

Joint Relay Beamforming and Transceiver Processing
in Multiuser Relay Network

by

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Abstract

In this thesis, we focus on joint relay beamforming and transceiver processing in multi-user relay network. First of all, we consider the scenario in which multiple source-to-destination (S-D) pairs intend to communicate with the help of multiple distributed amplify-and-forward (AF) relays. A rank-two beamforming Alamouti scheme is proposed at the sources and relays, and we aim to minimize maximal individual relay power subject to pre-defined SINR requirements. The resulting non-convex optimization problem is solved by ordinary semi-definite relaxation (SDR) and separable SDR approaches. Compared to conventional rank-one scheme, proposed rank-two methods provide one more degree of freedom in optimal solution, and have significantly better performance in terms of min-max per-relay power and optimality gap.

Secondly, we consider the scenario where multiple users exchange information with each other via a multi-way multi-antenna relaying. Our objective is to jointly design both relay beamforming and receiver linear processing to maximize the minimum signal-to-interference-and-noise ratio (SINR) under a relay power budget. The joint optimization problem is iteratively solved by designing relay beam matrix and receiver processing matrix. For the latter, both maximum-ratio-combining (MRC) receiver and zero-forcing (ZF) receiver are designed. The MRC receiver leads to the optimal

iterative design while the ZF receiver has lower computational complexity. We also use successive interference cancellation (SIC) as our decoding strategy to further enhance sum-rate. Simulation results show that the proposed iterative algorithm yields higher achievable sum-rate than the existing partial ZF (PZF) method which uses sum-rate maximization as the design objective.

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Chapter 1

Introduction

1.1 Overview

The massive growth of users in wireless network and the increasing demand for high data rate services, like video conference and virtual reality, have inspired the explosive growth of research in wireless communication. To cater the need of a high quality, low cost, and easy-to-access wireless network, numerous technologies have been developed to exploit the diversity of time, frequency, code and space. And recently, another type of diversity called cooperative diversity has attracted the attention of many researchers.

To achieve cooperative diversity, users of a wireless network cooperate by relaying each others messages and forming a virtual multi-antenna system for joint transmission and reception. It is highly beneficial in wireless communication systems in various aspects, including communication range extension, energy efficiency improvement and capacity enhancement [1–4]. Such relaying structure is also widely adopted in many current wireless systems, such as LTE [5] for 4G cellular networks and bluetooth. Also, research has found out that by using multiple-input-multiple-output (MIMO) systems, the performance of wireless networks can be improved significantly [6, 7].

It is then reasonable to consider using MIMO systems in combination with relaying. By accommodating multiple antennas at the relay nodes, higher diversity gain, more degrees of freedom and better performance can be achieved [8–10].

In this thesis, we first focus on distributed relay beamforming design in a multi-user peer-to-peer (MUP2P) relay network. We propose a joint transmit and relaying strategy via Alamouti scheme for rank-two relay beamforming to minimize maximal per-relay power consumption under the quality of service (QoS) constraints. Next, we consider multi-way multi-antenna (MWMA) relaying for multi-user communications. A joint design of relay beamforming and receiver processing is proposed to maximize the minimal SINR under the total relay power budget.

1.2 Cooperative Relay Network

A relay network is a broad class of network topology used in current wireless networks, where the source and destination are interconnected by relays to help data transmission between them, as shown in Fig 1.1. In such a network, the source and the destination may not communicate to each other directly due to distance limitation and signal quality. Hence the help of relays is needed to forward the replicas of signal from source to the destination, thereby not only to improve communication range and energy efficiency, but also to increase transmission diversity and signal quality [8, 11, 12].

Various relay schemes have been proposed, which can be categorized as amplify-and-forward (AF), decode-and-forward (DF), compress-and-forward (CF). Generally, in an AF relay scheme, relay amplifies what it receives, and send the amplified signal

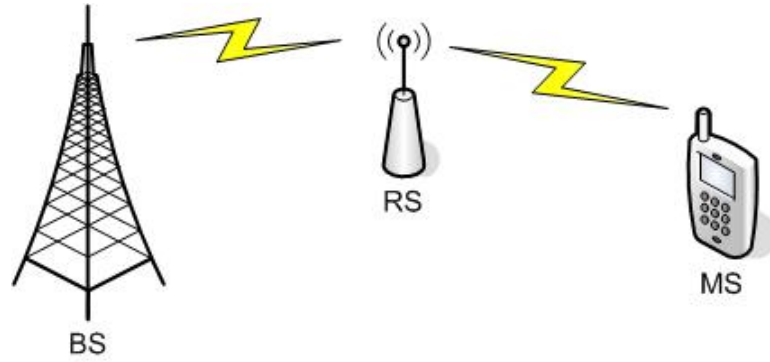


Figure 1.1: Cooperative relay network.

to the other end. It has been extensively used in practice due to its simplicity. While in a more complex DF scheme, relay station first decodes the received signal and then retransmits the decoded and regenerated symbols. DF scheme is also known as regenerative approach, and its performance is largely affected by the coding scheme applied at the relay. The CF scheme allows relay station to compress the received signal from source node and forward it to destination without decoding the signal. The receiver can then combine the two observations from source and relay and exploit the correlation between them at the destination.

Relay networks can also be categorized into different types by using different standards. By the duplexity of transmission, networks can be classified into one-way, two-way and multi-way relay networks (MWRNs), where the first is half-duplex and the rest two utilize full-duplex transmission. Especially in MWRNs, multiple users exchange information with each other under the help of one cooperative relay node. By carefully leveraging user interference, MWRNs are able to significantly improve spectral efficiency in wireless communication systems [13]. In addition, based on the number of source-destination (S-D) pairs, relay networks can be sorted into single-pair

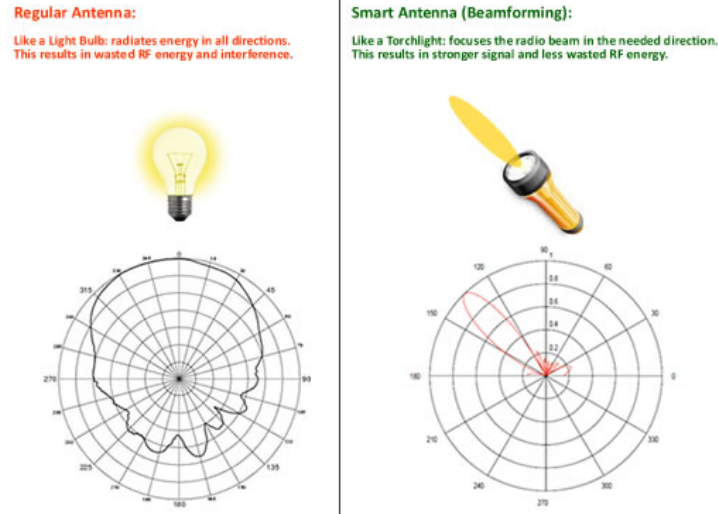


Figure 1.2: Beamforming technique

and MUP2P relay networks. The latter are able to accommodate more user pairs, and the role of relays here is to mitigate the cross-link interference and establish wireless connections between designated sources and destinations. In this thesis, we focus on the multi-user relaying scenarios, including MUP2P relaying and multi-way relaying.

1.3 Relay Beamforming Technique

The term *beamforming* originates from the fact that early spatial filters are designed to form pencil beams in order to receive signal radiating from a specific direction and to attenuate signals from other directions [14]. In a multi-antenna wireless system, beamforming is a low complexity technique for obtaining the spatial diversity provided by multiple antennas. Through concentrating power to the channel direction, beamforming has the ability to enhance the desired signal and reject interference, as shown in Fig. 1.2. The advantage of using beamforming includes improved signal-to-noise ratio (SNR), reduced power usage and extended transmission distance [15–17], thus it is widely applied in radar, sonar, and many wireless communication systems.

For relay beamforming, a set of relays form a virtual antenna array to forward signal from sources. By cooperating with each other, relays can focus power on the direction of desired destination, thus increasing diversity gain without the need of multiple antennas on each node. Depending on the information sharing scheme, relay beamforming can be categorized into centralized relay beamforming and distributed relay beamforming.

Centralized relay beamforming: In this scheme, signal is first processed over multi-antennas before forwarded by relays, then sent through independent paths, at last added coherently together at destination. It requires multiple antennas at the relay or multiple relays capable of information sharing, which may be hard to implement due to size and processing power of some mobile wireless devices. On the other hand, with the same number of antennas, centralized relay beamforming will provide equal or better performance compared with distributed beamforming [8,9]. Many existing works have studied the capacity, SNR performance and power consumption of centralized relay beamforming scheme [3, 9, 10, 18–22].

Distributed relay beamforming: In the distributed scheme, multiple independent relays (antennas) simultaneously transmit the same signal with controlled phase at the same frequency, so that signal can be constructively combined at a destination. It needs neither multiple antennas nor signal level cooperation among multiple single antenna nodes, and thus it is easy to implement. However, there can be some performance loss as compared with centralized relay beamforming due to less degree of freedom in the beamforming design. The diversity, capacity, robustness and power consumption under different constraints of distributed relay beamforming have been

studied in [12, 23–28]. In our work, we propose a combination of distributed relay beamforming technique and Alamouti coding scheme, aiming at further decreasing relay power consumption under the target QoS requirements.

1.4 Motivation and Objective

The peer-to-peer (P2P) distributed relay network is well studied in existing works for both single S-D pair and multiple S-D pairs scenarios. The optimal design for relay beamforming matrix has been proposed to minimize individual or total relay power under pre-defined QoS requirement. Aiming to minimize individual relay power, problem formulation always leads to a non-convex max-min-fair (MMF) problem where semi-definite relaxation (SDR) technique can be applied to relax the problem and find a solution through semidefinite programming (SDP) solvers. However, the solution of the relaxed problem can not always be used to recover the optimal rank-one solution for the original MMF problem. As a result, a randomization procedure is needed to generate a feasible but suboptimal solution. How to increase the likelihood of obtaining the optimal solution through SDR is an area of active research by many. Some recent results show that rank-two beamforming with Alamouti scheme is practical in multicast relaying and multi-group multicast transmission to enhance receiver performance and shrink performance gap between the optimal solution of SDR and the sub-optimal solution generated by randomization following SDR.

Considering the above, in this thesis, we plan to apply Alamouti coding scheme in P2P distributed relay network for source and relay transmission design. Our goal is to minimize individual relay power subject to target signal-to-interference-and-noise

ratio (SINR) requirements at destinations, at the same time, to increase the chance of obtaining the global optimal solution by taking advantage of Alamouti coding scheme.

In a MWMA relay network, multiple transmission slots are required for information exchange among users. In many existing studies, relay beamforming, such as ZF, minimum mean square error (MMSE) and match filter (MF), is separately designed in each broadcast (BC) phases with special structures, to enhance the sum-rate among all users. Also in some works, joint relay beamforming design and pair transmission strategies are proposed to further increase the network capacity. However, none of these works consider joint relay and receiver processing design and optimal relay beamforming. In the second part of this thesis, we propose a joint design of relay processing over multiple BC phases and receiver processing at each user aiming at further performance improvement of MWMA relay network. The receiver processing is based on all received signals from multiple BC phases to optimize the SINR performance.

1.5 Thesis Contribution

In this thesis, we investigate the joint design of relay beamforming and transceiver processing in an MUP2P network and an MWMA relay network. For the former, we jointly consider the transmission design of sources and relays, while for the latter we study the joint design of relay beam matrices and receiver processing matrices.

Multi-user peer-to-peer network: In this scenario, we consider communication between multiple S-D pairs assisted by several distributed AF relays. By applying rank-two beamforming with Alamouti scheme, we propose a joint transmission strategy at sources and relays aiming to minimize maximal individual relay power subject

to SINR constraint for each S-D pair. Two approaches, ordinary SDR and separable SDR, have been considered to solve the problem. Our solution structure shows that we can achieve two optimal rank-one beam weights for relay processing. Comparing with the conventional rank-one beamforming scheme, this rank-two optimal solution leads to a lower power consumption among relays. In addition, numerical results show that our proposed scheme can significantly enhance the chance for achieving optimality. At last, we prove that the worst-case approximation accuracy of proposed scheme scales on the order of $\sqrt{K} \log M$, where K is the number of S-D pairs in the network and M is the number of distributed relays.

Multi-way multi-antenna relaying network: In an MWMA relay network, multiple users exchange information through a multi-antenna relay. We formulate the joint optimization of relay and receiver processing problem to maximize minimal SINR for detecting symbols at all users. An iterative algorithm is proposed to solve the joint optimization problem. For receiver processing, two receiver linear processing structures, maximum-ratio-combining (MRC) and zero-forming (ZF), have been considered. Our proposed iterative algorithm with MRC receiver structure provides the optimal receiver processing design, while the ZF receiver incurs lower computational complexity. Although maximizing minimal SINR is our design objective, simulation shows that our proposed algorithm yields a higher achievable sum-rate than the existing state-of-art partial ZF (PZF) method which uses the sum-rate as maximization objective. To further improve the receiver decoding performance, we apply successive interference cancellation (SIC) technique as our decoding algorithm, which further enhances receiving sum-rate performance. Finally, we investigate the effect of imper-

fect channel state information (CSI) on the performance by analyzing performance loss due to CSI quantization.

1.6 Thesis Organization

The remainder of this thesis is organized as follows. In Chapter 2, a literature survey of related works on the related topics is provided. In Chapter 3, a power optimization problem for MUP2P relay network is formulated, in which a rank-two beamforming design is applied through Alamouti-based joint source and relay transmission strategy. Two approaches are proposed for the optimization problem. In Chapter 4, a joint design for relay beamforming and receiving processing is considered in MWMA relay network aiming at higher achievable SINR. An iterative algorithm is proposed for the joint optimization problem. The conclusion is provided in Chapter 5.

1.7 Notation

In the thesis, trace, Hermitian, transpose, and conjugate of \mathbf{A} are denoted by $\text{tr}[\mathbf{A}]$, \mathbf{A}^H , \mathbf{A}^T , and \mathbf{A}^* , respectively. The Kronecker product is denoted as \otimes . Notation $\text{vec}(\mathbf{A})$ means to vectorize $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_N]$ to $[\mathbf{a}_1^T, \dots, \mathbf{a}_N^T]^T$. A semi-definite matrix \mathbf{A} is denoted as $\mathbf{A} \succcurlyeq 0$, and \mathbf{I}_M denotes the $M \times M$ identity matrix. Notation $[\mathbf{A}]_{i,j}$ denote the (i, j) th entry of \mathbf{A} . Notation $\text{diag}(\mathbf{a})$ denotes a diagonal matrix, with the entries of the vector \mathbf{a} being its diagonal elements, and $\text{blkdiag}([\mathbf{A}_1, \dots, \mathbf{A}_M])$ denotes the block diagonal matrix formed from matrices $\mathbf{A}_1, \dots, \mathbf{A}_M$. Notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{a}, \mathbf{Y})$ means that \mathbf{x} is drawn from the complex Gaussian distribution with mean \mathbf{a} and covariance matrix \mathbf{Y} .

Chapter 2

Literature Review

2.1 One-Way Relay Networks

A one-way relay network, in general, consists of at least one source node, one relay node and one destination node. It is used to relay information from the source to the destination. There are extensive studies have been done for this scenario under the help of either a MIMO relay or several distributed relays, on the topics of power allocation, transmission protocols, receiving QoS, etc..

Considering single S-D pair setting, an optimal design of the processing matrix for a multi-antenna relay has been studied under different performance criteria, such as capacity, diversity gain, SNR maximization, and relay power minimization [9, 18–21]. Paper [19] studies the use of CSI at the relay station for the optimal beamforming design in non-regenerative cooperative schemes under a fixed power constraint. When CSI is fully available, the linear processing at the relay can be found analytically to increase mutual information. Additionally, it turns out that the mutual information maximizing solution is only achievable when the direct channel is known. With the absence of direct link, paper [9] develops the optimal non-regenerative MIMO relay matrix that maximizes the capacity between the source and the destination. Instead

of maximization of the relay channel capacity, paper [20] intend to maximize the SNR under relay power budget. A closed-form solution is proposed by using general rank beamforming approach [29]. They also show that for the case of statistically independent channels, the general rank beamforming approach results in a rank-one solution for the beamforming matrix regardless of the rank of channel correlation matrices. In [21], an optimal relay processing matrix has been designed to minimize the maximum per-antenna power budget subject to receiving SNR constraint. Due to the inherent complexity and non-convexity of power minimization problem, authors in [21] turn to Lagrangian dual domain and a semi-closed form solution is then obtained with low computational complexity.

In many situations, due to limited size and processing power, it is not practical to equip multiple antennas at a node. In this case, cooperative transmission via several distributed single-antenna nodes [23–26, 30, 31] can be used as an alternative. With channel information only known at the receiver, the non-coherent AF protocol is studied in [23], and distributed space-time coding is considered in [30]. In paper [24], authors deal with beamforming in relay networks with perfect CSI at the relays, receiver, and transmitter. Assuming every node has its own power constraint, it allows transmitter and relays to adaptively adjust not only beam directions but also their transmit power to improve the network performance. A multi-antenna transceiver of MIMO relay network is considered in [25]. Considering both perfect CSI and second-order statistics of CSI, it develops a linear processing scheme to satisfy a pre-defined QoS requirement with minimum relay transmit power. The more general case in one-way relay is MUP2P, which contains multiple S-D pairs in relay network. The

literature review on this topic can be found in Section 2.4.

2.2 Two-Way Relay Network

In two-way relaying schemes, one or multiple relays are deployed to establish a reliable bidirectional communication between the two transceivers. There are three approaches to implement a two-way relaying scheme: a four-step method which consists of two one-way relaying schemes, the three-step time division broadcast (TDBC) method, and the two-step multiple access broadcast (MABC) approach. The MABC technique requires only two time slots to exchange two symbols between the transceivers, and thus, is more bandwidth-efficient compared to the other two.

A lot of existing works [27, 32–36] study two-way relay network in different perspective. In [32], an analogue network coding (ANC) scheme has been employed to help devise an optimal relay beamforming structure for maximizing the smaller one of the receiving SNR under given power threshold. In [33], authors present an optimal joint relay selection and power allocation scheme to achieve the maximization of SNR in two-way relaying network. Applying linear beamforming techniques, joint source and relay beamforming is investigated in [34, 35]. To deal with relay processing complexity and imperfect channel information, a beamforming and combining based scheme has been proposed in [36] aiming at lower the symbol error rate with estimated channel gain.

2.3 Multi-Way Relay Network

In recent years, a new concept called MWRNs [1] has been proposed, where multiple users, without direct communication links among them, exchange information with each other under the help of one cooperative relay node. With intelligent relaying strategies, along with careful transceiver processing design, multi-way relaying can significantly improve spectral efficiency of a wireless system [13]. Generally, the transmission protocol for MWRN takes one or multiple multiple access (MAC) phases to convey the signal from users to relay, and relay then forwards processed signal among user in multiple BC phases. Depending on how many antennas the relay is equipped, each kind of phases will take one or multiple time slots. Hence we classify MWRN into single-antenna and multiple-antenna multi-way relay network.

2.3.1 Single-Antenna Relay

In single antenna MWRN, most existing literatures deal with half-duplex relaying mode [37–41]. A joint network and superposition coding scheme is proposed in [37] for the simultaneous transmission of multiple data streams over a relay network. Through half-duplex relaying mode, this scheme can expand achievable rate region with fewer transmission time slots. In [39], a so-called functional-decode-forward (FDF) coding strategy is proposed for the scheme where multiple users exchange information through a single relay at a common rate via AWGN multi-way channels. The coding scheme constructs a function which saves the relay from decoding individual messages before it broadcasts the functions back to the users. The authors in [41] study multi-way relaying when channel conditions are asymmetric. A pairwise transmission

is considered to maximize the achievable rate. Optimal user pairing is given for both DF and FDF protocols. Also they show that the achievable common rate for the network depends on the order in which the users are paired.

In full-duplex mode, both the users and relay can transmit and receive signals simultaneously. It has been investigated in [1, 13, 42] for multi-way relaying. The full-duplex data exchange model is studied by the author in [42]. They provide upper bounds on the symmetric capacity of the symmetric Gaussian MWRNs and evaluate the achievable symmetric rate for AF, DF and CF protocols. The capacity region of a class of multi-way relay channels is derived in [13], where the channel inputs and outputs take values over finite fields.

2.3.2 Multi-Antenna Relay

With multiple antennas considered at the relay in a MWRN, extra wireless channels will be created between the relay and the users. Benefiting from this, the required communication time slots can be considerably reduced, and further performance improvement is achievable.

Depending on whether relay try to decode and re-encode the data streams of the nodes or not, a relay node applying ANC can be classified into regenerative and non-regenerative cases. Regenerative multi-way relaying is studied in [43, 44]. In [43], authors propose an N -phase regenerative multi-way relaying for N nodes and design a transceiver strategy at relay which enables the relay to transmit in each BC phase with the achievable MAC rate while having minimum power. The same authors also expand the current results to multi-group multi-way relaying [44].

Non-regenerative multi-way relaying is studied in [45–48]. Under the scenario in which AF multi-antenna relay node assisting multiple nodes to communicate to each other, relaying protocol consists of a single MAC phase and multiple BC phases. Space-time analog network coding transmission and repetition transmission have been presented in [46] when CSI is not available at the relay. The authors of [45] devise three low complexity linear transceiver beamformers based on ZF, MMSE and MSNR criteria for N -phase multi-way relaying, and analyzes their performance in terms of sum-rate. While [45, 46] are focusing on one-group multi-way relaying, multi-group multi-way relaying is studied in [47, 48] where each group consists of multiple half-duplex nodes and each node wants to share its data with all other nodes within its group. ZF relay processing combined with pairwise and non-pairwise transmission scheme for MWMA relay network is studied [49], using the sum-rate objective. A relay PZF method is proposed in [50] to exploit degree of freedom of MWRN and a higher sum-rate is achieved compared with previous ZF method.

Although many special relay beamforming structures or pairwise transmission strategies are designed, the achieved results are still suboptimal. In this thesis, we will investigate joint design of relay beamforming and receiver processing, aiming to further enhance receiving performance and achieve optimality.

2.4 Multi-User Peer-to-Peer Relay Network

In MUP2P relay network, multiple pairwise users communicate through the assistance of relays. In such scheme, received signal contains interference caused by other sources. By smartly leveraging user interference instead of completely avoiding it, MUP2P

relay network is able to increase quality of received signal and improve power allocation efficiency.

Communication between each pair of users assisted by a MIMO relay has been studied in [3, 10, 22, 51]. In this scenario, users can benefit from MIMO structure to support high-data rates meanwhile combat fading and interference.

In [10], a typical P2P MIMO relay scenario is considered. Assuming that only imperfect CSI is available at the MIMO relay, it designs a MIMO relay beamformer in which the worst-case relay transmit power is minimized while keeping the worst-case SINR for all destinations above a certain threshold. An improvement on robustness is verified via this design. Papers [3, 22] discuss the same scenario under presence of full CSI. The former aims to minimize the sum mean-squared-error (MSE) by joint optimum relay and destination, while the latter proposes two different designs for the ZF beamforming matrix pursuing maximum sum-rate by either equally allocating the relay power for all data streams or adjusting the relay weights.

Comparing with distributed relay network, although MIMO relay provides much higher degree of freedom in beamforming design and greater diversity gain, it is limited in implementation due to design complexity and practical issue. Therefore, the analysis is later extended to a P2P distributed relay network where the relay node is located distributively thus generally no information sharing between each other.

In a P2P distributed relay network where communication is assisted by several distributed relays, beamforming design is presented in [12, 28, 52–54]. Assuming the transmitter, receiver, and relay nodes all use a single antenna, above works intend to obtain beamforming weights through minimizing total relay transmit power while

SINR requirement at destinations is promised. SDR has been applied in [12] for addressing non-convex minimization problem. The solution shows that significant power reduction can be achieved via space division multiplexing scheme, especially for high network data rates. Unlike the previous works, a joint optimization of the source power allocation and relay beamforming weights is developed in [28]. It aims at minimizing total transmit power from all sources and relays while guaranteeing the prescribed QoS requirement of each S-D pair. The proposed iterative feasibility search algorithm and constrained concave convex procedure based algorithms promise complexity reduction and performance enhancement.

In a P2P distributed relay network, typically source intends to forward one symbol to the specific destination, which makes it possible to consider the beamforming with Alamouti coding structure at transmitter and relays, in order to achieve a better receiver performance.

2.5 Rank-Two Beamforming with Alamouti Scheme

In physical-layer multicast relaying and multi-group multicast transmission, using SDR has been a popular approach for seeking a rank-one solution. When using SDR, rank-one beam weight needs to be recovered from optimal SDR solution, which may not be always available. In that case, we need to generate a sub-optimal rank-one feasible solution for original problem which will induce approximation loss. Considering a combination of beamforming and Alamouti space-time block code, rank-two beamformed Alamouti scheme has been studied in above scenarios [55–59]. By using Alamouti code, it reveals that the approximation accuracy degrades slower than

rank-one case.

For multi-user multi-input single-output (MISO) downlink channel, rank-two beamforming scheme has been proposed in [55, 56]. An analysis on the worst-case approximation shows that the accuracy declines at a rate of \sqrt{M} , slower than rank-one SDR scheme where M is the number of user served in the network. In addition, rank-two method has been adopted in [57] for multicasting relay networks to design a four time-slot transmission protocol which maximizes minimal individual SNR under network power constraints. Simulation results demonstrate significant performance improvement.

Rank-two beamforming with Alamouti scheme also works well in multi-group multicasting networks [58, 59]. The work [58] achieves further performance gain in multi-group multicast cognitive radio systems via a transmit beamformed Alamouti scheme, whose corresponding problem can be formulated as a rank-two constrained fractional semidefinite program. Authors in [59] consider a multi-antenna relay network. By applying Alamouti space-time code structure, the relays adopt two rank-one weights to convey signals in two time slots, which gives the beamformer one more degree of freedom compared with traditional beamforming schemes.

Above existing works only consider the multicasting scheme, with or without relay assistance. In an MUP2P network under the help of distributed relays, the rank-two Alamouti beamforming technique can be applied to provide more degree of freedom in relay beamforming design, and to increase QoS at receiver.

2.6 Limited CSI Feedback

In relay beamforming, transmitter adapts its transmission scheme depending on its CSI feedback from receiver. Perfect CSI (or instantaneous CSI) is required at transmitter for optimal relay beamforming. However, such information is not always available at transmitter at all times, and beamforming designs with limited CSI are then studied. There are two types of widely used limited CSI: quantized instantaneous CSI and channel statistics (means and covariance). The first type is studied in [60–62]. In [60], a beamforming scheme based on relay selection is proposed for limited feedback AF relay network. The performance gap between unlimited and limited feedback is found analytically, which grows rapidly with the number of relays. A generalized Lloyd algorithm is used in [61] to design the quantizer of the feedback information specifically to minimize the bit error rate (BER) of an AF relay network. Achievable bounds for SNR and BER are also derived for the method. For relay network using DF scheme, a similar topic is discussed in [62] where an optimal beamforming vector maximizing the receiver SNR is proposed together with an performance upper bound.

Some researchers focus on cases with second order channel statistics [26, 63, 64]. Distributed beamforming with second order statistics at relay is studied at [26]. This paper addresses two beamforming design approaches: minimize total transmit power with QoS constraint and maximize receiver SNR subject to total power constraint and per relay power constraint. In [63], a closed form distributed space-time coding with adaptive relay power control is proposed for a two-relay network, where pairwise error probability is minimized under separate relay power constraints.

Chapter 3

Relay Beamforming Design in Peer-to-Peer Relay Network

In this chapter, we consider multiple S-D pairs communicating through multiple distributed AF relays. We apply Alamouti precoding scheme at the sources and the relays aiming at minimizing relay powers under required SINR targets at destinations.

3.1 System Model

Let us consider a network with K S-D pairs communicating under the help of M relays. The sources, the relays and the destinations are single-antenna devices, as shown in Fig. 3.1. The channels between transmitting and receiving nodes are assumed to be frequency flat and constant over time slots for each S-D transmission. No direct link between S-D pairs is considered.

With AF relay strategy, we apply Alamouti scheme [65] to jointly design transmitting and processing scheme at sources and relays. Transmission protocol is designed within four consecutive time slots for relaying two symbols from a source to the corresponding destination. Basically, in the first two time slots, each source transmits two symbols to relay nodes. Through Alamouti encoding, relays convey the processed

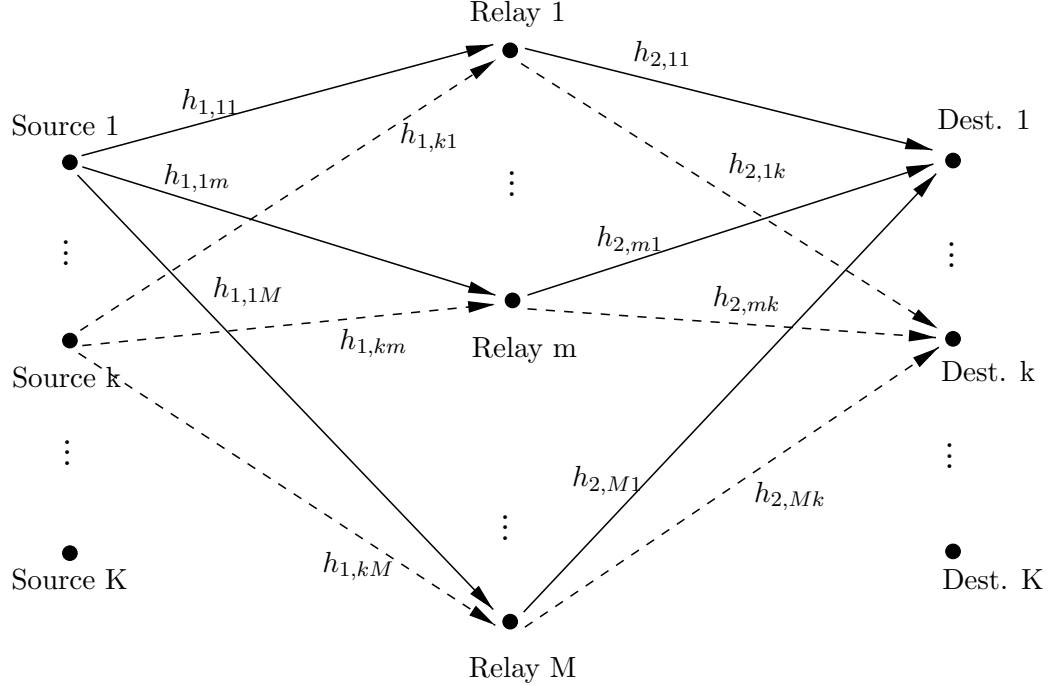


Figure 3.1: The multi-user P2P distributed relay network

signals to the destinations in next two time slots.

3.2 Alamouti Transmission and Relaying Scheme

In the first two time slots, K sources forward their own information symbols to relays. Let $h_{1,km}$ be channel coefficient from user k to relay m , which follows complex Gaussian distribution with zero-mean and variance σ_h^2 . Then channel vector from source k to the relays can be denoted as $\mathbf{h}_{1,k} \triangleq [h_{1,k1}, \dots, h_{1,kM}]^T$. In the first and second time slots, source k respectively transmits symbols $s_{k,1}$ and $s_{k,2}^*$ to relays with power $E|s_{k,1}|^2 = E|s_{k,2}^*|^2 = 1$, for $k = 1, \dots, K$. Let P_o denote the transmit power at each source. Then $M \times 1$ received signal vector \mathbf{r}_1 and \mathbf{r}_2 in the first and the second time slots at M relays are respectively given by

$$\mathbf{r}_1 = \sqrt{P_o} \sum_{k=1}^K \mathbf{h}_{1,k} s_{k,1} + \mathbf{n}_{r,1}, \quad \mathbf{r}_2 = \sqrt{P_o} \sum_{k=1}^K \mathbf{h}_{1,k} s_{k,2}^* + \mathbf{n}_{r,2} \quad (3.1)$$

where $\mathbf{n}_{r,1} \triangleq [n_{r,11}, \dots, n_{r,M1}]^T$ and $\mathbf{n}_{r,2} \triangleq [n_{r,12}, \dots, n_{r,M2}]^T$ are noise vectors at the relay receiver in the first two time slots, which is white Gaussian with covariance matrix being $\sigma_R^2 \mathbf{I}_M$.

Relays then encode \mathbf{r}_1 and \mathbf{r}_2 with Alamouti precoding scheme and transmit the processed signal in the third and fourth time slots. For relay m , define beam weights $w_{1,m}$, $w_{2,m}$. Then $\mathbf{w}_1 = [w_{1,1}, \dots, w_{1,M}]^T$ and $\mathbf{w}_2 = [w_{2,1}, \dots, w_{2,M}]^T$ are $M \times 1$ beamforming vectors at relays. Let $\mathbf{x}_1 \triangleq [x_{1,1}, \dots, x_{1,M}]^T$ and $\mathbf{x}_2 \triangleq [x_{2,1}, \dots, x_{2,M}]^T$ denote processed signal vectors at relays for transmission, they are given by

$$\mathbf{x}_1 = \mathbf{W}_1 \mathbf{r}_1 + \mathbf{W}_2 \mathbf{r}_2^*, \quad \mathbf{x}_2 = -\mathbf{W}_2 \mathbf{r}_1^* + \mathbf{W}_1 \mathbf{r}_2 \quad (3.2)$$

where $\mathbf{W}_1 \triangleq \text{diag}(\mathbf{w}_1)$, $\mathbf{W}_2 \triangleq \text{diag}(\mathbf{w}_2)$. Note that in (3.2), using two beam weight vectors \mathbf{w}_1 and \mathbf{w}_2 , the relays transmit vectors \mathbf{x}_1 and \mathbf{x}_2 are linear combinations of received signals and their conjugates. These two beam vectors will increase the degrees of freedom in the distributed beamforming design. This precoding scheme is similar to Alamouti scheme for OSTBC design in point-to-point transmission, where two data symbols are transmitted over two channels.

Let $y_{k,1}$ and $y_{k,2}$ be received signals at destination k in the third and the fourth

time slots, respectively. They are given by

$$\begin{aligned}
y_{k,1} &= \mathbf{h}_{2,k}^T \mathbf{x}_1 + n_{d,k1} \\
&= \sqrt{P_o} \mathbf{h}_{2,k}^T (\mathbf{W}_1 \mathbf{h}_{1,k} s_{k,1} \\
&\quad + \mathbf{W}_2 \mathbf{h}_{1,k}^* s_{k,2}) + \sqrt{P_o} \mathbf{h}_{2,k}^T (\mathbf{W}_1 \sum_{l=1, l \neq k}^K \mathbf{h}_{1,l} s_{l,1} + \mathbf{W}_2 \sum_{l=1, l \neq k}^K \mathbf{h}_{1,l}^* s_{l,2}) \\
&\quad + \mathbf{h}_{2,k}^T (\mathbf{W}_1 \mathbf{n}_{r,1} + \mathbf{W}_2 \mathbf{n}_{r,2}^*) + n_{d,k1}
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
y_{k,2} &= \mathbf{h}_{2,k}^T \mathbf{x}_2 + n_{d,k2} \\
&= \sqrt{P_o} \mathbf{h}_{2,k}^T (-\mathbf{W}_2 \mathbf{h}_{1,k}^* s_{k,1}^* + \mathbf{W}_1 \mathbf{h}_{1,k} s_{k,2}^*) \\
&\quad + \sqrt{P_o} \mathbf{h}_{2,k}^T (-\mathbf{W}_2 \sum_{l=1, l \neq k}^K \mathbf{h}_{1,l}^* s_{l,1}^* + \mathbf{W}_1 \sum_{l=1, l \neq k}^K \mathbf{h}_{1,l} s_{l,2}^*) \\
&\quad + \mathbf{h}_{2,k}^T (-\mathbf{W}_2 \mathbf{n}_{r,1}^* + \mathbf{W}_1 \mathbf{n}_{r,2}^*) + n_{d,k2}
\end{aligned} \tag{3.4}$$

where $\mathbf{h}_{2,k} \triangleq [h_{2,1k}, \dots, h_{2,Mk}]^T$ is the channel vector between M relays and destination k , $n_{d,k1}$ and $n_{d,k2}$ are the receiver noise at destination k in the third and fourth time slots, respectively, which are AWGN with variance σ_D^2 .

Define $\mathbf{y}_k \triangleq [y_{k,1}, y_{k,2}]^T$ as the received signal vector at destination k . Let $\mathbf{s}_k \triangleq [s_{1,k}, s_{2,k}]^T$, $\mathbf{n}_r \triangleq [\mathbf{n}_{r,1}^T, \mathbf{n}_{r,2}^H]^T$ and $\mathbf{n}_{d,k} \triangleq [n_{d,k1}, n_{d,k2}^*]^T$. From (3.3) and (3.4), we have

$$\mathbf{y}_k = \sqrt{P_o} \mathbf{H}_{2,k} \mathbf{W} \mathbf{H}_{1,k} \mathbf{s}_k + \sqrt{P_o} \mathbf{H}_{2,k} \mathbf{W} \sum_{l=1, l \neq k}^K \mathbf{H}_{1,l} \mathbf{s}_l + \mathbf{H}_{2,k} \mathbf{W} \mathbf{n}_r + \mathbf{n}_{d,k} \tag{3.5}$$

where $\mathbf{H}_{2,k} \triangleq \begin{bmatrix} \mathbf{h}_{2,k}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_{2,k}^H \end{bmatrix}$, $\mathbf{W} \triangleq \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 \\ -\mathbf{W}_2^* & \mathbf{W}_1^* \end{bmatrix}$, and $\mathbf{H}_{1,k} \triangleq \begin{bmatrix} \mathbf{h}_{1,k} & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_{1,k}^* \end{bmatrix}$. Note that the first term in (3.5) contains desired signal vector for destination k , the second term is the interference from other sources, and the third and fourth terms are the relay amplified noise and receiving noise, respectively.

3.3 Problem Formulation

From (3.5), the received SINR for k th S-D pair can be written as a function of \mathbf{W} ,

$$\text{SINR}_k(\mathbf{W}) = \frac{P_o \text{tr}(\mathbf{H}_{1,k}^H \mathbf{W}^H \mathbf{H}_{2,k}^H \mathbf{H}_{2,k} \mathbf{W} \mathbf{H}_{1,k})}{\mathcal{I}_{\text{int}} + \sigma_R^2 \text{tr}(\mathbf{W}^H \mathbf{H}_{2,k}^H \mathbf{H}_{2,k} \mathbf{W}) + 2\sigma_D^2} \quad (3.6)$$

where $\mathcal{I}_{\text{int}} \triangleq P_o \sum_{l=1, l \neq k}^K \text{tr}(\mathbf{H}_{1,l}^H \mathbf{W}^H \mathbf{H}_{2,k}^H \mathbf{H}_{2,k} \mathbf{W} \mathbf{H}_{1,l})$, for $k = 1, \dots, K$.

The power usage at relay m , in the third and fourth time slots, denoted as $P_{m,1}$ and $P_{m,2}$, respectively, are given by

$$\begin{aligned} P_{m,1} &= \mathbf{E}|x_{1,m}|^2 = \mathbf{E}|w_{1,m}r_{1,m} + w_{2,m}r_{2,m}^*|^2, \\ P_{m,2} &= \mathbf{E}|x_{2,m}|^2 = \mathbf{E}|-w_{2,m}r_{1,m}^* + w_{1,m}r_{2,m}|^2. \end{aligned} \quad (3.7)$$

Let P_m be the larger transmitted power at relay m in these two time slots, *i.e.*, $P_m \triangleq \max\{P_{m,1}, P_{m,2}\}$.

Our goal is to design relay beam weight $\mathbf{w}_1, \mathbf{w}_2$ (*i.e.*, \mathbf{W}) to minimize maximal relay power consumption, while satisfying the received SINR requirement at each destination. Let γ_k be received SINR target at destination k . The optimization problem is formulated as

$$\begin{aligned} \min_{\mathbf{W}} \max_{m \in \{1, \dots, M\}} P_m & \quad (3.8) \\ \text{s.t. SINR}_k(\mathbf{W}) \geq \gamma_k, \quad k = 1, \dots, K. & \end{aligned}$$

The above optimization problem for MUP2P network is non-convex and difficult to solve. In following, we first simplify the SINR and the relay power expressions, and then adopt ordinary SDR and separable SDR methods to solve the optimization problem.

Next, to simplify SINR expression in (3.6), it is noticed that both signal and interference components in (3.6) contains term $\mathbf{H}_{2,k}\mathbf{W}\mathbf{H}_{1,l}$. Expanding this term, we have

$$\mathbf{H}_{2,k}\mathbf{W}\mathbf{H}_{1,l} = \begin{bmatrix} \mathbf{h}_{2,k}^T \mathbf{W}_1 \mathbf{h}_{1,l} & \mathbf{h}_{2,k}^T \mathbf{W}_2 \mathbf{h}_{1,l}^* \\ -\mathbf{h}_{2,k}^H \mathbf{W}_2^* \mathbf{h}_{1,l} & \mathbf{h}_{2,k}^H \mathbf{W}_1^* \mathbf{h}_{1,l}^* \end{bmatrix}.$$

Taking advantage of this special structure, we have

$$\mathbf{H}_{1,l}^H \mathbf{W}^H \mathbf{H}_{2,k}^H \mathbf{H}_{2,k} \mathbf{W} \mathbf{H}_{1,l} = (|\mathbf{h}_{2,k}^T \mathbf{W}_1 \mathbf{h}_{1,l}|^2 + |\mathbf{h}_{2,k}^T \mathbf{W}_2 \mathbf{h}_{1,l}^*|^2) \mathbf{I}_2. \quad (3.9)$$

Since \mathbf{W}_1 and \mathbf{W}_2 are diagonal matrices, we can rewrite norm square in (3.9) as

$$|\mathbf{h}_{2,k}^T \mathbf{W}_1 \mathbf{h}_{1,l}|^2 = |\mathbf{w}_1^T \hat{\mathbf{H}}_{2,k} \mathbf{h}_{1,l}|^2 = |\mathbf{w}_1^H \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,l}^*|^2 \quad (3.10)$$

where $\hat{\mathbf{H}}_{2,k} \triangleq \text{diag}(\mathbf{h}_{2,k})$.

From (3.9) and (3.10), the received signal power, as the numerator in SINR expression (3.6) can be rewritten as

$$\begin{aligned} & P_o \text{tr} \left(\mathbf{H}_{1,k}^H \mathbf{W}^H \mathbf{H}_{2,k}^H \mathbf{H}_{2,k} \mathbf{W} \mathbf{H}_{1,k} \right) \\ &= 2P_o \left(|\mathbf{w}_1^H \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k}^*|^2 + |\mathbf{w}_2^H \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k}|^2 \right) \\ &= 2P_o \left(\mathbf{w}_1^H \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k}^* \mathbf{h}_{1,k}^T \hat{\mathbf{H}}_{2,k} \mathbf{w}_1 + \mathbf{w}_2^H \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k} \mathbf{h}_{1,k}^H \hat{\mathbf{H}}_{2,k} \mathbf{w}_2 \right) \\ &= 2P_o \mathbf{w}^H \mathcal{H}_{kk} \mathbf{w} \end{aligned} \quad (3.11)$$

where $\mathcal{H}_{kk} \triangleq \text{blkdiag}[\hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k}^* \mathbf{h}_{1,k}^T \hat{\mathbf{H}}_{2,k}, \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k} \mathbf{h}_{1,k}^H \hat{\mathbf{H}}_{2,k}]$ is the compound channel between source k and destination k , and $\mathbf{w} \triangleq [\mathbf{w}_1^T, \mathbf{w}_2^T]^T$ is relay processing vectors for the consecutive two time slots.

Similarly, the interference term in (3.6) can be expressed as

$$\mathcal{I}_{\text{int}} = P_o \sum_{l=1, l \neq k}^K \text{tr} \left(\mathbf{H}_{1,l}^H \mathbf{W}^H \mathbf{H}_{2,k}^H \mathbf{H}_{2,k} \mathbf{W} \mathbf{H}_{1,l} \right) = 2P_o \mathbf{w}^H \left(\sum_{l=1, l \neq k}^K \mathcal{H}_{lk} \right) \mathbf{w} \quad (3.12)$$

where $\mathcal{H}_{lk} \triangleq \text{blkdiag}[\hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,l}^* \mathbf{h}_{1,l}^T \hat{\mathbf{H}}_{2,k}, \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,l} \mathbf{h}_{1,l}^H \hat{\mathbf{H}}_{2,k}]$. The relay amplified noise term in (3.6) can be rewritten as

$$\begin{aligned}
& \sigma_R^2 \text{tr} \left(\mathbf{W}^H \mathbf{H}_{2,k}^H \mathbf{H}_{2,k} \mathbf{W} \right) \\
&= \sigma_R^2 \text{tr} \left\{ \begin{bmatrix} \mathbf{W}_1^H \mathbf{h}_{2,k}^* & -\mathbf{W}_2^T \mathbf{h}_{2,k} \end{bmatrix} \begin{bmatrix} \mathbf{h}_{2,k}^T \mathbf{W}_1 & \mathbf{h}_{2,k}^T \mathbf{W}_2 \\ -\mathbf{h}_{2,k}^H \mathbf{W}_2^* & \mathbf{h}_{2,k}^H \mathbf{W}_1^* \end{bmatrix} \right\} \\
&= \sigma_R^2 \text{tr} \left\{ \begin{bmatrix} \mathbf{W}_1^H \mathbf{h}_{2,k}^* \mathbf{h}_{2,k}^T \mathbf{W}_1 & \mathbf{W}_1^H \mathbf{h}_{2,k}^* \mathbf{h}_{2,k}^T \mathbf{W}_2 \\ \mathbf{W}_2^H \mathbf{h}_{2,k}^* \mathbf{h}_{2,k}^T \mathbf{W}_1 & \mathbf{W}_2^H \mathbf{h}_{2,k}^* \mathbf{h}_{2,k}^T \mathbf{W}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{W}_2^T \mathbf{h}_{2,k} \mathbf{h}_{2,k}^H \mathbf{W}_2^* & -\mathbf{W}_2^T \mathbf{h}_{2,k} \mathbf{h}_{2,k}^H \mathbf{W}_1^* \\ -\mathbf{W}_1^T \mathbf{h}_{2,k} \mathbf{h}_{2,k}^H \mathbf{W}_2^* & \mathbf{W}_1^T \mathbf{h}_{2,k} \mathbf{h}_{2,k}^H \mathbf{W}_1^* \end{bmatrix} \right\} \\
&= \sigma_R^2 \left[\text{tr}(\mathbf{W}_1^H \mathbf{h}_{2,k}^* \mathbf{h}_{2,k}^T \mathbf{W}_1) + \text{tr}(\mathbf{W}_2^H \mathbf{h}_{2,k}^* \mathbf{h}_{2,k}^T \mathbf{W}_2) + \text{tr}(\mathbf{W}_2^T \mathbf{h}_{2,k} \mathbf{h}_{2,k}^H \mathbf{W}_2^*) + \text{tr}(\mathbf{W}_1^T \mathbf{h}_{2,k} \mathbf{h}_{2,k}^H \mathbf{W}_1^*) \right] \\
&= \sigma_R^2 \left[\text{tr}(\hat{\mathbf{H}}_{2,k}^* \mathbf{w}_1^* \mathbf{w}_1^T \hat{\mathbf{H}}_{2,k}) + \text{tr}(\hat{\mathbf{H}}_{2,k} \mathbf{w}_2 \mathbf{w}_2^H \hat{\mathbf{H}}_{2,k}^*) + \text{tr}(\hat{\mathbf{H}}_{2,k}^* \mathbf{w}_2^* \mathbf{w}_2^T \hat{\mathbf{H}}_{2,k}) + \text{tr}(\hat{\mathbf{H}}_{2,k} \mathbf{w}_1 \mathbf{w}_1^H \hat{\mathbf{H}}_{2,k}^*) \right] \\
&= \sigma_R^2 \left[\mathbf{w}_1^T \hat{\mathbf{H}}_{2,k} \hat{\mathbf{H}}_{2,k}^* \mathbf{w}_1^* + \mathbf{w}_2^H \hat{\mathbf{H}}_{2,k}^* \hat{\mathbf{H}}_{2,k} \mathbf{w}_2 + \mathbf{w}_2^T \hat{\mathbf{H}}_{2,k} \hat{\mathbf{H}}_{2,k}^* \mathbf{w}_2^* + \mathbf{w}_1^H \hat{\mathbf{H}}_{2,k}^* \hat{\mathbf{H}}_{2,k} \mathbf{w}_1 \right] \\
&= 2\sigma_R^2 \mathbf{w}^H (\mathbf{I}_2 \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}) \mathbf{w}. \tag{3.13}
\end{aligned}$$

From (3.11), (3.12), (3.13), note that these components in the SINR expression contain factor 2 which comes from joint processing during two time slots in Alamouti scheme.

By (3.11)-(3.13), (3.6) can be written as a function of \mathbf{w}

$$\text{SINR}_k(\mathbf{w}) = \frac{P_o \mathbf{w}^H \mathcal{H}_{kk} \mathbf{w}}{P_o \mathbf{w}^H \left(\sum_{l \neq k} \mathcal{H}_{lk} \right) \mathbf{w} + \sigma_R^2 \mathbf{w}^H \left(\mathbf{I}_2 \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k} \right) \mathbf{w} + \sigma_D^2}. \tag{3.14}$$

The transmit power at relay m in the third time slot can be expressed as

$$\begin{aligned}
P_{m,1} &= E |w_{1,m} r_{1,m} + w_{2,m} r_{2,m}^*|^2 \\
&= E \left| w_{1,m} \sqrt{P_o} \sum_{k=1}^K h_{1,km} s_{k,1} + w_{1,m} n_{r,m1} + w_{2,m} \sqrt{P_o} \sum_{k=1}^K h_{1,km}^* s_{k,2} + w_{2,m} n_{r,m2}^* \right|^2 \\
&= P_o |w_{1,m}|^2 \sum_{k=1}^K |h_{1,km}|^2 + \sigma_R^2 |w_{1,m}|^2 + P_o |w_{2,m}|^2 \sum_{k=1}^K |h_{1,km}|^2 + \sigma_R^2 |w_{2,m}|^2 \\
&= (|w_{1,m}|^2 + |w_{2,m}|^2) \left(P_o \sum_{k=1}^K |h_{1,km}|^2 + \sigma_R^2 \right) \\
&= \mathbf{w}_1^H \mathbf{D}_m \mathbf{w}_1 + \mathbf{w}_2^H \mathbf{D}_m \mathbf{w}_2 \\
&= \mathbf{w}^H \tilde{\mathbf{D}}_m \mathbf{w}.
\end{aligned}$$

where $\tilde{\mathbf{D}}_m = \mathbf{I}_2 \otimes \mathbf{D}_m$, \mathbf{D}_m is a diagonal matrix defined as $\mathbf{D}_m \triangleq (P_o \sum_{k=1}^K |h_{1,km}|^2 + \sigma_R^2) \mathbf{E}_m$, with $\mathbf{E}_m = \text{diag}(\mathbf{e}_m)$. Through the same derivation procedure, we have $P_{m,2}(\mathbf{w}) = \mathbf{w}^H \tilde{\mathbf{D}}_m \mathbf{w}$, thereby, we know that $P_m = P_{m,1} = P_{m,2}$.

Thus, optimization problem (3.8) can be reformulated w.r.t. \mathbf{w}

$$\min_{\mathbf{w}} \max_{m \in \{1, \dots, M\}} \mathbf{w}^H \tilde{\mathbf{D}}_m \mathbf{w} \quad (3.15)$$

$$\text{s.t. SINR}_k(\mathbf{w}) \geq \gamma_k, \quad k = 1, \dots, K. \quad (3.16)$$

3.4 Rank-two Relay Beamforming Design with Alamouti Structure

In this section, we discuss the approach to solve optimization problem (3.15). The minmax problem is non-convex and NP-hard. We will first check its feasibility, and solve the problem via the SDR method [56]. Then we transform the problem into a separable SDP problem [66], and apply rank-constrained processing method to improve likelihood to achieve optimal solution and reduce computing complexity.

3.4.1 Feasibility

The existence of \mathbf{w} while satisfying SINR constraint in (3.16) depends on source transmission power P_o , receiving SINR target γ_k and channel conditions characterized by $\{\mathbf{h}_{1,k}\}$ and $\{\mathbf{h}_{2,k}\}$. We derive a feasibility condition for problem (3.15) below.

An upper bound on $\text{SINR}_k(\mathbf{w})$ for destination k is given by:

$$\begin{aligned} \text{SINR}_{k,\text{up}}(\mathbf{w}) &= \frac{P_o \mathbf{w}^H \mathcal{H}_{kk} \mathbf{w}}{P_o \mathbf{w}^H \left(\sum_{l \neq k} \mathcal{H}_{l,k} \right) \mathbf{w} + \sigma_R^2 \mathbf{w}^H \left(\mathbf{I}_2 \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k} \right) \mathbf{w}} \\ &= \frac{P_o \mathbf{w}^H \mathcal{H}_{kk} \mathbf{w}}{\mathbf{w}^H \mathcal{G}_k \mathbf{w}} \geq \text{SINR}_k(\mathbf{w}), \quad \forall k \end{aligned} \quad (3.17)$$

where $\mathcal{G}_{k^-} \triangleq \sum_{l \neq k} \mathcal{H}_{l,k} + \mathbf{I}_2 \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}$. Let λ_k denotes the principle eigenvalue of \mathcal{H}_{kk} , \mathbf{v}_k stands for the corresponding eigenvector. And let $\mathcal{G}_{k^-}^\dagger$ denote the pseudo-inverse of \mathcal{G}_{k^-} . When $\mathbf{w} = \mathcal{G}_{k^-}^\dagger \mathbf{u}_k$, where $\mathbf{u}_k = \sqrt{\lambda_k} \mathbf{v}_k$, the LHS of (3.17) is maximized, and its maximal value is obtained as $P_o \mathbf{u}_k^H \mathcal{G}_{k^-}^\dagger \mathbf{u}_k$. Thus, a necessary condition for problem (3.15) to be feasible is that source transmission power P_o , SINR requirement γ_k and channel information $\{\mathbf{h}_{1,k}\}$ and $\{\mathbf{h}_{2,k}\}$ should satisfy

$$P_o \mathbf{u}_k^H \mathcal{G}_{k^-}^\dagger \mathbf{u}_k \geq \gamma_k, \quad \forall k.$$

3.4.2 Ordinary SDR Approach

To solve the problem (3.15) using SDR method, we first rewrite SINR constraint (3.16) as

$$\begin{aligned} \text{SINR}_k(\mathbf{w}) &\geq \gamma_k \\ \Rightarrow P_o \mathbf{w}^H \mathcal{H}_{kk} \mathbf{w} &\geq (P_o \mathbf{w}^H \sum_{l \neq k} \mathcal{H}_{lk} \mathbf{w} + \sigma_R^2 \mathbf{w}^H (\mathbf{I}_{2 \times 2} \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}) \mathbf{w} + \sigma_D^2) \gamma_k \\ \Rightarrow \mathbf{w}^H \mathcal{F}_k \mathbf{w} &\geq \sigma_D^2 \gamma_k \end{aligned}$$

where $\mathcal{F}_k \triangleq P_o \mathcal{H}_{kk} - P_o \gamma_k \sum_{l \neq k} \mathcal{H}_{lk} - \sigma_R^2 \gamma_k (\mathbf{I}_{2 \times 2} \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k})$.

Then we introduce an auxiliary variable $P_r \triangleq \max_{m \in \{1, \dots, M\}} \text{tr}(\mathbf{w}^H \tilde{\mathbf{D}}_m \mathbf{w})$ as the maximal power among relays to transform problem into the joint minimization problem.

$$\begin{aligned} \min_{P_r, \mathbf{w}} \quad & P_r & (3.18) \\ \text{s.t.} \quad & P_r > 0 \\ & \text{tr}(\mathbf{w}^H \tilde{\mathbf{D}}_m \mathbf{w}) \leq P_r, \quad m = 1, \dots, M \\ & \text{tr}(\mathbf{w}^H \mathcal{F}_k \mathbf{w}) \geq \sigma_D^2 \gamma_k, \quad k = 1, \dots, K. \end{aligned}$$

Notice that \mathcal{H}_{lk} and $\mathbf{I}_{2 \times 2} \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}$ are all block diagonal. Hence, define $\mathbf{X}_1 \triangleq \mathbf{w}_1 \mathbf{w}_1^H$ and $\mathbf{X}_2 \triangleq \mathbf{w}_2 \mathbf{w}_2^H$, then $\mathbf{X} \triangleq \text{blkdiag}(\mathbf{X}_1, \mathbf{X}_2)$. We further rewrite problem (3.18) in the following form w.r.t. \mathbf{X} and P_r .

$$\begin{aligned}
& \min_{P_r, \mathbf{X}} P_r & (3.19) \\
& \text{s.t. } P_r > 0 \\
& \text{tr}(\mathbf{X} \tilde{\mathbf{D}}_m) \leq P_r, \quad m = 1, \dots, M \\
& \text{tr}(\mathbf{X} \mathcal{F}_k) \geq \sigma_D^2 \gamma_k, \quad k = 1, \dots, K \\
& \mathbf{X} \succeq 0, \text{rank}(\mathbf{X}_1) = 1, \text{rank}(\mathbf{X}_2) = 1.
\end{aligned}$$

By removing the rank constraints in problem (3.19), the above non-convex optimization problem is relaxed to the following SDR [56] form:

$$\begin{aligned}
& \min_{P_r, \mathbf{X}} P_r & (3.20) \\
& \text{s.t. } P_r > 0 \\
& \text{tr}(\mathbf{X} \tilde{\mathbf{D}}_m) \leq P_r, \quad m = 1, \dots, M \\
& \text{tr}(\mathbf{X} \mathcal{F}_k) \geq \sigma_D^2 \gamma_k, \quad k = 1, \dots, K \\
& \mathbf{X} \succeq 0.
\end{aligned}$$

We can solve problem (3.20) by SDP programming for optimal solution \mathbf{X}^* using standard SDP solvers [67].

Once \mathbf{X}^* is obtained, we remove the off-diagonal blocks of \mathbf{X}^* . Since all matrices $\tilde{\mathbf{D}}_m, \mathcal{F}_k$ are block diagonal, this operation does not change either the optimality or feasibility of the resulting solution [64]. Thus, without loss of generality, we can treat $\mathbf{X}^* = \text{blkdiag}[\mathbf{X}_1^*, \mathbf{X}_2^*]$, which contains two optimal solutions \mathbf{X}_1^* and \mathbf{X}_2^* .

Algorithm 1 Gaussian randomization procedure

Input: \mathbf{X}_1^* , \mathbf{X}_2^* , \mathcal{F}_k , $\tilde{\mathbf{D}}_m, \forall k, m$, number of randomization $N \geq 1$

- 1: **for** $j = 1$ to N **do**
- 2: **if** $\text{rank}(\mathbf{X}_1^*) > 1$ and $\text{rank}(\mathbf{X}_2^*) \leq 1$ **then**
- 3: generate an random vectors $\boldsymbol{\xi}_j \sim \mathcal{CN}(\mathbf{0}, \mathbf{X}_1^*)$.
- 4: construct $\mathbf{w}_j = [\boldsymbol{\xi}_j^T, \mathbf{w}_2^{*T}]^T$
- 5: **end if**
- 6: **if** $\text{rank}(\mathbf{X}_1^*) \leq 1$ and $\text{rank}(\mathbf{X}_2^*) > 1$ **then**
- 7: generate an random vectors $\boldsymbol{\eta}_j \sim \mathcal{CN}(\mathbf{0}, \mathbf{X}_2^*)$.
- 8: construct $\mathbf{w}_j = [\mathbf{w}_1^{*T}, \boldsymbol{\eta}_j^T]^T$
- 9: **end if**
- 10: **if** $\text{rank}(\mathbf{X}_1^*) > 1$ and $\text{rank}(\mathbf{X}_2^*) > 1$ **then**
- 11: generate an random vectors $\boldsymbol{\xi}_j \sim \mathcal{CN}(\mathbf{0}, \mathbf{X}_1^*)$, $\boldsymbol{\eta}_j \sim \mathcal{CN}(\mathbf{0}, \mathbf{X}_2^*)$.
- 12: construct $\mathbf{w}_j = [\boldsymbol{\xi}_j^T, \boldsymbol{\eta}_j^T]^T$
- 13: **end if**
- 14: let $\hat{\mathbf{w}}_j = \mathbf{w}_j / \sqrt{\min_{k \in \{1, \dots, K\}} \frac{1}{\sigma_D^2 \gamma_k} \mathbf{w}_j^H \mathcal{F}_k \mathbf{w}_j}$.
- 15: **end for**
- 16: let $j^* = \arg \min_{j \in \{1, \dots, N\}} \max_{m \in \{1, \dots, M\}} \text{tr}(\hat{\mathbf{w}}_j^H \tilde{\mathbf{D}}_m \hat{\mathbf{w}}_j)$.

Output: $\hat{\mathbf{w}} = \hat{\mathbf{w}}_{j^*}$.

For \mathbf{X}_1^* and \mathbf{X}_2^* , we need to extract beam vectors \mathbf{w}_1^* and \mathbf{w}_2^* . If $\text{rank}(\mathbf{X}_1^*) = 1$, we can directly obtain optimal \mathbf{w}_1^* as $\mathbf{X}_1^* = \mathbf{w}_1^* \mathbf{w}_1^{*H}$. The same for \mathbf{X}_2^* . If $\text{rank}(\mathbf{X}_1^*) > 1$ or $\text{rank}(\mathbf{X}_2^*) > 1$, we apply a Gaussian randomization procedure [56] to generate feasible solution as \mathbf{w}_1^s or \mathbf{w}_2^s , where $\hat{\mathbf{w}} = [\mathbf{w}_1^{sT}, \mathbf{w}_2^{sT}]^T$. The details of randomization procedure is shown in Algorithm 1.

3.4.3 Rank-Constrained Separable SDR Approach

When solving problem (3.20) with previous approach, it has following issues. First of all, the computational complexity for SDP solver is really high, as it desires a $2M \times 2M$ matrix under $K + M$ constraints. In addition, when the SDR approach is applied, it is with a high probability that we cannot extract the optimal solution of the original problem from the optimal solution of relaxed problem. Therefore, we need

to rely on randomization procedure to generate a feasible solution, but that solution is always suboptimal. To deal with that, we want to find a way which can increase the chance of recovering the optimal beam vectors from the SDR solution.

Thus, in this section, we consider a rank-constrained separable SDR approach to solve problem (3.15), which will further reduce the ranks of \mathbf{X}_1^* and \mathbf{X}_2^* solved from (3.20). At the same time, it will increase the chance for obtaining two rank-one optimal solutions \mathbf{X}_1^* and \mathbf{X}_2^* . We rewrite the relay power and SINR expression w.r.t. \mathbf{X}_1 and \mathbf{X}_2 . From (3.15), the relay power P_m can be expressed as

$$P_m(\mathbf{X}_1, \mathbf{X}_2) = \text{tr}(\mathbf{D}_m \mathbf{X}_1) + \text{tr}(\mathbf{D}_m \mathbf{X}_2). \quad (3.21)$$

Since \mathcal{H}_{lk} and $\mathbf{I}_2 \otimes \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}$ are both block diagonal in SINR expression (3.14), SINR constraint can be expressed as (3.22)

$$\begin{aligned} \text{SINR}_k(\mathbf{X}_1, \mathbf{X}_2) &\geq \gamma_k \\ \Rightarrow \text{tr}(\mathbf{A}_{k1} \mathbf{X}_1) + \text{tr}(\mathbf{A}_{k2} \mathbf{X}_2) &\geq \gamma_k \sigma_D^2 \end{aligned} \quad (3.22)$$

where

$$\mathbf{A}_{k1} \triangleq P_o \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k}^* \mathbf{h}_{1,k}^T \hat{\mathbf{H}}_{2,k} - P_o \gamma \sum_{l \neq k} \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,l}^* \mathbf{h}_{1,l}^T \hat{\mathbf{H}}_{2,k} - \gamma \sigma_R^2 \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k} \quad (3.23)$$

$$\mathbf{A}_{k2} \triangleq P_o \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k} \mathbf{h}_{1,k}^H \hat{\mathbf{H}}_{2,k} - P_o \gamma \sum_{l \neq k} \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,l} \mathbf{h}_{1,l}^H \hat{\mathbf{H}}_{2,k} - \gamma \sigma_R^2 \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}. \quad (3.24)$$

Using (3.21) and (3.22), we equivalently rewrite the maximal power minimization problem (3.15) w.r.t. \mathbf{X}_1 and \mathbf{X}_2 and as

$$\min_{\mathbf{X}_1, \mathbf{X}_2} \max_{m \in \{1, \dots, M\}} \text{tr}(\mathbf{D}_m \mathbf{X}_1) + \text{tr}(\mathbf{D}_m \mathbf{X}_2) \quad (3.25)$$

$$\text{s.t. } \text{tr}(\mathbf{A}_{k1} \mathbf{X}_1) + \text{tr}(\mathbf{A}_{k2} \mathbf{X}_2) \geq \gamma_k \sigma_D^2, \quad k = 1, \dots, K$$

$$\mathbf{X}_1 \succcurlyeq 0, \mathbf{X}_2 \succcurlyeq 0, \text{rank}(\mathbf{X}_1) = 1, \text{rank}(\mathbf{X}_2) = 1.$$

Also we introduce P_r with the same definition in 3.4.2 and drop the rank constraints, min-max problem (3.25) can be reformulated as following

$$\begin{aligned}
& \min_{\mathbf{X}_1, \mathbf{X}_2, P_r} P_r & (3.26) \\
& \text{s.t. } P_r > 0 \\
& \text{tr}(\mathbf{D}_m \mathbf{X}_1) + \text{tr}(\mathbf{D}_m \mathbf{X}_2) \leq P_r, \quad m = 1, \dots, M \\
& \text{tr}(\mathbf{A}_{k1} \mathbf{X}_1) + \text{tr}(\mathbf{A}_{k2} \mathbf{X}_2) \geq \gamma_k \sigma_D^2, \quad \forall k = 1, \dots, K \\
& \mathbf{X}_1 \succcurlyeq 0, \quad \mathbf{X}_2 \succcurlyeq 0.
\end{aligned}$$

The problem (3.26) is a separable SDP problem [66], and the following result gives condition for which the problem has optimal solution.

Theorem 1. [66, Theorem 1] *Suppose that the separable SDP below and its dual are solvable.*

$$\begin{aligned}
& \min_{\mathbf{X}_1, \dots, \mathbf{X}_L} \sum_{l=1}^L \text{tr}(\mathbf{C}_l \mathbf{X}_l) & (3.27) \\
& \text{s.t. } \sum_{l=1}^L \text{tr}(\mathbf{A}_{ml} \mathbf{X}_l) \succeq_m b_m, m = 1, \dots, M \\
& \mathbf{X}_l \succcurlyeq 0, l = 1, \dots, L
\end{aligned}$$

where $\mathbf{C}_l, \mathbf{A}_{ml}$ are all Hermitian matrices, b_m is real number, $\succeq_m \in \{\geq, =, \leq\}$.

Then the problem has always an optimal solution $(\mathbf{X}_1^*, \dots, \mathbf{X}_L^*)$ such that

$$\sum_{l=1}^L \text{rank}^2(\mathbf{X}_l^*) \leq M \quad (3.28)$$

The general form of separable SDP problem in (3.27) contains $M + 1$ limited conditions, including 1 objective and M constraints. Theorem 1 concludes that the rank of the optimal solution promises (3.28). Then comparing with our problem

Algorithm 2 Rank-constrained solution procedure for problem (3.26)

Input: $\mathbf{D}_m, \forall m, \mathbf{A}_{k1}, \mathbf{A}_{k2}, \gamma, \forall k \in \{1, \dots, \}, \sigma_D^2$

1: solve the problem (3.26) to find $\mathbf{X}_1, \mathbf{X}_2$ with arbitrary ranks.

2: evaluate $R_l = \text{rank}(\mathbf{X}_l), l = 1, 2$, and $U = \sum_{l=1}^2 R_l^2$

3: **while** $U > M + K - 1$ **do**

4: decompose $\mathbf{X}_l = \mathbf{V}_l \mathbf{V}_l^H, l = 1, 2$.

5: find a non zero solution (Δ_1, Δ_2) of $\text{tr}(\sum_{l=1}^2 \mathbf{V}_l^H \mathbf{A}_{kl} \mathbf{V}_l \Delta_l) = 0, \forall k$, where Hermitian matrix $\Delta_l \in \mathbb{C}^{R_l \times R_l}, \forall l$.

6: evaluate eigenvalues $\delta_{l1}, \dots, \delta_{lR_l}$ for δ_l with $l = 1, 2$.

7: determine l_0 and k_0 such that $|\delta_{l_0 k_0}| = \max\{|\delta_{lk}| : 1 \leq k \leq R_l, l = 1, 2\}$.

8: compute $\mathbf{X}_l = \mathbf{V}_l (\mathbf{I}_{R_l} - (1/\delta_{l_0 k_0}) \Delta_l) \mathbf{V}_l^H, l = 1, 2$.

9: evaluate $R_l = \text{rank}(\mathbf{X}_l), l = 1, 2$, and $U = \sum_{l=1}^2 R_l^2$.

10: **end while**

Output: $(\mathbf{X}_1^*, \mathbf{X}_2^*)$ with $\text{rank}^2(\mathbf{X}_1^*) + \text{rank}^2(\mathbf{X}_2^*) \leq M + K - 1$

(3.26), which holds $K + M$ limited conditions, the rank of the optimal solution \mathbf{X}_1^* and \mathbf{X}_2^* should satisfy the inequality below

$$\sum_{l=1}^L \text{rank}^2(\mathbf{X}_l^*) \leq K + M - 1. \quad (3.29)$$

According to Theorem 1, we apply Algorithm 2 to produce a rank-constrained solutions \mathbf{X}_1^* and \mathbf{X}_2^* of (3.26), which will guarantee rank constraint (3.29). However, the produced solutions may not be rank-one solutions. We propose the following procedure to obtain a solution to problem (3.15).

1) Solve problem (3.26) by an SDP solver and obtain arbitrary rank solutions.

2) Apply the rank-constrained procedure [66] to our problem (3.26), as described in Algorithm 2, to obtain the optimal, but rank-reduced solutions \mathbf{X}_1^* and \mathbf{X}_2^* .

3) If $\text{rank}(\mathbf{X}_1^*) = 1$, then extract optimal \mathbf{w}_1^* . Otherwise, apply Gaussian randomization procedure in Algorithm 1 to obtain sub-optimal solution \mathbf{w}_1^s which satisfy $\text{rank}(\hat{\mathbf{X}}_1^*) = 1$. The same applies to \mathbf{X}_2^* .

In each iteration of Algorithm 2, an undetermined system of linear equations

$\text{tr}(\sum_{l=1}^2 \mathbf{V}_l^H \mathbf{A}_{kl} \mathbf{V}_l \Delta_l) = 0$ in Algorithm 2 step 5 must be solved. To do this, we require the following conditions:

- 1) \mathbf{A}_{k1} and \mathbf{A}_{k2} are both Hermitian matrices.
- 2) Δ_1 and Δ_2 are required to be Hermitian matrices.

The first condition is satisfied through (3.23) and (3.24). Furthermore, it will guarantee matrices $\mathbf{V}_1^H \mathbf{A}_{k1} \mathbf{V}_1$ and $\mathbf{V}_2^H \mathbf{A}_{k2} \mathbf{V}_2$ are Hermitian as well. For the second condition, note that if the $\{\mathbf{Y}_1, \mathbf{Y}_2\}$ are the arbitrary solution of linear system $\text{tr}(\sum_{l=1}^2 \mathbf{V}_l^H \mathbf{A}_{kl} \mathbf{V}_l \Delta_l) = 0$, $\{\mathbf{Y}_1^H, \mathbf{Y}_2^H\}$ and $\{\mathbf{Y}_1 + \mathbf{Y}_1^H, \mathbf{Y}_2 + \mathbf{Y}_2^H\}$ are also its solution. Obviously, $\mathbf{Y}_1 + \mathbf{Y}_1^H$ and $\mathbf{Y}_2 + \mathbf{Y}_2^H$ are Hermitian matrices. Therefore, as long as we have non-Hermitian solution $\{\mathbf{Y}_1, \mathbf{Y}_2\}$ for the linear system as $\text{tr}(\sum_{l=1}^2 \mathbf{V}_l^H \mathbf{A}_{kl} \mathbf{V}_l \Delta_l) = 0, \forall k$, solution $\{\mathbf{Y}_1 + \mathbf{Y}_1^H, \mathbf{Y}_2 + \mathbf{Y}_2^H\}$ can be constructed as desired solution Δ_1 and Δ_2 for such linear system.

3.4.4 Performance Loss due to Randomization

Algorithm 1 is a generalization of the Gaussian randomization procedure used to generate rank-one solution for SDR-based and rank-constrained beamforming approaches. The produced solution, uniformly denoted as $\hat{\mathbf{X}}_1$ and $\hat{\mathbf{X}}_2$, no matter which approach is applied, is obviously sub-optimal. Thus, it is necessary to analyze the upper bound of the performance loss is, and the probability of its occurrence. Using the result in [56], we have the following proposition on its worst-case approximation performance.

Proposition 1. *With probability at least $1 - (\frac{7}{8})^L$, the solution $\hat{\mathbf{w}}$ returned by Algorithm 1 satisfies*

$$P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \leq [16\sqrt{K}(3 \log(8M) + 2)]P_m(\mathbf{X}_1^*, \mathbf{X}_2^*)$$

or

$$\text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \geq \frac{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)}{16\sqrt{K}(3\log(8M) + 2)}$$

where $\hat{\mathbf{w}} = [\hat{\mathbf{w}}_1^T, \hat{\mathbf{w}}_2^T]^T$, and $\hat{\mathbf{X}}_1 \triangleq \hat{\mathbf{w}}_1 \hat{\mathbf{w}}_1^H$, $\hat{\mathbf{X}}_2 \triangleq \hat{\mathbf{w}}_2 \hat{\mathbf{w}}_2^H$.

Proof. : See Appendix A. ■

Proposition 1 implies that the two beamforming approaches in 3.4.2 and 3.4.3 may suffer from a performance loss in terms of relay power or SINR where the loss increases logarithmically with the number of relays M and linearly with square root of the number of pairs \sqrt{K} . Hence, these two beamforming solutions are only effective when there are not too many relays or S-D pairs.

3.4.5 Computational Complexity Analysis

In this part, we plan to discuss the computational complexity of these two rank-two approaches, *i.e.*, ordinary SDR and separable SDR. As they both involve solving an SDR problem, we can do the analysis according to the size of unknown matrices, and the numbers of problem constraints [68]. In problem (3.20), we solve for a $2M \times 2M$ matrix which containing desired beam weight, with in total $K + M$ linear constraints related to SINR requirements and power allocation. It results to computational complexity as $\mathcal{O}((2M)^2(K + M))$. While in problem (3.26), the rewritten separable SDR problem aims to two $M \times M$ matrices under the same number of constraints. Therefore, the complexity for SDP solver is roughly reduced to $\mathcal{O}(2M^2(K + M))$.

From above discussion, we can see the time complexity is not only decided by the number of relays involved, but also related to how many users are accommodated in

the network. However, number of relays still play main role of computational cost for SDP solver. Additionally, separable SDR method theoretically would halve the complexity of ordinary SDR, but they are still at the same orders of magnitude.

3.5 Simulation Results

In this section, we study the performance, optimality and complexity of the two proposed rank-two solutions, namely, ordinary SDR and separable SDR, for per-relay power minimization problem (3.18). We also compare our performance with traditional rank-one beamforming without Alamouti scheme. The details of problem formulation can be found in Appendix B.

3.5.1 Per-Antenna Power Comparison

We assume the channel vector $\mathbf{h}_{1,k}$ and $\mathbf{h}_{2,k}$ are i.i.d Gaussian with unit variance $\sigma_h^2 = 1$, and set noise variance at the relays and the destination receivers to be equal to $\sigma_R^2 = \sigma_D^2 = 1$. The source transmit power over noise power is set to be $P_o/\sigma_R^2 = 10$ dB. The received SINR target γ_k are equal for all k , $\gamma_k = \gamma$. The number of channel realizations used is 1000.

We consider the following three cases:

- 1) $K = 3, M = 4, 6$.
- 2) $K = 4, M = 4, 6$.
- 3) $K = 6, M = 4, 6, 8$.

In the above cases, we plot per-relay power versus the target SINR γ . First of all, through Fig. 3.2, Fig. 3.3 and Fig. 3.4, we noticed that both proposed rank-two meth-

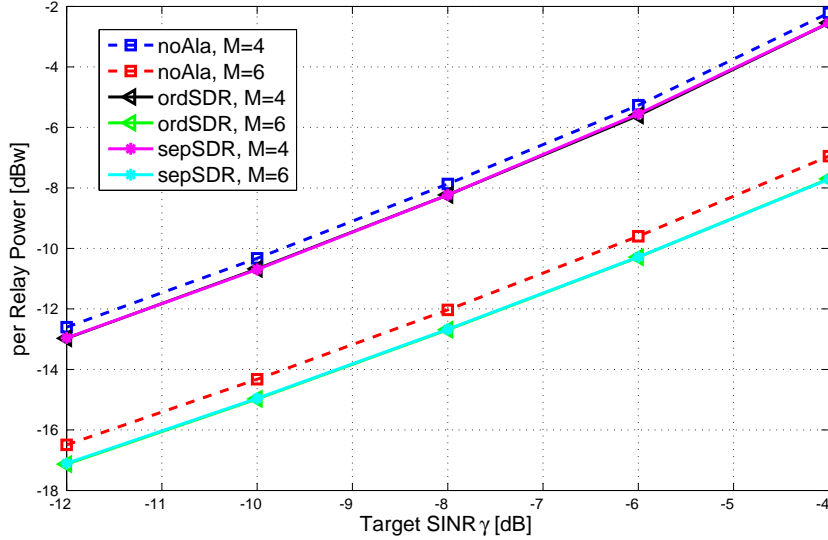


Figure 3.2: Comparison of rank-one and rank-two BF performance ($K = 3, M = 4, 6$) ods outperform rank-one method, which is denoted as "noAla" in these figures. Also, the ordinary SDR approach, "ordSDR" in figures, provides very close performance as seperable SDR method, "sepSDR" in figures. Secondly, given the SINR requirement γ , per-relay power is effectively reduced as M increases. Additionally, with the same number of relays, having more users in the network results in higher per-relay power consumption for given SINR target.

3.5.2 Gap Comparison

We then look at the performance gap of these two approaches for per-relay power minimization problem, and compare with rank-one approach where Alamouti scheme is not applied. Let G^{SDR} denote the gap between optimal solution from relaxed problem and sub-optimal solution produced by randomly generated beam vectors. Define $G^{\text{SDR}} \triangleq P_m(\mathbf{X}_1^*, \mathbf{X}_2^*) - P_m(\mathbf{w}_1^s, \mathbf{w}_2^s)$ in dB domain.

We first set $K = 2 : 4, M = 4, \gamma_k = -4\text{dB}, \forall k$. The same set of 10000 channel

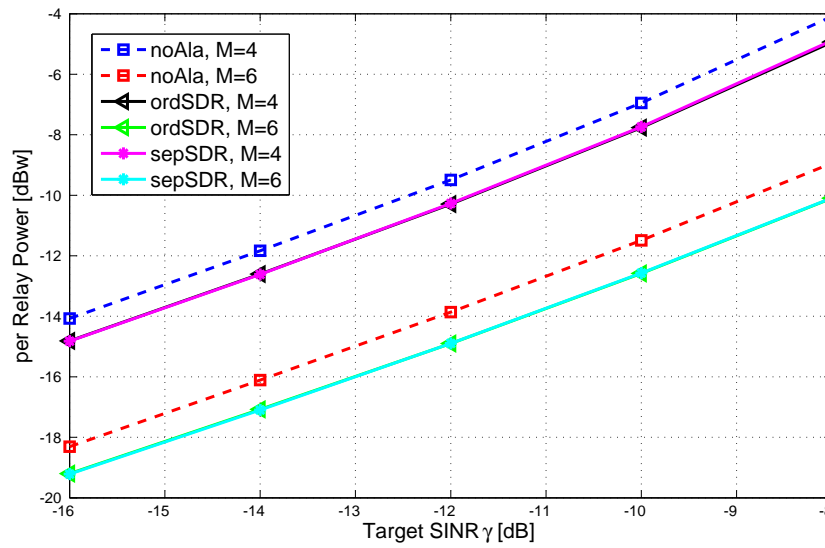


Figure 3.3: Comparison of rank-one and rank-two BF performance ($K = 4, M = 4, 6$)

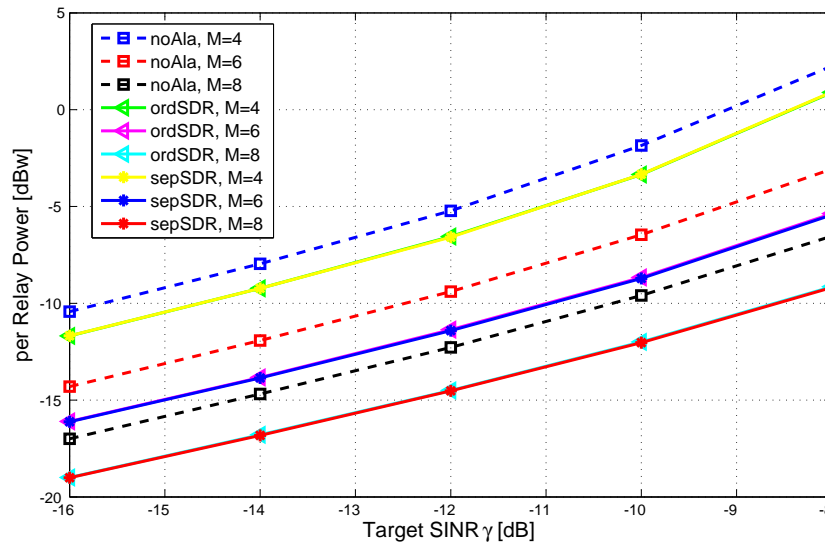
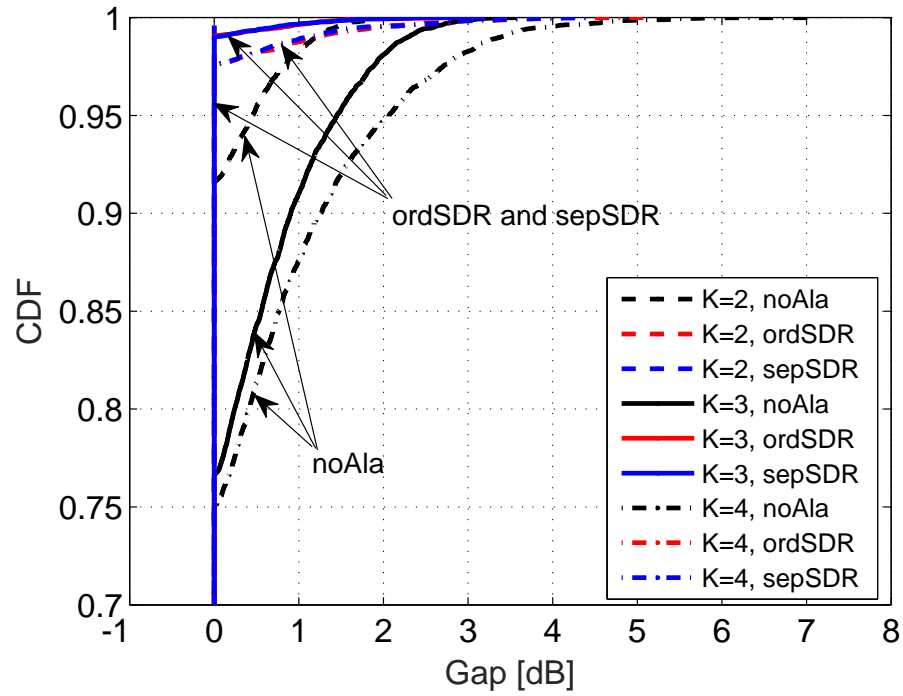
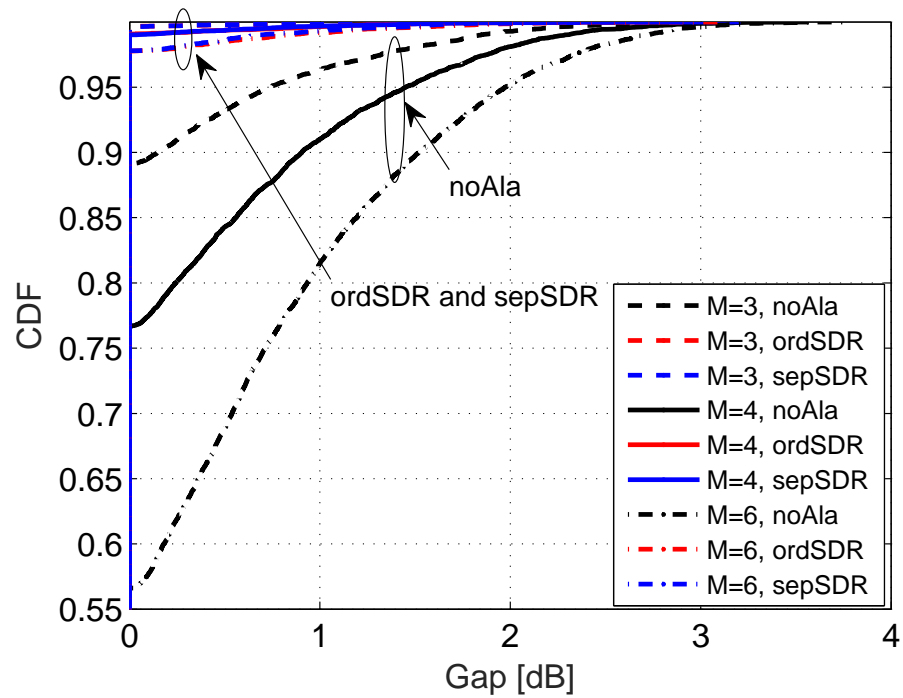


Figure 3.4: Comparison of rank-one and rank-two BF performance ($K = 6, M = 4, 6, 8$)

realizations are used for each method. Taking rank-one scheme, shorted as noAla in figures, as comparison, the CDF of G^{SDR} for ordinary SDR and separable SDR approaches are plotted in Fig. 3.5. The gap being 0 dB indicates the optimal solution is obtained. We can see that the percentage of 0 dB gap in rank-two approaches are identical, which means these two methods have almost the same potential to achieve optimal solution. Also those rank-two methods achieve significantly higher percentage than rank-one method for optimality. Especially for the case $K = 2, M = 4$, optimality can be always achieved through rank-two approaches. In addition, when the solution is suboptimal, we notice that the tail distribution of G^{SDR} for rank-two schemes is tighter than rank-one. Therefore, the two rank-two approaches produce a tighter approximate solution than rank-one in those cases. In addition, we can see for given number of relays, the more S-D pairs are contained in the network, the looser approximate solution will be.

Next, we set $K = 3, M = 3, 4, 6$ and plot CDF versus Gap in Fig. 3.6 to discuss the gap performance with given number of S-D pairs. We notice that optimality cannot be guaranteed in these cases. Beyond that, more optimality will be lost along with the increasing number of relay nodes in the network. Because more relay nodes means more power constraints in optimization problem.

Figure 3.5: Gap CDF ($M = 4, \gamma = -4$ dB)Figure 3.6: Gap CDF ($K = 3, \gamma = -4$ dB)

Chapter 4

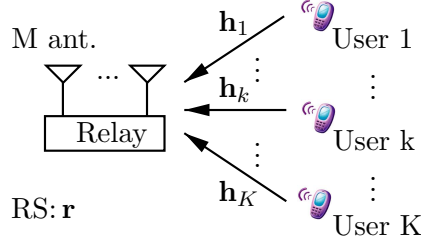
Joint Relay Beamforming and Receiver Processing Design of MWMA Relay Network

In this chapter, we aim to further improving performance of MWMA relaying by jointly designing relay processing matrices over multiple BC phases and receiver processing matrix at each user, where receiver processing is based on all received signals from multiple BC phases to optimize the performance. We formulate the joint optimization problem to maximize the minimum SINR for detected symbols at all users, and solve it through iterative optimization of relay beamforming matrix and receiver processing matrix.

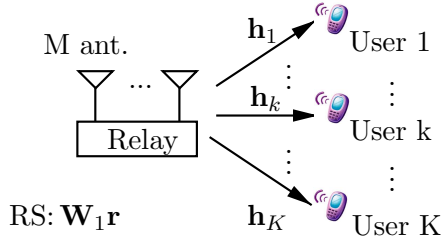
4.1 System Model

We consider a MWMA relay network with K users and one relay node. Each user is equipped with single antenna while the relay node has M antennas. With AF relaying strategy, the multi-way relaying protocol consists of one MAC phase and $K - 1$ BC phases, as shown in Fig. 4.1. In the MAC phase, K users transmit their own information symbols simultaneously to the relay. Let $\mathbf{h}_k \triangleq [h_{k1}, \dots, h_{kM}]^T$ denote the

MAC phase



BC phase 1



BC phase K-1

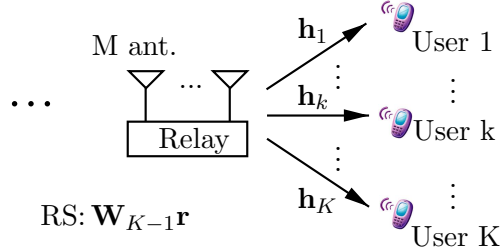


Figure 4.1: The Multi-Way Multi-Antenna relay network model.

channel vector between user k and the relay, and s_k denote the transmitted symbol from user k with $E|s_k|^2 = 1$, for $k = 1, \dots, K$. Let P_o denote the transmit power at each user. The $M \times 1$ received signal vector \mathbf{r} at the relay is given by

$$\mathbf{r} = \sum_{k=1}^K \sqrt{P_o} \mathbf{h}_k s_k + \mathbf{n}_r = \sqrt{P_o} \mathbf{H} \mathbf{s} + \mathbf{n}_r \quad (4.1)$$

where $\mathbf{H} \triangleq [\mathbf{h}_1, \dots, \mathbf{h}_K]$, $\mathbf{s} \triangleq [s_1, \dots, s_K]^T$, and $\mathbf{n}_r \triangleq [n_{r,1}, \dots, n_{r,M}]^T$ is the Gaussian noise vector at the relay receiver with covariance matrix $\sigma_R^2 \mathbf{I}_M$.

In the BC phase i , for $i = 1, \dots, K-1$, the multi-antenna relay processes received signal vector \mathbf{r} with an $M \times M$ beam matrix \mathbf{W}_i , and forwards the processed signal vector to all K users. We assume channel reciprocity in MAC and BC phases and the channel matrix \mathbf{H} is unchanged in K -slot multi-way relaying. The received signal $y_{i,k}$

at user k in BC phases i is given by

$$\begin{aligned} y_{i,k} &= \mathbf{h}_k^T \mathbf{W}_i \mathbf{r} + n_{d,ik} \\ &= \sqrt{P_o} \sum_{j=1}^K \mathbf{w}_i^T (\mathbf{h}_k \otimes \mathbf{h}_j) s_j + \mathbf{w}_i^T (\mathbf{h}_k \otimes \mathbf{n}_r) + n_{d,ik}, \forall k \end{aligned}$$

where $\mathbf{w}_i \triangleq \text{vec}(\mathbf{W}_i^T)$, $n_{d,ik}$ is the receiver noise at user k in BC phase i , which is Gaussian with variance σ_D^2 , and we have applied the property $\text{vec}(\mathbf{ABC}) = (\mathbf{A} \otimes \mathbf{C}^T) \text{vec}(\mathbf{B}^T)$ in deriving the last equation. We assume perfect knowledge of CSI at the relay and each user.

Define $\mathbf{y}_k \triangleq [y_{1,k}, \dots, y_{K-1,k}]^T$ as the received signal vector for user k in all $(K-1)$ BC phases. From the above, we have

$$\mathbf{y}_k = \sqrt{P_o} \mathbf{W} (\mathbf{h}_k \otimes \mathbf{h}_k) s_k + \sqrt{P_o} \mathbf{W} \sum_{j \in \mathcal{S}_{k-}} (\mathbf{h}_k \otimes \mathbf{h}_j) s_j + \mathbf{W} (\mathbf{h}_k \otimes \mathbf{I}_M) \mathbf{n}_r + \mathbf{n}_{d,k} \quad (4.2)$$

where $\mathbf{W} \triangleq [\mathbf{w}_1, \dots, \mathbf{w}_{K-1}]^T$, $\mathcal{S}_{k-} \triangleq \{1, \dots, K\} \setminus \{k\}$, and $\mathbf{n}_{d,k} \triangleq [n_{d,1k}, \dots, n_{d,(K-1)k}]^T$.

The first term in (4.2) is the self-interference for user k , which is known to user k and can be subtracted. The residual signal vector after self-interference cancellation, denoted by $\tilde{\mathbf{y}}_k$, is given by

$$\tilde{\mathbf{y}}_k = \sqrt{P_o} \mathbf{W} \sum_{j \in \mathcal{S}_{k-}} (\mathbf{h}_k \otimes \mathbf{h}_j) s_j + \mathbf{W} (\mathbf{h}_k \otimes \mathbf{I}_M) \mathbf{n}_r + \mathbf{n}_{d,k}. \quad (4.3)$$

At user k , we apply a receiver processing matrix to $\tilde{\mathbf{y}}_k$ to obtain the decision variables to decode s_j 's, for $j \in \mathcal{S}_{k-}$. Define a $(K-1) \times (K-1)$ receiver processing matrix \mathbf{G}_k for user k by $\mathbf{G}_k \triangleq [\mathbf{g}_{k1}, \dots, \mathbf{g}_{k(k-1)}, \mathbf{g}_{k(k+1)}, \dots, \mathbf{g}_{kK}]^H$, where \mathbf{g}_{kj} is the combining vector for decoding s_j , for $j \in \mathcal{S}_{k-}$. Define the output vector after processing by $\mathbf{z}_k \triangleq [z_{k1}, \dots, z_{k(k-1)}, z_{k(k+1)}, \dots, z_{kK}]^T$, where z_{kj} denotes the post-processed

signal from user j at user k . We have

$$\mathbf{z}_k = \mathbf{G}_k \tilde{\mathbf{y}}_k. \quad (4.4)$$

Substituting the expression of $\tilde{\mathbf{y}}_k$ in (4.3) into above, we have the post-processed signal z_{kj} for s_j as

$$\begin{aligned} z_{kj} = & \sqrt{P_o} \mathbf{g}_{kj}^H \mathbf{W} (\mathbf{h}_k \otimes \mathbf{h}_j) s_j + \sqrt{P_o} \mathbf{g}_{kj}^H \mathbf{W} \sum_{l \in \mathcal{S}_k - \{j\}} (\mathbf{h}_k \otimes \mathbf{h}_l) s_l \\ & + \mathbf{g}_{kj}^H \mathbf{W} (\mathbf{h}_k \otimes \mathbf{I}_M) \mathbf{n}_r + \mathbf{g}_{kj}^H \mathbf{n}_{d,k} \end{aligned} \quad (4.5)$$

where the first term contains s_j of user j to be decoded, the second term contains the cross interference caused by other users except j , and the third and fourth terms are the post processed relay amplified noise and receiver noise, respectively.

4.2 Problem Formulation

Based on (4.5), the received SINR for s_j at user k after post processing, as a function of \mathbf{W} and \mathbf{g}_{kj} , is given by

$$\text{SINR}_{kj}(\mathbf{W}, \mathbf{g}_{kj}) = \frac{P_o |\mathbf{g}_{kj}^H \mathbf{W} (\mathbf{h}_k \otimes \mathbf{h}_j)|^2}{\mathcal{I}_{\text{int}} + \sigma_R^2 \|\mathbf{g}_{kj}^H \mathbf{W} (\mathbf{h}_k \otimes \mathbf{I}_M)\|^2 + \sigma_D^2 \|\mathbf{g}_{kj}^H\|^2} \quad (4.6)$$

where $\mathcal{I}_{\text{int}} \triangleq P_o \sum_{l \in \mathcal{S}_k - \{j\}} |\mathbf{g}_{kj}^H \mathbf{W} (\mathbf{h}_k \otimes \mathbf{h}_l)|^2$, for $k \in \{1, \dots, K\}$, $j \in \mathcal{S}_k$.

Our objective is to jointly design relay beam matrices $\{\mathbf{W}_1, \dots, \mathbf{W}_{K-1}\}$ for $K-1$ BC phases and processing matrices $\{\mathbf{G}_1, \dots, \mathbf{G}_K\}$ at all K users to maximize the minimal SINR among users, under the relay power budget P_r . The transmit power at the relay in BC phase i , denoted as P_i , is given by

$$P_i = E\{\|\mathbf{W}_i \mathbf{r}\|^2\} = \text{tr}[\mathbf{W}_i (P_o \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M) \mathbf{W}_i^H]. \quad (4.7)$$

Thus, the optimization problem is formulated as **P0**.

$$\begin{aligned}
(\mathbf{P0}) : \quad & \max_{\{\mathbf{W}_i\}, \{\mathbf{G}_k\}} \min_{k, j \in \mathcal{S}_k^-} \text{SINR}_{kj}(\mathbf{W}, \mathbf{g}_{kj}) \\
& \text{s.t. } P_i \leq P_r, \forall i
\end{aligned}$$

The above joint optimization problem for MWMA is non-convex and difficult to solve.

4.3 Joint Relay Beamforming and Receiver Processing Design

In the following, we propose specific receiver processing structure and develop an approach for the joint optimization problem.

4.3.1 Vectorization of \mathbf{W}

To facilitate the derivation of our solution, we first rewrite SINR expression in (4.6) by vectorizing the processing matrix \mathbf{W} . Defining $\mathbf{w} \triangleq \text{vec}(\mathbf{W}^T)$ and applying property $\text{vec}(\mathbf{ABC}) = (\mathbf{A} \otimes \mathbf{C}^T)\text{vec}(\mathbf{B}^T)$, the desired signal power in SINR expression (4.6) can be rewritten as

$$\begin{aligned}
P_o |\mathbf{g}_{kj}^H \mathbf{W}(\mathbf{h}_k \otimes \mathbf{h}_j)|^2 &= P_o |\text{vec}[\mathbf{g}_{kj}^H \mathbf{W}(\mathbf{h}_k \otimes \mathbf{h}_j)]|^2 \\
&= P_o |[\mathbf{g}_{R,kj}^H \otimes (\mathbf{h}_k^T \otimes \mathbf{h}_j^T)] \text{vec}(\mathbf{W}^T)|^2 \\
&= P_o \mathbf{w}^H [\mathbf{g}_{kj} \otimes (\mathbf{h}_k^* \otimes \mathbf{h}_j^*)] [\mathbf{g}_{kj}^H \otimes (\mathbf{h}_k^T \otimes \mathbf{h}_j^T)] \mathbf{w} \\
&= P_o \mathbf{w}^H [(\mathbf{g}_{kj} \mathbf{g}_{kj}^H) \otimes (\mathbf{h}_k^* \mathbf{h}_k^T \otimes \mathbf{h}_j^* \mathbf{h}_j^T)] \mathbf{w} \\
&= \mathbf{w}^H \mathbf{A}_{k,j}(\mathbf{g}_{kj}) \mathbf{w}
\end{aligned}$$

where $\mathbf{A}_{kj}(\mathbf{g}_{kj}) \triangleq P_o [(\mathbf{g}_{kj}\mathbf{g}_{kj}^H) \otimes (\mathbf{h}_k^*\mathbf{h}_k^T \otimes \mathbf{h}_j^*\mathbf{h}_j^T)]$ is the matrix for signal power depending on receiving vector \mathbf{g}_{kj} .

The interference \mathcal{I}_{int} from other saucers can be reformulated as

$$\begin{aligned}
\mathcal{I}_{\text{int}} &= P_o \sum_{l \in \mathcal{S}_k - \{j\}} |\mathbf{g}_{kj}^H \mathbf{W}(\mathbf{h}_k \otimes \mathbf{h}_l)|^2 \\
&= P_o \sum_{l \in \mathcal{S}_k - \{j\}} |\text{vec}[\mathbf{g}_{kj}^H \mathbf{W}(\mathbf{h}_k \otimes \mathbf{h}_l)]|^2 \\
&= P_o \sum_{l \in \mathcal{S}_k - \{j\}} |[\mathbf{g}_{kj}^H \otimes (\mathbf{h}_k^T \otimes \mathbf{h}_l^T)] \text{vec}(\mathbf{W}^T)|^2 \\
&= P_o \sum_{l \in \mathcal{S}_k - \{j\}} \left\{ \mathbf{w}^H [\mathbf{g}_{kj} \otimes (\mathbf{h}_k^* \otimes \mathbf{h}_l^*)] [\mathbf{g}_{kj}^H \otimes (\mathbf{h}_k^T \otimes \mathbf{h}_l^T)] \mathbf{w} \right\} \\
&= P_o \mathbf{w}^H \left[(\mathbf{g}_{kj}\mathbf{g}_{kj}^H) \otimes (\mathbf{h}_k^*\mathbf{h}_k^T \otimes \sum_{l \in \mathcal{S}_k - \{j\}} \mathbf{h}_l^*\mathbf{h}_l^T) \right] \mathbf{w} \\
&= \mathbf{w}^H \mathbf{B}_{k,j}(\mathbf{g}_{kj}) \mathbf{w}
\end{aligned} \tag{4.8}$$

where to arrive (4.8), we use properties $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD})$ and $(\mathbf{A} \otimes \mathbf{B}) + (\mathbf{A} \otimes \mathbf{C}) = \mathbf{A} \otimes (\mathbf{B} + \mathbf{C})$ of kronecker product. In addition, $\mathbf{B}_{kj}(\mathbf{g}_{kj}) \triangleq P_o [(\mathbf{g}_{kj}\mathbf{g}_{kj}^H) \otimes (\mathbf{h}_k^*\mathbf{h}_k^T \otimes \sum_{l \in \mathcal{S}_k - \{j\}} \mathbf{h}_l^*\mathbf{h}_l^T)]$ denotes interference matrix w.r.t. \mathbf{g}_{kj} .

Finally, we apply vectorization for \mathbf{W} in amplified noise power expression.

$$\begin{aligned}
\sigma_R^2 \|\mathbf{g}_{kj}^H \mathbf{W}(\mathbf{h}_k \otimes \mathbf{I}_M)\|^2 &= \sigma_R^2 \|\text{vec}[\mathbf{g}_{kj}^H \mathbf{W}(\mathbf{h}_k \otimes \mathbf{I}_M)]\|^2 \\
&= \sigma_R^2 \|[\mathbf{g}_{kj}^H \otimes (\mathbf{h}_k^T \otimes \mathbf{I}_M)] \text{vec}(\mathbf{W}^T)\|^2 \\
&= \sigma_R^2 \mathbf{w}^H [\mathbf{g}_{kj} \otimes (\mathbf{h}_k^* \otimes \mathbf{I}_M)] [\mathbf{g}_{kj}^H \otimes (\mathbf{h}_k^T \otimes \mathbf{I}_M)] \mathbf{w} \\
&= \sigma_R^2 \mathbf{w}^H [(\mathbf{g}_{kj}\mathbf{g}_{kj}^H) \otimes (\mathbf{h}_k^*\mathbf{h}_k^T \otimes \mathbf{I}_M)] \mathbf{w} \\
&= \mathbf{w}^H \mathbf{C}_{kj}(\mathbf{g}_{kj}) \mathbf{w}
\end{aligned}$$

where $\mathbf{C}_{kj}(\mathbf{g}_{kj}) = \sigma_R^2 [(\mathbf{g}_{kj}\mathbf{g}_{kj}^H) \otimes (\mathbf{h}_k^*\mathbf{h}_k^T \otimes \mathbf{I}_M)]$.

Now the SINR expression for user k required symbol j can be expressed w.r.t. \mathbf{w}

and \mathbf{g}_{kj} in (4.9).

$$\text{SINR}_{kj}(\mathbf{w}, \mathbf{g}_{kj}) = \frac{\mathbf{w}^H \mathbf{A}_{kj}(\mathbf{g}_{kj}) \mathbf{w}}{\mathbf{w}^H \mathbf{B}_{kj}(\mathbf{g}_{kj}) \mathbf{w} + \mathbf{w}^H \mathbf{C}_{kj}(\mathbf{g}_{kj}) \mathbf{w} + \sigma_D^2 \|\mathbf{g}_{kj}\|^2}. \quad (4.9)$$

Furthermore, using \mathbf{w} , we rewrite the relay power in (4.7) w.r.t. \mathbf{w} as

$$\begin{aligned} P_i &= \text{tr} \left[\mathbf{W}_i (P_0 \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M) \mathbf{W}_i^H \right] \\ &= \text{tr} \left[\mathbf{W}_i^H \mathbf{W}_i (P_0 \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M) \right] \\ &= \text{tr} \left[\left(\sum_{m=1}^M \mathbf{w}_{i,m}^* \mathbf{w}_{i,m}^T \right) (P_0 \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M) \right] \\ &= \left[\sum_{m=1}^M \mathbf{w}_{i,m}^T (P_0 \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M) \mathbf{w}_{i,m}^* \right] \\ &= (\text{vec}(\mathbf{W}_i^T))^T [\mathbf{I}_M \otimes (P_0 \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M)] (\text{vec}(\mathbf{W}_i^T))^* \\ &= \mathbf{w}_i^T [\mathbf{I}_M \otimes (P_0 \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M)] \mathbf{w}_i^* \\ &= \mathbf{w}^T \{ \mathbf{E}_i \otimes [\mathbf{I}_M \otimes (P_0 \mathbf{H} \mathbf{H}^H + \sigma_R^2 \mathbf{I}_M)] \} \mathbf{w}^* \end{aligned}$$

where $\mathbf{D}_i \triangleq \mathbf{E}_i \otimes [\mathbf{I}_M \otimes (P_0 \mathbf{H}^* \mathbf{H}^T + \sigma_R^2 \mathbf{I}_M)]$, in which $\mathbf{E}_i \triangleq \text{diag}(\mathbf{e}_i)$, with \mathbf{e}_i being a $(K-1) \times 1$ unitary vector with the i th entry being 1 and others all 0's. Then the power constraint in **P0** will be expressed as $\mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_r$.

The joint relay beamforming and receiver processing optimization problem **P0** can now be rewritten as

$$\begin{aligned} (\mathbf{P1}) : \quad & \max_{\mathbf{w}, \{\mathbf{G}_k\}} \min_{k, j \in \mathcal{S}_k^-} \text{SINR}_{kj}(\mathbf{w}, \mathbf{g}_{kj}) \\ & \text{s.t. } \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_r, \quad \forall i \end{aligned}$$

To solve this joint optimization problem, in the following, we first consider the sub-problem w.r.t. the beam vector \mathbf{w} and processing vector \mathbf{g}_{kj} separately, then iteratively solve the problem.

4.3.2 Relay Beamforming Matrix Design

When $\{\mathbf{G}_k\}$ is fixed, the optimization problem **P1** is only w.r.t. \mathbf{w} and can be reformulated into the following maxmin problem **P2**.

$$\begin{aligned}
 (\mathbf{P2}) : \quad & \max_{\mathbf{w}} \min_{k,j \in \mathcal{S}_{k-}} \text{SINR}_{kj}(\mathbf{w}, \mathbf{g}_{kj}) \\
 & \text{s.t. } \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_r, \quad \forall i.
 \end{aligned}$$

Inducing auxiliary variable t , problem **P2** can be rewritten as following. The SINR constraint in **P3** is not jointly convex w.r.t. both \mathbf{w} and t .

$$\begin{aligned}
 (\mathbf{P3}) : \quad & \min_{\mathbf{w}, t} t \\
 & \text{s.t. } t > 0 \\
 & \text{SINR}_{kj}(\mathbf{w}, \mathbf{g}_{kj}) \geq 1/t \quad j \in \mathcal{S}_{k-}, \forall k \\
 & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_r, \quad \forall i.
 \end{aligned}$$

To solve it, we first reformulate **P3**. Define $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$ and use SINR expression in (4.9), we transform the problem to **P4**.

$$\begin{aligned}
 (\mathbf{P4}) : \quad & \min_{\mathbf{X}, t} t \\
 & \text{s.t. } t > 0 \\
 & \text{tr}(\mathbf{D}_i \mathbf{X}) \leq P_r, \quad \forall i \\
 & \text{tr} \left[(t\mathbf{A}_{kj} - \mathbf{B}_{kj} - \mathbf{C}_{kj}) \mathbf{X} \right] \geq \sigma_D^2 \|\mathbf{g}_{kj}\|^2, j \in \mathcal{S}_{k-}, \forall k \\
 & \mathbf{X} \succeq 0, \text{rank}(\mathbf{X}) \leq 1
 \end{aligned}$$

Although the above optimization problem is not jointly convex w.r.t \mathbf{X} and t , it can be solved using the SDR approach when t is fixed. Thus, we can apply the 1D bi-

section search over t along with the SDR approach to find \mathbf{X} . Specifically, given t , after dropping the rank constraint, the problem becomes an SDP feasibility problem w.r.t. \mathbf{X} as

$$\text{Find } \mathbf{X} \tag{4.10}$$

$$\text{s.t. } \text{tr}(\mathbf{D}_i \mathbf{X}) \leq P_r, \quad \forall i$$

$$\text{tr} \left[(t\mathbf{A}_{kj} - \mathbf{B}_{kj} - \mathbf{C}_{kj}) \mathbf{X} \right] \geq \sigma_D^2 \|\mathbf{g}_{kj}\|^2, j \in \mathcal{S}_{k^-}, \forall k.$$

The SDP problem can be solved efficiently using interior-point methods [69] with standard SDP solvers. To recover \mathbf{w} from the optimal solution \mathbf{X}^* , if $\text{rank}(\mathbf{X}^*) = 1$, we can directly obtain the optimal beam vector \mathbf{w}^s from $\mathbf{X}^* = \mathbf{w}^s \mathbf{w}^{sH}$. Otherwise, a randomization technique [56] can be applied to find suboptimal rank-one solution \mathbf{w}^s .

4.3.3 Receiver Processing Structures

At user k 's receiver, we consider two specific structures, MRC and ZF, in designing the linear processing matrix \mathbf{G}_k . In the following, we discuss each design.

4.3.2.1 MRC Receiver

At user k , the MRC combining vector $\mathbf{g}_{kj}^{\text{MRC}}$ intends to maximize receiving SINR for each transmitted symbol s_j , $j \in \mathcal{S}_{k^-}$. Rewriting the SINR expression in (4.9) as a function of \mathbf{g}_{kj} , we have

$$\text{SINR}_{kj}(\mathbf{w}, \mathbf{g}_{kj}) = \frac{\mathbf{g}_{kj}^H \mathbf{x}_{kj} \mathbf{x}_{kj}^H \mathbf{g}_{kj}}{\mathbf{g}_{kj}^H \mathbf{F}_{kj} \mathbf{g}_{kj}} \quad j \in \mathcal{S}_{k^-}, \forall k \tag{4.11}$$

where

$$\mathbf{x}_{kj} \triangleq \sqrt{P_o} \mathbf{W}(\mathbf{h}_k \otimes \mathbf{h}_j),$$

$$\mathbf{F}_{kj} \triangleq P_o \mathbf{W}(\mathbf{h}_k \mathbf{h}_k^H \otimes \sum_{l \in \mathcal{S}_{k-} \setminus \{j\}} \mathbf{h}_l \mathbf{h}_l^H) \mathbf{W}^H + \sigma_R^2 \mathbf{W}(\mathbf{h}_k \mathbf{h}_k^H \otimes \mathbf{I}_M) \mathbf{W}^H + \sigma_D^2 \mathbf{I}_{K-1}.$$

The MRC combining vector $\mathbf{g}_{kj}^{\text{MRC}}$ is to maximize SINR in (4.11) w.r.t. s_j , given by

$$\mathbf{g}_{kj}^{\text{MRC}} = \arg \max_{\mathbf{g}_{kj}} \frac{\mathbf{g}_{kj}^H \mathbf{x}_{kj} \mathbf{x}_{kj}^H \mathbf{g}_{kj}}{\mathbf{g}_{kj}^H \mathbf{F}_{kj} \mathbf{g}_{kj}} \quad (4.12)$$

which is a generalized eigenvalue problem [70]. Since \mathbf{F}_{kj} is invertible, the solution to (4.12) is given by

$$\mathbf{g}_{kj}^{\text{MRC}} = \mathbf{F}_{kj}^{-1} \mathbf{x}_{kj} \quad j \in \mathcal{S}_{k-}, \forall k. \quad (4.13)$$

4.3.3.2 ZF Receiver

For an ZF receiver, we use processing matrix \mathbf{G}_k^{ZF} to cancel the interference in z_{kj} caused by signals from users other than user j , before send it to decoder to decode s_j , *i.e.*, the second term of z_{kj} in (4.5) should be zero. Rewrite $\tilde{\mathbf{y}}_k$ in (4.3) as

$$\tilde{\mathbf{y}}_k = \sqrt{P_o} \mathbf{W}(\mathbf{h}_k \otimes \mathbf{H}_{k-}) \mathbf{s}_{k-} + \mathbf{W}(\mathbf{h}_k \otimes \mathbf{I}_M) \mathbf{n}_r + \mathbf{n}_{d,k} \quad (4.14)$$

where \mathbf{H}_{k-} is an $M \times (K-1)$ matrix defined as matrix \mathbf{H} with column k removed, and \mathbf{s}_{k-} is a $(K-1) \times 1$ vector defined as transmitted symbol vector \mathbf{s}_k with k th entry removed. We design $\mathbf{g}_{kj}^{\text{ZF}}$ to maximize $\text{SINR}_{kj}(\mathbf{w}, \mathbf{g}_{kj})$, subject to the interference cancellation constraint, for $k = 1, \dots, K$ and $j \in \mathcal{S}_{k-}$. Using SINR expression in (4.11) and removing interference term, the optimization problem is given by

$$\begin{aligned} \max_{\mathbf{g}_{kj}} & \frac{\mathbf{g}_{kj}^H \mathbf{x}_{kj} \mathbf{x}_{kj}^H \mathbf{g}_{kj}}{\mathbf{g}_{kj}^H \mathbf{Q}_k \mathbf{g}_{kj}} \\ \text{s.t.} & \quad \mathbf{g}_{kj}^H \mathbf{X}_k = \mathbf{e}_j^T \end{aligned} \quad (4.15)$$

where $\mathbf{Q}_k \triangleq \sigma_R^2 \mathbf{W}(\mathbf{h}_k \mathbf{h}_k^H \otimes \mathbf{I}_M) \mathbf{W}^H + \sigma_D^2 \mathbf{I}_{K-1}$, and $\mathbf{X}_k \triangleq \mathbf{W}(\mathbf{h}_k \otimes \mathbf{H}_{k-})$. Note that under the interference cancellation constraint in (4.15), we have $\mathbf{g}_{kj}^H \mathbf{x}_{kj} \mathbf{x}_{kj}^H \mathbf{g}_{kj} = P_o$.

Then, the problem in (4.15) is equivalent to minimizing the noise power (relay amplified noise and receiver noise). Thus, it can be reformulated to

$$\begin{aligned} \min_{\mathbf{g}_{kj}} \quad & \mathbf{g}_{kj}^H \mathbf{Q}_k \mathbf{g}_{kj} \\ \text{s.t.} \quad & \mathbf{g}_{kj}^H \mathbf{X}_k = \mathbf{e}_j^T. \end{aligned} \quad (4.16)$$

Note that for the interference cancellation condition in (4.16) to hold for all k and $j \in \mathcal{S}_{k^-}$, we require \mathbf{X}_k to be full rank. In general, under fading condition and for physically separated users, channel matrix \mathbf{H} is typically full rank. This means the relay beam matrix \mathbf{W} needs to be full rank. In addition, in order to be able to cancel interference from all other users, the number of relay antennas should be no less than the number of interferers, *i.e.*, $M \geq K - 1$. Thus, we have the following assumptions under ZF receiver:

- A1) Relay beam matrix \mathbf{W} is full rank;
- A2) $M \geq K - 1$.

Under assumptions A1 and A2, we solve (4.16) by Lagrange multiplier technique. Lagrange cost function is set-up as (4.17), with multiplier $\lambda = [\lambda_1, \dots, \lambda_{K-1}]^T$.

$$\mathcal{L}(\mathbf{g}_{kj}, \lambda) = \mathbf{g}_{kj}^H \mathbf{Q}_k \mathbf{g}_{kj} - (\mathbf{g}_{kj}^H \mathbf{X}_k - \mathbf{e}_j^T) \lambda. \quad (4.17)$$

Taking derivative w.r.t. \mathbf{g}_{kj}^H , and setting the result to zero, we have $(\mathbf{Q}_k \mathbf{g}_{kj})^T - (\mathbf{X}_k \lambda)^T = 0$. \mathbf{g}_{kj} and multiplier λ can be solved as $\mathbf{g}_{kj} = \mathbf{Q}_k^{-1} \mathbf{X}_k \lambda$, $\lambda = (\mathbf{X}_k^H \mathbf{Q}_k^{-1} \mathbf{X}_k)^{-1} \mathbf{e}_j^*$, respectively. Substituting λ into expression of \mathbf{g}_{kj} , we have the close-form solution for $\mathbf{g}_{kj}^{\text{ZF}}$ as follows

$$\mathbf{g}_{kj}^{\text{ZF}} = \mathbf{Q}_k^{-1} \mathbf{X}_k \left[\mathbf{X}_k^H \mathbf{Q}_k^{-1} \mathbf{X}_k \right]^{-1} \mathbf{e}_j \quad (4.18)$$

where the inversion of $\mathbf{X}_k^H \mathbf{Q}_k^{-1} \mathbf{X}_k$ exists under the assumptions A1 and A2. As a result, we have ZF receiving structure as

$$\mathbf{G}_k^{\text{ZF}} = \left[\mathbf{X}_k^H \mathbf{Q}_k^{-1} \mathbf{X}_k \right]^{-1} \mathbf{X}_k^H \mathbf{Q}_k^{-1}.$$

4.3.4 Iterative Algorithm for Joint Design

For the joint optimization problem **P0** (or **P1**), we perform iteratively optimization over the relay beam matrix \mathbf{w} and receiver processing matrices $\{\mathbf{G}_k\}$.

Note that with the MRC receiver, $\mathbf{g}_{kj}^{\text{MRC}}$ in (4.13) is obtained by maximizing $\text{SINR}_{kj}(\mathbf{w}, \mathbf{g}_{kj})$. Thus, the resulting $\mathbf{G}_k^{\text{MRC}}$ is the optimal solution of problem **P1** with fixed \mathbf{w} , *i.e.*, $\mathbf{G}_k^o = \mathbf{G}_k^{\text{MRC}}$. Thus, we can iteratively solve the original problem **P0** by the solutions in Section 4.3.2 and 4.3.3.

Obtaining $\mathbf{g}_{kj}^{\text{MRC}}$ in (4.13), for each k and $j \in \mathcal{S}_{k^-}$, involves higher computational complexity. Alternatively, we can consider using ZF processing matrix \mathbf{G}_k^{ZF} , which can be obtained with much lower complexity but is suboptimal due to the ZF constraint in (4.15) imposed.

The proposed iterative optimization algorithm for joint relay beamforming and receiver processing in the MWMA relaying is summarized in Algorithm 3.

4.4 Successive Interference Cancellation Decoding

Until now, we have been focusing on the joint design of relay and receiver processing. At each user k , the receiver decoder extracts $K - 1$ required symbols from received vector $\tilde{\mathbf{y}}_k$. In our design above, the receiver processing vector \mathbf{g}_{kj} is proposed by maximizing SINR_{kj} for each source j , $j \neq k$, assuming that the signal from each source

Algorithm 3 Proposed joint relay and receiver design for MWMA relaying

Input: \mathbf{w} , \mathbf{D}_i , P_r , \mathbf{g}_{kj} , number of iterations N , \mathcal{E}_{th}

- 1: **Initialization:** Randomly generate $\mathbf{w}^{(0)}$ and scale it to ensure power constraint in **P1** satisfied.
- 2: **for** $n = 1$ to N **do**
- 3: Obtain $\mathbf{g}_{kj}^{(n)}$ in (4.13), $j \in \mathcal{S}_{k-}$, $\forall k$, for MRC receiver; Or obtain $\mathbf{G}_k^{(n)}$ for ZF receiver.
- 4: Fix $\mathbf{g}_{kj}^{(n)}$, solving SDP problem (4.10) with 1D bi-section search to reconstruct $\mathbf{w}^{(n)}$ from \mathbf{X}^* .
- 5: Obtain $\text{SINR}_{\min}^{(n)} \triangleq \min_{k,j \in \mathcal{S}_{k-}} \text{SINR}_{kj}^{(n)}$
- 6: Compute $\mathcal{E}_{\text{SINR}} = (\text{SINR}_{\min}^{(n)} - \text{SINR}_{\min}^{(n-1)}) / \text{SINR}_{kj}^{*(n-1)}$
- 7: **if** $\mathcal{E}_{\text{SINR}} \leq \mathcal{E}_{\text{th}}$ **then**
- 8: break
- 9: **end if**
- 10: **end for**
- 11: $\mathbf{w}^s = \mathbf{w}^{(n)}$, $\mathbf{G}_k^s = \mathbf{G}_k^{(n)}$.

Output: \mathbf{w}^s , \mathbf{G}_k^s

j is decoded independently. Note that, to decode each intended source signal, received vector $\tilde{\mathbf{y}}_k$ in (4.3) contains not only the required symbol, but also cross interference components. Therefore, to further improve the sum-rate performance, SIC [71] can be applied for receiver decoding. And the essential idea of SIC is through the best decoding sequence for all $j \in \mathcal{S}_{k-}$ to improve receiving performance.

Generally speaking, for SIC, we need $K-1$ recursive rounds for each user to decode these $K-1$ symbols one by one. In each round, we determine which source symbol is to be decoded and perform decoding. And using the decoded symbol, we remove the corresponding cross-interference component from the received vector $\tilde{\mathbf{y}}_k$. To explain how SIC works in details, we first define two sets, \mathcal{S}_{k-} and \mathcal{S}_k^D for un-decoded symbols and decoded symbols, respectively. \mathcal{S}_{k-} is initialized as $\mathcal{S}_{k-} = \{1, \dots, K\} \setminus \{k\}$, and \mathcal{S}_k^D is \emptyset . The SIC procedure is depicted in Fig.4.2.

According to Fig. 4.2, first of all, at user k , once $\tilde{\mathbf{y}}_k$ and optimal \mathbf{w}^s are successfully

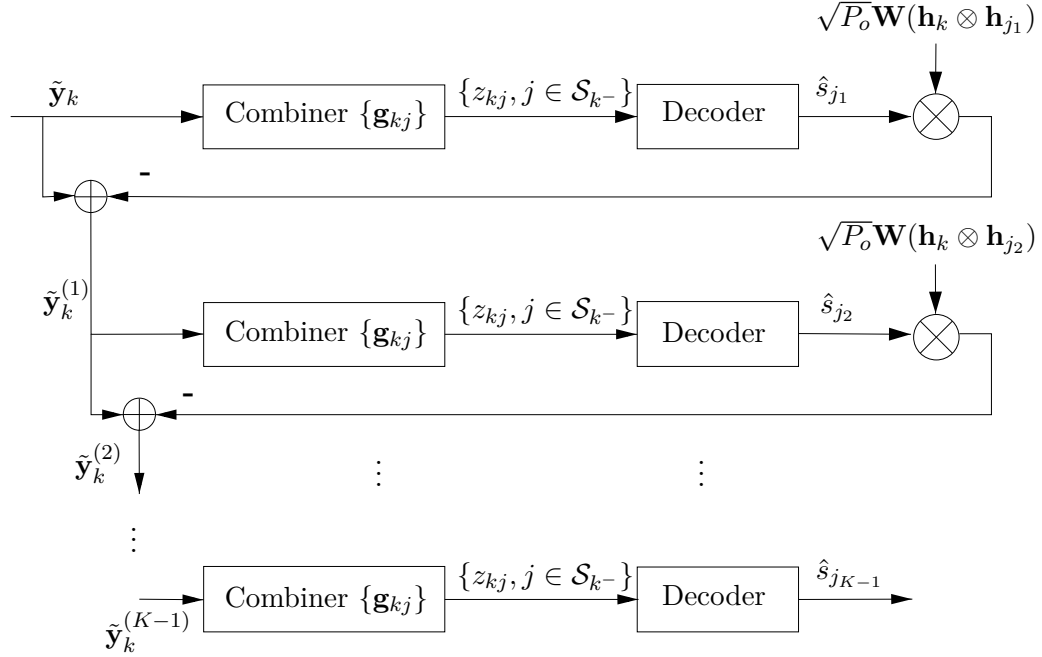


Figure 4.2: Successive Interference Cancellation Procedure

obtained, the receiver starts computing combining vectors $\mathbf{g}_{kj}, j \in \mathcal{S}_{k^-}$. The output of the combiner is a sequence of symbols $\{z_{kj}, j \in \mathcal{S}_{k^-}\}$. Next, the decoder will pick the one in $\{z_{kj}, j \in \mathcal{S}_{k^-}\}$, which $j = \arg \max_{j \in \mathcal{S}_{k^-}} \text{SINR}_{kj}$, whose index is denoted by j_1 , and decode the corresponding symbol s_{j_1} . Then with the decoded symbol \hat{s}_{j_1} , the related portion $\sqrt{P_o} \mathbf{W}(\mathbf{h}_k \otimes \mathbf{h}_j) \hat{s}_{j_1}$ is subtracted from $\tilde{\mathbf{y}}_k$ to remove the interference caused by s_{j_1} (in the case of no decoding error), and we have results updated as $\tilde{\mathbf{y}}_k^{(1)}$. It is then used as the input of the combiner in the next round. The last step of this round is to update \mathcal{S}_{k^-} and \mathcal{S}_k^D by moving index j_1 from \mathcal{S}_{k^-} to \mathcal{S}_k^D . This procedure then repeats until all $K - 1$ symbols are decoded. The SIC algorithm is briefly described in Algorithm 4.

Algorithm 4 SIC Algorithm

Input: \mathbf{w}^s , channel matrix \mathbf{H} , receiving vector $\tilde{\mathbf{y}}_k$, number of users K

1: **Initialization:** For each user k , define the empty set \mathcal{S}_k^D for all decoded symbols

2: **for** $i = 1$ to $K - 1$ **do**

3: Update $\mathcal{S}_{k^-} = \mathcal{S}_{k^-} \setminus \mathcal{S}_k^D$

4: Calculate $\mathbf{g}_{kj}, j \in \mathcal{S}_{k^-}$ with \mathbf{w}^s according to (4.13) for $\tilde{\mathbf{y}}_k$

5: Evaluate $\text{SINR}_{kj}, j \in \mathcal{S}_{k^-}$ as (4.6)

6: Decode symbol \hat{s}_j as $j = \arg \max_{j \in \mathcal{S}_{k^-}} \text{SINR}_{kj}$

7: Update $\tilde{\mathbf{y}}_k$ by $\tilde{\mathbf{y}}_k = \tilde{\mathbf{y}}_k - \sqrt{P_o} \mathbf{W}(\mathbf{h}_k \otimes \mathbf{h}_j) \hat{s}_j$

8: Update $\mathcal{S}_k^D = \mathcal{S}_k^D \cup \{j\}$

9: **end for**

Output: Decoding symbols $\hat{s}_j, j \in \mathcal{S}_k^D$

4.5 Receiver Processing under Partial CSI

In above discussion on the combining and decoding design, we ideally assume that CSI between the relay and all users is perfectly known for each user. In practice, for each user, only full CSI between relay and user itself is available, while CSI between the relay and other users needs to be broadcasted by the relay node. Considering limited bandwidth for the feedback channel, the relay is only capable of sending quantized CSIs. As a result, it would definitely cause performance loss in terms of sum-rate due to this quantization used in combining and decoding.

Intuitively, the performance loss can be reduced if the code book (for quantized CSIs) of a larger size is applied. However, more bits for quantization means more feedback bits are required. Therefore, in order to balance the requirement of performance and limitation of bandwidth, there is a trade-off between performance and feedback bits used.

To investigate the effect of quantized CSI on the performance, we assume that beam weight matrix \mathbf{W} is perfectly known at each user. For each user k , CSI \mathbf{h}_k ,

between the relay and the user itself, is perfectly known as well, but other CSIs $\mathbf{h}_j, j \in \mathcal{S}_{k^-}$ are not available. Instead, the quantized version $\hat{\mathbf{h}}_j$ will be used for $j \in \mathcal{S}_{k^-}$. The method of quantization of CSI is described as follows:

1) *Generate code book*: Assume we need a code book with each code having n bits. First, for a Gaussian random variable x with zero mean and unit variance, we generate a large number of realizations and divide the range of x into $N = 2^n$ bins, with the middle point of each bin as $x_i = \frac{2x_{\max}}{N}(i + \frac{1}{2}), i = -N/2, -N/2 + 1, \dots, N/2 - 1$ where x_{\max} is the maximal value of x in those realizations. Under this quantization, we have a codebook of size n for Gaussian r.v..

2) *Scaling by channel variance σ_h^2* : For a complex Gaussian distributed channel coefficient with variance σ_h^2 , we multiply a scalar $\sqrt{\sigma_h^2/2}$ to whole code book.

3) *Quantize \mathbf{h}_j* : For each entry in \mathbf{h}_j , we quantize its real and image parts, respectively, by quantizing it to the nearest x_i . This produces the quantized channel vector denoted as $\hat{\mathbf{h}}_j$.

The receiver at user k compute the combining vectors $\{\mathbf{g}_{kj}\}$ using quantized CSI $\hat{\mathbf{h}}_j, j \in \mathcal{S}_{k^-}$. Taking the MRC combiner as an example, the combining vector $\mathbf{g}_{kj}^{\text{MRC}}$ for $j \in \mathcal{S}_{k^-}, \forall k$ will be expressed as

$$\mathbf{g}_{kj}^{\text{MRC}} = \left\{ \mathbf{W} [P_o(\mathbf{h}_k \mathbf{h}_k^H \otimes \sum_{l \in \mathcal{S}_{k^-} \setminus \{j\}} \hat{\mathbf{h}}_l \hat{\mathbf{h}}_l^H) + \sigma_R^2(\mathbf{h}_k \mathbf{h}_k^H \otimes \mathbf{I}_M)] \mathbf{W}^H + \sigma_D^2 \mathbf{I}_{K-1} \right\}^{-1} \sqrt{P_o} \mathbf{W}(\mathbf{h}_k \otimes \hat{\mathbf{h}}_j)$$

where \mathbf{h}_k is perfectly known, but \mathbf{h}_j is replaced by its quantized version $\hat{\mathbf{h}}_j, j \in \mathcal{S}_{k^-}$.

Similarly, when using the SIC decoder structure, quantized CSIs will be used in computing \mathbf{g}_{kj} in each successive decoding round, and will be subsequently substituted into SINR expression in (4.6).

4.6 Simulation Results

In this section we study the performance of the proposed iterative algorithm with MRC and ZF receiver structures for a MWMA relay network which consists of K single antenna user and one relay node equipped with M antennas. We assume each channel coefficient h_{km} is independent and identically distributed following the complex Gaussian distribution with zero-mean and variance σ_h^2 , and it is unchanged in K time slots. We set power of each transmitted symbol P_o is always identical to total relay power for each BC phase P_r . Noise variance at the relay and each receiver is set as $\sigma_R^2 = \sigma_D^2 = 1$. In addition, we define $\overline{\text{SNR}} = \sigma_h^2 / \sigma_D^2$ as the nominal average SNR over the channel between the relay and each user when transmit power is 1. It is an indication of channel quality between the relay and a user.

4.6.1 Initialization and Convergence of Iterative Algorithm

In this part, we analyze initialization and convergence behavior of our proposed Algorithm 3. Iteration starts from a randomly generated beamforming matrix \mathbf{W} , and terminated when outcome SINR difference ratio $\mathcal{E}_{\text{SINR}}$ is less than certain threshold. Thus, we set $K = M = 3$, $P_o = P_r$, channel quality $\overline{\text{SNR}} = \{10, 20, 30\}$, and develop simulation for 30 Monte-Carlo runs with same channel realization but different initial beamforming matrix \mathbf{W} under MRC and ZF receiver structure. Minimum SINR evaluated at the end of each iteration will be saved.

4.6.1.1 Initialization Analysis

Minimum SINR outcome at the end of each Monte-Carlo run will be used to show the performance under each initialization. We plot the sorted SINR outcomes over 30

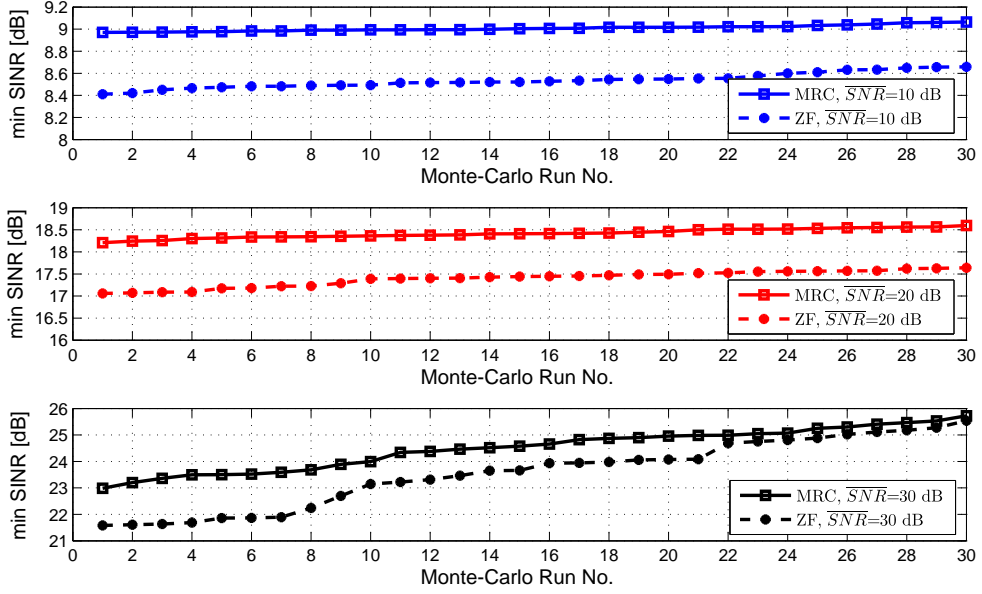


Figure 4.3: Different initial \mathbf{W} comparison ($M = K = 3$)

initializations to see how large the difference is, as shown in Fig. 4.3. It shows that the final SINR outcome becomes more and more sensitive to initial matrix \mathbf{W} along with the increasing of channel quality $\overline{\text{SNR}}$. Therefore, for a better performance, we should use several initializations and then pick the one with the best SINR outcome further improve the performance.

4.6.1.2 Convergence Analysis

We provide the flow diagram of our proposed iterative algorithm in Fig. 4.4. We compare the minimum SINR over all required symbols at points A, B and C for iterative round l , where points A, B, and C are described as follows:

- 1) *Point A*: $\text{SINR}^{(l_A)}$ is evaluated by $1/t$, where t is the output of bi-section search.
- 2) *Point B*: $\text{SINR}^{(l_B)}$ is evaluated by $\mathbf{w}^{(l)}$ and mismatched $\mathbf{G}_k^{(l-1)}$.
- 3) *Point C*: $\text{SINR}^{(l_C)}$ is evaluated by $\mathbf{w}^{(l)}$ and updated $\mathbf{G}_k^{(l)}$.

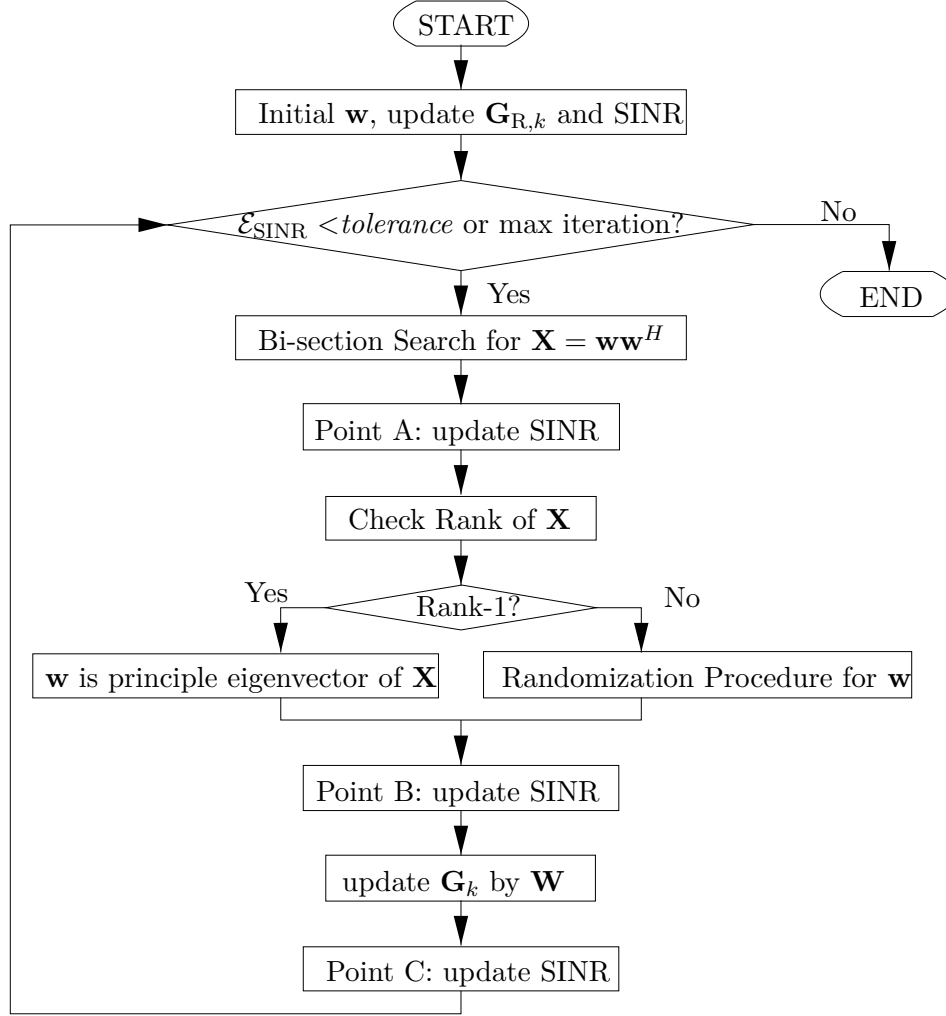


Figure 4.4: Iteration Algorithm Flow Diagram

Note that, ZF receiver structure for \mathbf{G}_k is not consistent with the max-min problem **P1** for \mathbf{w} . Therefore, using the iterative algorithm, we cannot guarantee that the SINR at point B and C will keep increasing in each iteration.

For the MRC combiner, we apply the method to update \mathbf{w} and \mathbf{G}_k following the same criteria, maximize the minimum SINR, thus, theoretically, the SINR increases until it converges to some value. In the simulation, the SINR values at point A and C keep increasing, but from point A to point B, the SINR will decrease due to following reasons:

1) After bi-section search, if \mathbf{X} is rank-1, then we take principle eigenvector as \mathbf{w} . In this case, the power at other direction will be ignored, which results in $\text{SINR}^{(l_B)}$ based on \mathbf{w} is smaller than $\text{SINR}^{(l_A)}$.

2) If \mathbf{X} is not rank-1, randomization procedure will be applied to find a rank-1 \mathbf{w} . On this occasion, $\text{SINR}^{(l_B)}$ can be either smaller or greater than $\text{SINR}^{(l_A)}$. And it is checked that for the case $\text{SINR}^{(l_B)} > \text{SINR}^{(l_A)}$, $\text{SINR}^{(l_B)} - \text{SINR}^{(l_A)}$ is always less than $1/t_l - 1/t_h$.

Next, we use output SINR at point B and C of each iteration to discuss convergence behavior of MRC and ZF, shown in Fig. 4.5 and Fig. 4.6, respectively. We set the maximum number of iterations to be 20. Minimum SINR related to half and integer points of X-axis denotes the point B and C outcomes of that iteration, respectively. We can see these two receiver structures have acceptable convergent feature in numerical iteration.

4.6.2 Performance Comparison

Next, we utilize Algorithm 3 to solve our optimization problem with MRC and ZF receiving structures and compare the performance.

4.6.2.1 Min SINR comparison under various relay power

We first compare the minimum SINR performance vs. relay power budget P_r . For 3 user case ($K=3$), we set $M = 2, 3, 4$. And for 4 user case ($K = 4$), relay is equipped with $M = 3, 4$ antennas. We set transmit power such that $P_o/\sigma_D^2 = 10$ dB, channel variance $\sigma_h^2 = 1$. Simulation with MRC and ZF combiner is running with 100 different channel realizations. For specific channel, initial \mathbf{w} is randomly generated

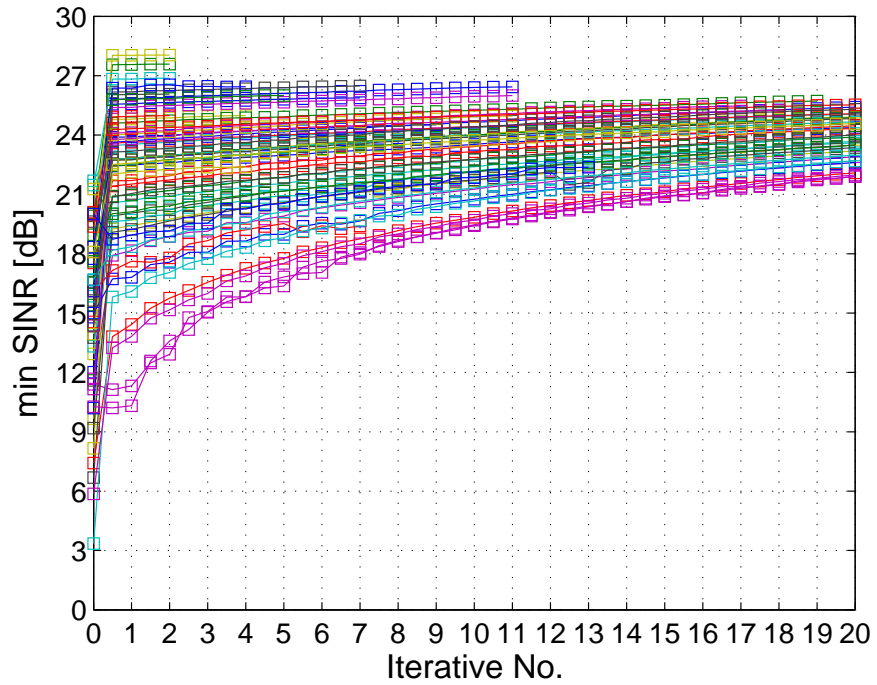


Figure 4.5: Convergence behavior ($K = M = 3, \overline{\text{SINR}} = 30$ dB, MRC)

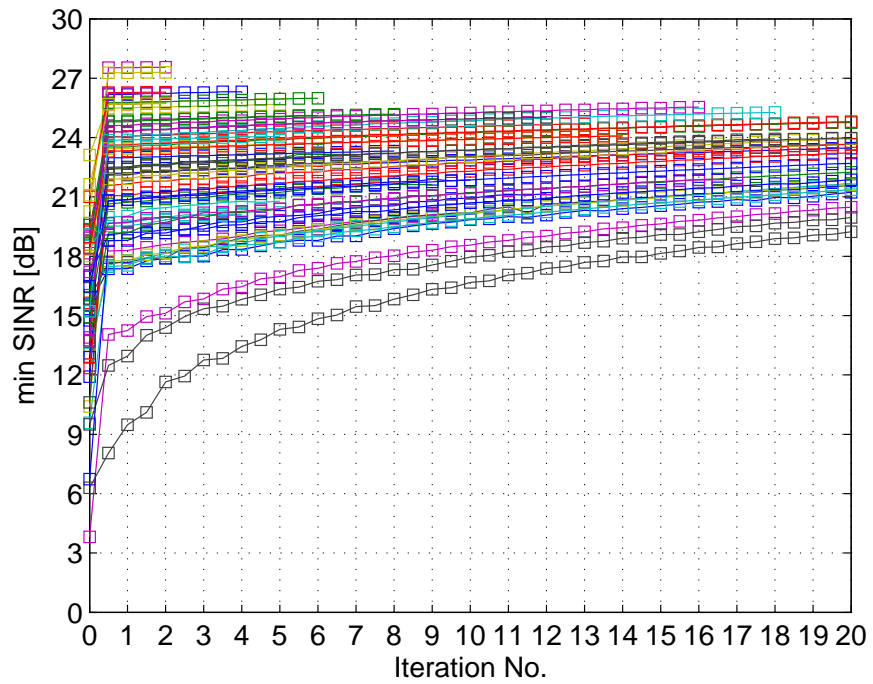


Figure 4.6: Convergence behavior ($K = M = 3, \overline{\text{SINR}} = 30$ dB, ZF)

and recursively iterated for at most 20 times for output SINR.

Fig. 4.7 illustrates the minimum SINR performance for $K = 3$. As we see, increasing relay power budget can improve the minimum SINR among all users. However, the improvement is gradually saturated as P_r becomes much higher. For different receiver structure, we see that MRC provides slightly better performance than ZF, with larger performance gap at relatively lower relay power P_r , and negligible difference at higher P_r . This is because ZF is only effective when noise is relatively low. We repeat the experiment for $K = 4$ as shown in Fig. 4.8. Note that by Assumption A2, we need $M \geq 3$. Similar observation is seen with a bigger gap as the number of users increases, demonstrating the suboptimality of the proposed ZF receiver.

4.6.2.2 Min SINR comparison under various CSI quality

After that, we change variable into CSI quality $\overline{\text{SNR}}$, and assume $P_o = P_r = 1$ to redo the simulation. Fig. 4.9 shows the minimum SINR versus $\overline{\text{SNR}}$ for $K = 3$, and $M = 2, 3, 4$. As expected, the minimal SINR performance increases with the channel quality improves, as well as with more relay antennas for beamforming gain. Comparing the two receiver structure, we see that the MRC receiver outperforms the ZF receiver, especially at lower $\overline{\text{SNR}}$. This again shows the suboptimality of the proposed ZF receiver. Nonetheless, we see that for higher M , the performance loss under the ZF receiver is smaller. In Fig. 4.10, it provides the minimum SINR performance versus $\overline{\text{SNR}}$ for $K = 4$. The same as $K = 3$ case, ZF receiver achieves lower SINR compared with MRC. And for lower $\overline{\text{SNR}}$, gap between MRC and ZF is even larger when we M is one less than K implying that degree of freedom supplied by antennas is not enough to cancel interference and deal with noise at the same time.

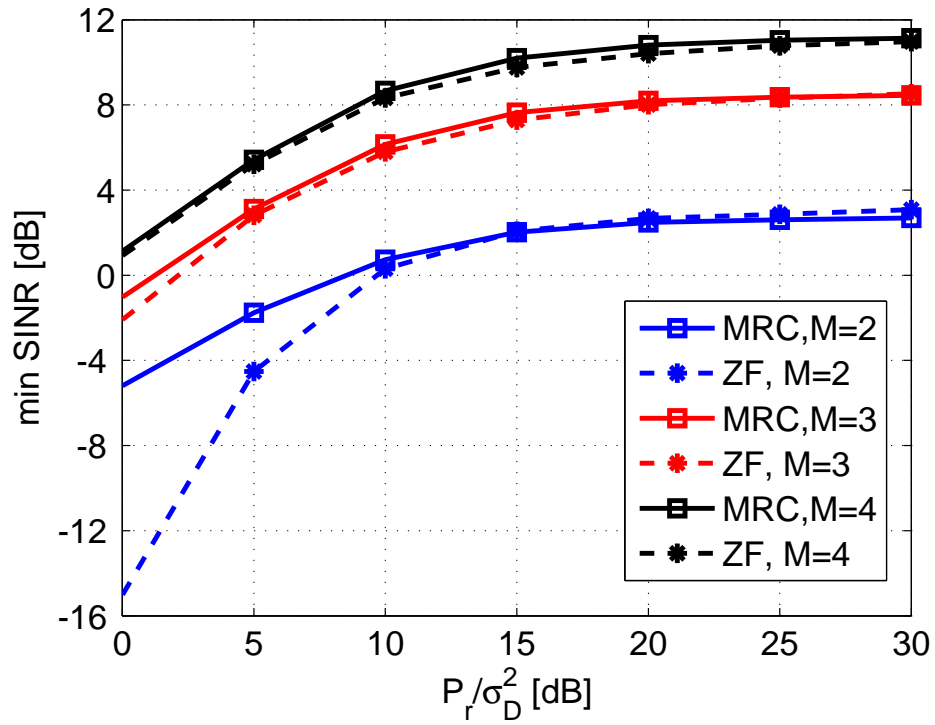


Figure 4.7: Minimum SINR vs. P_r ($K = 3$).

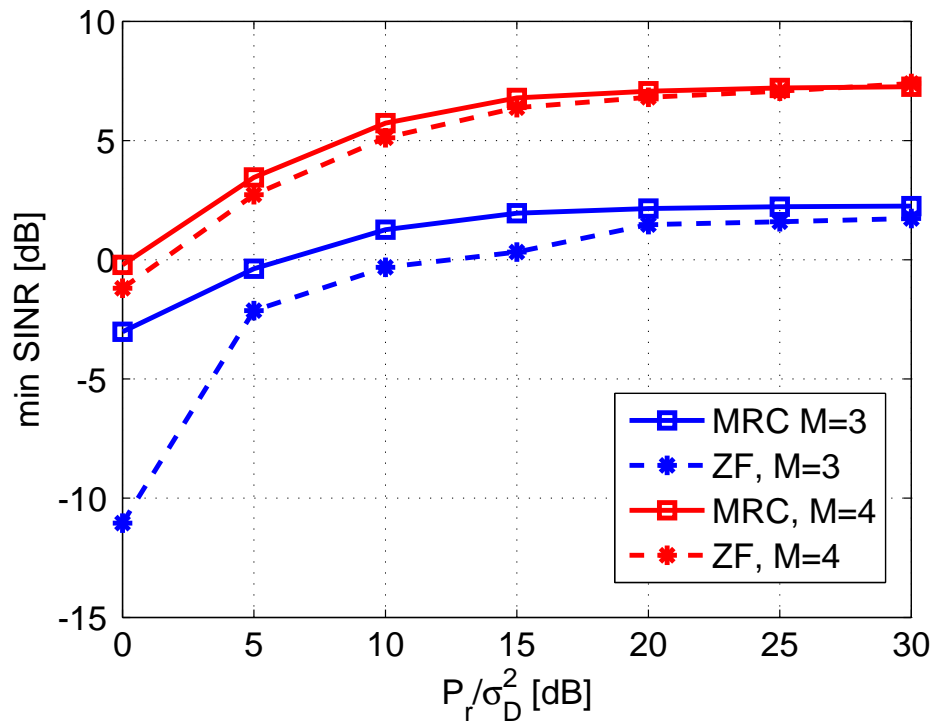


Figure 4.8: Minimum SINR vs. P_r ($K = 4$).

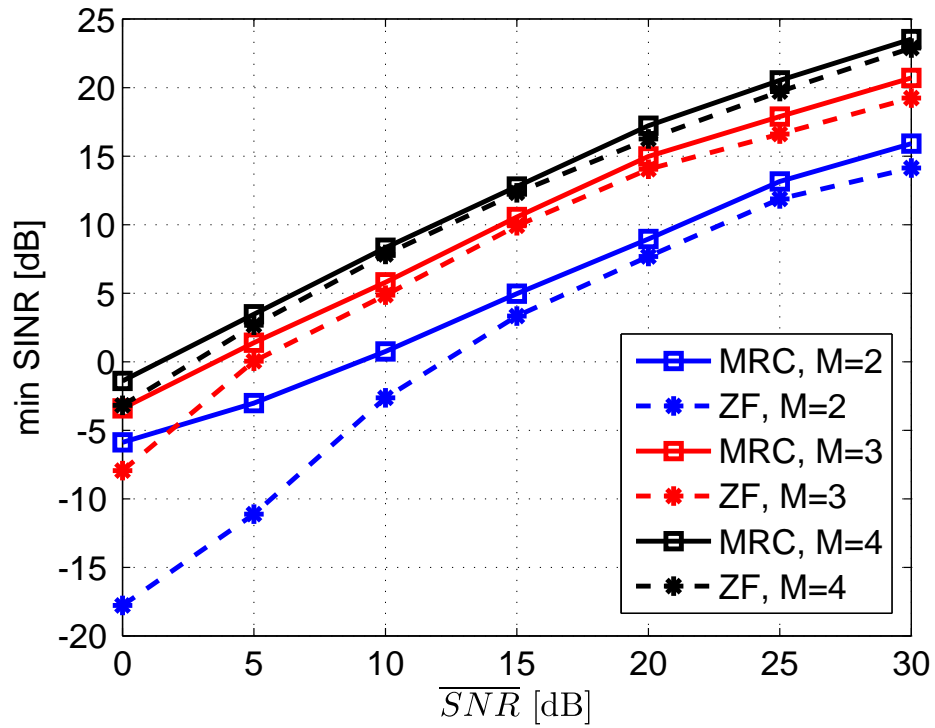


Figure 4.9: Minimum SINR vs. $\overline{\text{SNR}}$ ($K = 3$).

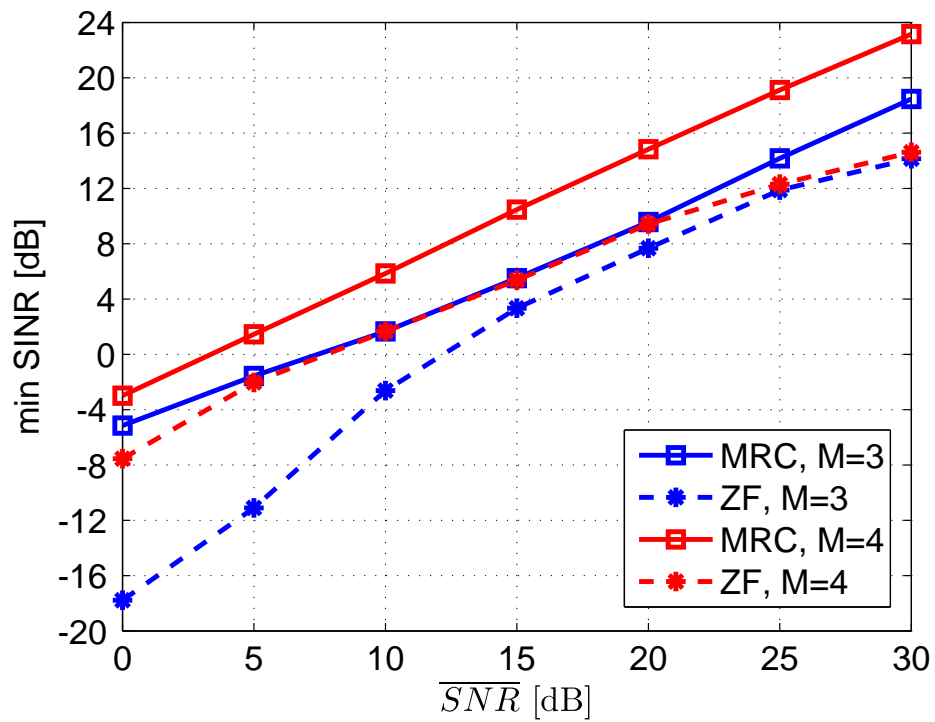


Figure 4.10: Minimum SINR vs. $\overline{\text{SNR}}$ ($K = 4$).

4.6.2.3 Sum-rate comparison

The achievable rate from user j to user k , denoted as R_{kj} , is given by $R_{kj} = \log(1 + \text{SINR}_{kj})$. Define the data rate from user j to all other users with a common rate as $R_j \triangleq \min_{k \neq j} R_{kj}$. The achievable sum-rate R_{sum} is then defined as $R_{\text{sum}} = \frac{K-1}{K} \sum_{j=1}^N R_j$. In sum-rate comparison, we assume transmitted power to be the same as total relay power, *i.e.*, $P_o = P_r = 1$ dBw, and variable is channel signal-to-noise ratio $\overline{\text{SNR}} = \sigma_h^2 / \sigma_D^2$. We apply 10 different initial beamforming matrix \mathbf{W} for each channel realization and study the achieved sum-rate performance in following three cases.

1) SIC decoder performance: In this part, we chose $K = M = 3$ and $K = M = 4$ cases with MRC receiver to develop simulation. Fig. 4.11 plots average sum-rate versus $\overline{\text{SNR}}$ observed at input and output of decoder. Applied SIC algorithm, the sum-rate performance will be effectively increased, especially at the higher $\overline{\text{SNR}}$ end. Furthermore, for higher K , sum-rate performance gain via SIC is even bigger.

2) Average sum-rate v.s. $\overline{\text{SNR}}$: Shown in Fig. 4.12 and Fig. 4.13, we compare decoding sum-rate for MRC and ZF receiver structures for 3 and 4 users cases. As expected, MRC structure outperforms ZF, because the latter can only provide the suboptimal solution for the joint problem **P1**, while MRC aims for global optimal solution to maximize minimum SINR. Besides, for more relay antennas, higher sum-rate can be achieved for both receiver structures. But we noticed sum-rate performance can not keep going up at higher $\overline{\text{SNR}}$ end in ZF, especially when $K = 4$. Because within that region, transmission environment is dominated by cross-interference, so that sub-optimality caused by interference cancellation condition in (4.16) will carry

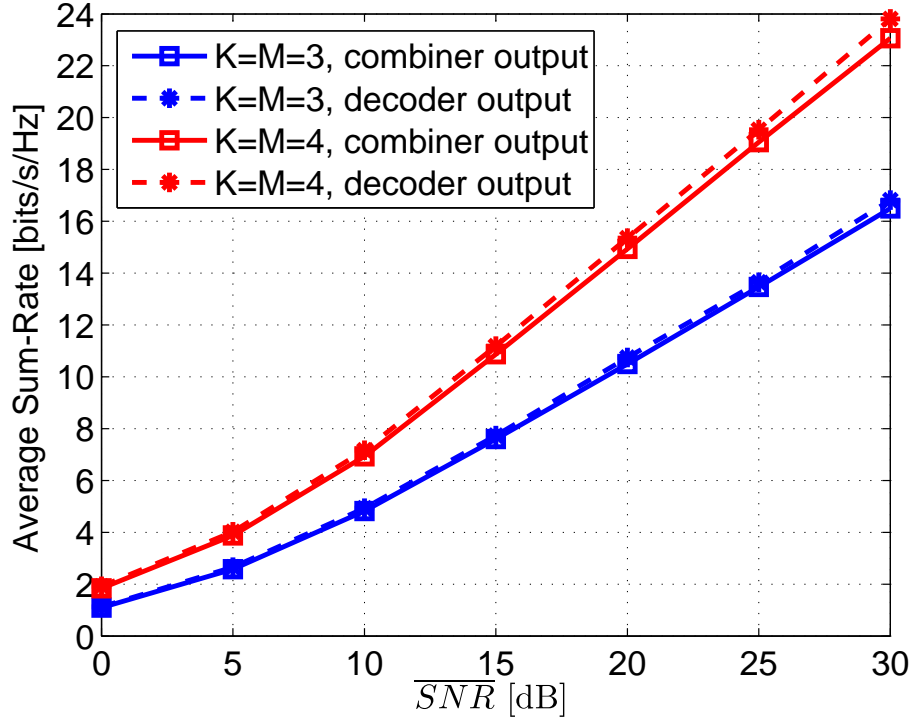
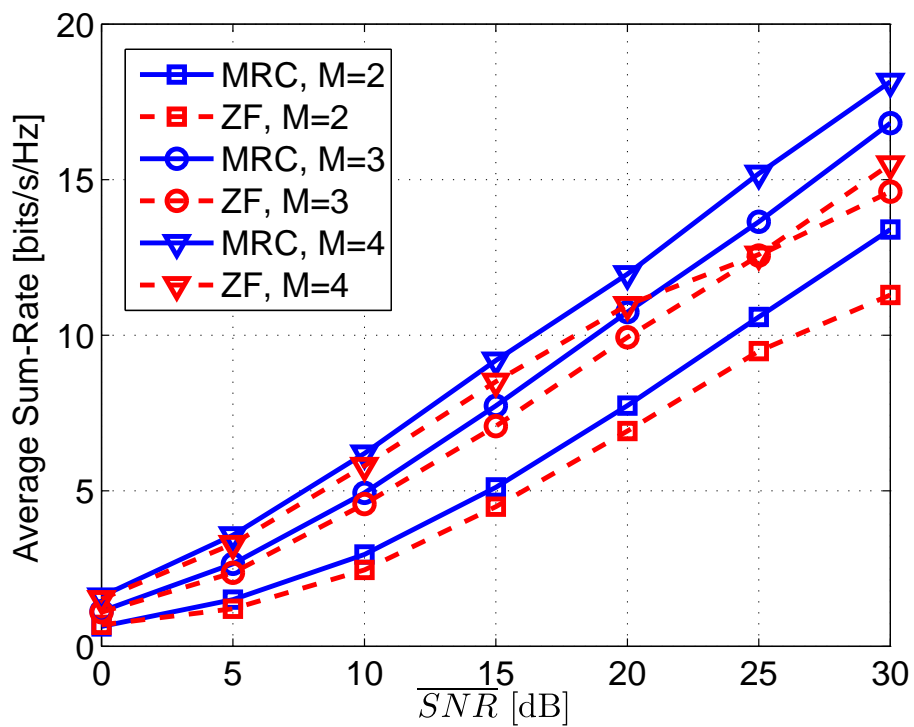
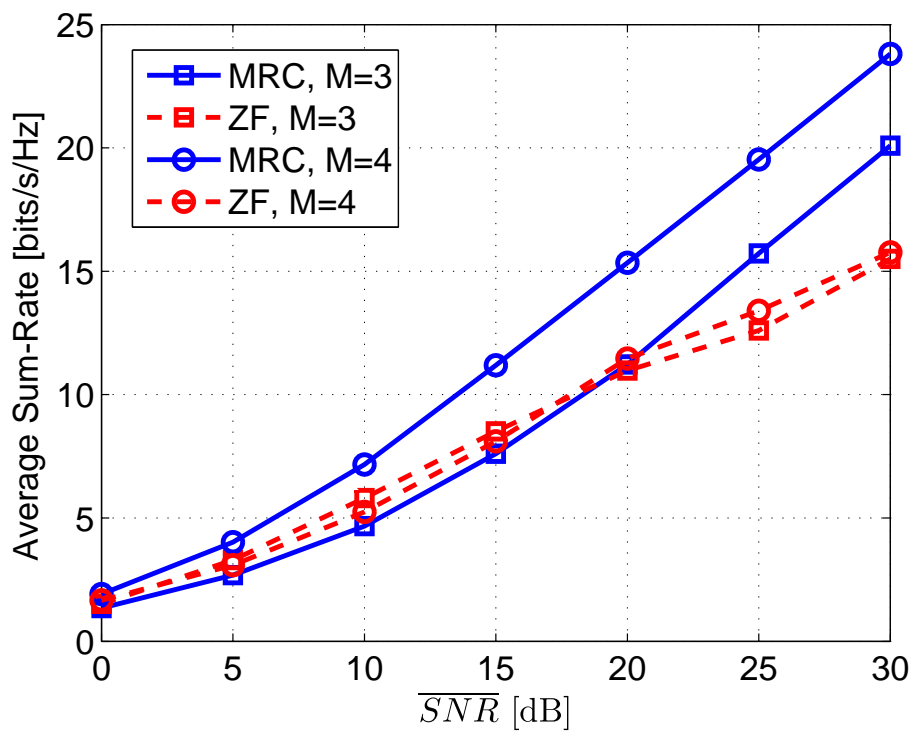


Figure 4.11: Sum-Rate vs. $\overline{\text{SNR}}$ for SIC decoder gain

out more impact on eventual outcome.

3) Comparison with existing algorithms: In addition, we compare our proposed algorithm with two existing algorithms, one is partial Zero-Forcing (PZF) algorithm given in [50], the other is linear ZF beamforming presented in [45]. Both of them are suboptimal algorithm proposed to maximizing the sum-rate R_{sum} . We plot the sum-rate versus $\overline{\text{SNR}}$ under joint designed MRC and ZF receiver structures in Fig 4.14 when $K = M = 3$, and Fig 4.15 for $K = M = 4$. We see that, even though sum-rate is not the objective of our design, our proposed algorithm with MRC receiver still outperforms the PZF and linear ZF algorithms in terms of sum-rate. For joint designed ZF receiver, our proposed algorithm outperforms those two existing algorithms except for higher $\overline{\text{SNR}}$.

Figure 4.12: Sum-Rate vs. $\overline{\text{SNR}}$ ($K = 3$)Figure 4.13: Sum-Rate vs. $\overline{\text{SNR}}$ ($K = 4$)

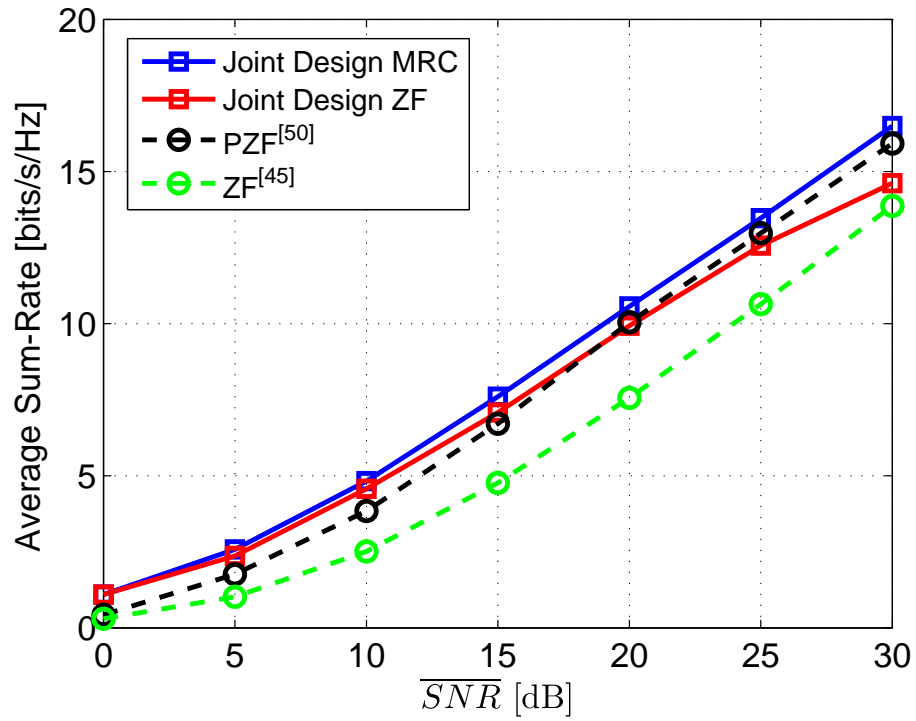


Figure 4.14: Sum-Rate vs. $\overline{\text{SNR}}$ ($K=3, M=3$)

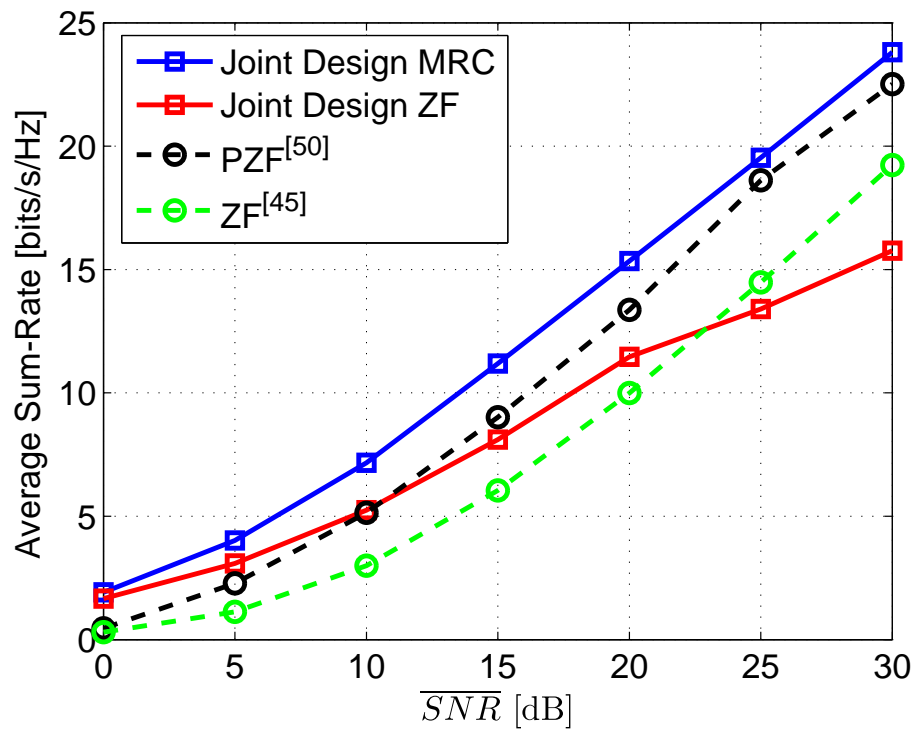
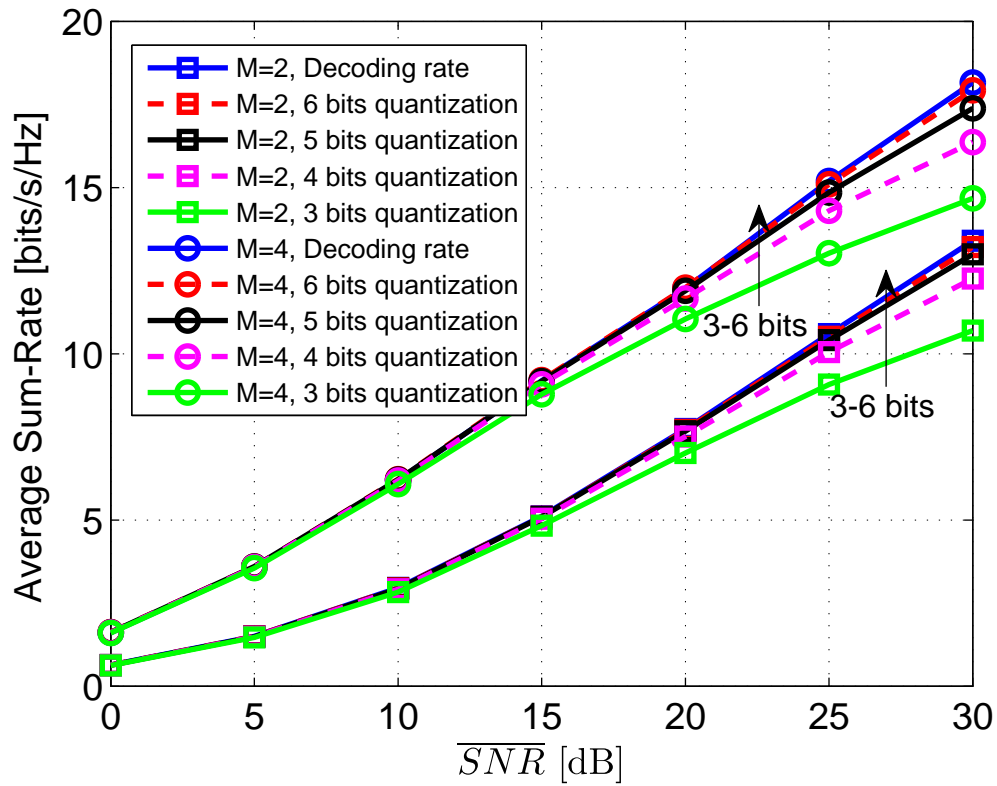
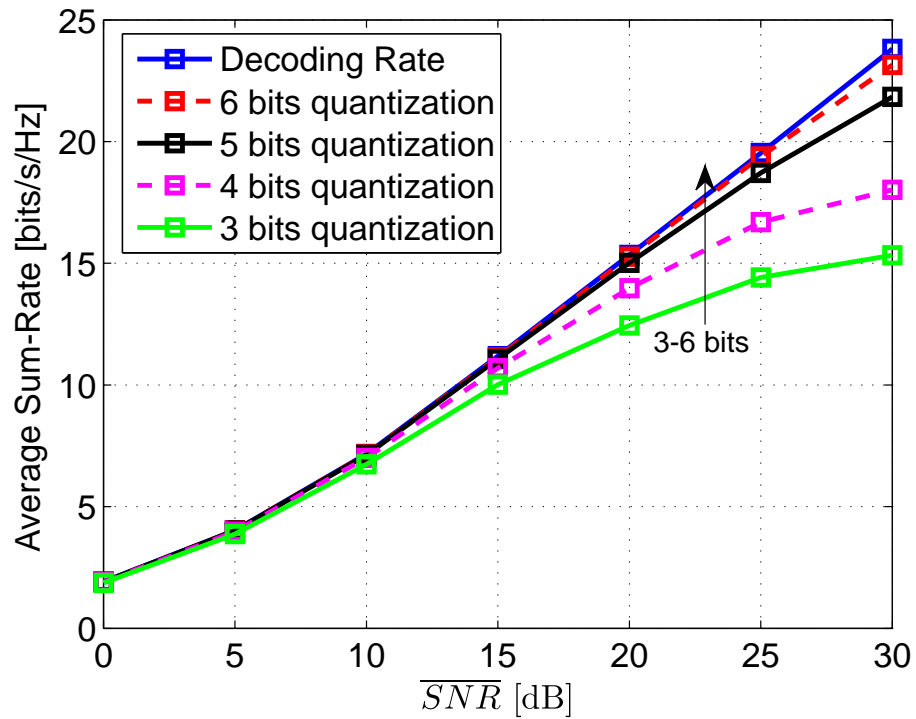


Figure 4.15: Sum-Rate vs. $\overline{\text{SNR}}$ ($K=4, M=4$)

4.6.3 Performance Loss due to Partial CSI

In this part, we study quantization performance loss under different code book size. we set $P_o = P_r = 1$ dBw, $\sigma_R^2 = \sigma_D^2 = 1$. $\overline{\text{SNR}}$ is varying from 0 to 30 dB. Using quantized channel information, in Fig. 4.16 and Fig. 4.17, we re-evaluate the resulting decoding outcome in terms of sum-rate and plot the sum-rate versus $\overline{\text{SNR}}$ under $K = M = 3$ and $K = M = 4$ schemes with joint designed MRC receiver . We noticed that partial CSI has greater impact in sum-rate performance when $\overline{\text{SNR}}$ is higher. Considering the trade-off between sum-rate performance and bandwidth, we can conclude that 5 bits code book is precisely enough for channel quantization when $K = 3$, while for $K = 4$, 6 bits code book is needed.

Figure 4.16: Sum-Rate vs. $\overline{\text{SNR}}$ ($K = 3, M = 2, 4$)Figure 4.17: Sum-Rate vs. $\overline{\text{SNR}}$ ($K = 4, M = 4$)

Chapter 5

Conclusion

In this thesis, we proposed a rank-two beamforming scheme with Alamouti code for MUP2P AF relay network. By jointly designing transmitter processing and relay beamforming, we aim to minimize maximal per-relay power under the prescribed receiving SINR constraints. Two approaches, namely ordinary SDR and separable SDP have been proposed to optimize the solution. Comparing with rank-one method, both approaches significantly decrease the per-relay power consumption and provide one more degree of freedom for optimal solution. When the solution is suboptimal, numerical performance illustrates that both two rank-two approaches have similar capability to effectively shrink the optimality gap and increase the chance of achieving optimal solution.

In addition, we considered a MWMA relay network for multi-user communications. Aiming at maximizing the minimum received symbol SINR at users under the relay power budget, we jointly designed a sequence of processing matrices at relay for multiple BC phases and receiver processing matrix at each user. An iterative algorithm is proposed to find the optimal solution by considering relay processing design and receiver processing design separately. Both MRC and ZF receivers are

derived, where the former leads to the optimal solution and the latter provides a simpler receiver implementation. Also we applied SIC as decoding algorithm at receiver. The numerical results demonstrated that the proposed algorithm with MRC receiver yields better performance than ZF receiver, and both of them provide better sum-rate performance as compared with the existing PZF method which is designed using the sum-rate objective. SIC decoder brings further performance increase in terms of sum-rate. When the CSI is not perfectly known, the uncertainty of CSI will induce considerably performance loss.

Appendix A

Proof of Proposition 1

Proof. : To prove Proposition 1, we first provide the following Lemma.

Lemma 1. *Given that \mathbf{D} is a Hermitian positive semidefinite matrix with $\text{rank}(\mathbf{D}) \geq 1$, let $\xi \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}^*)$, be independent random vectors. Consider the matrix $\mathbf{W} = \xi\xi^H$, and $\text{rank}(\mathbf{W}) \leq d$, $d \geq 2$ for any $\gamma \geq d/(d-1)$, we have*

$$\text{Prob} \left(\text{tr}(\mathbf{D}\mathbf{W}) \geq \gamma \text{tr}(\mathbf{D}\mathbf{W}^*) \right) \leq 2 \exp \left(-\frac{\gamma}{2} \right).$$

Lemma 2. *Given Hermitian positive semidefinite matrices $\mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_1, \mathbf{B}_2$, let $\xi \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_1^*)$, $\eta \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}_2^*)$ be independent random vectors. Consider the matrix $\mathbf{W}_1 = \xi\xi^H$ and $\mathbf{W}_2 = \eta\eta^H$. Then for any $\beta \leq 1$,*

$$\text{Prob} \left[\frac{\text{tr}(\mathbf{A}_1\mathbf{W}_1 + \mathbf{A}_2\mathbf{W}_2)}{\text{tr}(\mathbf{B}_1\mathbf{W}_1 + \mathbf{B}_2\mathbf{W}_2) + 1} \leq \beta \frac{\text{tr}(\mathbf{A}_1\mathbf{W}_1^* + \mathbf{A}_2\mathbf{W}_2^*)}{\text{tr}(\mathbf{B}_1\mathbf{W}_1^* + \mathbf{B}_2\mathbf{W}_2^*) + 1} \right] \leq \left(\frac{5\beta}{\alpha - 2\beta} \right)^2$$

where $\alpha \leq 1/\text{rank}[(\mathbf{W}_1^*)^{1/2}\mathbf{A}_1(\mathbf{W}_1^*)^{1/2} + (\mathbf{W}_2^*)^{1/2}\mathbf{A}_2(\mathbf{W}_2^*)^{1/2}]$, and $0 < \beta < \frac{\alpha}{2}$.

Consider a fixed j in Algorithm 1 and let $\hat{\mathbf{X}}_1 = \xi_j\xi_j^H$, $\hat{\mathbf{X}}_2 = \eta_j\eta_j^H$ for $j = 1, \dots, N$. Define events:

$$E_m^\gamma : P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \geq \gamma P_m(\mathbf{X}_1^*, \mathbf{X}_2^*), \forall m \in \{1, \dots, M\}$$

$$F_k^\beta : \text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \leq \beta \text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*), \forall k \in \{1, \dots, K\}$$

According to Lemma 1, we have

$$\begin{aligned}
\text{Prob}(E_m^\gamma) &= \text{Prob}(\text{tr}(\tilde{\mathbf{D}}_m \hat{\mathbf{X}}) \geq \gamma \text{tr}(\tilde{\mathbf{D}}_m \mathbf{X}^*)) \\
&= \text{Prob} \left[\text{tr}(\mathbf{D}_m \hat{\mathbf{X}}_1 + \mathbf{D}_m \hat{\mathbf{X}}_2) \geq \gamma \text{tr}(\mathbf{D}_m \mathbf{X}_1^* + \mathbf{D}_m \mathbf{X}_2^*) \right] \\
&\geq 2 \exp\left(-\frac{\gamma}{2}\right), \quad \forall m
\end{aligned}$$

where

$$\gamma \geq 2, \quad \tilde{\mathbf{D}}_m = \mathbf{I}_2 \otimes \mathbf{D}_m, \quad \mathbf{D}_m = (P_0 \sum_{k=1}^K |h_{1,km}|^2 + \sigma_R^2) \mathbf{E}_m.$$

Based on Lemma 2, we bound $\text{Prob}(F_k^\beta)$ as

$$\begin{aligned}
\text{Prob}(F_k^\beta) &= \text{Prob} \left\{ \frac{\text{tr}(\mathcal{A}_{k1} \hat{\mathbf{X}}_1 + \mathcal{A}_{k2} \hat{\mathbf{X}}_2)}{\text{tr}(\mathcal{B}_{k1} \hat{\mathbf{X}}_1 + \mathcal{B}_{k2} \hat{\mathbf{X}}_2) + 1} \leq \beta \frac{\text{tr}(\mathcal{A}_{k1} \mathbf{X}_1^* + \mathcal{A}_{k2} \mathbf{X}_2^*)}{\text{tr}(\mathcal{B}_{k1} \mathbf{X}_1^* + \mathcal{B}_{k2} \mathbf{X}_2^*) + 1} \right\} \\
&\leq \left(\frac{5\beta}{\alpha - 2\beta} \right)^2 \quad \forall k \in \{1, \dots, K\}
\end{aligned}$$

where \mathcal{A}_{k1} , \mathcal{A}_{k2} , \mathcal{B}_{k1} and \mathcal{B}_{k2} are all Hermitian matrices given as

$$\begin{aligned}
\frac{1}{\alpha} &\geq \text{rank}[(\mathbf{X}_1^*)^{1/2} \mathcal{A}_{k1} (\mathbf{X}_1^*)^{1/2} + (\mathbf{X}_2^*)^{1/2} \mathcal{A}_{k2} (\mathbf{X}_2^*)^{1/2}] = 2, \quad 0 \leq \beta \leq \frac{\alpha}{2} \\
\mathcal{A}_{k1} &= \frac{P_o}{\sigma_D^2} \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k}^* \mathbf{h}_{1,k}^T \hat{\mathbf{H}}_{2,k}, \quad \mathcal{A}_{k2} = \frac{P_o}{\sigma_D^2} \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,k} \mathbf{h}_{1,k}^H \hat{\mathbf{H}}_{2,k} \\
\mathcal{B}_{k1} &= \frac{P_o}{\sigma_D^2} \sum_{l \neq k} \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,l}^* \mathbf{h}_{1,l}^T \hat{\mathbf{H}}_{2,k} + \frac{\sigma_R^2}{\sigma_D^2} \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k} \\
\mathcal{B}_{k2} &= \frac{P_o}{\sigma_D^2} \sum_{l \neq k} \hat{\mathbf{H}}_{2,k}^H \mathbf{h}_{1,l} \mathbf{h}_{1,l}^H \hat{\mathbf{H}}_{2,k} + \frac{\sigma_R^2}{\sigma_D^2} \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}.
\end{aligned}$$

Now, let $E = \cup_m E_m^\gamma$, $F = \cup_k F_k^\beta$. Upon choosing $\gamma = 2 \log(16M)$, $\alpha = 1/2$, $\beta =$

$1/(16\sqrt{K})$ and applying union bound, we have

$$\begin{aligned}
\text{Prob}(E) &= \text{Prob}(\cup_m E_m^\gamma) \leq \sum_m \text{Prob}(E_m^\gamma) \leq M [2 \exp(-\frac{\gamma}{2})] < M \times \frac{1}{8M} = \frac{1}{8} \\
\text{Prob}(F) &= \text{Prob}(\cup_k F_k^\beta) \leq \sum_k \text{Prob}(F_k^\beta) \leq K \left(\frac{5\beta}{1/2 - 2\beta} \right)^2 < \frac{3}{4}.
\end{aligned}$$

Thus, for the events E^c and F^c ,

$$E^c = \left\{ P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \leq \gamma P_m(\mathbf{X}_1^*, \mathbf{X}_2^*), \forall m \right\},$$

$$F^c = \left\{ \text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \geq \beta \text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*), \forall k \right\}.$$

we conclude that

$$\begin{aligned} \text{Prob}(E^c \cap F^c) &= \text{Prob}[(E \cup F)^c] = 1 - \text{Prob}(E \cup F) \\ &= 1 - \text{Prob}(E) - \text{Prob}(F) + \text{Prob}(E \cap F) \\ &\geq 1 - \text{Prob}(E) - \text{Prob}(F) \\ &> 1 - \frac{1}{8} - \frac{3}{4} \\ &= \frac{1}{8}, \end{aligned}$$

i.e., , with probability at least $1/8$, we have the result:

$$\begin{aligned} \frac{P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2)}{\text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2)} &\leq \frac{\gamma}{\beta} \frac{P_m(\mathbf{X}_1^*, \mathbf{X}_2^*)}{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)} = 16\sqrt{K}[2\log(16M)] \frac{P_m(\mathbf{X}_1^*, \mathbf{X}_2^*)}{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)} \\ \Rightarrow P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) &\leq 16\sqrt{K}[2\log(16M)] P_m(\mathbf{X}_1^*, \mathbf{X}_2^*) \frac{\text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2)}{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)} \\ \Rightarrow P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) &\leq 16\sqrt{K}[2\log(16M)] P_m(\mathbf{X}_1^*, \mathbf{X}_2^*), \end{aligned}$$

or

$$\begin{aligned} \frac{\text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2)}{P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2)} &\geq \frac{\beta}{\gamma} \frac{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)}{P_m(\mathbf{X}_1^*, \mathbf{X}_2^*)} \\ \Rightarrow \text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) &\geq \frac{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)}{16\sqrt{K}(2\log(16M))} \frac{P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2)}{P_m(\mathbf{X}_1^*, \mathbf{X}_2^*)} \\ \Rightarrow \text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) &\geq \frac{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)}{16\sqrt{K}(2\log(16M))}. \end{aligned}$$

Therefore, the events

$$\left\{ \exists j : P_m(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \leq 16\sqrt{K}[2\log(16M)] P_m(\mathbf{X}_1^*, \mathbf{X}_2^*), \forall m \right\}$$

and

$$\left\{ \exists j : \text{SINR}_k(\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2) \geq \frac{\text{SINR}_k(\mathbf{X}_1^*, \mathbf{X}_2^*)}{16\sqrt{K}(2\log(16M))}, \forall k \right\}$$

occur with probability at least $1 - (7/8)^N$. ■

Appendix B

Rank-One Multi-User Peer-to-Peer Problem Formulation

In rank-one beamforming scheme, it needs two time slots to complete the transmission between S-D pair k via assistance of M relays. In the first time slot, K source forwards their own information symbols to relays. Under the same definition of channel status, the received signal at relays can be denoted as $\mathbf{r} = \sqrt{P_o} \sum_{k=1}^K \mathbf{h}_{1,k} s_k + \mathbf{n}_r$, where \mathbf{n}_r is noise vector at relays. Then relay m processes received signal with beam weight w_m . Define $\mathbf{w} \triangleq [w_1, \dots, w_M]^T$ as a $M \times 1$ beamforming vector and $\mathbf{x} \triangleq [x_1, \dots, x_M]^T$ as processed signal vector at relays for transmission, then the relays operation can be denoted as $\mathbf{x} = \mathbf{W}\mathbf{r}$, where $\mathbf{W} \triangleq \text{diag}(\mathbf{w})$. In the second time slot, relays send processed signal \mathbf{x} to destinations. For destination k , the received signal can be denoted as

$$\begin{aligned} y_k &= \mathbf{h}_{2,k}^T \mathbf{x} + n_{d,k} \\ &= \sqrt{P_o} \mathbf{h}_{2,k}^T \mathbf{W} \mathbf{h}_{1,k} s_k + \sqrt{P_o} \mathbf{h}_{2,k}^T \mathbf{W} \sum_{l=1, l \neq k}^K \mathbf{h}_{1,l} s_l + \mathbf{h}_{2,k}^T \mathbf{W} \mathbf{n}_r + n_{d,k} \end{aligned} \quad (\text{B.1})$$

Derive SINR expression for S-D pair k according to (B.1), we have

$$\text{SINR}_k(\mathbf{W}) = \frac{P_o |\mathbf{h}_{2,k}^T \mathbf{W} \mathbf{h}_{1,k}|^2}{P_o \sum_{l=1, l \neq k}^K |\mathbf{h}_{2,k}^T \mathbf{W} \mathbf{h}_{1,l}|^2 + \sigma_R^2 \|\mathbf{h}_{2,k}^T \mathbf{W}\|^2 + \sigma_D^2} \quad (\text{B.2})$$

Let P_m denote transmission power for relay m , it can be expressed as

$$P_m = E|x_m|^2 = E|w_m r_m|^2 \quad (\text{B.3})$$

Then we formulate the problem to minimize maximum per-relay transmission power P_m , while promising received SINR requirement γ_k at destination k , as shown in (B.4).

$$\begin{aligned} \min_{\mathbf{W}} \max_{m \in \{1, \dots, M\}} P_m \quad (\text{B.4}) \\ \text{s.t. SINR}_k(\mathbf{W}) \geq \gamma_k, \quad k = 1, \dots, K \end{aligned}$$

For simplification, the transmit power at relay m and received SINR at destination k can be expressed w.r.t. \mathbf{w} as following

$$P_m(\mathbf{w}) = \mathbf{w}^H \mathbf{D}_m \mathbf{w} \quad (\text{B.5})$$

$$\text{SINR}_k(\mathbf{w}) = \frac{P_o \mathbf{w}^H \hat{\mathbf{H}}_{2,k} \mathbf{h}_{1,k}^* \mathbf{h}_{1,k}^T \hat{\mathbf{H}}_{2,k} \mathbf{w}}{P_o \sum_{l=1, l \neq k}^K \mathbf{w}^H \hat{\mathbf{H}}_{2,k} \mathbf{h}_{1,l}^* \mathbf{h}_{1,l}^T \hat{\mathbf{H}}_{2,k} \mathbf{w} + \sigma_R^2 \mathbf{w}^H \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k} \mathbf{w} + \sigma_D^2} \quad (\text{B.6})$$

where $\mathbf{D}_m \triangleq (P_o \sum_{k=1}^K |h_{1,km}|^2 + \sigma_R^2) \mathbf{E}_m$.

Then we reformulate problem (B.4) w.r.t \mathbf{w} as

$$\begin{aligned} \min_{\mathbf{w}} \max_{m \in \{1, \dots, M\}} \mathbf{w}^H \mathbf{D}_m \mathbf{w} \quad (\text{B.7}) \\ \text{s.t. SINR}_k(\mathbf{w}) \geq \gamma_k, \quad k = 1, \dots, K \end{aligned}$$

The above problem is non-convex and NP-hard, so that SDR approach will be applied to it. We first rewrite SINR constraints (B.6) as

$$\begin{aligned} \text{SINR}_k &\geq \gamma_k \\ \Rightarrow P_o \mathbf{w}^H \hat{\mathbf{H}}_{2,k} \mathbf{h}_{1,k}^* \mathbf{h}_{1,k}^T \hat{\mathbf{H}}_{2,k} \mathbf{w} &\geq \gamma_k (P_o \sum_{l=1, l \neq k}^K \mathbf{w}^H \hat{\mathbf{H}}_{2,k} \mathbf{h}_{1,l}^* \mathbf{h}_{1,l}^T \hat{\mathbf{H}}_{2,k} \mathbf{w} + \sigma_R^2 \mathbf{w}^H \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k} \mathbf{w} + \sigma_D^2) \\ \Rightarrow \mathbf{w}^H \mathcal{R}_k \mathbf{w} &\geq \gamma_k \sigma_D^2 \quad (\text{B.8}) \end{aligned}$$

where $\mathcal{R}_k \triangleq P_o \hat{\mathbf{H}}_{2,k} \mathbf{h}_{1,k}^* \mathbf{h}_{1,k}^T \hat{\mathbf{H}}_{2,k} - \gamma_k P_o \sum_{l=1, l \neq k}^K \hat{\mathbf{H}}_{2,k} \mathbf{h}_{1,l}^* \mathbf{h}_{1,l}^T \hat{\mathbf{H}}_{2,k} - \gamma_k \sigma_R^2 \hat{\mathbf{H}}_{2,k}^H \hat{\mathbf{H}}_{2,k}$.

Define $\mathbf{X} \triangleq \mathbf{w} \mathbf{w}^H$, we transform problem (B.7) to the following form

$$\begin{aligned} \min_{\mathbf{X}} \max_{m \in \{1, \dots, m\}} \text{tr}(\mathbf{X} \mathbf{D}_m) & \quad (\text{B.9}) \\ \text{s.t. } \text{tr}(\mathbf{X} \mathcal{R}_k) \geq \gamma_k \sigma_D^2, \quad k = 1, \dots, K \\ \mathbf{X} \succcurlyeq 0, \text{rank}(\mathbf{X}) = 1. \end{aligned}$$

By removing the rank constraint in (B.9) and introducing an auxiliary variable $P_r \triangleq \max_{m \in \{1, \dots, M\}} \text{tr}(\mathbf{X} \mathbf{D}_m)$ be the maximum power among relays, the above non-convex optimization problem can be rewritten as the following SDP problem

$$\begin{aligned} \min_{P_r, \mathbf{X}} P_r & \quad (\text{B.10}) \\ \text{s.t. } P_r \geq 0 \\ \text{tr}(\mathbf{X} \mathbf{D}_m) \leq P_r, \quad m = 1, \dots, M \\ \text{tr}(\mathbf{X} \mathcal{R}_k) \geq \gamma_k \sigma_D^2, \quad k = 1, \dots, K \\ \mathbf{X} \succcurlyeq 0. & \quad (\text{B.11}) \end{aligned}$$

The problem (B.10) can be solved by SDP programming for optimal solution \mathbf{X}^* . If $\text{rank}(\mathbf{X}^*) = 1$, we extract optimal \mathbf{w}^* as $\mathbf{X}^* = \mathbf{w}^* \mathbf{w}^{*H}$. Otherwise we apply a Gaussian randomization procedure to generate a suboptimal solution as \mathbf{w}^* . ■

Appendix C

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