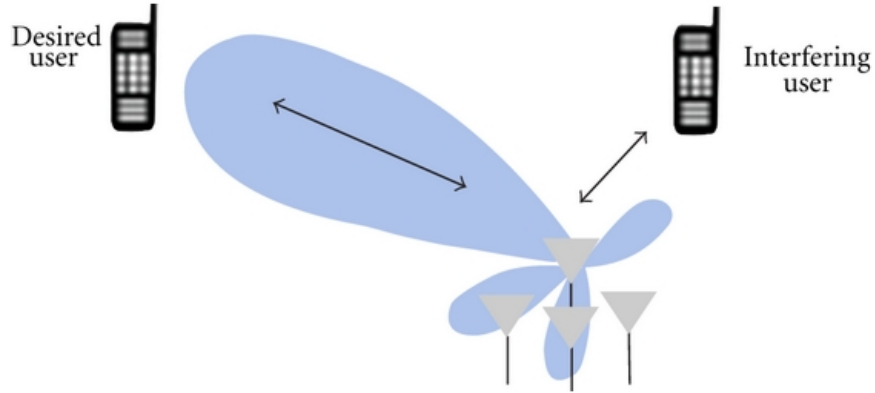


Figure 2.2: Beamforming technique

sharing information between antennas, beamforming technique can be classified into distributed and centralized beamforming.

Distributed beamforming is a beamforming technique in which multiple independent antennas simultaneously transmit the same signal with controlled phase and synchronized frequency, so that signal can be constructively combined at a destination. Depending on design objectives and constraints, advantages like energy efficiency, improved received SNR and security against eavesdropping can be achieved [23, 28–30]. For relay network, distributed beamforming can be used in both AF and DF schemes.

Centralized Beamforming is a centralized technique that utilizes multiple antennas to process signal. When a signal is transmitted to a destination, it is first processed over multi-antennas and then sent through independent paths, and at last added coherently together at destination. This approach also allows transmitter to use certain processing procedures, like ZF or MRC [31, 32]. For relay beamforming, if the total number of antennas in a relay network is fixed, centralized relay beamforming will always have a better or equivalent performance as comparing with distributed

beamforming [33, 34].

2.3 Single-Cluster Relay Beamforming Design

In this section, we review relay beamforming design in both single S-D pair and MUP2P scenarios.

2.3.1 Single S-D Pair Relay Beamforming

Many existing works have studied the optimal design of the AF relay processing matrix in a single S-D pair environment with a multi-antenna relay [22, 35–39]. In [35], an optimal weighting matrix for multiple-input multiple-output (MIMO) relay is found to maximize capacity under a total relay power constraint. Paper [22] considers MIMO relay beamforming problem in both full channel state information (CSI) and receiver CSI only cases. It aims to maximize receive SNR at destination under source and relay power constraint. An optimal beamforming scheme is provided for the full CSI case, while a quantized beamforming scheme is proposed for receiver CSI case using Grassmannian codebooks. Authors in [37] use receiver SNR as criterion, and develop a general rank beamforming matrix based on the second order statistics of the channel coefficients subject to constrained relay transmission power. They also show that a closed-form solution can be found with correlated source-relay and relay-destination channels.

Multiple single-antenna relays forming distributed beamforming has also been discussed in [12, 23, 40, 41]. By assuming perfect CSI at relay, a distributed beamformer with closed form solution has been obtained in [12] which maximizes SNR at receiver

under per-relay power constraint. Paper [40] assumes both the relay beamforming weights and the transceiver transmit powers to be the design parameters, and achievable beamforming rate region is characterized under a total (network) transmit power constraint. Authors of [41] develop relay power allocation algorithms for non-coherent and coherent AF relay networks. A robust method is proposed in this paper which takes the absence of global CSI into consideration, and the goal is to minimize the total relay transmission power under individual relay power constraints, while satisfying quality of service (QoS) requirement.

The cost of relay cooperation, such as exchanging control and data signals among relays, or system-level synchronization can outweigh the benefits, thus relay selection scheme is proposed and studied in [42–49]. Most of the works focus on single relay selection, either using AF scheme [42–44] or DF scheme [45–47], where only one relay cooperates. Multiple relay selection is discussed in [48] assuming perfect CSI and power constraint at each relay. Selection schemes which achieve full diversity order are proposed in [48] for both single and multi-relay selection problems, with complexity linear and quadratic to the number of relays, respectively. A joint optimization of multi-relay selection and beamforming problem is studied in [49], where a subset of relays and their joint linear beamformer are found based on second order channel statistics, so that the receiver SNR is maximized subject to per-relay power requirement. The NP-hard mix-integer problem is solved by using the SDR approach with a guaranteed approximation performance.

2.3.2 Multi-user Peer-to-Peer Relay Beamforming

For relay networks with multiple S-D pairs, many studies have been done by considering S-D pairs sharing common relays and forming a single cluster. In this scenario, the problems are typically non-convex and difficult to solve. Numerical algorithms have been developed to obtain approximate solutions.

For total relay power minimization among relays or among all network nodes, relay beamforming design has been studied in [15, 24, 28, 50, 51]. Paper [50] designs beamforming weights for distributed relays through the minimization of total relay power while satisfying the SINR targets at the destinations. SDR is used to convert the non-convex minimization problem into a semidefinite programming (SDP) problem, which can be efficiently solved numerically using standard SDP software, such as SeDuMi [52, 53]. A joint optimization of source power allocation and relay beamforming is considered [51] in a distributed MUP2P AF relay networks. It minimizes total power transmitted from all sources and relays while meeting SINR constraints of each source-destination pair. An iterative feasibility search algorithm is proposed to help find the optimal solution of the problem.

Apart from the total relay power minimization, per relay power minimization and per-antenna power minimization are considered recently in [54, 55] for MUP2P AF relay networks, under target QoS requirements. Multi-user multi-channel cooperative relay beamforming with single antenna relay is considered in [54], where semi-closed form relay beam vectors are obtained through Lagrange dual approach. Paper [55] considers multi-antenna relay, and a low-complexity approximate solution using La-

grange dual approach is developed with semi-closed-form structure. A combination of these two methods is also proposed as a trade-off between performance and complexity.

2.4 Multi-cluster Relay Beamforming Design

Unlike single cluster scenario, very few studies have directly addressed the problem of relay beamforming in multi-cluster relay networks. However, such scenario often arises in practical systems such as cellular networks, where S-D pairs may be at neighboring cells and do not share the common relays. Also, it may be beneficial to divide a large relay network into multiple clusters, either geographically or functionally, to improve the transmission efficiency, as the per-node through output of capacity of a wireless ad-hoc network reduces rapidly with an increasing network size [4].

In a multi-cluster relay network, communication suffers from both inter-cluster and intra-cluster interference. The subject of relay interference networks have been studied in [56–59]. [56] uses a deterministic two-stage interference channel model to study the characteristic of relay-interference networks, and shows the necessity of using interference-targeted transmission scheme, such as interference suppression, interference alignment and interference separation for relay-interference networks. An approximate characterization of the capacity region of a two-stage relay-interference network is studied in [58] using a new interference management scheme term interference neutralization. Paper [59] investigates the degrees of freedom of the interference channel in the presence of a dedicated MIMO relay. By using a hybrid coding strategy that exploits both direct link and MIMO relay, it achieves full degree of freedom for

the network.

Recently, the multi-cluster relay beamforming has been considered in [60] to minimize the total relay power of all clusters while meeting SINR requirements at destinations. It assumes channel second order statistics are available at relay. The NP-hard non-convex problem is then relaxed into an SDP problem, and a semi-distributed iterative method has been proposed to solve it.

2.5 Relay Beamforming Design Objectives

Different designing philosophies and various optimization goals have been suggested in the study of relay beamforming problem, and among them the more commonly used ones are relay power minimization and the maximization of minimum SINR among users.

2.5.1 Relay Power Minimization

Relay power minimization, including per-antenna, per-relay and total relay power minimization, minimizes relay transmission power subject to QoS constraints under different assumptions of channel knowledge. They have been commonly used as design objectives since the power functions are quadratic and the problem is tractable. However, these optimization problems also have a common feasibility issue, as the satisfaction of QoS constraints also depends on the transmit power and channel conditions. It may not always be possible to satisfy any given QoS target.

In aforementioned papers, [35, 40, 41, 50, 51, 60] consider total relay power minimization, while per antenna and per relay power minimization problems are considered

in [33, 38, 39, 55]. The SDR approach is often used to solve relay power minimization problems. Because of the quadratic objective functions, some times dual approach can be used as a complement with performance close to that of SDR approach but much lower computational complexity [33, 39, 55].

2.5.2 Maximization of Minimum SINR Among Users

The maximization of minimum SINR criterion maximizes the minimal SINR among destinations for MUP2P case, or simply SNR for single user case, while satisfying different relay power constraints. It improves the worst user QoS for given network conditions, and the optimization problem is always feasible. It's a less commonly used formulation as compared with the power minimization problem, because of the SINR (or SNR) objective function is a non-convex function with respect to (w.r.t.) beam vectors or matrices. Thus these problems are often hard to solve.

In aforementioned works, [12, 23, 37, 49] consider maximizing minimum SNR in a single S-D pair network, with distributed relays [12, 23, 49] or a centralized relay [37]. The SDR approach is often used to solve these problems. In some special cases, e.g. with a centralized relay or distributed relay with per-relay power constraint, closed form beamformer designs are also available [12, 37].

2.6 Optimization Approach

There are several optimization methods used in literature to solve the relay beamforming problems. The SDR approach is perhaps the most commonly used one, as these problems are usually non-convex. Additionally, many closed form solutions may exist

for centralized relay cases with much lower complexity than SDR. As mentioned in Section 2.5.1, the dual approach can be used as a complement for power minimization problem.

2.6.1 SDR Approach

The SDR technique has been at very popular in the study of signal processing and communications in recent yeas. It is a powerful, computationally efficient approximation technique which can be applied to many non-convex quadratically constrained quadratic programs (QCQP) [16, 61, 62].

As mentioned in Section 2.5, in relay beamforming SDR approach can be used to solve either relay power minimization [50] or max-min SINR [23] problems. By expressing the original problem in quadratically constraint quadratic problem (QCQP) form and then transforming it into an SDP, it can then be solved using standard SDP software, such as SeDuMi [52, 53], in polynomial time. After obtaining the solution of the SDP, the result may not be rank one in general thus solutions can't be extracted directly from the result. For these cases, randomization techniques like [63] are needed to extract a rank-one solution from the result.

It is worth mentioning that because of the exitance of non-rank-one results and the randomization process used to exert a solution, SDR in many cases only offers a sub-optimal solution. Furthermore, although it can be solved in polynomial time, the complexity of SDP is affected by problem size and number of constraints [64], thus it can still be relative high when we have a big network or a large number of users.

2.6.2 Closed-Form Beamformer Design

When we have a multi-antenna relay or multiple relays capable of information sharing forming a virtual multi-antenna, beamforming schemes can benefit from spatial processing at the nodes. By exploiting more degrees of freedom in beamforming matrix brought by multi-antennas, many beamformers with closed form solutions can be found [22, 32, 37, 65]. These methods aim at different optimization goals and are highly computationally efficient.

In [22], a non-regenerative multiple-antenna relaying strategy is developed which maximizes the capacity between a multi-antenna source and the multi-antenna destination. The resulting beamforming matrix turns out to be full rank. Similar problem is also addressed in [65] with SNR maximization as design objective. Assuming second-order channel statistics, a general rank beamforming approach is proposed [37] for a single antenna S-D relay network, where the receive SNR is maximized under the total relay power requirement.

Linear transmit processing criteria for MIMO systems [31], like ZF, minimum mean square error (MMSE, or Wiener filter) and match filter (MF), can also be used in multi-antenna relay beamforming. In [32], all 3 criteria mentioned above are used in a non-regenerative multi-group multi-way (MGMW) relay network. It aims at maximizing the sum rate of the MGMW network. ZF, MMSE and MF are used to design generalized low-complexity transceive beamforming schemes for N-phase multi-way relaying. It is worth mentioning that all these schemes are sub-optimal, as can be seen from [32] that their optimization goals often do not fully align with the original

objectives.

Chapter 3

Single Pair Relay Beamforming Design for Multi-cluster Relay Interference Networks

In this chapter, we consider a relay beamforming design in a multi-cluster AF relay network with a single pair and M relays per cluster. We aim to maximize the minimum SINR at destinations subject to a total relay power budget within each cluster.

3.1 System Model

We consider a two-hop relay network with K S-D pairs, where each pair communicates through a set of M dedicated relays, as shown in Fig. 3.1. They form K clusters of nodes for half-duplex relaying. This scenario could arise in a cellular environment where each cell may contain such relaying clusters, or in an ad-hoc environment where multiple peer-to-peer S-D pairs communicate through relays forming multiple relaying clusters. Let S_i and D_i denote the source and destination nodes for S-D pair i , each equipped with a single antenna. We assume that the dedicated relay(s) for each pair can be either a multi-antenna relay or multiple relays capable of signal sharing to form virtual multi-antenna, and we generally consider them as a relay cluster. Let

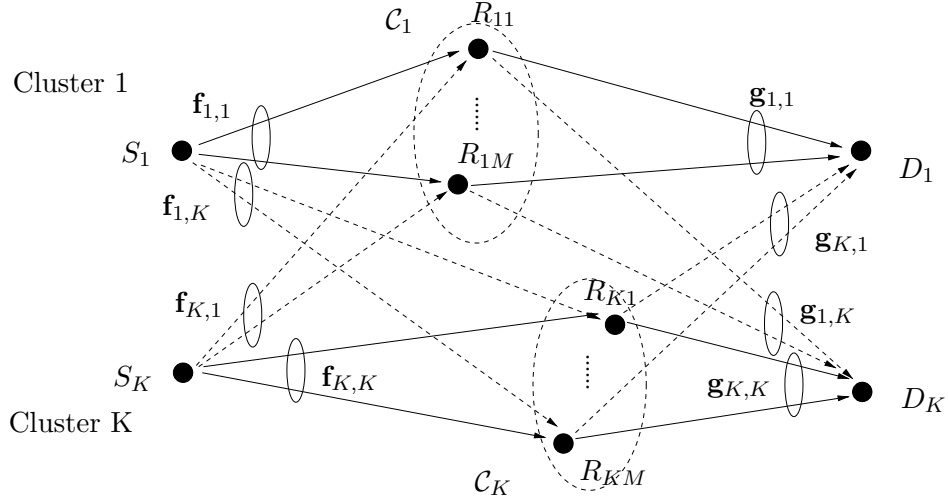


Figure 3.1: A relaying network with K clusters.

$\mathcal{C}_i = \{R_{i1}, \dots, R_{iM}\}$ denote M relay antennas in the relay cluster for S-D pair i , for $i = 1, \dots, K$. We assume $K \leq M$. Let $f_{i,jm}$ denote the complex channel coefficient between S_i and $R_{jm} \in \mathcal{C}_j$, and $g_{jm,i}$ the complex channel coefficient between $R_{jm} \in \mathcal{C}_j$ and D_i .

We assume that the AF relaying protocol is used, and the direct links between the source and the destination nodes are ignored. In the first phase, all sources transmit their signals to the relays. The received signal vector at relay cluster \mathcal{C}_i is given by

$$\mathbf{r}_i = \sum_{j=1}^K \sqrt{P_0} \mathbf{f}_{j,i} s_j + \mathbf{v}_i, \quad i = 1, \dots, K \quad (3.1)$$

where $\mathbf{f}_{j,i} \triangleq [f_{j,i1}, \dots, f_{j,iM}]^T$ is the channel vector between S_j and relay cluster \mathcal{C}_i , s_j is the transmitted signal from S_j with $E|s_j|^2 = 1$ and $E[s_i s_j] = 0, \forall i \neq j$, P_0 is the transmit power¹, and $\mathbf{v}_i \in \mathbb{C}^{M \times 1}$ is the additive white Gaussian noise (AWGN) vector at relay cluster \mathcal{C}_i with covariance matrix $\sigma_v^2 \mathbf{I}$.

¹For simplicity, we assume the same transmit power P_0 for all sources. Extension to different transmit power at different source is straightforward.

In the second phase, received signal vector \mathbf{r}_i is processed with a beam matrix $\mathbf{W}_i \in \mathbb{C}^{M \times M}$, and the received signal at D_i is given by

$$z_i = \sum_{j=1}^K \mathbf{g}_{j,i}^T \mathbf{W}_j \mathbf{r}_j + n_i$$

where $\mathbf{g}_{j,i} \triangleq [g_{j1,i}, \dots, g_{jM,i}]^T$ denotes the channel vector between relay cluster \mathcal{C}_j and destination D_i , and n_i denotes the AWGN noise at D_i with variance σ_n^2 . Note that only the signal from S_i to D_i via its own relay cluster \mathcal{C}_i is regarded as the intended signal for decoding. All other received signals are considered as interference. Due to the presence of multiple relay clusters, the received signal z_i can be decomposed into the following components

$$\begin{aligned} z_i = & \sqrt{P_0} \mathbf{g}_{i,i}^T \mathbf{W}_i \mathbf{f}_{i,i} s_i + \sqrt{P_0} \sum_{j=1, j \neq i}^K \mathbf{g}_{j,i}^T \mathbf{W}_j \mathbf{f}_{i,j} s_i \\ & + \sqrt{P_0} \sum_{j=1, j \neq i}^K \sum_{k=1}^K \mathbf{g}_{k,i}^T \mathbf{W}_k \mathbf{f}_{j,k} s_j + \sum_{j=1}^K \mathbf{g}_{j,i}^T \mathbf{W}_j \mathbf{v}_j + n_i \end{aligned} \quad (3.2)$$

where the first term is the intended signal from S_i ; the second term is the interference from other relay cluster \mathcal{C}_j , $j \neq i$, that is originated from S_i ; the third term is the interference from S_j , $j \neq i$, and the fourth term is the amplified noise forwarded by all relay clusters. Based on (3.2), the receive SINR at D_i is given by

$$\text{SINR}_i = \frac{P_0 |\mathbf{g}_{i,i}^T \mathbf{W}_i \mathbf{f}_{i,i}|^2}{I_i + \sigma_v^2 \sum_{j=1}^K \|\mathbf{g}_{j,i}^T \mathbf{W}_j\|_2^2 + \sigma_n^2}, \quad (3.3)$$

where I_i is the interference power at D_i , given by

$$\begin{aligned}
I_i &\triangleq P_0 \mathbb{E} \left\{ \sum_{j=1, j \neq i}^K \mathbf{g}_{j,i}^T \mathbf{W}_j \mathbf{f}_{i,j} s_i \left(\sum_{j'=1, j' \neq i}^K \mathbf{g}_{j',i}^T \mathbf{W}_{j'} \mathbf{f}_{i,j'} s_i \right)^* \right\} \\
&\quad + P_0 \mathbb{E} \left\{ \sum_{j=1, j \neq i}^K \sum_{k=1}^K \mathbf{g}_{k,i}^T \mathbf{W}_k \mathbf{f}_{j,k} s_j \left(\sum_{j'=1, j' \neq i}^K \sum_{k'=1}^K \mathbf{g}_{k',i}^T \mathbf{W}_{k'} \mathbf{f}_{j',k'} s_{j'} \right)^* \right\} \\
&= P_0 \sum_{j=1, j \neq i}^K \mathbf{g}_{j,i}^T \mathbf{W}_j \mathbf{f}_{i,j} \left(\sum_{j'=1, j' \neq i}^K \mathbf{g}_{j',i}^T \mathbf{W}_{j'} \mathbf{f}_{i,j'} \right)^* \mathbb{E}[s_i s_i^*] \\
&\quad + P_0 \sum_{j=1, j \neq i}^K \sum_{k=1}^K \mathbf{g}_{k,i}^T \mathbf{W}_k \mathbf{f}_{j,k} \left(\sum_{k'=1}^K \mathbf{g}_{k',i}^T \mathbf{W}_{k'} \mathbf{f}_{j,k'} \right)^* \mathbb{E}[s_j s_j^*]
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
&= P_0 \left(\left| \sum_{j=1, j \neq i}^K \mathbf{g}_{j,i}^T \mathbf{W}_j \mathbf{f}_{i,j} \right|^2 + \sum_{j=1, j \neq i}^K \left| \sum_{k=1}^K \mathbf{g}_{k,i}^T \mathbf{W}_k \mathbf{f}_{j,k} \right|^2 \right).
\end{aligned} \tag{3.5}$$

Let P_r denote the total relay power budget at each relay cluster. The outputs of \mathcal{C}_i should satisfy $\mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r$.

Our goal is to design $\{\mathbf{W}_i\}$ for the relay clusters to maximize the minimum SINR at the destinations, while satisfying relay power constraint P_r , given by

$$\begin{aligned}
&\max_{\{\mathbf{W}_i\}} \min_i \text{SINR}_i \\
&\text{s.t. } \mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r, \quad i = 1, \dots, K.
\end{aligned} \tag{3.6}$$

The above max-min SINR problem can be equivalently expressed as follows

$$\begin{aligned}
(\text{OP}_1) \quad &\max_{\{\mathbf{W}_i\}, \gamma} \gamma \\
&\text{s.t. } \mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r, \\
&\text{SINR}_i \geq \gamma, \quad i = 1, \dots, K.
\end{aligned} \tag{3.7}$$

OP₁ is non-convex due to the non-convex SINR constraints w.r.t. $\{\mathbf{W}_i\}$ in (3.7). Finding an optimal solution for OP₁ directly is difficult. The focus is to find an algorithm with good performance and also being computationally efficient. Typical approaches to solve this problem could be exploring some convex relaxation or

convexification techniques to obtain a suboptimal solution $\{\mathbf{W}_i\}$. However, these numerically obtained beam matrices are usually difficult to analyze or to provide any insight into the structures of the solution. Instead, in this chapter, we construct the beam matrix using a linear combination of two specific types of beam matrices, and study the optimal weight for each type of beam matrix. Not only can our solution be efficiently obtained with low complexity, but also it reveals how the weights on two types of beam matrices shift as the topology of the multiple relay clusters changes.

3.2 Low Complexity Multi-cluster Relay Beamforming Design

In this section, we consider constructing the relay beam matrix using a weighted sum of two specific types of beam matrices, and then study the optimal weight for each type of beam matrix.

3.2.1 Structured Beam Matrix

As we know, when there is only a single S-D pair with no interference, the beam matrix \mathbf{W}_i should be designed to maximize the beamforming gain for the source signal. When there are other relaying clusters causing inter-cluster interference to the intended signal, the beam matrix should be designated to reduce or cancel the interference. Based on this, instead of finding the beam matrix \mathbf{W}_i in OP_1 directly, we construct \mathbf{W}_i by a weighted sum of two types of beam matrices, *i.e.*, ZF beam matrix \mathbf{W}_i^{ZF} and MRC beam matrix $\mathbf{W}_i^{\text{MRC}}$, as follows

$$\mathbf{W}_i = \alpha_i \mathbf{W}_i^{\text{ZF}} + \beta_i \mathbf{W}_i^{\text{MRC}}, \quad i = 1, \dots, K, \quad (3.8)$$

where α_i and β_i are the weights for \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MRC}}$, respectively. Specifically, \mathbf{W}_i^{ZF} is designed based on ZF criterion to cancel interference from and to the other clusters, while $\mathbf{W}_i^{\text{MRC}}$ is designed to only maximize SNR for the signal forwarded within its own cluster without considering interference. Given the structure of \mathbf{W}_i in (3.8), OP_1 is reformulated into the following problem

$$\begin{aligned}
(\text{OP}_2) \quad & \max_{\{\alpha_i, \beta_i\}} \gamma \\
& \text{s.t.} \quad \mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r, \\
& \text{SINR}_i \geq \gamma, \\
& \mathbf{W}_i = \alpha_i \mathbf{W}_i^{\text{ZF}} + \beta_i \mathbf{W}_i^{\text{MRC}}, \quad i = 1, \dots, K.
\end{aligned}$$

Note that since both \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MRC}}$ can be determined using their respective design criteria, the remaining parameters to be optimized are only weights $\{\alpha_i, \beta_i\}$, $i = 1, \dots, K$.

3.2.2 ZF Beam Matrix Design

We now design ZF beam matrix \mathbf{W}_i^{ZF} at the relay cluster i to cancel interference from/to other clusters. The ZF beam matrix is given by the following structure

$$\mathbf{W}_i^{\text{ZF}} = \mathbf{w}_{\text{T},i}^{\text{ZF}} \mathbf{w}_{\text{R},i}^{\text{ZF}H},$$

where $\mathbf{w}_{\text{R},i}^{\text{ZF}}$ and $\mathbf{w}_{\text{T},i}^{\text{ZF}}$ are the $M \times 1$ receive ZF beam vector and transmit ZF beam vector, respectively. Since only s_i is the desired signal for cluster i , the receive ZF beam vector $\mathbf{w}_{\text{R},i}^{\text{ZF}}$ should be designed to cancel interference originated from other sources s_j , $j \neq i$. This is given by

$$\mathbf{w}_{\text{R},i}^{\text{ZF}H} \mathbf{r}_i = \sqrt{P_o} s_i \quad | \mathbf{v}_i = \mathbf{0}, \quad (3.9)$$

which means when $\mathbf{v}_i = \mathbf{0}$, received signal after the processing of $\mathbf{w}_{R,i}^{\text{ZF}H}$ should be exactly equal to $\sqrt{P_o}s_i$. Rewrite received signal vector \mathbf{r}_i at relay cluster \mathcal{C}_i in (3.1) as

$$\mathbf{r}_i = \sqrt{P_o}\mathbf{F}_i\mathbf{s} + \mathbf{v}_i$$

where $\mathbf{F}_i \triangleq [\mathbf{f}_{1,i}, \dots, \mathbf{f}_{K,i}] \in \mathbb{C}^{M \times K}$ is channel state matrix from all sources to \mathcal{C}_i , and $\mathbf{s} \triangleq [s_1, \dots, s_K]^T$. ZF constraint in (3.9) is equivalent to

$$\mathbf{F}_i^H \mathbf{w}_{R,i}^{\text{ZF}} = \mathbf{e}_i, \quad (3.10)$$

where \mathbf{e}_i is the unit vector with the i th entry being 1 and the elsewhere 0's. Denote the received signal after receive ZF processing by $y_i \triangleq \mathbf{w}_{R,i}^{\text{ZF}H} \mathbf{r}_i$. The receive ZF problem can be formulated as

$$\mathbf{w}_{R,i}^{\text{ZF}} = \arg \min_{\mathbf{w}} \mathbb{E}\{|y_i - \sqrt{P_o}s_i|^2\} \quad (3.11)$$

$$\text{s.t. } \mathbf{F}_i^H \mathbf{w} = \mathbf{e}_i \quad (3.12)$$

The objective function in (3.11) can be rewritten as

$$\begin{aligned} \mathbb{E}\{|y_i - \sqrt{P_o}s_i|^2\} &= \mathbb{E}\{|\mathbf{w}^H(\sqrt{P_o}\mathbf{F}_i\mathbf{s} + \mathbf{v}_i) - \sqrt{P_o}s_i|^2\} \\ &= \mathbb{E}\{|\sqrt{P_o}(\mathbf{w}^H\mathbf{F}_i - \mathbf{e}_i^T)\mathbf{s} + \mathbf{w}^H\mathbf{v}_i|^2\}. \end{aligned}$$

Substitute (3.12) into the above equation, we have

$$\begin{aligned} \mathbb{E}\{|y_i - \sqrt{P_o}s_i|^2\} &= \mathbb{E}\{|\mathbf{w}^H\mathbf{v}_i|^2\} \\ &= \sigma_v^2 \mathbf{w}^H \mathbf{w}, \end{aligned} \quad (3.13)$$

where $\mathbb{E}[\mathbf{v}_i\mathbf{v}_i^H] = \sigma_v^2\mathbf{I}$. The receive ZF problem has a quadratic and convex objective function (3.13), subject to a linear equality constraint (3.12). Thus it is convex and

can be solved by using the Lagrange multiplier method [66]. In this case, the Lagrange multiplier function for (3.11) can be formulated as

$$\begin{aligned}
L(\mathbf{w}, \lambda) &\triangleq \sigma_v^2 \mathbf{w}^H \mathbf{w} - \sum_{j=1}^K \lambda_j [\mathbf{F}_i^H \mathbf{w} - \mathbf{e}_i]_j \\
&= \sigma_v^2 \mathbf{w}^H \mathbf{w} - \sum_{j=1}^K [\boldsymbol{\lambda}]_j [\mathbf{F}_i^H \mathbf{w} - \mathbf{e}_i]_j \\
&= \sigma_v^2 \mathbf{w}^H \mathbf{w} - \sum_{j=1}^K \boldsymbol{\lambda}^T \mathbf{e}_j \mathbf{e}_j^T (\mathbf{F}_i^H \mathbf{w} - \mathbf{e}_i) \\
&= \sigma_v^2 \mathbf{w}^H \mathbf{w} - \sum_{j=1}^K \text{tr}(\mathbf{e}_j \mathbf{e}_j^T (\mathbf{F}_i^H \mathbf{w} - \mathbf{e}_i) \boldsymbol{\lambda}^T) \\
&= \sigma_v^2 \mathbf{w}^H \mathbf{w} - \text{tr}((\mathbf{F}_i^H \mathbf{w} - \mathbf{e}_i) \boldsymbol{\lambda}^T), \tag{3.14}
\end{aligned}$$

where λ_j is the Lagrangian multiplier, and $\boldsymbol{\lambda} \triangleq [\lambda_1, \dots, \lambda_K]^T \in \mathbb{R}^K$. Differentiate $L(\mathbf{w}, \lambda)$ w.r.t. \mathbf{w} and equating it to zero, we have

$$\frac{\partial L(\mathbf{w}, \lambda)}{\partial \mathbf{w}} = (\sigma_v^2 \mathbf{w}^H)^T - (\boldsymbol{\lambda}^T \mathbf{F}_i^H)^T = 0.$$

From above we obtain that

$$\mathbf{w}_{\text{R},i}^{\text{ZF}} = \mathbf{F}_i (\mathbf{F}_i^H \mathbf{F}_i)^{-1} \mathbf{e}_i. \tag{3.15}$$

Note that since $K \leq M$, and entries of \mathbf{F}_i are independent channel coefficients, $\mathbf{F}_i^H \mathbf{F}_i$ is invertible.

After receive ZF at relays, the transmit ZF beam vector $\mathbf{w}_{\text{T},i}^{\text{ZF}}$ is then applied. The received signals at all destinations from relay cluster \mathcal{C}_i is given by

$$\mathbf{z} = \mathbf{G}_i \mathbf{w}_{\text{T},i}^{\text{ZF}} y_i + \mathbf{n},$$

where $\mathbf{G}_i \triangleq [\mathbf{g}_{i,1}, \dots, \mathbf{g}_{i,K}]^T \in \mathbb{C}^{K \times M}$ is the channel state matrix containing channels from \mathcal{C}_i to all destinations, $\mathbf{z} \triangleq [z_1, \dots, z_K]^T$, and $\mathbf{n} \triangleq [n_1, \dots, n_K]^T$. The transmit

ZF beamforming design is to suppress transmit signals to all other clusters, while maximize the signal power intended to the destination D_i within its own cluster. The transmit ZF beamforming problem can be written as

$$\mathbf{w}_{T,i}^{\text{ZF}} = \arg \max_{\mathbf{w}} \mathbb{E}\{|\mathbf{g}_{i,i}^T \mathbf{w} y_i|^2\} \quad (3.16)$$

$$\text{s.t. } \mathbb{E}\{\|\mathbf{w} y_i\|^2\} \leq P_r \quad (3.17)$$

$$\mathbf{G}_{i-} \mathbf{w} y_i = \mathbf{0} \quad (3.18)$$

where \mathbf{G}_{i-} is the $(K-1) \times M$ matrix obtained from \mathbf{G}_i by removing the i th row $\mathbf{g}_{i,i}^T$. ZF constraint (3.18) implies that

$$\mathbf{G}_i \mathbf{w}_{T,i}^{\text{ZF}} = \kappa_{\text{ZF},i} \mathbf{e}_i, \quad (3.19)$$

for some scalar $\kappa_{\text{ZF},i}$. To solve the optimization problem, we first introduce the following Lemma.

Lemma 3.1. *The objective function (3.16) is maximized when $\kappa_{\text{ZF},i}$ is set such that constraint (3.17) is met with equality.*

Proof. Assume (3.17) is met with inequality when (3.16) is maximized with β , equivalently we have

$$\mathbb{E}\{\|\mathbf{w} y_i\|^2\} = \alpha P_r,$$

for $0 < \alpha < 1$ and some κ . It is straight forward to see that if we let $\kappa' = \kappa/\alpha$, we will have $\mathbf{w}' = \frac{1}{\alpha} \mathbf{w}$, and the objective function will become

$$\begin{aligned} \mathbb{E}\{|\mathbf{g}_{i,i}^T \mathbf{w}' y_i|^2\} &= \frac{1}{\alpha^2} \mathbb{E}\{|\mathbf{g}_{i,i}^T \mathbf{w} y_i|^2\} \\ &= \frac{\beta}{\alpha^2}, \end{aligned}$$

which is bigger than β , and in the mean time power constraint (3.17) is met with equality. ■

Also, the objective function (3.16) can be rewritten as

$$\begin{aligned}\mathbb{E}\{|\mathbf{g}_{i,i}^T \mathbf{w} y_i|^2\} &= \mathbb{E}\{|\mathbf{e}_i^T \mathbf{G}_i \mathbf{w} y_i|^2\} \\ &= \text{tr}(\mathbf{e}_i^T \mathbf{G}_i \mathbf{w} R_{y_i} \mathbf{w}^H \mathbf{G}_i^H \mathbf{e}_i),\end{aligned}\quad (3.20)$$

where $R_{y_i} \triangleq \mathbb{E}[y_i y_i^*]$, and (3.20) is convex w.r.t. \mathbf{w} . Given the linear equality constraint in (3.17) and (3.19), the transmit ZF problem is convex and thus can be solved using the Lagrangian multiplier method with it's Lagrangian written as

$$\begin{aligned}L(\mathbf{w}, \lambda) &\triangleq \text{tr}(\mathbf{e}_i^T \mathbf{G}_i \mathbf{w} R_{y_i} \mathbf{w}^H \mathbf{G}_i^H \mathbf{e}_i) - \sum_{j=1}^K \lambda_j [\mathbf{G}_i \mathbf{w} - \kappa \mathbf{e}_i]_j \\ &= \text{tr}(\mathbf{e}_i^T \mathbf{G}_i \mathbf{w} R_{y_i} \mathbf{w}^H \mathbf{G}_i^H \mathbf{e}_i) - \text{tr}((\mathbf{G}_i \mathbf{w} - \kappa \mathbf{e}_i) \boldsymbol{\lambda}^T),\end{aligned}$$

where λ_j is the Lagrangian multiplier, $\boldsymbol{\lambda} \triangleq [\lambda_1, \dots, \lambda_K]^T \in \mathbb{R}^K$. Differentiate $L(\mathbf{w}, \lambda)$ w.r.t. \mathbf{w} and equating it to zero, we have

$$\frac{\partial L(\mathbf{w}, \lambda)}{\partial \mathbf{w}} = R_{y_i} (\mathbf{w}^H \mathbf{G}_i^H \mathbf{e}_i \mathbf{e}_i^T \mathbf{G}_i)^T - (\boldsymbol{\lambda}^T \mathbf{G}_i)^T = \mathbf{0}$$

From above we can obtain that

$$\mathbf{w}_{\text{T},i}^{\text{ZF}} = \kappa_{\text{ZF},i} \mathbf{G}_i^H (\mathbf{G}_i \mathbf{G}_i^H)^{-1} \mathbf{e}_i. \quad (3.21)$$

with

$$\kappa_{\text{ZF},i}^2 = \frac{P_r}{[(\mathbf{G}_i \mathbf{G}_i^H)^{-1}]_{i,i} (P_0 + \sigma_v^2 \|\mathbf{w}_{\text{R},i}^{\text{ZF}}\|^2)}, \quad i = 1, \dots, K.$$

From $\mathbf{w}_{\text{R},i}^{\text{ZF}}$ in (3.15), we have

$$\kappa_{\text{ZF},i}^2 = \frac{P_r}{[(\mathbf{G}_i \mathbf{G}_i^H)^{-1}]_{i,i} (P_0 + \sigma_v^2 [(\mathbf{F}_i^H \mathbf{F}_i)^{-1}]_{i,i})}.$$

Thus, using (3.15) and (3.21), we have ZF beam matrix \mathbf{W}_i^{ZF} , for $i = 1, \dots, K$, as

$$\mathbf{W}_i^{\text{ZF}} = \kappa_{\text{ZF},i} \mathbf{G}_i^H (\mathbf{G}_i \mathbf{G}_i^H)^{-1} \mathbf{e}_i \mathbf{e}_i^T (\mathbf{F}_i^H \mathbf{F}_i)^{-1} \mathbf{F}_i^H.$$

3.2.3 MRC Beam Matrix Design

For designing MRC beam matrix $\mathbf{W}_i^{\text{MRC}}$, we are only concerned about the signal forwarded by its own relay cluster \mathcal{C}_i . The goal is to maximize received SNRs at the relays in \mathcal{C}_i and at destination D_i , respectively. Thus, we set the MRC beam matrix structure as

$$\mathbf{W}_i^{\text{MRC}} = \mathbf{w}_{\text{T},i}^{\text{MRC}} \mathbf{w}_{\text{R},i}^{\text{MRC}H},$$

where $\mathbf{w}_{\text{R},i}^{\text{MRC}}$ and $\mathbf{w}_{\text{T},i}^{\text{MRC}}$ are the $M \times 1$ receive beam vector and transmit beam vector, respectively. The signal received at relay cluster \mathcal{C}_i from its own intended source S_i , denoted by $\mathbf{r}_{i,i}$, is given by

$$\mathbf{r}_{i,i} = \sqrt{P_0} \mathbf{f}_{i,i} s_i + \mathbf{v}_i.$$

Receive beam vector $\mathbf{w}_{\text{R},i}^{\text{MRC}}$ is given by the following expression

$$\mathbf{w}_{\text{R},i}^{\text{MRC}} = \arg \max_{\mathbf{w}} \frac{P_0 \mathbb{E}\{|\mathbf{w}^H \mathbf{f}_{i,i} s_i|^2\}}{\mathbb{E}\{|\mathbf{w}^H \mathbf{v}_i|^2\}} \quad (3.22)$$

The objective function can be simplified as

$$\begin{aligned} \frac{P_0 \mathbb{E}\{|\mathbf{w}^H \mathbf{f}_{i,i} s_i|^2\}}{\mathbb{E}\{|\mathbf{w}^H \mathbf{v}_i|^2\}} &= \frac{P_0 \mathbf{w}^H \mathbf{f}_{i,i} \mathbb{E}[s_i s_i^*] \mathbf{f}_{i,i}^H \mathbf{w}}{\mathbf{w}^H \mathbb{E}[\mathbf{v}_i \mathbf{v}_i^H] \mathbf{w}} \\ &= \frac{\mathbf{w}^H \mathbf{f}_{i,i} \mathbf{f}_{i,i}^H \mathbf{w}}{\sigma_v^2 \mathbf{w}^H \mathbf{w}}. \end{aligned}$$

Since (3.22) is a maximization problem without constraint, it can be solved by taking derivation of the objective function w.r.t. \mathbf{w} and set it to zero, which has only one

root. It is straightforward to obtain $\mathbf{w}_{R,i}^{\text{MRC}}$ as

$$\mathbf{w}_{R,i}^{\text{MRC}} = \mathbf{f}_{i,i} / \|\mathbf{f}_{i,i}\|. \quad (3.23)$$

After applying $\mathbf{w}_{R,i}^{\text{MRC}}$ to the actual received signal vector \mathbf{r}_i , we have

$$\begin{aligned} y_i &= \mathbf{w}_{R,i}^{\text{MRC}H} \mathbf{r}_i \\ &= \sqrt{P_o} \|\mathbf{f}_{i,i}\| s_i + \sqrt{P_o} \sum_{j=1, j \neq i}^K \frac{\mathbf{f}_{i,i}^H \mathbf{f}_{j,i}}{\|\mathbf{f}_{i,i}\|} s_j + \frac{\mathbf{f}_{i,i}^H}{\|\mathbf{f}_{i,i}\|} \mathbf{v}_i. \end{aligned} \quad (3.24)$$

Applying transmit beam vector, the received signal at D_i with out considering interference from relays in other cluster is given by

$$z_{i,i} = \mathbf{g}_{i,i}^T \mathbf{w}_{T,i}^{\text{MRC}} y_i + n_i.$$

The received SINR for $z_{i,i}$ is then given by

$$\text{SINR}_{i,i} \triangleq \frac{P_o \|\mathbf{f}_{i,i}\|^2 |\mathbf{g}_{i,i}^T \mathbf{w}_{T,i}^{\text{MRC}}|^2}{\sigma_v^2 |\mathbf{g}_{i,i}^T \mathbf{w}_{T,i}^{\text{MRC}}|^2 + \sum_{j=1, j \neq i}^K P_o \|\mathbf{f}_{j,i}\|^2 |\mathbf{g}_{i,i}^T \mathbf{w}_{T,i}^{\text{MRC}}|^2 + \sigma_n^2}, \quad (3.25)$$

where we consider the 2nd term in (3.24) as additive noise. The transmit beam vector $\mathbf{w}_{T,i}^{\text{MRC}}$ is designed to maximize $\text{SNR}_{i,i}$ under the relay power constraint. The problem is formulated as

$$\begin{aligned} \max_{\mathbf{w}_{T,i}^{\text{MRC}}} \quad & \text{SINR}_{i,i} \\ \text{s.t.} \quad & \mathbb{E}\{\|\mathbf{w}_{T,i}^{\text{MRC}} y_i\|^2\} \leq P_r. \end{aligned} \quad (3.26)$$

Maximizing (3.25) equals to maximizing $|\mathbf{g}_{i,i}^T \mathbf{w}_{T,i}^{\text{MRC}}|^2$, as the objective function is monotonically increasing w.r.t $|\mathbf{g}_{i,i}^T \mathbf{w}_{T,i}^{\text{MRC}}|$. Also, following similar steps in Lemma 3.1, it's straight forward to prove that (3.26) is met with equality when $\text{SNR}_{i,i}$ is maximized.

Thus the transmit MRC problem can be reformulated as

$$\begin{aligned} \mathbf{w}_{T,i}^{\text{MRC}} &= \arg \max_{\mathbf{w}} |\mathbf{g}_{i,i}^T \mathbf{w}|^2 \\ \text{s.t. } & \mathbb{E}\{\|\mathbf{w}y_i\|^2\} = P_r, \end{aligned}$$

which is convex and thus can be solved with the Lagrangian multiplier method. It's Lagrangian can then be written as

$$L(\mathbf{w}, \lambda) \triangleq \text{tr}(\mathbf{g}_{i,i}^T \mathbf{w} \mathbf{w}^H \mathbf{g}_{i,i}^*) - \lambda(\text{tr}(\mathbf{w} R_{y_i} \mathbf{w}^H) - P_r),$$

where $R_{y_i} \triangleq \mathbb{E}[y_i y_i^*]$ and λ is the Lagrangian multiplier. Taking derivative w.r.t. \mathbf{w} and setting it to zero we have

$$\frac{\partial L(\mathbf{w}, \lambda)}{\partial \mathbf{w}} = (\mathbf{w}^H \mathbf{g}_{i,i}^* \mathbf{g}_{i,i}^T)^T - \lambda(\mathbf{w}^H R_{y_i})^T = \mathbf{0}.$$

From above we can obtain that

$$\mathbf{w}_{T,i}^{\text{MRC}} = \kappa_{\text{MRC},i} \mathbf{g}_{i,i}^* / \|\mathbf{g}_{i,i}\|, \quad (3.27)$$

where

$$\kappa_{\text{MRC},i}^2 = \frac{P_r \|\mathbf{f}_{i,i}\|^2}{\mathbf{f}_{i,i}^H (P_0 \mathbf{F}_i \mathbf{F}_i^H + \sigma_v^2 \mathbf{I}) \mathbf{f}_{i,i}}.$$

Finally, using (3.23) and (3.27), we have MRC beam matrix $\mathbf{W}_i^{\text{MRC}}$, for $i = 1, \dots, K$, as

$$\mathbf{W}_i^{\text{MRC}} = \kappa_{\text{MRC},i} \frac{\mathbf{g}_{i,i}^*}{\|\mathbf{g}_{i,i}\|} \frac{\mathbf{f}_{i,i}^H}{\|\mathbf{f}_{i,i}\|}, \quad i = 1, \dots, K. \quad (3.28)$$

3.2.4 Optimization of $\{\alpha_i, \beta_i\}$ via SDR Approach

With the optimal solution of the two beam matrices \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MRC}}$, we now focus on solving OP₂ to obtain the optimal weights α_i, β_i , to determine \mathbf{W}_i , for $i = 1, \dots, K$.

We begin with vectorizing beam matrix \mathbf{W}_i for OP₂. Define $\mathbf{w}_i \triangleq \text{vec}(\mathbf{W}_i^H)$. From (3.8), we have

$$\mathbf{w}_i = \alpha_i^* \mathbf{w}_i^{\text{ZF}} + \beta_i^* \mathbf{w}_i^{\text{MRC}}, \quad i = 1, \dots, K, \quad (3.29)$$

where $\mathbf{w}_i^{\text{ZF}} \triangleq \text{vec}(\mathbf{W}_i^{\text{ZF}H})$, $\mathbf{w}_i^{\text{MRC}} \triangleq \text{vec}(\mathbf{W}_i^{\text{MRC}H})$. We further rewrite the SINR expression in (3.3) w.r.t. the vector form \mathbf{w}_i using the property $\text{vec}(\mathbf{ABC}) = (\mathbf{A} \otimes \mathbf{C}^T) \text{vec}(\mathbf{B}^T)$, for some matrices \mathbf{A}, \mathbf{B} and \mathbf{C} . A single entry from the first term of interference power I_i in (3.4) can then be written as

$$\begin{aligned} \mathbf{g}_{k,i}^T \mathbf{W}_k \mathbf{f}_{j,k} &= (\mathbf{g}_{k,i}^T \otimes \mathbf{f}_{j,k}^T) \text{vec}(\mathbf{W}_k^T) \\ &= ((\mathbf{g}_{k,i} \otimes \mathbf{f}_{j,k})^T) \text{vec}(\mathbf{W}_k^T) \\ &= ((\mathbf{h}_{j,i}^{(k)H} \mathbf{w}_k))^*, \end{aligned}$$

where $\mathbf{h}_{i,k}^{(j)} \triangleq \mathbf{g}_{j,k} \otimes \mathbf{f}_{i,j} \in \mathbb{C}^{M^2 \times 1}$ is defined as the compound channel from S_i to D_k through relay cluster \mathcal{C}_j . In this way, the second term of I_i in (3.4) can be rewritten as

$$\begin{aligned} I_{i,S_j} &\triangleq \sum_{j=1, j \neq i}^K \left| \sum_{k=1}^K \mathbf{g}_{k,i}^T \mathbf{W}_k \mathbf{f}_{j,k} \right|^2 \\ &= P_0 \sum_{j=1, j \neq i}^K \sum_{k=1}^K \mathbf{g}_{k,i}^T \mathbf{W}_k \mathbf{f}_{j,k} \sum_{k'=1}^K (\mathbf{g}_{k',i}^T \mathbf{W}_{k'} \mathbf{f}_{j,k'})^* \\ &= P_0 \sum_{j=1, j \neq i}^K \sum_{k=1}^K (\mathbf{h}_{j,i}^{kH} \mathbf{w}_k)^* \sum_{k'=1}^K (\mathbf{h}_{j,i}^{k'H} \mathbf{w}_{k'}), \end{aligned}$$

where I_{i,S_j} is the power of interference originates from $S_j, j \neq i$. Define $\mathbf{h}_{j,i} \triangleq [\mathbf{h}_{j,i}^{1H}, \dots, \mathbf{h}_{j,i}^{KH}]^H \in \mathbb{C}^{KM^2 \times 1}, j \neq i$ as the channel vector from S_j to D_i through all relay clusters. Also define $\mathbf{w} \triangleq [\mathbf{w}_1^H, \dots, \mathbf{w}_K^H]^H \in \mathbb{C}^{KM^2 \times 1}$ as the vectorized beam-

former of all clusters, we have

$$I_{i,S_j} = P_0 \sum_{j=1, j \neq i}^K |\mathbf{h}_{j,i}^H \mathbf{w}|^2 = \mathbf{w}^H \left(\sum_{j=1, j \neq i}^K P_0 \mathbf{h}_{j,i} \mathbf{h}_{j,i}^H \right) \mathbf{w}. \quad (3.30)$$

Similarly, for the first term of I_i in (3.4), let I_{i,S_i} be the power of interference originates from S_i itself, we have

$$\begin{aligned} I_{i,S_i} &= P_0 \left| \sum_{j=1, j \neq i}^K \mathbf{g}_{j,i}^T \mathbf{W}_j \mathbf{f}_{i,j} \right|^2 \\ &= P_0 \left| \sum_{j=1, j \neq i}^K (\mathbf{h}_{i,i}^j)^H \mathbf{w}_j \right|^2 \\ &= \mathbf{w}^H P_0 \mathbf{h}_{i,i} \mathbf{h}_{i,i}^H \mathbf{w}, \end{aligned} \quad (3.31)$$

where $\mathbf{h}_{i,i} = [\mathbf{h}_{i,i}^1, \dots, \mathbf{h}_{i,i}^{i-1}, \mathbf{0}, \mathbf{h}_{i,i}^{i+1}, \dots, \mathbf{h}_{i,i}^K]^H \in \mathbb{C}^{KM^2 \times 1}$ denotes the channel vector from S_i to D_i via all the relay clusters $\mathcal{C}_j, j \neq i$. Thus I_i in (3.4) can be rewritten as

$$\mathbf{I}_i = \mathbf{I}_{i,S_i} + \mathbf{I}_{i,S_j} = \mathbf{w}^H \left(\sum_{j=1}^K P_0 \mathbf{h}_{j,i} \mathbf{h}_{j,i}^H \right) \mathbf{w}. \quad (3.32)$$

Also the desired signal part in (3.3) can be rewritten as

$$P_0 |\mathbf{g}_{i,i}^T \mathbf{W}_i \mathbf{f}_{i,i}|^2 = P_0 |\mathbf{h}_{i,i}^i)^H \mathbf{w}_i|^2 = \mathbf{w}^H \mathbf{D}(\mathbf{A}_i) \mathbf{w}, \quad (3.33)$$

where $\mathbf{A}_i \triangleq P_0 \mathbf{h}_{i,i}^{(i)} \mathbf{h}_{i,i}^{(i)H}$, and $\mathbf{D}(\mathbf{A}_i)$ to be block diagonal matrix with the i th diagonal block being \mathbf{A}_i and the rest $\mathbf{0}$'s. For the amplified noise term in SNR expression (3.3), note that

$$\begin{aligned} \text{vec}(\mathbf{g}_{j,i}^T \mathbf{W}_j) &= (\mathbf{g}_{j,i} \otimes \mathbf{I})^T \text{vec}(\mathbf{W}_j)^T \\ &= ((\mathbf{g}_{j,i} \otimes \mathbf{I})^H \mathbf{w}_j)^*. \end{aligned}$$

Using this, the amplified noise term (3.3) can be rewritten as

$$\begin{aligned}
\sigma_v^2 \sum_{j=1}^K \|\mathbf{g}_{j,i}^T \mathbf{W}_j\|^2 &= \sum_{j=1}^K \mathbf{w}_j^H (\mathbf{g}_{j,i} \mathbf{g}_{j,i}^H \otimes \mathbf{I} \sigma_v^2) \mathbf{w}_j \\
&= \sum_{j=1}^K \mathbf{w}_j^H \mathbf{B}_{j,i} \mathbf{w}_j \\
&= \mathbf{w}^H \mathbf{B}_i \mathbf{w},
\end{aligned} \tag{3.34}$$

where the property $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$ is used to obtain the right hand side of the first equation. Also we define $\mathbf{B}_{j,i} \triangleq (\mathbf{g}_{j,i} \mathbf{g}_{j,i}^H) \otimes \mathbf{I} \sigma_v^2$ as the amplified noise covariance matrix from \mathcal{C}_j to D_i and $\mathbf{B}_i \triangleq \text{bldg}(\mathbf{B}_{1,i}, \dots, \mathbf{B}_{K,i}) \in \mathbb{C}^{KM^2 \times KM^2}$ as the $K \times K$ block diagonal amplified noise covariance matrix from all $\mathcal{C}_j, j = 1, \dots, K$ to D_i . In this way, we now can rewrite SINR in (3.3) as

$$\begin{aligned}
\text{SINR}_i &= \frac{\mathbf{w}^H \mathbf{D}(\mathbf{A}_i) \mathbf{w}}{\mathbf{w}^H (\sum_{j=1}^K P_0 \mathbf{h}_{j,i} \mathbf{h}_{j,i}^H) \mathbf{w} + \mathbf{w}^H \mathbf{B}_i \mathbf{w} + \sigma_n^2} \\
&= \frac{\mathbf{w}^H \mathbf{D}(\mathbf{A}_i) \mathbf{w}}{\mathbf{w}^H \mathbf{C}_i \mathbf{w} + \sigma_n^2},
\end{aligned} \tag{3.35}$$

where $\mathbf{C}_i \triangleq \sum_{j=1}^K P_0 \mathbf{h}_{j,i} \mathbf{h}_{j,i}^H + \mathbf{B}_i$. For power constraint (3.6) we have

$$\begin{aligned}
\text{vec}(\mathbf{W}_i \mathbf{r}_i) &= \text{vec}(\mathbf{I} \mathbf{W}_i \mathbf{r}_i) \\
&= (\mathbf{I} \otimes \mathbf{r}_i^T) \text{vec}(\mathbf{W}_i^T).
\end{aligned}$$

Thus, power constraint (3.6) can be rewritten as

$$\begin{aligned}
\mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} &= \mathbf{w}_i^H \mathbb{E}\{(\mathbf{I} \otimes \mathbf{r}_i)(\mathbf{I}^H \otimes \mathbf{r}_i^H)\} \mathbf{w}_i \\
&= \mathbf{w}_i^H (\mathbf{I} \otimes \mathbb{E}[\mathbf{r}_i \mathbf{r}_i^H]) \mathbf{w}_i \\
&= \mathbf{w}_i^H \mathbf{E}_i \mathbf{w}_i
\end{aligned} \tag{3.36}$$

$$\leq P_r, \tag{3.37}$$

where $\mathbf{E}_i \triangleq \mathbf{I} \otimes \mathbb{E}[\mathbf{r}_i \mathbf{r}_i^H] = P_o \mathbf{F}_i \mathbf{F}_i^H + \sigma_v^2 \mathbf{I}$.

Define $\overline{\mathbf{W}}_i \triangleq [\mathbf{w}_i^{\text{ZF}}, \mathbf{w}_i^{\text{MRC}}]$, and $\mathbf{a}_i \triangleq [\alpha_i, \beta_i]^H$, for $i = 1, \dots, K$. From (3.29), it follows that $\mathbf{w}_i = \overline{\mathbf{W}}_i \mathbf{a}_i$. Similarly, define $\overline{\mathbf{W}} \triangleq \text{bldg}(\overline{\mathbf{W}}_1, \dots, \overline{\mathbf{W}}_K)$ and $\mathbf{x} \triangleq [\mathbf{a}_1^H, \dots, \mathbf{a}_K^H]^H$, we have

$$\begin{aligned} \mathbf{w} &= \begin{bmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_K \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{W}}_1 \mathbf{a}_1 \\ \vdots \\ \overline{\mathbf{W}}_K \mathbf{a}_K \end{bmatrix} = \text{bldg}(\overline{\mathbf{W}}_1, \dots, \overline{\mathbf{W}}_K) \begin{bmatrix} \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_K \end{bmatrix} \\ &= \overline{\mathbf{W}} \mathbf{x}. \end{aligned}$$

SINR expression in (3.35) can now be rewritten as a function of \mathbf{x} as

$$\text{SINR}_i = \frac{\mathbf{x}^H \mathbf{D}(\tilde{\mathbf{A}}_i) \mathbf{x}}{\mathbf{x}^H \tilde{\mathbf{C}}_i \mathbf{x} + \sigma_n^2} \quad (3.38)$$

where $\mathbf{D}(\tilde{\mathbf{A}}_i) \triangleq \overline{\mathbf{W}}^H \mathbf{D}(\mathbf{A}_i) \overline{\mathbf{W}}$ and $\tilde{\mathbf{C}}_i \triangleq \overline{\mathbf{W}}^H \mathbf{C}_i \overline{\mathbf{W}}$. Also, the right hand side of (3.36) can be rewritten as

$$\mathbf{w}_i^H \mathbf{E}_i \mathbf{w}_i = \mathbf{a}_i^H \tilde{\mathbf{E}}_i \mathbf{a}_i = \mathbf{x}^H \mathbf{D}(\tilde{\mathbf{E}}_i) \mathbf{x} \quad (3.39)$$

where $\tilde{\mathbf{E}}_i \triangleq \overline{\mathbf{W}}_i^H \mathbf{E}_i \overline{\mathbf{W}}_i$, and $\mathbf{D}(\tilde{\mathbf{E}}_i)$ is defined in the same way as $\mathbf{D}(\mathbf{A}_i)$. With (3.38) and (3.39), we now reformulate OP₂ into the following equivalent problem

$$\begin{aligned} (\text{OP}_3) \quad & \max_{\mathbf{x}, \gamma} \quad \gamma \\ & \text{s.t.} \quad \mathbf{x}^H \left(\frac{1}{\gamma} \mathbf{D}(\tilde{\mathbf{A}}_i) - \tilde{\mathbf{C}}_i \right) \mathbf{x} \geq \sigma_n^2, \quad i = 1, \dots, K \\ & \quad \mathbf{x}^H \mathbf{D}(\tilde{\mathbf{E}}_i) \mathbf{x} \leq P_r, \quad i = 1, \dots, K. \end{aligned}$$

Define $\mathbf{X} \triangleq \mathbf{x} \mathbf{x}^H$, and remove the rank-one constraint on \mathbf{X} , we relax OP₃ into the

Algorithm 1 Gaussian randomization procedure for OP₃

Let L be an integer.

- 1: For $l = 1, \dots, L$, generate random vector $\boldsymbol{\xi}_l \in \mathbb{C}^{2K}$ from complex Gaussian distribution $\mathcal{CN}(\mathbf{0}, \mathbf{X}_{\text{opt}})$, and let

$$\mathbf{x}_l = \boldsymbol{\xi}_l / \max_{1 < i < K} \sqrt{\frac{1}{P_r} \boldsymbol{\xi}_l^H \mathbf{D}(\tilde{\mathbf{E}}_i) \boldsymbol{\xi}_l}.$$

- 2: Let

$$\mathbf{x} = \arg \max_{\mathbf{x}_l, l=1, \dots, L} \min_{1 < i < K} \frac{\mathbf{x}^H \mathbf{D}(\tilde{\mathbf{A}}_i) \mathbf{x}}{\mathbf{x}^H \tilde{\mathbf{C}}_i \mathbf{x} + \sigma_n^2},$$

be the approximate solution for OP₃

following problem

$$\begin{aligned} (\text{OP}_4) \quad & \max_{\mathbf{X}, \gamma} \quad \gamma \\ & \text{s.t.} \quad \text{tr} \left[\left(\frac{1}{\gamma} \mathbf{D}(\tilde{\mathbf{A}}_i) - \tilde{\mathbf{C}}_i \right) \mathbf{X} \right] \geq \sigma_n^2, \quad i = 1, \dots, K \\ & \quad \text{tr}[\mathbf{D}(\tilde{\mathbf{E}}_i) \mathbf{X}] \leq P_r, \quad i = 1, \dots, K \\ & \quad \mathbf{X} \succcurlyeq 0. \end{aligned}$$

Note that OP₄ is not jointly convex w.r.t. \mathbf{X} and γ . For a fixed γ , OP₄ is an SDP feasibility problem. Thus, we can solve OP₄ efficiently using bisection search over γ along with an SDP feasibility problem. The optimal solution \mathbf{X}_{opt} may not be rank one in general. If \mathbf{X}_{opt} is rank one, then \mathbf{x} can be extracted directly from it. Otherwise, we use randomization methods [63] to generate \mathbf{x} and finally obtain $\{\alpha_i, \beta_i\}$. The procedure is shown in Algorithm 1. Note that OP₄ always has $2K \times 2K$ variables and $2K$ constraints. The problem size does not grow with M .

3.3 Optimization through Direct SDR Approach

To compare with our proposed solution, we consider obtaining $\{\mathbf{W}_i\}$ by solving OP_1 using the SDR approach directly. Based on SINR expression in (3.35), we can rewrite OP_1 w.r.t. \mathbf{w} into the following problem

$$\begin{aligned} \max_{\mathbf{w}, \gamma} \quad & \gamma \\ \text{s.t.} \quad & \mathbf{w}^H \left(\frac{1}{\gamma} \mathbf{D}(\mathbf{A}_i) - \mathbf{C}_i \right) \mathbf{w} \geq \sigma_n^2, \quad i = 1, \dots, K \\ & \mathbf{w}^H \mathbf{D}(\mathbf{E}_i) \mathbf{w} \leq P_r, \quad i = 1, \dots, K. \end{aligned}$$

The above optimization problem is not jointly convex w.r.t. \mathbf{w} and γ . Define $\mathbf{Y} \triangleq \mathbf{w}\mathbf{w}^H$, and remove the rank-one constraint on \mathbf{Y} , we can relax it into the following problem

$$\begin{aligned} (\text{OP}_5) \quad & \max_{\mathbf{Y}, \gamma} \quad \gamma \\ \text{s.t.} \quad & \text{tr} \left[\left(\frac{1}{\gamma} \mathbf{D}(\mathbf{A}_i) - \mathbf{C}_i \right) \mathbf{Y} \right] \geq \sigma_n^2, \quad i = 1, \dots, K \\ & \text{tr}[\mathbf{D}(\mathbf{E}_i) \mathbf{Y}] \leq P_r, \quad i = 1, \dots, K; \\ & \mathbf{Y} \succeq 0 \end{aligned}$$

where $\mathbf{D}(\mathbf{E}_i)$ is defined in the same way as $\mathbf{D}(\mathbf{A}_i)$. Again, the optimal solution \mathbf{Y}_{opt} may not be rank one in general, and randomization methods [63] are used to generate \mathbf{y} and to obtain $\{\mathbf{W}_i\}$.

3.4 Complexity Analysis and Comparison

Comparing our proposed solution and the direct SDR approach, we see that both involve solving an SDP problem. The SDP can be solved efficiently using interior-point

methods with standard SDP solvers. However, the difference in problem size results in a significant difference in computational complexity. Based on the complexity analysis of the standard SDP form [64], for the direct SDR approach, the SDP problem OP_5 has $(KM^2)^2$ variables and $2K$ constraints. The complexity to solve the SDP is about $\mathcal{O}(K^5M^8)$. For our proposed solution, OP_4 has the variable size of $(2K)^2$ and $2K$ constraints, with complexity being $\mathcal{O}(K^5)$, which only depends on K and independent of M .² Thus, it is clear that our proposed solution is computationally efficient with significantly lower complexity than the direct SDP approach.

3.5 Simulation Results

We consider a relay network consists of two clusters ($K = 2$) with 4 relays ($M = 4$) per cluster, as shown in Fig.3.2. The distance between S_i and D_i is set to $d_{\text{SD}} = 1$. We assume the two S-D pairs are parallel to each other, and relays are located in the middle point of each S-D pair ($d_{\text{SR}} = d_{\text{RD}} = 0.5d_{\text{SD}}$). Let d_0 denote the distance between the two clusters, measured between the center of two relay clusters. Channel vectors $\mathbf{f}_{i,j}$ and $\mathbf{g}_{j,i}$ are assumed to be i.i.d. Gaussian. We use the nominal SNR from source to relay to indicate the average channel strength over this link, defined as $\overline{\text{SNR}}_{\text{SR}} = P_0K_0(d_{\text{SR}})^{-3.5}/\sigma_n^2$, with K_0 being the pathloss constant. We set $P_0/\sigma_n^2 = 1$. Since $d_{\text{SR}} = d_{\text{RD}}$, we have $\overline{\text{SNR}}_{\text{SR}} = \overline{\text{SNR}}_{\text{RD}}$. We set $\sigma_v^2 = \sigma_n^2$.

²The computation of \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MRC}}$ using the closed-forms incurs negligible computational complexity as compared with the SDP complexity

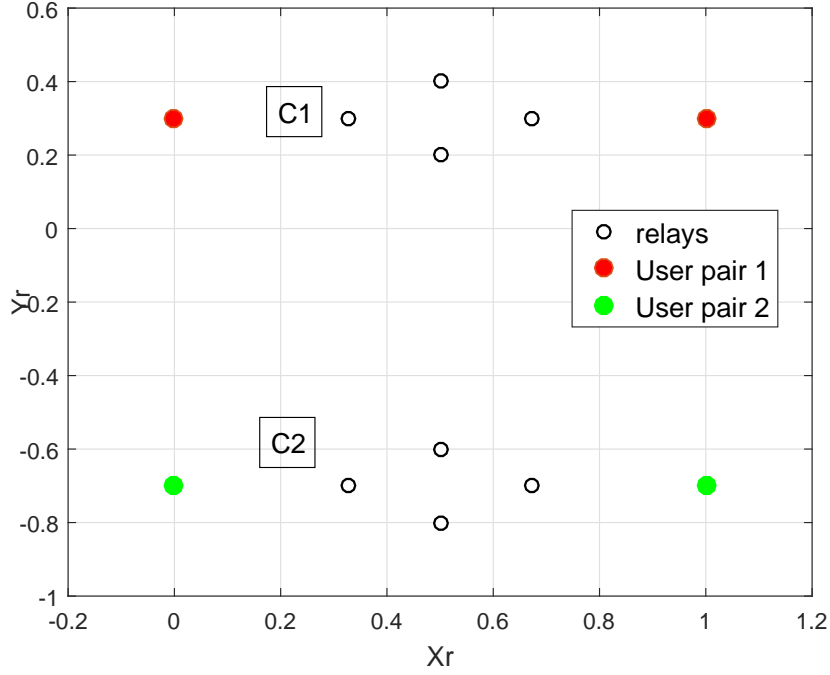


Figure 3.2: Relay layout, $K = 2$, $M = 4$, $d_0 = 1.0$.

3.5.1 SINR Performance

First, we compare the minimum SINR performance of our proposed solution with that of the direct SDR approach in OP_5 . From Figs. 3.3 to 3.5, we plot the minimum SINR vs. \overline{SNR}_{SR} , for $d_0 = 1.0, 1.6$ and 2.2 , respectively. We set $M = 4$, and $P_r = MP_0$. Besides the two aforementioned methods, we also plot the optimal objective values of the SDP problems OP_4 and OP_5 , which serve as performance upper bounds for our proposed approach and direct SDR approach, respectively. As can be seen, our proposed solution with structured beam matrix \mathbf{W}_i provides a very close performance as compared with the direct SDR approach in all values of d_0 . Also, the gap between 2 methods becomes smaller as d_0 becomes bigger. A larger SINR gap is observed at higher \overline{SNR}_{SR} which is less than 1dB for $d_0 = 1.0$ when the two clusters are relatively

close, while the gap is negligible for $d_0 = 2.2$ when the two clusters are further apart. Furthermore, both solutions have very small performance loss as compared with their upper bounds.

Next, we compare the SINR performance of multi-antenna relays against separate relays with the same number of total antennas. A 4-antenna relay is then put in the middle of each cluster in Fig. 3.2 instead of the original relays, where each channel is identically distributed with the same pathloss. Fig. 3.6 to 3.8 show the minimum SINR vs. $\overline{\text{SNR}}_{\text{SR}}$ with the multi-antenna relay, for $M = 4$, $P_r = MP_0$, and for $d_0 = 1.0, 1.6$ and 2.2 , respectively. The performance of separate relays under the same setting is also plotted here as a comparison. As can be seen from Fig. 3.6 to 3.8, there is about 1 dB gain of using separate relays over centralized multi-antenna relay with the same number of antenna in each plot. The results suggest that heterogeneous relays (each with different channel variance) help improve the relay beamforming performance in a relay interference network.

3.5.2 Power Allocation

Next, based on our structured beam matrix \mathbf{W}_i , we study how the relay power is allocated to the two types of beam matrices \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MRC}}$, and how the inter-cluster interference affects the power allocation. Define $P_i \triangleq \mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\}$ as the actual power consumption of each relay cluster, and note that $P_i \leq P_r$. The power used for each type of beam matrix relative to P_i can be expressed as $\rho_i^{\text{ZF}} \triangleq \frac{\mathbb{E}\|\alpha_i \mathbf{W}_i^{\text{ZF}} \mathbf{r}_i\|_2^2}{P_i}$ and $\rho_i^{\text{MRC}} \triangleq \frac{\mathbb{E}\|\beta_i \mathbf{W}_i^{\text{MRC}} \mathbf{r}_i\|_2^2}{P_i}$. It indicates the portion of relay power allocated to the specific type of beam matrix. Figs. 3.9 and 3.10 show relay power allocation $\{\rho_i^{\text{ZF}}, \rho_i^{\text{MRC}}\}$ vs.

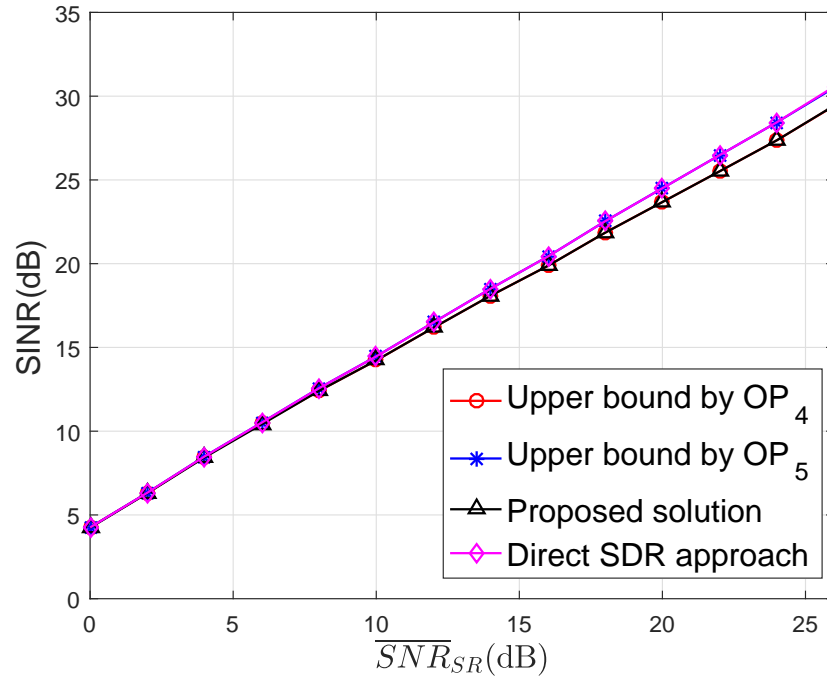


Figure 3.3: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 1.0, Pr = MP_0$).

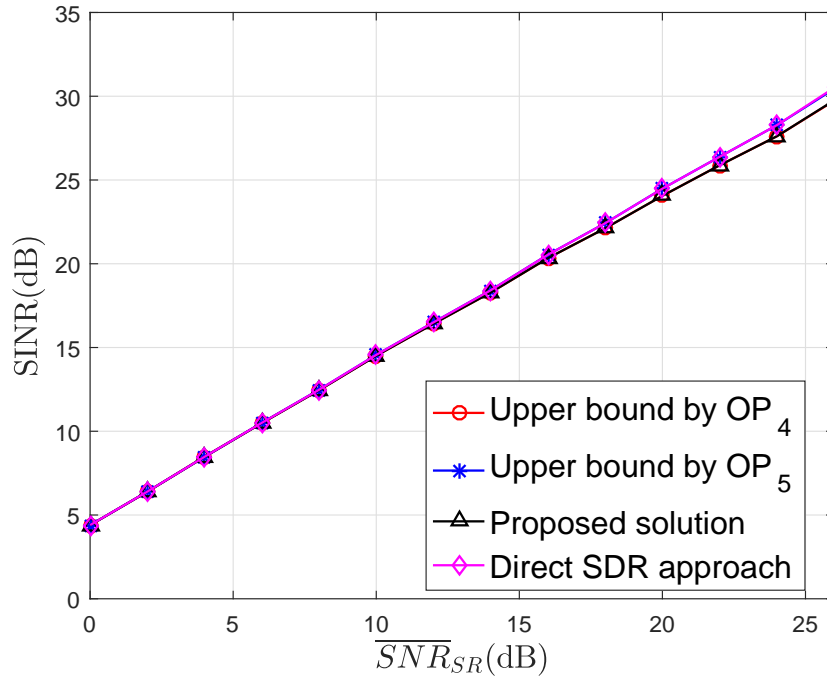


Figure 3.4: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 1.6, Pr = MP_0$).

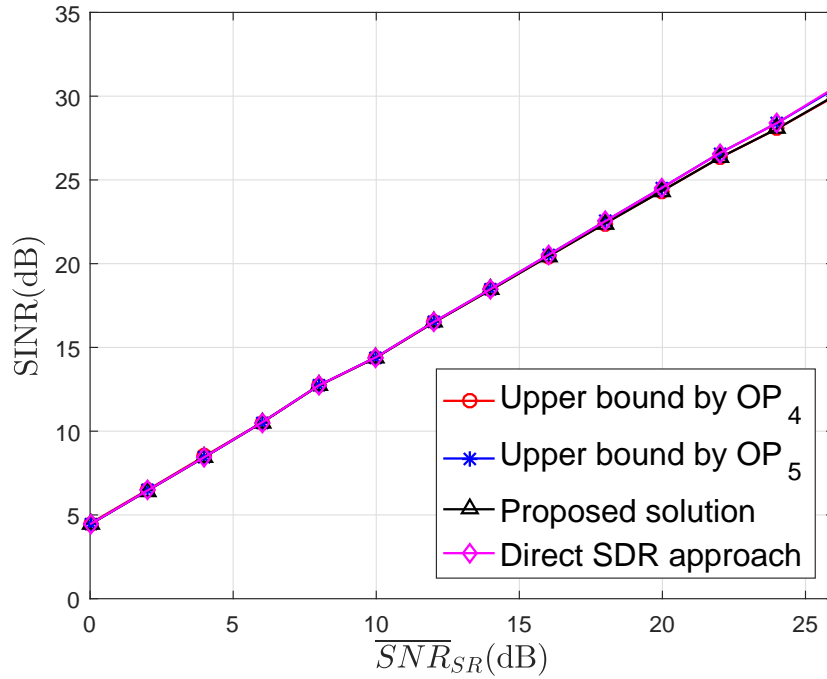


Figure 3.5: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 2.2, Pr = MP_0$).

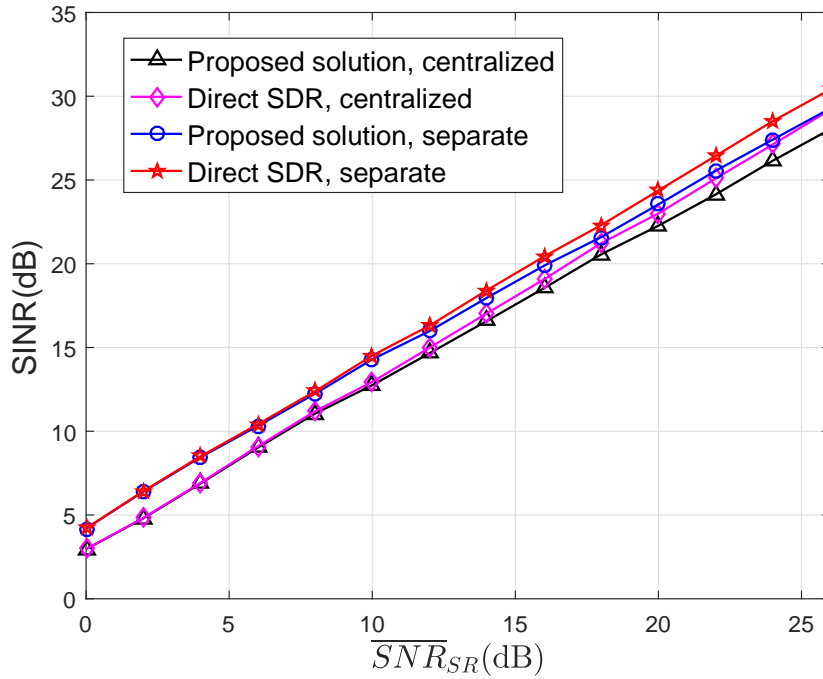


Figure 3.6: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 1.0, Pr = MP_0$).

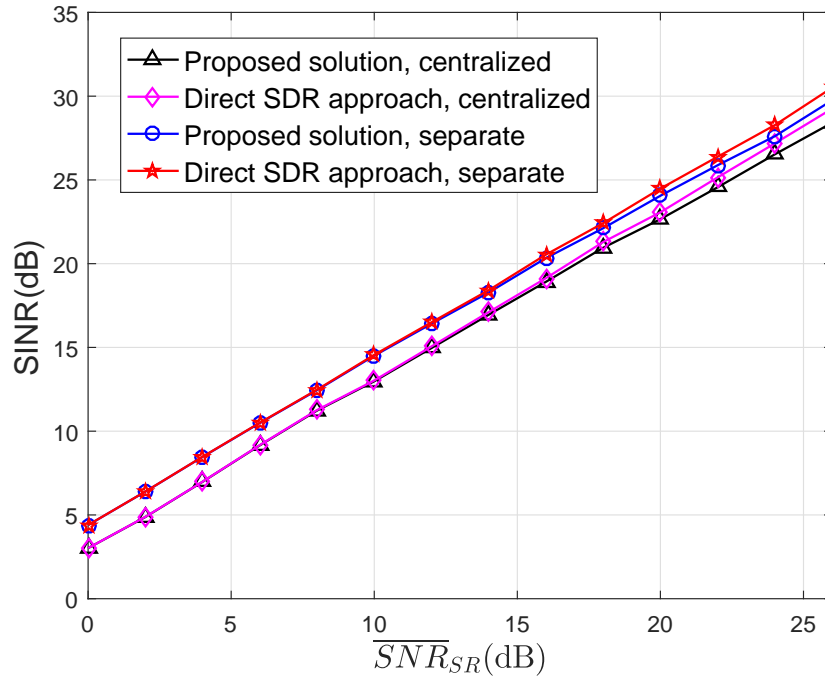


Figure 3.7: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 1.6, Pr = MP_0$).

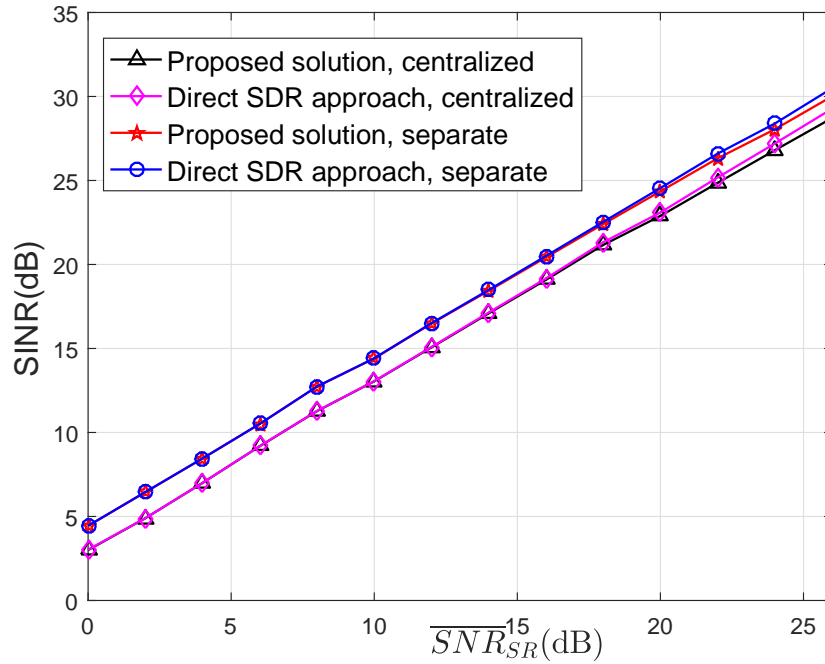


Figure 3.8: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 2.2, Pr = MP_0$).

$\overline{\text{SNR}}_{\text{SR}}$ at different distance $d_0 = 1.0, 1.6, 2.2$, for relay clusters 1 and 2, respectively. We see that as $\overline{\text{SNR}}_{\text{SR}}$ becomes higher, ρ_i^{MRC} decreases, and a larger portion of P_i is allocated to \mathbf{W}_i^{ZF} , $i = 1, 2$. This is because the interference channel becomes stronger as well as $\overline{\text{SNR}}_{\text{SR}}$ increases. As a result, the interference becomes stronger and dominant at each destination. Also, we see clearly that with the same $\overline{\text{SNR}}_{\text{SR}}$, when d_0 increases, the interference between two clusters reduces, and more portion of P_i is allocated to $\mathbf{W}_i^{\text{MRC}}$ to focus on increasing beamforming gain within the cluster. Thus, from our proposed structure of \mathbf{W}_i , we can observe clearly the power shift between interference suppression between clusters and beamforming gain maximization within the cluster.

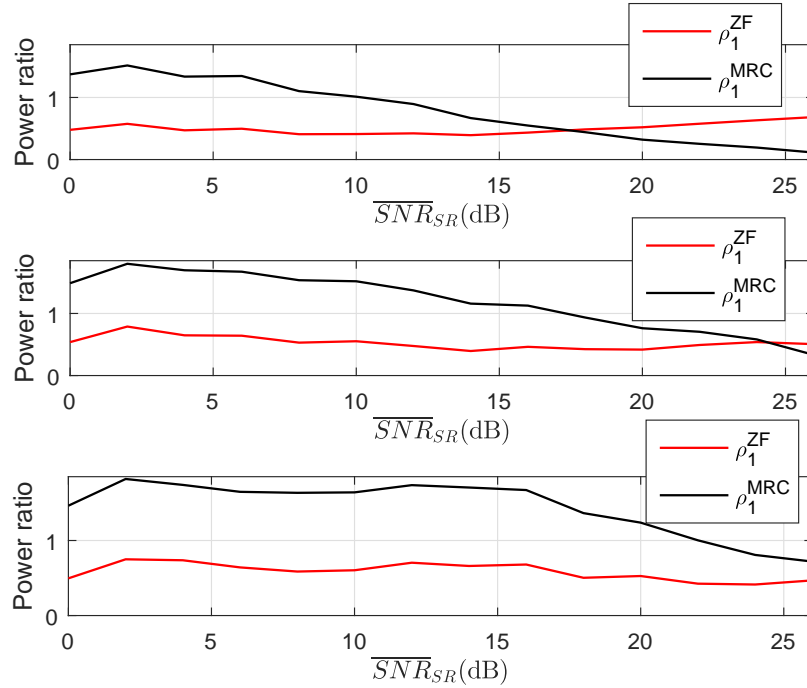


Figure 3.9: Power distribution vs. $\overline{\text{SNR}}_{\text{SR}}$ for Cluster 1 ($M = 4$). Top: $d_0 = 1.0$; Middle: $d_0 = 1.6$; Bottom: $d_0 = 2.2$.

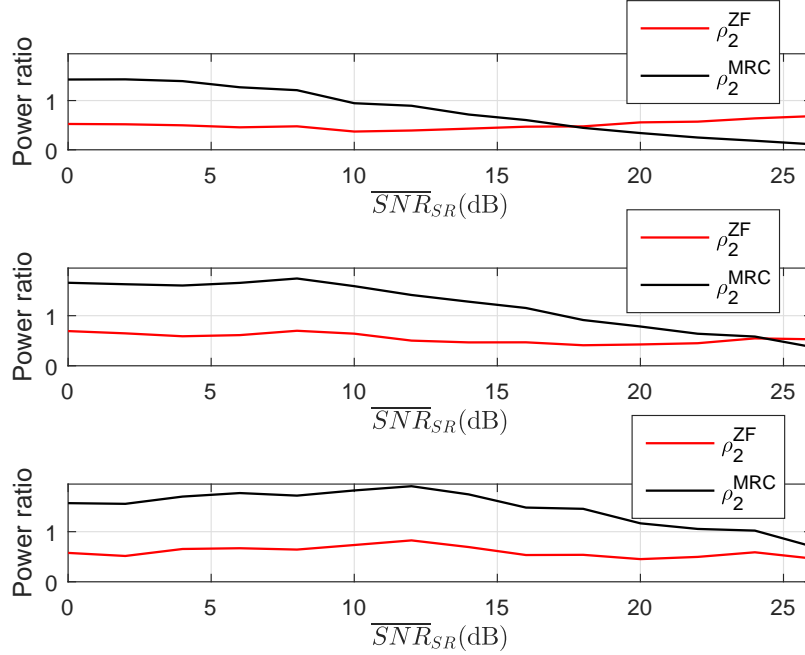


Figure 3.10: Power distribution vs. \overline{SNR}_{SR} for Cluster 2 ($M = 4$). Top: $d_0 = 1.0$; Middle: $d_0 = 1.6$; Bottom: $d_0 = 2.2$.

3.5.3 Computational Time

Given the similar performance, we also compare the complexity of our solution with that of the direct SDR approach. For simulation settings, we have one Intel Core i7-4770 processor with 3.40 GHz, also the CVX tool box [67] and Matlab 2015 are used here to solve the SDP problem. Only the core processing time of each algorithm, including the solving of SDP and randomization process is calculated here. Fig. 3.11 plots the average computation time of each method versus M . As explained earlier, the computation time in our solution does not depend on M , and thus remains constant as M grows. On the other hand, the computation time of the direct SDR approach grows with M significantly.

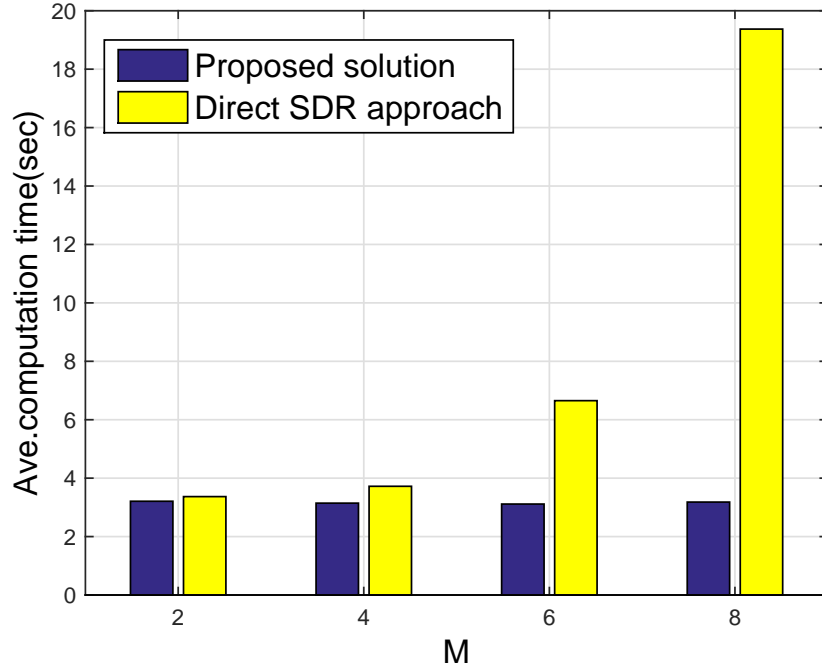


Figure 3.11: Average processing time vs. M ($d_0 = 1.0$).

3.6 Summary

In this chapter, we have designed relay beam matrices for the multi-cluster relay interference network, where each cluster causes interference to other clusters. We have proposed a structured beam matrix which is a weighted sum of ZF beam matrix and MRC beam matrix. With the goal of maximizing the minimum SINR, the proposed beam matrix structure simplifies the optimization problem to one over the scalar weights assigned to each type of beam matrices. The optimal beam matrix for each type has been obtained in closed-form. Through transforming the max-min SINR problem, we have applied the SDR approach to obtain the weight to each beam matrix. For our proposed approach, the size of the SDR problem only depends on the number of relay clusters, not the cluster size, and thus is highly computational

efficient. Comparing with the direct SDR approach, our approach offers both similar performance and significant computational complexity reduction. Furthermore, the structured beam matrix clearly revealed the power shift between interference suppression among clusters and beamforming gain maximization within the cluster as the distance among clusters changes.

Chapter 4

Multi-user Relay Beamforming Design for Multi-cluster Relay Interference Networks

In this chapter, we consider the relay beamforming design in a multi-user multi-cluster AF relay network with N S-D pairs and M relays in each cluster. Similar to Chapter 3, we aim at maximizing the minimum SINR among destinations in all clusters subject to a total relay power budget within each cluster.

4.1 System Model

We consider a two-hop relay network with K clusters each serving N peer-to-peer S-D pairs. In each cluster, the N S-D pairs communicate through a set of dedicated M relays, as shown in Fig. 4.1. Each source and destination is equipped with a single antenna. Let S_{ij} and D_{ij} denote the source and destination nodes for the j th S-D pair in cluster i . The M relays in each cluster can be either a multi-antenna relay or multiple relays capable of signal sharing to form virtual multi-antenna. Let $\mathcal{C}_i = \{R_{i1}, \dots, R_{iM}\}$ denote M relay antennas in the relay cluster for S-D pairs in cluster i , for $i = 1, \dots, K$. We assume $KN \leq M$. Let $f_{ij,mn}$ denote the complex

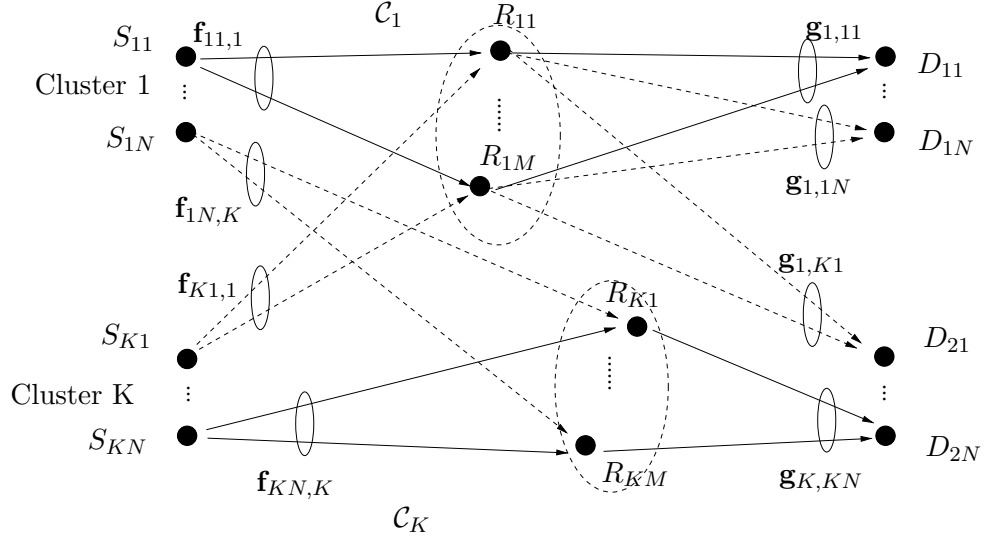


Figure 4.1: A relaying network with K clusters.

channel coefficient between S_{ij} and R_{mn} , and $g_{mn,ij}$ the complex channel coefficient between R_{mn} and D_{ij} .

We again assume that the AF relaying protocol is used, and ignore the direct links between source and destination nodes. In the first phase, the received signal vector at relay cluster \mathcal{C}_m is a superposition of signals from all sources in all clusters, given as

$$\mathbf{r}_m = \sum_{k=1}^K \sum_{n=1}^N \sqrt{P_0} \mathbf{f}_{kn,m} s_{kn} + \mathbf{v}_m, \quad m = 1, \dots, K, \quad (4.1)$$

where $\mathbf{f}_{kn,m} \triangleq [f_{kn,m1}, \dots, f_{kn,mM}]^T$ is the channel vector between S_{kn} and relay cluster \mathcal{C}_m , s_{kn} is the transmitted signal from S_{kn} with $E|s_{kn}|^2 = 1$ and $E[s_{kn}s_{ij}] = 0, \forall kn \neq ij$, P_0 is the common transmit power of all sources, and $\mathbf{v}_m \in \mathbb{C}^{M \times 1}$ is the AWGN vector at relay cluster \mathcal{C}_m with covariance matrix $\sigma_v^2 \mathbf{I}$.

In the second phase, received signal vector \mathbf{r}_m is processed with a beam matrix

$\mathbf{W}_m \in \mathbb{C}^{M \times M}$, and the received signal at D_{ij} is given by

$$\begin{aligned} z_{ij} &= \sum_{m=1}^k \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{r}_m + n_{ij} \\ &= \sum_{m=1}^k \mathbf{g}_{m,ij}^T \mathbf{W}_m \left(\sum_{k=1}^K \sum_{n=1}^N \sqrt{P_0} \mathbf{f}_{kn,m} s_{kn} + \mathbf{v}_m \right) + n_{ij}, \end{aligned}$$

where $\mathbf{g}_{m,ij} \triangleq [g_{m1,ij}, \dots, g_{mM,ij}]^T$ denotes the channel vector between relay cluster \mathcal{C}_m and destination D_{ij} , and n_{ij} denotes the AWGN at D_{ij} with variance σ_n^2 . Note that the received signal at z_{ij} is a superposition of signals received from all relays of all clusters. Only the signal from S_{ij} to D_{ij} via its own relay cluster \mathcal{C}_i is regarded as the intended signal for decoding, and all other received signals are considered as interference. Similar to (3.2), due to the presence of multiple relay clusters, the received signal z_{ij} can be decomposed into the following components

$$\begin{aligned} z_{ij} &= \sqrt{P_0} \mathbf{g}_{i,ij}^T \mathbf{W}_i \mathbf{f}_{ij,i} s_{ij} + \sum_{m=1, m \neq i}^K \sqrt{P_0} \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{ij,m} s_{ij} + \sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \sum_{k,n=1}^K \sum_{kn \neq ij}^N \sqrt{P_0} \mathbf{f}_{kn,m} s_{kn} \\ &\quad + \sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{v}_m + n_{ij}, \quad i = 1 \dots K, \quad j = 1 \dots N, \end{aligned} \quad (4.2)$$

where the first term is the intended signal from S_{ij} , the second term is the interference from other relay cluster \mathcal{C}_m , $m \neq i$, that is originated from S_{ij} , the third term is the interference from all sources other than S_{ij} , and the fourth term is the amplified noise forwarded by all relay clusters. Based on (4.2), the receive SINR at D_{ij} is given by

$$\text{SINR}_{ij} = \frac{P_0 |\mathbf{g}_{i,ij}^T \mathbf{W}_i \mathbf{f}_{ij,i}|^2}{I_{ij} + \sigma_v^2 \sum_{m=1}^K \|\mathbf{g}_{m,ij}^T \mathbf{W}_m\|_2^2 + \sigma_n^2}, \quad (4.3)$$

where \mathbf{I}_{ij} denotes the interference power at D_{ij} , given by

$$\begin{aligned}
\mathbf{I}_{ij} &\triangleq P_0 \mathbb{E} \left\{ \sum_{m=1, m \neq i}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{ij,m} s_{ij} \left(\sum_{m'=1, m' \neq i}^K \mathbf{g}_{m',ij}^T \mathbf{W}_{m'} \mathbf{f}_{ij,m'} s_{ij} \right)^* \right\} \\
&\quad + P_0 \mathbb{E} \left\{ \sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \sum_{k,n=1}^K \sum_{kn \neq ij}^N \mathbf{f}_{kn,m} s_{kn} \left(\sum_{m'=1}^K \mathbf{g}_{m',ij}^T \mathbf{W}_{m'} \sum_{k',n'=1}^K \sum_{k'n' \neq ij}^N \mathbf{f}_{k'n',m'} s_{k'n'} \right)^* \right\} \\
&= P_0 \left(\sum_{m=1, m \neq i}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{ij,m} \left(\sum_{m'=1, m' \neq i}^K \mathbf{g}_{m',ij}^T \mathbf{W}_{m'} \mathbf{f}_{ij,m'} \right)^* \right) \mathbb{E}[s_{ij} s_{ij}^*] \\
&\quad + P_0 \sum_{k,n=1}^K \sum_{kn \neq ij}^N \left(\sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{kn,m} \left(\sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{kn,m} \right)^* \right) \mathbb{E}[s_{kn} s_{kn}^*] \\
&= P_0 \left(\left| \sum_{m=1, m \neq i}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{ij,m} \right|^2 + \sum_{k,n=1}^K \sum_{kn \neq ij}^N \left| \sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{kn,m} \right|^2 \right). \tag{4.4}
\end{aligned}$$

Let P_r denote the total relay power budget at each relay cluster. The outputs of \mathcal{C}_i should satisfy $\mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r$.

Our goal is to design $\{\mathbf{W}_i\}$ for the relay clusters to maximize the minimum SINR among all destinations in all clusters, while satisfying relay power constraint P_r , given by

$$\begin{aligned}
&\max_{\{\mathbf{W}_i\}} \min_{i,j} \text{SINR}_{ij} \\
&\text{s.t. } \mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r, \quad i = 1, \dots, K, j = 1, \dots, N. \tag{4.5}
\end{aligned}$$

The above max-min SINR problem can be equivalently expressed as follows

$$\begin{aligned}
(\text{OP}_6) \quad &\max_{\{\mathbf{W}_i\}, \gamma} \gamma \\
&\text{s.t. } \mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r, \\
&\text{SINR}_{ij} \geq \gamma, \quad i = 1, \dots, K, j = 1, \dots, N. \tag{4.6}
\end{aligned}$$

OP₆ is non-convex due to the non-convex SINR constraints with w.r.t. $\{\mathbf{W}_i\}$ in (4.6).

In order to solve OP₆, we follow the similar steps as in Chapter 3 to construct a beam

matrix using a weighted sum of two specific types of beam matrices, and study the optimal weight for each type of beam matrix. Not only can our solution be efficiently obtained with low complexity, but it also reveals how the weights on two types of beam matrices shift as the topology or the size of the relay clusters changes.

4.2 Low Complexity Multi-cluster Relay Beamforming Design

4.2.1 Structured Beam Matrix

In this section, we consider constructing a relay matrix using a weighted sum of two specific types of beam matrices. In the single pair scenario, we use ZF beam matrix \mathbf{W}_i^{ZF} to cancel interference and MRC beam matrix $\mathbf{W}_i^{\text{MRC}}$ to maximize the beamforming gain. Because of the presence of multiple S-D pairs, MRC is no longer applicable. Instead, here we use MMSE beam matrix $\mathbf{W}_i^{\text{MMSE}}$. We have

$$\mathbf{W}_i = \alpha_i \mathbf{W}_i^{\text{ZF}} + \beta_i \mathbf{W}_i^{\text{MMSE}}, \quad i = 1, \dots, K, \quad (4.7)$$

where α_i and β_i are the weights for \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MMSE}}$, respectively. Specifically, \mathbf{W}_i^{ZF} is designed based on ZF criterion to cancel interference from and to the other clusters, while $\mathbf{W}_i^{\text{MMSE}}$ is designed to minimize the mean square error between the received and transmitted signal within its own cluster without considering interference. In Section 4.2.3, we show that $\mathbf{W}_i^{\text{MMSE}}$ is identical to $\mathbf{W}_i^{\text{MRC}}$ in the single S-D pair scenario. Given

the structure of \mathbf{W}_i in (4.7), OP_6 is reformulated into the following problem

$$\begin{aligned}
(\text{OP}_7) \quad & \max_{\{\alpha_i, \beta_i\}} \gamma \\
\text{s.t.} \quad & \mathbb{E}\{\|\mathbf{W}_i \mathbf{r}_i\|_2^2\} \leq P_r, \\
& \text{SINR}_{ij} \geq \gamma, \\
& \mathbf{W}_i = \alpha_i \mathbf{W}_i^{\text{ZF}} + \beta_i \mathbf{W}_i^{\text{MMSE}}, \quad i = 1, \dots, K, \quad j = 1, \dots, N.
\end{aligned}$$

Note that since both \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MMSE}}$ can be determined using their respective design criteria, the remaining parameters to be optimized are only weights $\{\alpha_i, \beta_i\}$, $i = 1, \dots, K$.

4.2.2 ZF Beam Matrix Design

We now design ZF beam matrix \mathbf{W}_i^{ZF} at the relay cluster i to cancel interference from/to other clusters. The ZF beam matrix is given by the following structure

$$\mathbf{W}_i^{\text{ZF}} = \mathbf{W}_{\text{T},i}^{\text{ZF}} \mathbf{W}_{\text{R},i}^{\text{ZF}},$$

where $\mathbf{W}_{\text{R},i}^{\text{ZF}H}$ and $\mathbf{W}_{\text{T},i}^{\text{ZF}}$ are the $M \times N$ receive ZF beam matrix and transmit ZF beam matrix, respectively. Define $\mathbf{s}_i \triangleq [s_{i1}, \dots, s_{iN}]^T$ as the signal vector from sources in cluster i and $\mathbf{s} \triangleq [\mathbf{s}_1, \dots, \mathbf{s}_K]^T$ as the signal vector from all sources. Since only \mathbf{s}_i is the desired signal at \mathcal{C}_i , the receive ZF beam vector $\mathbf{W}_{\text{R},i}^{\text{ZF}}$ should be designed to cancel interference originated from sources in clusters j , for $j \neq i$. This is given by

$$\mathbf{W}_{\text{R},i}^{\text{ZF}} \mathbf{r}_i = \sqrt{P_0} \mathbf{s}_i \quad \mathbf{v}_i = \mathbf{0}, \quad (4.8)$$

which indicates when $\mathbf{v}_i = \mathbf{0}$, received signal after the processing of $\mathbf{W}_{\text{R},i}^{\text{ZF}}$ should be exactly equal to $\sqrt{P_0} \mathbf{s}_i$. From (4.8), we can see that the interference among sources

S_{i1}, \dots, S_{iN} in the same cluster is also cancelled. Rewrite received signal vector \mathbf{r}_i at relay cluster \mathcal{C}_i in (4.1) as

$$\mathbf{r}_i = \sqrt{P_0} \mathbf{F}_i \mathbf{s} + \mathbf{v}_i,$$

where $\mathbf{F}_i \triangleq [\mathbf{f}_{11,i}, \mathbf{f}_{12,i}, \dots, \mathbf{f}_{KN,i}] \in \mathbb{C}^{M \times KN}$ is channel state matrix containing channels from all sources to \mathcal{C}_i . ZF constraint in (4.8) is equivalent to

$$\mathbf{W}_{R,i}^{\text{ZF}} \mathbf{F}_i = \mathbf{\Pi}_i,$$

where $\mathbf{\Pi}_i = [\mathbf{0}, \dots, \mathbf{I}, \dots, \mathbf{0}]$ is a $1 \times K$ block matrix with the i th block being $N \times N$ identity matrix and the elsewhere $\mathbf{0}$'s. Denote the received signal after receive ZF processing by $\mathbf{y}_i \triangleq \mathbf{W}_{R,i}^{\text{ZF}} \mathbf{r}_i$. The receive ZF problem can be formulated as

$$\mathbf{W}_{R,i}^{\text{ZF}} = \arg \max_{\mathbf{W}} \mathbb{E}\{\|\mathbf{y}_i - \sqrt{P_0} \mathbf{s}_i\|_2^2\} \quad (4.9)$$

$$\text{s.t. } \mathbf{W} \mathbf{F}_i = \mathbf{\Pi}_i. \quad (4.10)$$

The objective function in (4.9) can be rewritten as

$$\begin{aligned} \mathbb{E}\{\|\mathbf{y}_i - \sqrt{P_0} \mathbf{s}_i\|_2^2\} &= \mathbb{E}\{\|\mathbf{W}(\sqrt{P_0} \mathbf{F}_i \mathbf{s} + \mathbf{v}_i) - \sqrt{P_0} \mathbf{s}_i\|_2^2\} \\ &= \mathbb{E}\{\|\sqrt{P_0}(\mathbf{W} \mathbf{F}_i - \mathbf{\Pi}_i) \mathbf{s} + \mathbf{W} \mathbf{v}_i\|_2^2\}. \end{aligned}$$

Substitute (4.10) into the above equation, we have

$$\begin{aligned} \mathbb{E}\{\|\mathbf{y}_i - \sqrt{P_0} \mathbf{s}_i\|_2^2\} &= \mathbb{E}\{\|\mathbf{W} \mathbf{v}_i\|_2^2\} \\ &= \sigma_v^2 \text{tr}(\mathbf{W} \mathbf{W}^H) \end{aligned} \quad (4.11)$$

where $\mathbb{E}[\mathbf{v}_i \mathbf{v}_i^H] = \sigma_v^2 \mathbf{I}$. The receive ZF problem has a quadratic and convex objective function (4.11), subject to a linear equality constraint (4.10). Thus it is convex and

can be solve by using the Lagrange multiplier method [66]. The Lagrangian for (4.9) can be formulated as

$$\begin{aligned} L(\mathbf{w}, \lambda) &\triangleq \sigma_v^2 \text{tr}(\mathbf{W}\mathbf{W}^H) - \sum_{n=1}^N \sum_{m=1}^{NK} \lambda_{nm} [\mathbf{W}\mathbf{F}_i - \mathbf{\Pi}_i]_{nm} \\ &= \sigma_v^2 \text{tr}(\mathbf{W}\mathbf{W}^H) - \text{tr}((\mathbf{W}\mathbf{F}_i - \mathbf{\Pi}_i)\mathbf{\Lambda}), \end{aligned} \quad (4.12)$$

where λ_{nm} is the Lagrangian multiplier, and $[\mathbf{\Lambda}]_{mn} = \lambda_{nm}$. Following the similar steps of solving $\mathbf{w}_{R,i}^{\text{ZF}}$ in Section 3.2.2, we obtain $\mathbf{W}_{R,i}^{\text{ZF}}$ as

$$\mathbf{W}_{R,i}^{\text{ZF}} = \mathbf{\Pi}_i (\mathbf{F}_i^H \mathbf{F}_i)^{-1} \mathbf{F}_i^H. \quad (4.13)$$

Note that since $KN \leq M$, and entries of \mathbf{F}_i are independent channel coefficients, $\mathbf{F}_i^H \mathbf{F}_i$ is invertible.

After receive ZF at relays, the transmit ZF beam matrix $\mathbf{W}_{T,i}^{\text{ZF}}$ is then applied. Define received signal vector at destinations in cluster i as $\mathbf{z}_i \triangleq [z_{i1}, \dots, z_{iN}]^T$ and at all destinations as $\mathbf{z} = [\mathbf{z}_1, \dots, \mathbf{z}_K]^T$. We have

$$\mathbf{z} = \mathbf{G}_i \mathbf{W}_{T,i}^{\text{ZF}} \mathbf{y}_i + \mathbf{n},$$

where $\mathbf{G}_i \triangleq [\mathbf{g}_{i,11}, \mathbf{g}_{i,12}, \dots, \mathbf{g}_{i,KN}]^T \in \mathbb{C}^{KN \times M}$ is channel state matrix from \mathcal{C}_i to all destinations, and noise vector $\mathbf{n} \triangleq [n_{11}, \dots, n_{KN}]^T$. The transmit ZF beamforming is designed to suppress transmit signals to all other clusters, while maximize the intended signal power to receivers in cluster i . Define $\mathbf{G}_{i,j} \triangleq [\mathbf{g}_{i,j1}, \dots, \mathbf{g}_{i,jN}]^T \in \mathbb{C}^{N \times M}$ as channel state matrix containing channels from \mathcal{C}_i to destinations in cluster j . We have $\mathbf{G}_i = [\mathbf{G}_{i,1}^T, \dots, \mathbf{G}_{i,K}^T]^T$. The transmit ZF beamforming problem can be written

as

$$\mathbf{W}_{T,i}^{\text{ZF}} = \arg \max_{\mathbf{W}} \mathbb{E}\{\|\mathbf{G}_{i,i} \mathbf{W} \mathbf{y}_i\|^2\} \quad (4.14)$$

$$\text{s.t. } \mathbb{E}\{\|\mathbf{W} \mathbf{y}_i\|^2\} \leq P_r \quad (4.15)$$

$$\mathbf{G}_{i-} \mathbf{W} \mathbf{y}_i = \mathbf{0}, \quad (4.16)$$

where \mathbf{G}_{i-} is the $(K-1) \times 1$ block matrix obtained from \mathbf{G}_i by removing the i th block $\mathbf{G}_{i,i}^T$. ZF constraint (4.16) implies that

$$\mathbf{G}_i \mathbf{W}_{T,i}^{\text{ZF}} = \kappa_{\text{ZF},i} \mathbf{\Pi}_i^H, \quad (4.17)$$

for some scalar $\kappa_{\text{ZF},i}$. Similar to Lemma 3.1, it's easy to prove that the objective function (4.14) is maximized when $\kappa_{\text{ZF},i}$ is set such that constraint (4.15) is met with equality. Also, the objective function (4.14) can be written in trace form as

$$\begin{aligned} \mathbb{E}\{\|\mathbf{G}_{i,i} \mathbf{W} \mathbf{y}_i\|^2\} &= \mathbb{E}\{\|\mathbf{\Pi}_i \mathbf{G}_i \mathbf{W} \mathbf{y}_i\|^2\} \\ &= \text{tr}(\mathbf{\Pi}_i \mathbf{G}_i \mathbf{W} \mathbf{R}_{\mathbf{y}_i} \mathbf{W}^H \mathbf{G}_i^H \mathbf{\Pi}_i^H) \end{aligned} \quad (4.18)$$

where $\mathbf{R}_{\mathbf{y}_i} \triangleq \mathbb{E}[\mathbf{y}_i \mathbf{y}_i^H]$, and (4.18) is convex w.r.t. \mathbf{W} . Given the linear equality constraints (4.15) and (4.17), the transmit ZF problem is convex and thus can be solved using the Lagrangian multiplier method. It's Lagrangian can then be written as

$$\begin{aligned} L(\mathbf{w}, \lambda) &\triangleq \text{tr}(\mathbf{\Pi}_i \mathbf{G}_i \mathbf{W} \mathbf{R}_{\mathbf{y}_i} \mathbf{W}^H \mathbf{G}_i^H \mathbf{\Pi}_i^H) - \sum_{m=1}^{KN} \sum_{n=1}^N \lambda_{mn} [\mathbf{G}_i \mathbf{W}_{T,i}^{\text{ZF}} - \kappa_{\text{ZF},i} \mathbf{\Pi}_i^H]_{mn} \\ &= \text{tr}(\mathbf{\Pi}_i \mathbf{G}_i \mathbf{W} \mathbf{R}_{\mathbf{y}_i} \mathbf{W}^H \mathbf{G}_i^H \mathbf{\Pi}_i^H) - \text{tr}((\mathbf{G}_i \mathbf{W}_{T,i}^{\text{ZF}} - \kappa_{\text{ZF},i} \mathbf{\Pi}_i^H) \mathbf{\Lambda}), \end{aligned}$$

where λ_{mn} is the Lagrangian multiplier, and $[\mathbf{\Lambda}]_{nm} = \lambda_{mn}$. Following the similar steps

of solving $\mathbf{w}_{\text{T},i}^{\text{ZF}}$ in Section 3.2.2, it's straight forward to obtain that

$$\mathbf{W}_{\text{T},i}^{\text{ZF}} = \kappa_{\text{ZF},i} \mathbf{G}_i^H (\mathbf{G}_i \mathbf{G}_i^H)^{-1} \mathbf{\Pi}_i^H, \quad (4.19)$$

with

$$\kappa_{\text{ZF},i}^2 = \frac{Pr}{\text{tr}(\mathbf{\Pi}_i (\mathbf{G}_i \mathbf{G}_i^H)^{-1} \mathbf{\Pi}_i^H \mathbf{R}_{\mathbf{y}_i})}, \quad i = 1 \cdots K,$$

where $\mathbf{R}_{\mathbf{y}_i} \triangleq P_0 \mathbf{I} + \sigma_v^2 \mathbf{W}_{\text{R},i}^{\text{ZF}} \mathbf{W}_{\text{R},i}^{\text{ZF}H}$. From $\mathbf{W}_{\text{R},i}^{\text{ZF}}$ in (4.13), we have

$$\kappa_{\text{ZF},i}^2 = \frac{P_r}{\text{tr}(\mathbf{\Pi}_i (\mathbf{G}_i \mathbf{G}_i^H)^{-1} \mathbf{\Pi}_i^H (P_0 \mathbf{I} + \sigma_v^2 \mathbf{\Pi}_i (\mathbf{F}_i^H \mathbf{F}_i)^{-1} \mathbf{\Pi}_i^H))}.$$

Thus, using (4.13) and (4.19), we have ZF beam matrix \mathbf{W}_i^{ZF} , for $i = 1, \cdots, K$, as

$$\begin{aligned} \mathbf{W}_i^{\text{ZF}} &= \mathbf{W}_{\text{T},i}^{\text{ZF}} \mathbf{W}_{\text{R},i}^{\text{ZF}} \\ &= \kappa_{\text{ZF},i} \mathbf{G}_i^H (\mathbf{G}_i \mathbf{G}_i^H)^{-1} \mathbf{\Pi}_i^H \mathbf{\Pi}_i (\mathbf{F}_i^H \mathbf{F}_i)^{-1} \mathbf{F}_i^H \end{aligned} \quad (4.20)$$

4.2.3 MMSE Beam Matrix Design

For designing MMSE beam matrix $\mathbf{W}_i^{\text{MMSE}}$, we are only concerned about the signal forwarded by its own relay cluster \mathcal{C}_i . The goal is to minimize the mean square error of received signal at \mathcal{C}_i and at destinations in cluster i , respectively. Thus, we set the MMSE beam matrix structure as

$$\mathbf{W}_i^{\text{MMSE}} = \mathbf{W}_{\text{T},i}^{\text{MMSE}} \mathbf{W}_{\text{R},i}^{\text{MMSE}},$$

where $\mathbf{W}_{\text{R},i}^{\text{MMSE}H}$ and $\mathbf{W}_{\text{T},i}^{\text{MMSE}}$ are the $M \times N$ receive and transmit beam matrix, respectively. The signal received at relay cluster \mathcal{C}_i from its own intended source $S_{ij}, j = 1, \cdots, N$, denoted by $\mathbf{r}_{i,i}$, is given by

$$\mathbf{r}_{i,i} = \sqrt{P_0} \mathbf{F}_{i,i} \mathbf{s}_i + \mathbf{v}_i,$$

where $\mathbf{F}_{i,i} = [\mathbf{f}_{i1,i}, \dots, \mathbf{f}_{iN,i}]$ is the channel state matrix containing channels from all sources in cluster i to \mathcal{C}_i . Denote the received signal after receive MMSE processing by $\mathbf{y}_{i,i} \triangleq \mathbf{W}_{R,i}^{\text{MMSE}} \mathbf{r}_{i,i}$. Receive beam matrix $\mathbf{W}_{R,i}^{\text{MMSE}}$ is given by the following expression

$$\mathbf{W}_{R,i}^{\text{MMSE}} = \arg \max_{\mathbf{W}} \mathbb{E}\{\|\mathbf{y}_{i,i} - \sqrt{P_0} \mathbf{s}_i\|^2\}, \quad (4.21)$$

which can be simplified into the following trace form

$$\begin{aligned} \mathbb{E}\{\|\mathbf{y}_{i,i} - \sqrt{P_0} \mathbf{s}_i\|^2\} &= \mathbb{E}\{\|\mathbf{W}(\sqrt{P_0} \mathbf{F}_{i,i} \mathbf{s}_i + \mathbf{v}_i) - \sqrt{P_0} \mathbf{s}_i\|^2\} \\ &= \text{tr}\left(\left(\mathbf{W}(\sqrt{P_0} \mathbf{F}_{i,i} \mathbf{s}_i + \mathbf{v}_i) - \sqrt{P_0} \mathbf{s}_i\right)\left(\mathbf{W}(\sqrt{P_0} \mathbf{F}_{i,i} \mathbf{s}_i + \mathbf{v}_i) - \sqrt{P_0} \mathbf{s}_i\right)^H\right) \\ &= \text{tr}(P_0 \mathbf{W} \mathbf{F}_{i,i} \mathbf{F}_{i,i}^H \mathbf{W}^H - P_0 \mathbf{W} \mathbf{F}_{i,i} + \sigma_v^2 \mathbf{W} \mathbf{W}^H - P_0 \mathbf{F}_{i,i}^H \mathbf{W}^H + P_0 \mathbf{I}). \end{aligned}$$

Since (4.21) is an unconstrained quadratic maximization problem, it can be solved by taking the first derivative w.r.t \mathbf{W} and set it to zero, it is straightforward to obtain

$\mathbf{W}_{R,i}^{\text{MMSE}}$ as

$$\mathbf{W}_{R,i}^{\text{MMSE}} = P_0 \mathbf{F}_{i,i}^H (P_0 \mathbf{F}_{i,i} \mathbf{F}_{i,i}^H + \sigma_v^2 \mathbf{I})^{-1}. \quad (4.22)$$

After applying $\mathbf{W}_{R,i}^{\text{MMSE}}$ to the actual received signal vector \mathbf{r}_i , we have

$$\mathbf{y}_i = \mathbf{W}_{R,i}^{\text{MMSE}} \mathbf{r}_i = P_0 \mathbf{F}_{i,i}^H (P_0 \mathbf{F}_{i,i} \mathbf{F}_{i,i}^H + \sigma_v^2 \mathbf{I})^{-1} (\sqrt{P_0} \mathbf{F}_i \mathbf{s} + \mathbf{v}_i). \quad (4.23)$$

Then the transmit beam matrix $\mathbf{W}_{T,i}^{\text{MMSE}}$ is applied. The received signal at all destinations in \mathcal{C}_i without considering interference from relays in other cluster is given by

$$\mathbf{z}_{i,i} = \mathbf{G}_{i,i} \mathbf{W}_{T,i}^{\text{MMSE}} \mathbf{y}_i + \mathbf{n}_i,$$

where $\mathbf{G}_{i,i} = [\mathbf{g}_{i,i1}, \dots, \mathbf{g}_{i,iN}]^T$ is the channel state matrix containing channels from \mathcal{C}_i to all the destinations in cluster i , and $\mathbf{n}_i \triangleq [n_{i1}, \dots, n_{iN}]^T$. The transmit MMSE

beamforming problem can be formulated as

$$\begin{aligned} \{\mathbf{W}_{T,i}^{\text{MMSE}}, \kappa_{\text{MMSE},i}\} &= \arg \min_{\mathbf{W}, \kappa} \mathbb{E}\{ \|\mathbf{y}_i - \kappa^{-1} \mathbf{z}_{i,i}\|^2 \} \\ \text{s.t. } \mathbb{E}\{ \|\mathbf{W}\mathbf{y}_i\|^2 \} &= P_r, \end{aligned} \quad (4.24)$$

where $\kappa_{\text{MMSE},i}$ can be viewed as an automatic gain control [31]; it can ensure that the MSE is minimized while fulfilling the power constraint. The above optimization question has a convex objective function and a convex equality constraint, thus can be solved using Lagrangian multiplier method. The Lagrangian associated with the it can be constructed as:

$$L(\mathbf{W}, \kappa, \lambda) = \mathbb{E}\{ \|\mathbf{y}_i - \kappa^{-1} \mathbf{z}_{i,i}\|^2 \} + \lambda \text{tr}(\mathbf{W}\mathbf{R}_{\mathbf{y}_i} \mathbf{W}^H - P_r), \quad (4.25)$$

where $\lambda \in \mathbb{R}$ is the Lagrangian multiplier. The cost function can then be rewritten into trace form

$$\begin{aligned} \mathbb{E}\{ \|\mathbf{y}_i - \kappa^{-1} \mathbf{z}_{i,i}\|^2 \} &= \mathbb{E}\{ \text{tr}((\mathbf{y}_i - \kappa^{-1} \mathbf{G}_{i,i} \mathbf{W}\mathbf{y}_i - \kappa^{-1} \mathbf{n}_i)(\mathbf{y}_i - \kappa^{-1} \mathbf{G}_{i,i} \mathbf{W}\mathbf{y}_i - \kappa^{-1} \mathbf{n}_i)^H) \} \\ &= \text{tr}(\mathbf{R}_{\mathbf{y}_i}) - \kappa^{-1} \text{tr}(\mathbf{R}_{\mathbf{y}_i} \mathbf{W}^H \mathbf{G}_{i,i}^H) - \kappa^{-1} \text{tr}(\mathbf{G}_{i,i} \mathbf{W} \mathbf{R}_{\mathbf{y}_i}) \\ &\quad + \kappa^{-2} \text{tr}(\mathbf{G}_{i,i} \mathbf{W} \mathbf{R}_{\mathbf{y}_i} \mathbf{W}^H \mathbf{G}_{i,i}^H + \sigma_n^2 \mathbf{I}), \end{aligned} \quad (4.26)$$

where $\mathbf{R}_{\mathbf{y}_i} \triangleq \mathbb{E}[\mathbf{y}_i \mathbf{y}_i^H]$. By taking the first derivative of (4.25) w.r.t. \mathbf{W} and κ , respectively, and setting them to zero, we have

$$\frac{\partial L(\mathbf{W}, \kappa, \lambda)}{\partial \mathbf{W}} = -\kappa^{-1} \mathbf{G}_{i,i}^T \mathbf{R}_{\mathbf{y}_i}^T + \kappa^{-2} (\mathbf{G}_{i,i}^T \mathbf{G}_{i,i}^* \mathbf{W}^* \mathbf{R}_{\mathbf{y}_i}^T) + \lambda (\mathbf{W}^* \mathbf{R}_{\mathbf{y}_i}^T) = \mathbf{0} \quad (4.27)$$

$$\frac{\partial L(\mathbf{W}, \kappa, \lambda)}{\partial \kappa} = \kappa \text{tr}(\text{Re}(\mathbf{G}_{i,i} \mathbf{W} \mathbf{R}_{\mathbf{y}_i})) - \text{tr}(\mathbf{G}_{i,i} \mathbf{W} \mathbf{R}_{\mathbf{y}_i} \mathbf{W}^H \mathbf{G}_{i,i}^H + \sigma_n^2 \mathbf{I}) = 0. \quad (4.28)$$

From (4.27), we obtain the structure of \mathbf{W}

$$\begin{aligned}\mathbf{W} &= \kappa(\mathbf{G}_{i,i}^H \mathbf{G}_{i,i} + \lambda\kappa^2 \mathbf{I})^{-1} \mathbf{G}_{i,i}^H \\ &= \kappa \tilde{\mathbf{W}},\end{aligned}\tag{4.29}$$

where $\tilde{\mathbf{W}} \triangleq (\mathbf{G}_{i,i}^H \mathbf{G}_{i,i} + \lambda\kappa^2 \mathbf{I})^{-1} \mathbf{G}_{i,i}^H$. Substitute (4.29) into power constraint (4.24),

we have

$$\kappa = \sqrt{\frac{P_r}{\text{tr}(\tilde{\mathbf{W}} \mathbf{R}_{y_i} \tilde{\mathbf{W}}^H)}}.$$

Note that $\text{tr}(\text{Re}(\mathbf{G}_{i,i} \tilde{\mathbf{W}} \mathbf{R}_{y_i})) = \text{tr}(\mathbf{G}_{i,i} \tilde{\mathbf{W}} \mathbf{R}_{y_i})$ since $\mathbf{G}_{i,i} \tilde{\mathbf{W}} \mathbf{R}_{y_i}$ is hermitian. And also from (4.29), we have $\text{tr}(\mathbf{G}_{i,i} \tilde{\mathbf{W}} \mathbf{R}_{y_i}) = \text{tr}(\tilde{\mathbf{W}}^H (\mathbf{G}_{i,i}^H \mathbf{G}_{i,i} + \lambda\kappa^2 \mathbf{I}) \tilde{\mathbf{W}} \mathbf{R}_{y_i})$. Substitute them together into (4.28), we have

$$\begin{aligned}\text{tr}(\tilde{\mathbf{W}}^H (\mathbf{G}_{i,i}^H \mathbf{G}_{i,i} + \lambda\kappa^2 \mathbf{I}) \tilde{\mathbf{W}} \mathbf{R}_{y_i}) - \text{tr}(\mathbf{G}_{i,i} \tilde{\mathbf{W}} \mathbf{R}_{y_i} \tilde{\mathbf{W}}^H \mathbf{G}_{i,i}^H + \sigma_n^2 \mathbf{I}) &= 0, \\ \text{tr}(\lambda\kappa^2 \mathbf{I} \tilde{\mathbf{W}} \mathbf{R}_{y_i}) - \sigma_n^2 N &= 0, \\ \lambda\kappa^2 &= \frac{\sigma_n^2 N}{P_r}.\end{aligned}$$

Therefore we have our transmit MMSE matrix as

$$\begin{aligned}\mathbf{W}_{T,i}^{\text{MMSE}} &= \kappa_{\text{MMSE},i} \mathbf{J}_i^{-1} \mathbf{G}_{i,i}^H \\ \kappa_{\text{MMSE},i}^2 &= \frac{P_r}{\text{tr}(\mathbf{J}_i^{-2} \mathbf{G}_{i,i}^H \mathbf{R}_{y_i} \mathbf{G}_{i,i})},\end{aligned}\tag{4.30}$$

where

$$\begin{aligned}\mathbf{J}_i &\triangleq \mathbf{G}_{i,i}^H \mathbf{G}_{i,i} + \frac{\sigma_n^2 N}{P_r} \mathbf{I}, \\ \mathbf{R}_{y_i} &\triangleq \mathbb{E}[\mathbf{y}_i \mathbf{y}_i^H] = \mathbf{W}_{R,i}^{\text{MMSE}} (P_0 \mathbf{F}_i \mathbf{F}_i^H + \sigma_v^2 \mathbf{I}) \mathbf{W}_{R,i}^{\text{MMSE}H}.\end{aligned}$$

Using (4.22) and (4.30), we have MMSE beam matrix $\mathbf{W}_i^{\text{MMSE}}$, for $i = 1, \dots, K$, as

$$\begin{aligned}\mathbf{W}_i^{\text{MMSE}} &= \mathbf{W}_{T,i}^{\text{MMSE}} \mathbf{W}_{R,i}^{\text{MMSE}} \\ &= \kappa_{\text{MMSE},i} P_0 \mathbf{J}_i^{-1} \mathbf{G}_{i,i}^H \mathbf{F}_{i,i}^H (P_0 \mathbf{F}_{i,i} \mathbf{F}_{i,i}^H + \sigma_v^2 \mathbf{I})^{-1}.\end{aligned}\quad (4.31)$$

4.2.3.1 Evidence of $\mathbf{W}_i^{\text{MMSE}}$ and $\mathbf{W}_i^{\text{MRC}}$ for $N = 1$

Next, we show that in the single S-D pair scenario, $\mathbf{W}_i^{\text{MMSE}}$ is equivalent to $\mathbf{W}_i^{\text{MRC}}$.

Thus our design in the multiple S-D pairs per cluster case for $N = 1$ is consistent with the design in Chapter 3.

Proposition 4.1. *For $N = 1$, the MMSE beamformer of each cluster can be expressed as*

$$\begin{aligned}\mathbf{W}_i^{\text{MMSE}} &= \mathbf{w}_{T,i}^{\text{MMSE}} \mathbf{w}_{R,i}^{\text{MMSE}H} \\ &= \kappa_{\text{MMSE},i} P_0 \mathbf{J}_i^{-1} \mathbf{g}_{i,i}^* \mathbf{f}_{i,i}^H (P_0 \mathbf{f}_{i,i} \mathbf{f}_{i,i}^H + \sigma_v^2 \mathbf{I})^{-1},\end{aligned}$$

where $\mathbf{f}_{i,i}$ is the channel vector between S_j and relay cluster C_i , and $\mathbf{g}_{i,i}$ denotes the channel vector between relay cluster C_j and destination D_i . Also for $\kappa_{\text{MMSE},i}$ we have

$$\kappa_{\text{MMSE},i} = \sqrt{\frac{P_r}{\text{tr}(\mathbf{J}_i^{-2} \mathbf{g}_{i,i}^* R_{y_i} \mathbf{g}_{i,i}^T)}},$$

where $\mathbf{J}_i \triangleq \mathbf{g}_{i,i}^* \mathbf{g}_{i,i}^T + \frac{\sigma_n^2}{P_r} \mathbf{I}$ and $R_{y_i} \triangleq \mathbb{E}[y_i y_i^*]$. y_i denotes the signal after the process of $\mathbf{w}_{R,i}^{\text{MMSE}}$.

And $\mathbf{W}_i^{\text{MMSE}}$ here is exactly the same with $\mathbf{W}_i^{\text{MRC}}$ which is given in (3.28)

$$\mathbf{W}_i^{\text{MRC}} = \kappa_{\text{MRC},i} \frac{\mathbf{g}_{i,i}^* \mathbf{f}_{i,i}^H}{\|\mathbf{g}_{i,i}\| \|\mathbf{f}_{i,i}\|}, \quad i = 1, \dots, K.$$

Proof. See Appendix 4.6.1.

4.2.4 Optimization of $\{\alpha_i, \beta_i\}$ via SDR Approach

With the optimal solution of the two beam matrices \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MMSE}}$, we now focus on solving OP₇ to obtain the optimal weights α_i, β_i , to determine \mathbf{W}_i , for $i = 1, \dots, K$.

Following similar steps of vectorization in 3.2.4, we have

$$\mathbf{w}_i = \alpha_i^* \mathbf{w}_i^{\text{ZF}} + \beta_i^* \mathbf{w}_i^{\text{MMSE}} \quad i = 1, \dots, K, \quad (4.32)$$

where $\mathbf{w}_i \triangleq \text{vec}(\mathbf{W}_i^H)$, $\mathbf{w}_i^{\text{ZF}} \triangleq \text{vec}(\mathbf{W}_i^{\text{ZF}H})$ and $\mathbf{w}_i^{\text{MMSE}} \triangleq \text{vec}(\mathbf{W}_i^{\text{MMSE}H})$. Define $\mathbf{h}_{kn,ij}^{(m)} \triangleq \mathbf{g}_{m,ij} \otimes \mathbf{f}_{kn,m} \in \mathbb{C}^{M^2 \times 1}$ as the compound channel from S_{kn} to D_{ij} through relay cluster \mathcal{C}_m . Using the same vectorization techniques in Section 3.2.4, we further rewrite SINR expression (4.3) w.r.t \mathbf{w}_i . The second term of \mathbf{I}_{ij} in (4.4) can be rewritten as

$$\begin{aligned} I_{ij, S_{kn}} &\triangleq P_0 \sum_{k,n=1}^K \sum_{kn \neq ij}^N \left| \sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{kn,m} \right|^2 \\ &= P_0 \sum_{k,n=1}^K \sum_{kn \neq ij}^N \left(\sum_{m=1}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{kn,m} \sum_{m'=1}^K (\mathbf{g}_{m',ij}^T \mathbf{W}_{m'} \mathbf{f}_{kn,m'})^* \right) \\ &= P_0 \sum_{k,n=1}^K \sum_{kn \neq ij}^N \left(\sum_{m=1}^K (\mathbf{h}_{kn,ij}^{(m)H} \mathbf{w}_m)^* \sum_{m'=1}^K (\mathbf{h}_{kn,ij}^{(m')H} \mathbf{w}_{m'}) \right), \end{aligned}$$

where $I_{ij, S_{kn}}$ is the power of interference originates from S_{kn} , $kn \neq ij$. Define $\mathbf{h}_{kn,ij} \triangleq [\mathbf{h}_{kn,ij}^{(1)H}, \dots, \mathbf{h}_{kn,ij}^{(K)H}]^H \in \mathbb{C}^{KM^2 \times 1}$ as the channel vector from S_{kn} to D_{ij} through all relay clusters. Also define $\mathbf{w} \triangleq [\mathbf{w}_1^H, \dots, \mathbf{w}_K^H]^H \in \mathbb{C}^{KM^2 \times 1}$ as the vectorized beamformer of all clusters, we have

$$\begin{aligned} I_{ij, S_{kn}} &= P_0 \sum_{k,n=1}^K \sum_{kn \neq ij}^N |\mathbf{h}_{kn,ij}^H \mathbf{w}|^2 \\ &= \mathbf{w}^H \left(\sum_{k,n=1}^K \sum_{kn \neq ij}^N P_0 \mathbf{h}_{kn,ij} \mathbf{h}_{kn,ij}^H \right) \mathbf{w}. \end{aligned} \quad (4.33)$$

Similarly, for the first term of \mathbf{I}_{ij} in (4.4), let $I_{ij,S_{ij}}$ be the power of interference originates from S_{ij} itself. We have

$$\begin{aligned}
I_{ij,S_{ij}} &\triangleq P_0 \left| \sum_{m=1, m \neq i}^K \mathbf{g}_{m,ij}^T \mathbf{W}_m \mathbf{f}_{ij,m} \right|^2 \\
&= P_0 \left| \sum_{m=1, m \neq i}^K (\mathbf{h}_{ij,ij}^{(m)H} \mathbf{w}_m) \right|^2 \\
&= \mathbf{w}^H P_0 \mathbf{h}_{ij,ij} \mathbf{h}_{ij,ij}^H \mathbf{w},
\end{aligned} \tag{4.34}$$

where $\mathbf{h}_{ij,ij} = [\mathbf{h}_{ij,ij}^{(1)H}, \dots, \mathbf{h}_{ij,ij}^{(i-1)H}, \mathbf{0}, \mathbf{h}_{ij,ij}^{(i+1)H}, \dots, \mathbf{h}_{ij,ij}^{(K)H}]^H \in \mathbb{C}^{KM^2 \times 1}$ denotes the channel vector from S_{ij} to D_{ij} via all the relay clusters $\mathcal{C}_j, j \neq i$. Thus \mathbf{I}_{ij} in (4.4) can be rewritten as

$$\begin{aligned}
\mathbf{I}_{ij} &= \mathbf{I}_{ij,S_{ij}} + \mathbf{I}_{ij,S_{kn}} \\
&= \mathbf{w}^H \left(\sum_{k=1}^K \sum_{n=1}^N P_0 \mathbf{h}_{kn,ij} \mathbf{h}_{kn,ij}^H \right) \mathbf{w}.
\end{aligned} \tag{4.35}$$

Perform the similar vectorization process on the desired signal part in (4.3)

$$\begin{aligned}
P_0 |\mathbf{g}_{i,ij}^T \mathbf{W}_i \mathbf{f}_{ij,i}|^2 &= P_0 |\mathbf{h}_{ij,ij}^{(i)H} \mathbf{w}_i|^2 \\
&= \mathbf{w}^H \mathbf{D}(\mathbf{A}_{ij}) \mathbf{w},
\end{aligned} \tag{4.36}$$

where $\mathbf{A}_{ij} \triangleq P_0 \mathbf{h}_{ij,ij}^{(i)} \mathbf{h}_{ij,ij}^{(i)H}$, and $\mathbf{D}(\mathbf{A}_{ij})$ is the block diagonal matrix with the i th diagonal block being \mathbf{A}_{ij} and the rest $\mathbf{0}$'s. For the amplified noise term in SINR expression (4.3), we have

$$\begin{aligned}
\sigma_v^2 \sum_{m=1}^K \|\mathbf{g}_{m,ij}^T \mathbf{W}_m\|_2^2 &= \sigma_v^2 \sum_{m=1}^K \mathbf{w}_m^H \mathbf{B}_{m,ij} \mathbf{w}_m \\
&= \mathbf{w}^H \mathbf{B}_{ij} \mathbf{w},
\end{aligned} \tag{4.37}$$

where $\mathbf{B}_{m,ij} \triangleq (\mathbf{g}_{m,ij} \mathbf{g}_{m,ij}^H) \otimes \mathbf{I}$ is the amplified noise covariance matrix from \mathcal{C}_m to D_{ij} and $\mathbf{B}_{ij} = \sigma_v^2 \text{diag}(\mathbf{B}_{1,ij}, \dots, \mathbf{B}_{K,ij}) \in \mathbb{C}^{KM^2 \times KM^2}$ is the $K \times K$ block diagonal

amplified noise matrix from all $\mathcal{C}_j, j = 1, \dots, K$ to D_{ij} . In this way, we now can rewrite SINR in (4.3) as

$$\begin{aligned} \text{SINR}_{ij} &= \frac{\mathbf{w}^H \mathbf{D}(\mathbf{A}_{ij}) \mathbf{w}}{\mathbf{w}^H (\sum_{k=1}^K \sum_{n=1}^N P_0 \mathbf{h}_{kn,ij} \mathbf{h}_{kn,ij}^H) \mathbf{w} + \mathbf{w}^H \mathbf{B}_{ij} \mathbf{w} + \sigma_n^2} \\ &= \frac{\mathbf{w}^H \mathbf{D}(\mathbf{A}_{ij}) \mathbf{w}}{\mathbf{w}^H \mathbf{C}_i \mathbf{w} + \sigma_n^2}, \end{aligned} \quad (4.38)$$

where $\mathbf{C}_{ij} \triangleq \sum_{k=1}^K \sum_{n=1}^N P_0 \mathbf{h}_{kn,ij} \mathbf{h}_{kn,ij}^H + \mathbf{B}_{ij}$. Similarly, power constraint (4.5) can be rewritten as

$$\mathbf{w}_i^H \mathbf{E}_i \mathbf{w}_i \leq P_r, \quad i = 1, \dots, K \quad (4.39)$$

where $\mathbf{E}_i \triangleq \mathbf{I} \otimes \mathbb{E}[\mathbf{r}_i \mathbf{r}_i^H] = P_o \mathbf{F}_i \mathbf{F}_i^H + \sigma_v^2 \mathbf{I}$.

Define $\overline{\mathbf{W}}_i \triangleq [\mathbf{w}_i^{\text{ZF}}, \mathbf{w}_i^{\text{MMSE}}]$, and $\mathbf{a}_i \triangleq [\alpha_i, \beta_i]^H$, for $i = 1, \dots, K$. From (4.32), it follows that $\mathbf{w}_i = \overline{\mathbf{W}}_i \mathbf{a}_i$. Similarly, define $\overline{\mathbf{W}} \triangleq \text{bldg}(\overline{\mathbf{W}}_1, \dots, \overline{\mathbf{W}}_K)$ and $\mathbf{x} \triangleq [\mathbf{a}_1^H, \dots, \mathbf{a}_K^H]^H$, we have $\mathbf{w} = \overline{\mathbf{W}} \mathbf{x}$. SINR expression in (4.38) can now be rewritten as a function of \mathbf{x} as

$$\text{SINR}_{ij} = \frac{\mathbf{x}^H \mathbf{D}(\tilde{\mathbf{A}}_{ij}) \mathbf{x}}{\mathbf{x}^H \tilde{\mathbf{C}}_{ij} \mathbf{x} + \sigma_n^2}, \quad (4.40)$$

where $\mathbf{D}(\tilde{\mathbf{A}}_i) \triangleq \overline{\mathbf{W}}^H \mathbf{D}(\mathbf{A}_i) \overline{\mathbf{W}}$ and $\tilde{\mathbf{C}}_{ij} \triangleq \overline{\mathbf{W}}^H \mathbf{C}_{ij} \overline{\mathbf{W}}$. Also, the left hand side of (4.39) can be rewritten as

$$\mathbf{w}_i^H \mathbf{E}_i \mathbf{w}_i = \mathbf{a}_i^H \tilde{\mathbf{E}}_i \mathbf{a}_i = \mathbf{x}^H \mathbf{D}(\tilde{\mathbf{E}}_i) \mathbf{x} \quad (4.41)$$

where $\tilde{\mathbf{E}}_i \triangleq \overline{\mathbf{W}}_i^H \mathbf{E}_i \overline{\mathbf{W}}_i$, and $\mathbf{D}(\tilde{\mathbf{E}}_i)$ is defined in as a block diagonal matrix with i th diagonal block being $\tilde{\mathbf{E}}_i$ and the rest $\mathbf{0}$'s. With (4.40) and (4.41), we now reformulate

OP₇ into the following equivalent problem

$$\begin{aligned}
(\text{OP}_8) \quad & \max_{\mathbf{x}, \gamma} \gamma \\
\text{s.t.} \quad & \mathbf{x}^H \left(\frac{1}{\gamma} \mathbf{D}(\tilde{\mathbf{A}}_{ij}) - \tilde{\mathbf{C}}_{ij} \right) \mathbf{x} \geq \sigma_n^2, \quad i = 1, \dots, K, \quad j = 1, \dots, N, \\
& \mathbf{x}^H \mathbf{D}(\tilde{\mathbf{E}}_i) \mathbf{x} \leq P_r, \quad i = 1, \dots, K.
\end{aligned}$$

Define $\mathbf{X} \triangleq \mathbf{x}\mathbf{x}^H$, and remove the rank-one constraint on \mathbf{X} , we relax OP₈ into the following problem

$$\begin{aligned}
(\text{OP}_9) \quad & \max_{\mathbf{X}, \gamma} \gamma \\
\text{s.t.} \quad & \text{tr} \left[\left(\frac{1}{\gamma} \mathbf{D}(\tilde{\mathbf{A}}_{ij}) - \tilde{\mathbf{C}}_{ij} \right) \mathbf{X} \right] \geq \sigma_n^2, \quad i = 1, \dots, K, \quad j = 1, \dots, N, \\
& \text{tr}[\mathbf{D}(\tilde{\mathbf{E}}_i) \mathbf{X}] \leq P_r, \quad i = 1, \dots, K; \\
& \mathbf{X} \succeq 0.
\end{aligned}$$

Note that OP₉ is not jointly convex w.r.t. \mathbf{X} and γ . Similar to OP₄, it can also be solved by performing bi-section search on γ and transforming it into a feasibility problem. Similar randomization method used in Algorithm 1 is needed if the optimal solution \mathbf{X}_{opt} is not rank one. Compared with OP₄ in single pair scenario, we now end up with a similar SDP problem with the same problem size of $2K \times 2K$, which is not growing with M . Also now we have KN SINR constraints instead of just K , and the number of power constraint remains the same.

As a comparison, we also consider the direct SDR approach to obtain $\{\mathbf{W}_i\}$. Rewrite OP₆ w.r.t \mathbf{w} using SINR expression in (4.38), and then relax it into SDP

form

$$\begin{aligned}
(\text{OP}_{10}) \quad & \max_{\mathbf{Y}, \gamma} \quad \gamma \\
\text{s.t.} \quad & \text{tr} \left[\left(\frac{1}{\gamma} \mathbf{D}(\mathbf{A}_{ij}) - \mathbf{C}_{ij} \right) \mathbf{Y} \right] \geq \sigma_n^2, \quad i = 1, \dots, K, \quad j = 1, \dots, N \\
& \text{tr}[\mathbf{D}(\mathbf{E}_i)\mathbf{Y}] \leq P_r, \quad i = 1, \dots, K \\
& \mathbf{Y} \succeq 0
\end{aligned}$$

where $\mathbf{Y} \triangleq \mathbf{w}\mathbf{w}^H$ and $\mathbf{D}(\mathbf{E}_i)$ is defined in the same way as $\mathbf{D}(\tilde{\mathbf{E}}_i)$. Solving OP_{10} follows the exact same way of solving OP_9 .

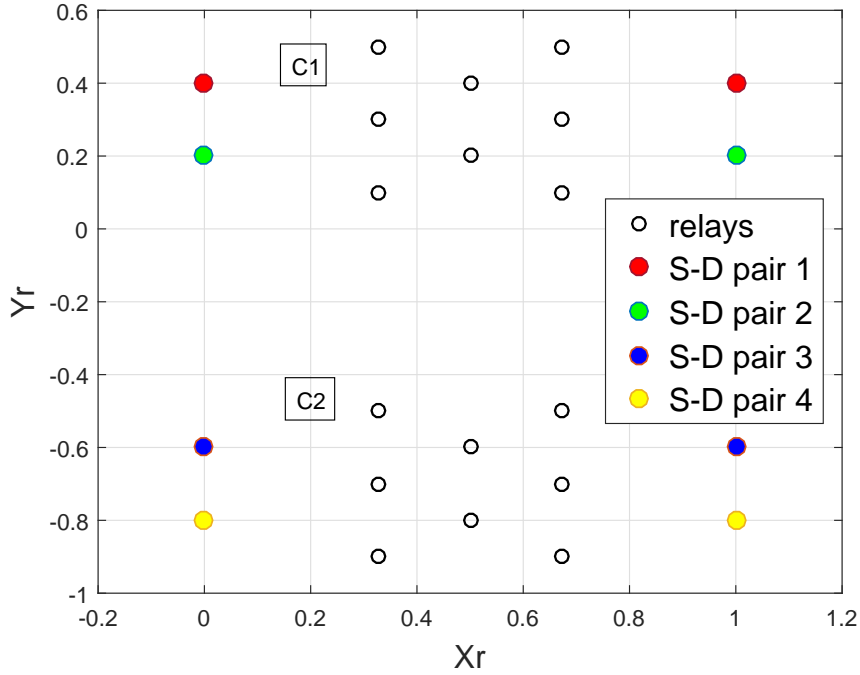
4.3 Complexity Analysis and Comparison

Comparing our proposed solution and the direct SDR approach, the former one still has the obvious advantage in computation complexity in multiple S-D pair scenario. Based on the complexity analysis of the standard SDP form [64], for the direct SDR approach, the SDP problem OP_{10} has $(KM^2)^2$ variables and $K(N+1)$ constraints. The complexity to solve the SDP is about $\mathcal{O}(K^5M^8N)$. For our proposed solution, OP_9 has the variable size of $(2K)^2$ and $K(N+1)$ constraints, with complexity being $\mathcal{O}(K^5N)$, which is also independent of M . By incorporating structural information $\overline{\mathbf{W}}$ into constraints, OP_9 reduces its problem size and results in significantly lower complexity than the direct SDP approach.

4.4 Simulation Results

For the multiple S-D pair scenario, we consider a relay network consists of two clusters ($K = 2$) with 8 relays ($M = 8$) and 2 S-D pairs ($N = 2$) per cluster, as shown in

Fig.4.2. Similar with Section 3.5, we assume parallel S-D pairs with relays located

Figure 4.2: Relay layout, $K = 2$, $M = 8$, $N = 2$, $d_0 = 1.0$.

in the middle point of each S-D pair ($d_{\text{SR}} = d_{\text{RD}} = 0.5d_{\text{SD}}$, $d_{\text{SD}} = 1$). Let d_0 denote the distance between the two clusters, measured between the centers of the two relay clusters. Channel vectors $\mathbf{f}_{ij,k}$ and $\mathbf{g}_{k,ij}$ are assumed to be i.i.d. Gaussian. Nominal SNR from source to relay are used to indicate the average channel strength over this link, defined as $\overline{\text{SNR}}_{\text{SR}} = P_0 K_0 (d_{\text{SR}})^{-3.5} / \sigma_n^2$, with K_0 being the pathloss constant. We set $P_0 / \sigma_n^2 = 1$. Since $d_{\text{SR}} = d_{\text{RD}}$, we have $\overline{\text{SNR}}_{\text{SR}} = \overline{\text{SNR}}_{\text{RD}}$. We set $\sigma_v^2 = \sigma_n^2$.

4.4.1 SINR Performance

Again, we first compare the minimum SINR performance of our proposed solution with that of the direct SDR approach in OP_{10} . From Figs. 4.3 and 4.5, we plot SINR vs. $\overline{\text{SNR}}_{\text{SR}}$ with $d_0 = 2.2$, for $M = 4, 6$ and 8 , respectively. In each figure, the

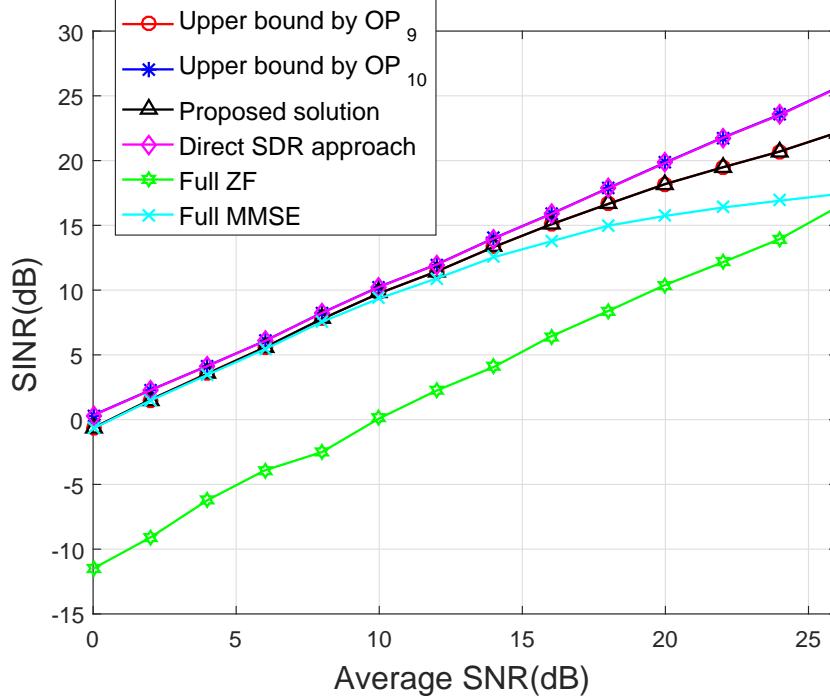


Figure 4.3: SINR vs. $\overline{\text{SNR}}_{\text{SR}}$ ($M = 4, d_0 = 2.2, P_r = MP_0$).

central 4, 6 and 8 relays shown in Fig. 4.2 are chosen, respectively. We set $P_r = MP_0$. Optimal objective values of the SDP problems OP_9 and OP_{10} are also plotted here, which serve as performance upper bounds for our proposed approach and direct SDR approach, respectively. Additionally, we exam SINR performance of \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MMSE}}$ in each setup when each cluster uses full relay power on ZF ($\alpha_i = 1, \beta_i = 0$) or MMSE ($\alpha_i = 0, \beta_i = 1$).

As can be seen from Figs. 4.3 to 4.5, our proposed solution provides similar SINR performance with that of direct approach, especially with larger number of M . The gap between two methods becomes bigger when M becomes smaller. A larger SINR gap is observed at higher $\overline{\text{SNR}}_{\text{SR}}$ which is about 3dB for $M = 4$, while the gap decreases to 1.2dB for $M = 8$. Further we can see that the performance of \mathbf{W}_i^{ZF} increases considerably with bigger M , from about 12dB lower than the direct

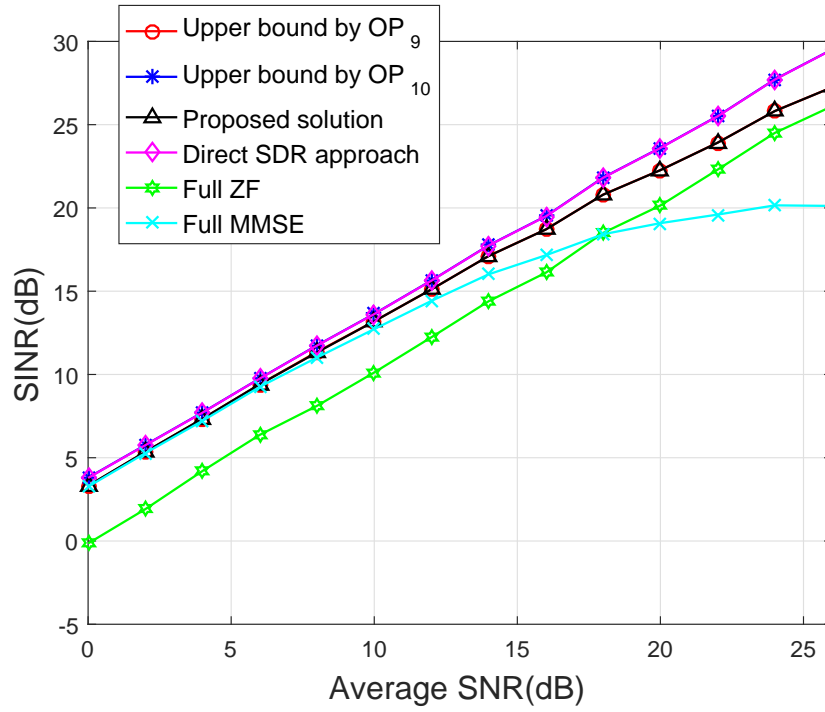


Figure 4.4: SINR vs. \overline{SNR}_{SR} ($M = 6, d_0 = 2.2, Pr = MP_0$).

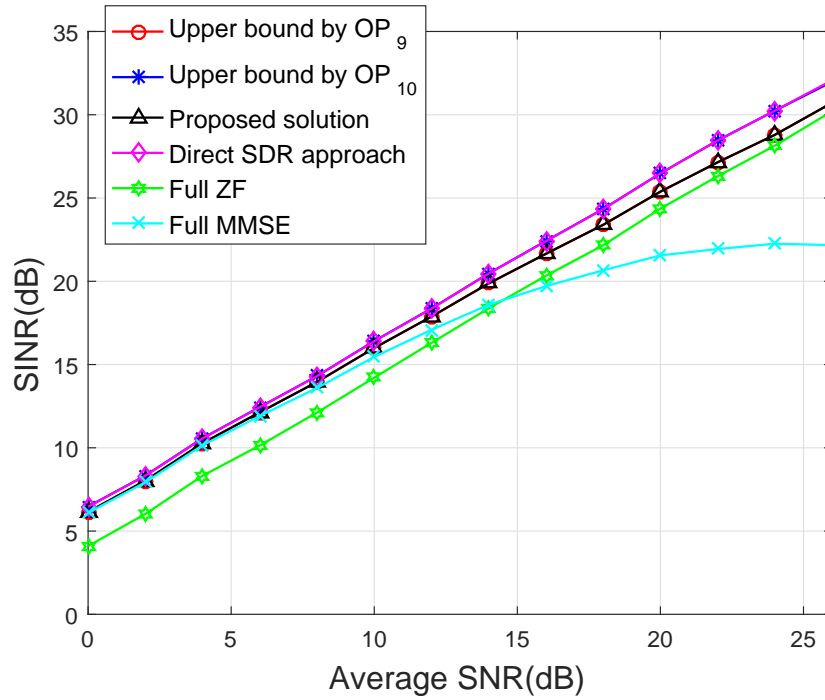


Figure 4.5: SINR vs. \overline{SNR}_{SR} ($M = 8, d_0 = 2.2, Pr = MP_0$).

approach when $M = 4$, to just 2dB below the direct approach at $M = 8$. This is because when $M = NK = 4$, there is no extra degree of freedom in \mathbf{W}_i^{ZF} to improve the received signal power, as can be seen in ZF constraints in (4.10) and (4.16). When M increases, extra degrees of freedom in beamformer allows us to focus signal power on the direction of the intended receivers, which results in a better SINR performance.

Next we compare the SINR performance of the 2 approaches with different d_0 . In Figs. 4.6 to 4.7, we plot the minimum SINR vs. $\overline{\text{SNR}}_{\text{SR}}$, for $d_0 = 1.6$ and 1.0, respectively. We set $M = 4$ by using the central 4 relays in each cluster (same as Fig 3.2), and $P_r = MP_0$. As can be seen from Figs. 4.3, 4.6 and 4.7, the gap between proposed solution and direct approach are bigger comparing with single S-D pair scenario in Figs. 3.3 to 3.5. This is because of the lack of degree of freedom in \mathbf{W}_i^{ZF} when $M = NK = 4$. In addition, the gap between two methods becomes larger as d_0 becomes smaller. A larger SINR gap is observed at higher $\overline{\text{SNR}}_{\text{SR}}$ which is about 6dB for $d_0 = 1.0$ when the two clusters are relatively close, while the gap decreases to 3dB for $d_0 = 2.2$ when the two clusters are further apart. Furthermore, we can see that in these three figures the performance of $\mathbf{W}_i^{\text{MMSE}}$ increases with increasing d_0 . This is because the inter-cluster interference reduces with longer distance between cells, thus SINR performance relies more on beamforming gain maximization when d_0 is relatively large.

4.4.2 Power Allocation

Next, based on our structured beam matrix \mathbf{W}_i , we study how the relay power is allocated to the two types of beam matrices \mathbf{W}_i^{ZF} and $\mathbf{W}_i^{\text{MMSE}}$, and how the inter-

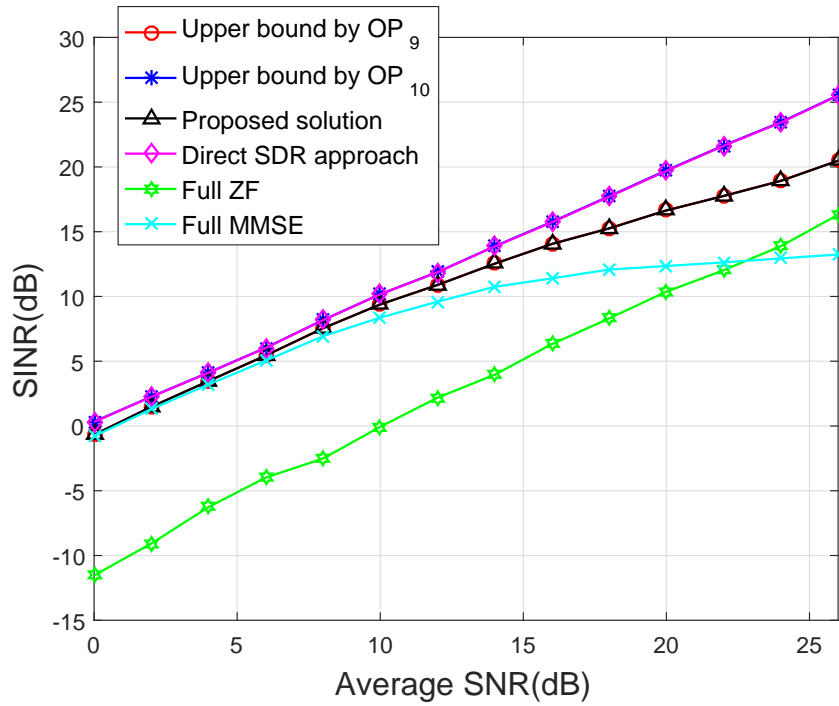


Figure 4.6: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 1.6, Pr = MP_0$).

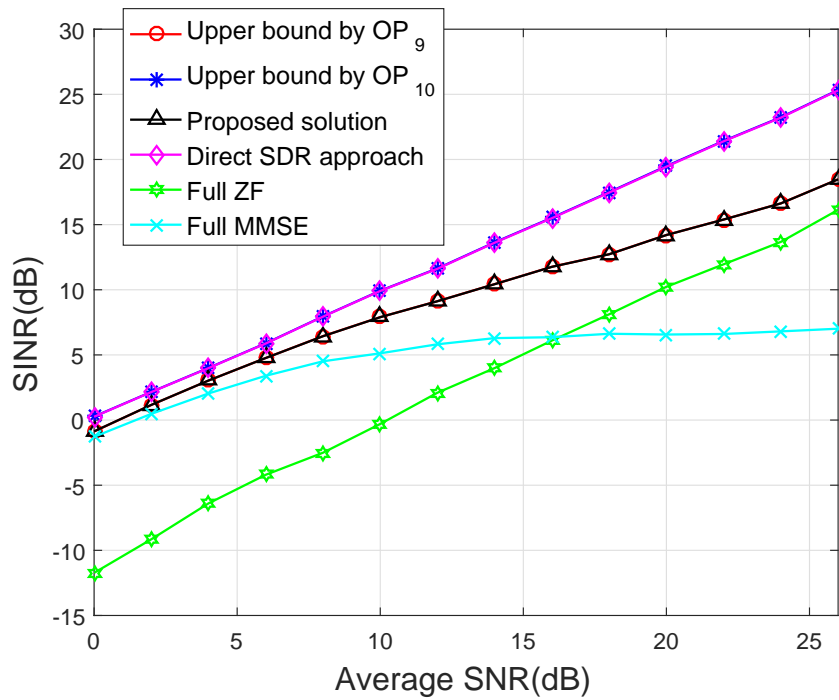


Figure 4.7: SINR vs. \overline{SNR}_{SR} ($M = 4, d_0 = 1.0, Pr = MP_0$).

cluster interference affects the power allocation. Similar definitions used in Section 3.5.2 are also adopted here with ρ_i^{ZF} and $\rho_i^{\text{MMSE}} \triangleq \frac{\mathbb{E}[\|\beta_i \mathbf{W}_i^{\text{MMSE}} \mathbf{r}_i\|_2^2]}{P_i}$ indicating the portion of actual relay power P_i allocated to the specific type of beam matrix. Figs. 4.8 and 4.9 show relay power allocation $\{\rho_i^{\text{ZF}}, \rho_i^{\text{MMSE}}\}$ vs. $\overline{\text{SNR}}_{\text{SR}}$ at different distance $d_0 = 1.0, 1.6, 2.2$ and with $M = 4$, for relay clusters 1 and 2, respectively. Similar trends with single S-D pair scenario are observed here. As $\overline{\text{SNR}}_{\text{SR}}$ becomes higher, ρ_i^{MMSE} decreases, and a larger portion of P_i is allocated to $\mathbf{W}_i^{\text{ZF}}, i = 1, 2$. This is because the interference channel becomes stronger as well as $\overline{\text{SNR}}_{\text{SR}}$ increases. As a result, the interference becomes stronger and dominant at each destination. Also, we see clearly that with the same $\overline{\text{SNR}}_{\text{SR}}$, when d_0 increases, the interference between two clusters reduces, and a bigger portion of P_i is allocated to $\mathbf{W}_i^{\text{MMSE}}$ to focus on increasing beamforming gain within the cluster.

Figs. 4.10 and 4.11 illustrate relay power allocation $\{\rho_i^{\text{ZF}}, \rho_i^{\text{MMSE}}\}$ vs. $\overline{\text{SNR}}_{\text{SR}}$ with different relay number $M = 4, 6, 8$ with $d_0 = 1.0$, for relay clusters 1 and 2, respectively. As can be seen from these figures, with the same $\overline{\text{SNR}}_{\text{SR}}$, when M increases, the interference between two clusters becomes stronger, and a bigger portion of P_i is allocated to $\mathbf{W}_i^{\text{ZF}}, i = 1, 2$, to focus on interference cancelation. Thus, from our proposed structure of \mathbf{W}_i , we can observe clearly the power shift between interference suppression between clusters and beamforming gain maximization within the cluster.

4.4.3 Computation Time

Given the performance and power distribution, we also compare the complexity of our solution with that of the direct SDR approach. Similar settings in Chapter 3.5.3 are

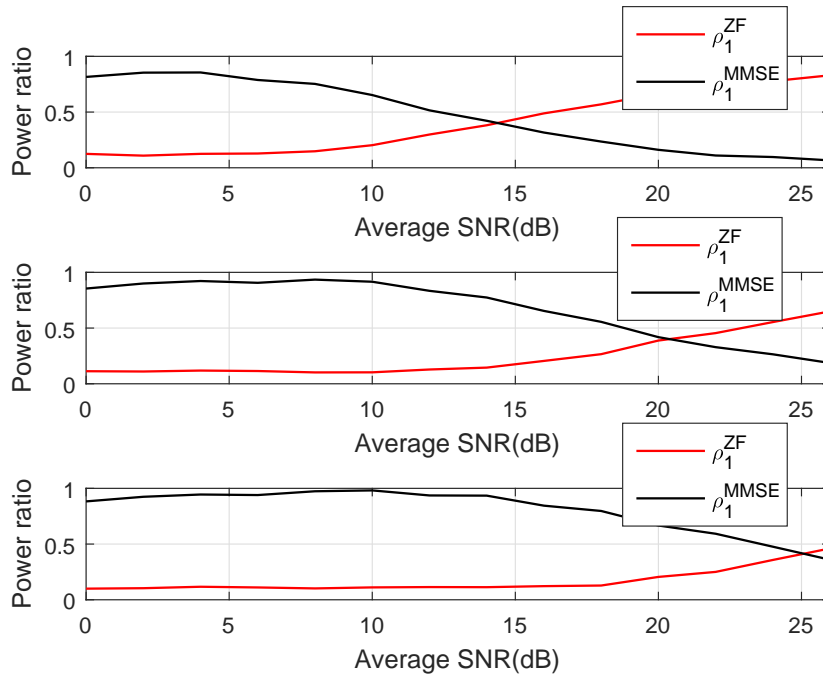


Figure 4.8: Power distribution vs. $\overline{\text{SNR}}_{\text{SR}}$ for Cluster 1 ($M = 4$). Top: $d_0 = 1.0$; Middle: $d_0 = 1.6$; Bottom: $d_0 = 2.2$.

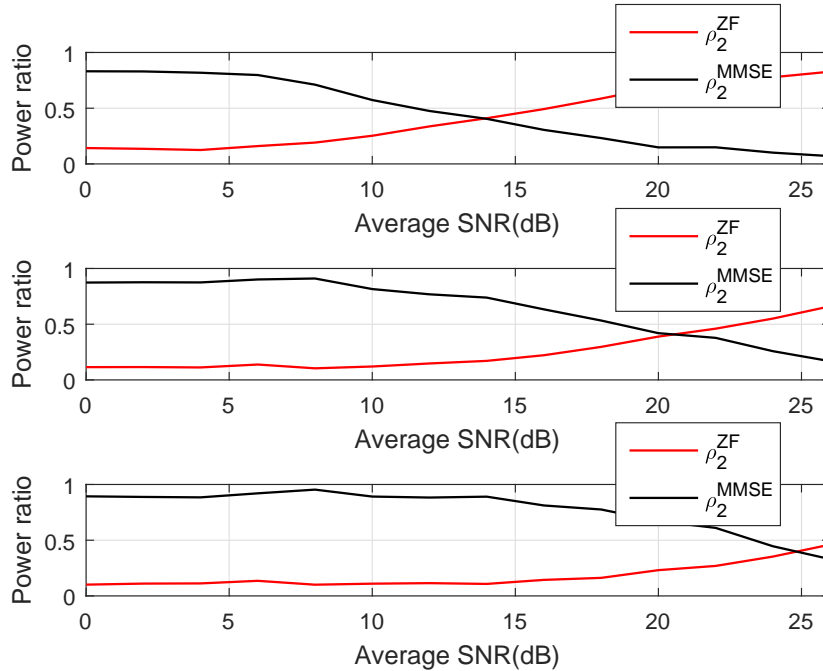


Figure 4.9: Power distribution vs. $\overline{\text{SNR}}_{\text{SR}}$ for Cluster 2 ($M = 4$). Top: $d_0 = 1.0$; Middle: $d_0 = 1.6$; Bottom: $d_0 = 2.2$.

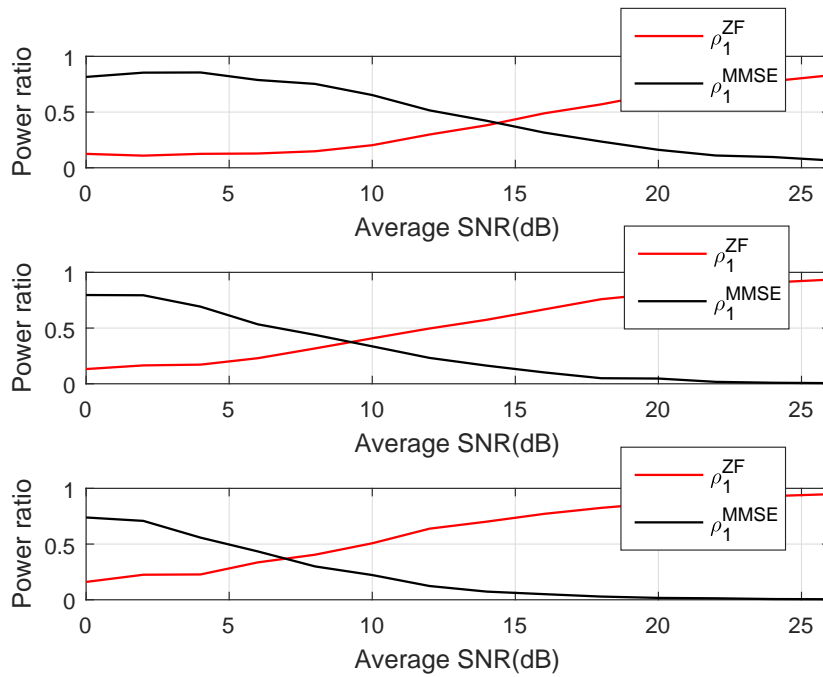


Figure 4.10: Power distribution vs. $\overline{\text{SNR}}_{\text{SR}}$ for Cluster 1 ($d_0 = 1.0$). Top: $M = 4$; Middle: $M = 6$; Bottom: $M = 8$.

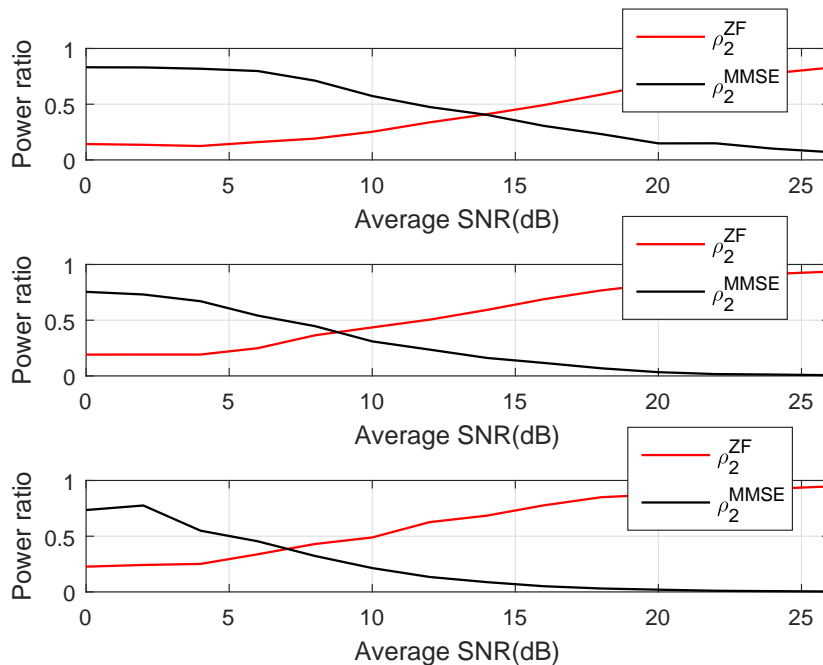


Figure 4.11: Power distribution vs. $\overline{\text{SNR}}_{\text{SR}}$ for Cluster 2 ($d_0 = 1.0$). Top: $M = 4$; Middle: $M = 6$; Bottom: $M = 8$.

also used here. Fig. 4.12 plots the average computation time of each method versus M . As explained earlier, the computation time in our solution does not depend on M , and thus remains constant as M grows. On the other hand, the computation time of the direct SDR approach grows with M significantly.

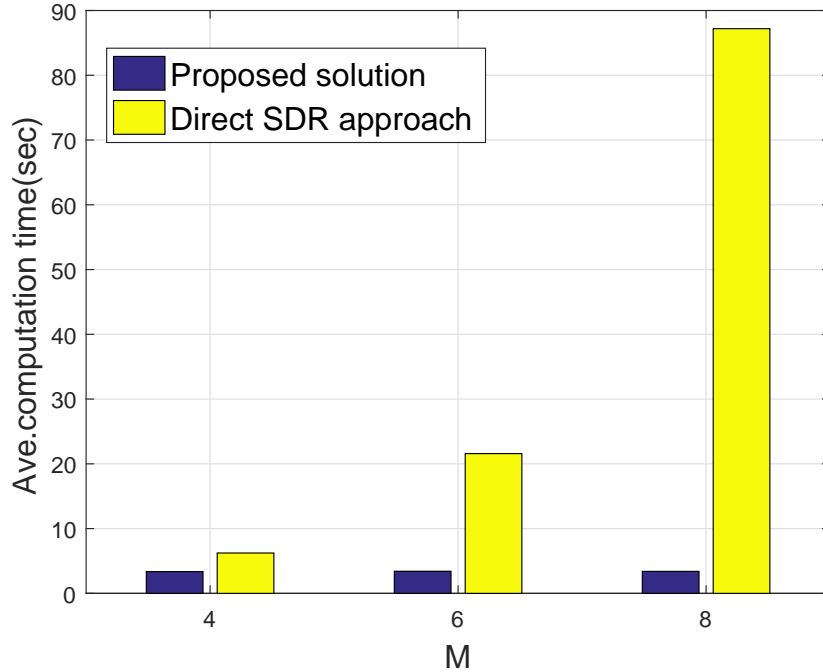


Figure 4.12: Average processing time vs. M ($d_0 = 1.0$).

4.5 Summary

In this chapter, we have designed relay beam matrices for the multi-user multi-cluster relay interference network, where each cluster causes interference to others. We have proposed a structured beam matrix which is a weighted sum of ZF beam matrix and MMSE beam matrix. With the goal of maximizing the minimum SINR, the proposed beam matrix structure simplifies the optimization problem to one over the scalar weights assigned to each type of beam matrices. The optimal beam matrix

for each type has been obtained in closed-form. Through transforming the max-min SINR problem, we have applied the SDR approach to obtain the weight to each beam matrix. For our proposed approach, the size of the SDR problem only depends on the number of relay clusters, not the cluster size, and thus is highly computational efficient. Simulations show that our proposed solution provides similar SINR performance with that of direct approach, and the performance gap decreases as the number of relays increases. Additionally, proposed approach has significant lower computational complexity. Furthermore, the structured beam matrix clearly revealed the power shift between interference suppression among clusters and beamforming gain maximization within the cluster as the distance among clusters or size of clusters changes.

4.6 Appendices

4.6.1 Proof of Proposition 4.1

Proof. We show the solution in each case below.

1) For receive MMSE vector $\mathbf{w}_{R,i}^{\text{MMSE}}$: The signal received at relay cluster \mathcal{C}_i from its own intended source S_i , denoted by $\mathbf{r}_{i,i}$, is given by

$$\mathbf{r}_{i,i} = \sqrt{P_0} \mathbf{f}_{i,i} s_i + \mathbf{v}_i.$$

The receive MMSE problem can then be formulated as

$$\mathbf{w}_{R,i}^{\text{MMSE}} = \arg \max_{\mathbf{w}} \mathbb{E}\{\|y_{i,i} - \sqrt{P_0} s_i\|_2^2\}, \quad (4.42)$$

where $y_{i,i} \triangleq \mathbf{w}_{R,i}^{\text{MMSE}H} \mathbf{r}_{i,i}$. Follow the same steps of solving (4.21), we obtain $\mathbf{w}_{R,i}^{\text{MMSE}}$ as

$$\mathbf{w}_{R,i}^{\text{MMSE}H} = P_0 \mathbf{f}_{i,i}^H (P_0 \mathbf{f}_{i,i} \mathbf{f}_{i,i}^H + \sigma_v^2 \mathbf{I})^{-1}$$

2) For transmit MMSE vector $\mathbf{w}_{T,i}^{MMSE}$: The received signal at the i th destination is given by

$$z_{i,i} = \mathbf{g}_{i,i}^T \mathbf{w}_{T,i}^{MMSE} y_i + n_i,$$

with

$$y_i \triangleq \mathbf{w}_{R,i}^{MMSEH} \mathbf{r}_i = \mathbf{w}_{R,i}^{MMSEH} (\sqrt{P_o} \mathbf{F}_i \mathbf{s} + \mathbf{v}_i),$$

where $\mathbf{F}_i \triangleq [\mathbf{f}_{1,i}, \dots, \mathbf{f}_{K,i}] \in \mathbb{C}^{M \times K}$, and $\mathbf{s} \triangleq [s_1, \dots, s_K]^T$. And the transmit MMSE problem can be formulated as

$$\begin{aligned} \{\mathbf{w}_{T,i}^{MMSE}, \kappa_{MMSE,i}\} &= \arg \min_{\mathbf{w}, \kappa} \mathbb{E}\{\|y_i - \kappa^{-1} z_{i,i}\|_2^2\} \\ \text{s.t. } &\mathbb{E}\{\|\mathbf{w} y_i\|_2^2\} = P_r. \end{aligned} \quad (4.43)$$

And solving (4.43) follows the same way of solving (4.24), thus we obtain $\mathbf{w}_{T,i}^{MMSE}$ as

$$\begin{aligned} \mathbf{w}_{T,i}^{MMSE} &= \kappa_{MMSE,i} \mathbf{J}_i^{-1} \mathbf{g}_{i,i}, \\ \kappa_{MMSE,i} &= \sqrt{\frac{P_r}{\text{tr}(\mathbf{J}_i^{-2} \mathbf{g}_{i,i}^* R_{y_i} \mathbf{g}_{i,i}^T)}}, \end{aligned}$$

where $\mathbf{J}_i \triangleq \mathbf{g}_{i,i}^* \mathbf{g}_{i,i}^T + \frac{\sigma_v^2}{P_r} \mathbf{I}$ and $R_{y_i} \triangleq \mathbb{E}[y_i y_i^*]$.

3) For MMSE matrix \mathbf{W}_i^{MMSE} : Using the Sherman Morrison formula $(\mathbf{A} + \mathbf{u}\mathbf{v}^H)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1} \mathbf{u} \mathbf{v}^H \mathbf{A}^{-1}}{1 + \mathbf{v}^H \mathbf{A}^{-1} \mathbf{u}}$, for matrix \mathbf{A} and vector \mathbf{u}, \mathbf{v} [68], we can rewrite $\mathbf{w}_{R,i}^{MMSEH}$ as

$$\begin{aligned} \mathbf{w}_{R,i}^{MMSEH} &= \mathbf{f}_{i,i}^H (\mathbf{f}_{i,i} \mathbf{f}_{i,i}^H + \frac{\sigma_v^2}{P_0} \mathbf{I})^{-1} \\ &= \mathbf{f}_{i,i}^H \left(\frac{P_0}{\sigma_v^2} \mathbf{I} - \frac{\frac{P_0}{\sigma_v^2} \mathbf{f}_{i,i} \mathbf{f}_{i,i}^H \frac{P_0}{\sigma_v^2} \mathbf{I}}{1 + \mathbf{f}_{i,i}^H \frac{P_0}{\sigma_v^2} \mathbf{f}_{i,i}} \right) \\ &= \frac{P_0}{\sigma_v^2} (\mathbf{f}_{i,i}^H - \frac{\mathbf{f}_{i,i}^H \mathbf{f}_{i,i} \mathbf{f}_{i,i}^H}{\frac{P_0}{\sigma_v^2} + \mathbf{f}_{i,i}^H \mathbf{f}_{i,i}}) \\ &= \lambda_i \mathbf{f}_{i,i}^H, \end{aligned} \quad (4.44)$$

where $\lambda_i = (\frac{\sigma_n^2}{P_0} + \mathbf{f}_{i,i}^H \mathbf{f}_{i,i})^{-1} \in \mathbb{R}$. Similar for $\mathbf{w}_{T,i}^{\text{MMSE}}$, we have

$$\begin{aligned} \mathbf{w}_{T,i}^{\text{MMSE}} &= \kappa_{\text{MMSE},i} \mathbf{J}_i^{-1} \mathbf{g}_{i,i}^* \\ &= \kappa_{\text{MMSE},i} (\mathbf{g}_{i,i}^* \mathbf{g}_{i,i}^T + \frac{\sigma_n^2}{P_r} \mathbf{I})^{-1} \mathbf{g}_{i,i}^* \\ &= \kappa_{\text{MMSE},i} \mu_i \mathbf{g}_{i,i}^*, \end{aligned} \quad (4.45)$$

where $\mu_i = (\frac{\sigma_n^2}{P_r} + \mathbf{g}_{i,i}^T \mathbf{g}_{i,i}^*)^{-1} \in \mathbb{R}$. Next we rewrite $\kappa_{\text{MMSE},i}$ as following

$$\begin{aligned} \kappa_{\text{MMSE},i}^2 &= \frac{P_r}{\text{tr}(\mathbf{J}_i^{-2} \mathbf{g}_{i,i}^* \mathbf{R}_{y_i} \mathbf{g}_{i,i}^T)} \\ &= \frac{P_r}{\text{tr}((\mathbf{J}_i^{-1} \mathbf{g}_{i,i}^*) \mathbf{R}_{y_i} (\mathbf{J}_i^{-1} \mathbf{g}_{i,i}^*)^H)} \\ &= \frac{P_r}{\text{tr}(\mathbf{R}_{y_i}) \mu_i^2 \|\mathbf{g}_{i,i}\|^2}. \end{aligned}$$

Also from 4.44 we have

$$\mathbf{R}_{y_i} = \mathbb{E}\{|\mathbf{w}_R^{\text{MMSE}H} \mathbf{r}_i|^2\} = \lambda_i^2 \mathbf{f}_{i,i}^H \mathbf{R}_{\mathbf{r}_i} \mathbf{f}_{i,i}. \quad (4.46)$$

Using (4.44), (4.45) (4.46), we can rewrite $\mathbf{W}_i^{\text{MMSE}}$ as

$$\begin{aligned} \mathbf{W}_i^{\text{MMSE}} &= \mathbf{w}_{T,i}^{\text{MMSE}} \mathbf{w}_{R,i}^{\text{MMSE}H} \\ &= \sqrt{\frac{P_r}{\text{tr}(\mathbf{f}_{i,i}^H \mathbf{R}_{\mathbf{r}_i} \mathbf{f}_{i,i}) \lambda_i^2 \mu_i^2 \|\mathbf{g}_{i,i}\|^2}} \mu_{i,i} \mathbf{g}_{i,i}^* \lambda_i \mathbf{f}_{i,i}^H \\ &= \sqrt{\frac{P_r \|\mathbf{f}_{i,i}\|^2}{\text{tr}(\mathbf{f}_{i,i}^H \mathbf{R}_{\mathbf{r}_i} \mathbf{f}_{i,i})}} \frac{\mathbf{g}_{i,i}^*}{\|\mathbf{g}_{i,i}\|} \frac{\mathbf{f}_{i,i}^H}{\|\mathbf{f}_{i,i}\|} \\ &= \mathbf{W}_i^{\text{MRC}} \end{aligned}$$

So we have proved that in single pair scenario, $\mathbf{W}_i^{\text{MMSE}}$ and $\mathbf{W}_i^{\text{MRC}}$ are the same. ■

Chapter 5

Conclusions

In this thesis, we consider a multi-cluster AF relay interference network and design relay beam matrix for each cluster to maximize the minimum SINR at destinations subject to the relay power budget within each cluster. We assume that the dedicated relay(s) for each pair can be either a multi-antenna relay or multiple relays capable of signal sharing to form virtual multi-antenna, and both single and multiple S-D pair scenarios are considered.

We first consider the problem in single pair scenario. A beam matrix structure as a weighted sum of two types of beam matrices is proposed: ZF beam matrix for inter-cluster interference suppression, and MRC beam matrix for beamforming gain maximization within a cluster. The optimal beam matrix for each type is obtained with a closed-form solution. We then obtain the optimal weights to each type of beam matrix by transforming the max-min SINR problem and solving it via the SDR approach. Comparing with applying the direct SDR approach to the original problem, our solution offers similar performance with significantly lower computational complexity. In addition, our proposed structured beam matrix clearly revealed the power shift between interference suppression among clusters and beamforming gain

maximization within the cluster as the distance among clusters changes.

We then extend our study to multiple S-D pair case in which different users in the same cluster also communicate interference to each other. A similar beam matrix as weighted sum of ZF and MMSE is proposed to solve the problem together with a direct SDR approach as a comparison. Simulations show that our proposed solution provides similar SINR performance with that of direct approach, and the performance gap decreases as the number of relays increases. Additionally, proposed approach also has significant lower computational complexity and the ability of revealing the power shift as the distance among clusters or size of clusters changes.

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