

## **Modeling reward expiry for loyalty programs in a competitive market<sup>1</sup>**

*Amirhossein Bazargan*

*PhD student, Ryerson University,  
350 Victoria St., Toronto, ON, Canada, M5B 2K3.*

*Salma Karray<sup>2</sup>*

*Associate professor, University of Ontario Institute of Technology,  
2000 Simcoe Street North, Oshawa, ON, Canada, L1H 7K4.*

*Saeed Zolfaghari*

*Professor, Ryerson University,  
Ryerson University,  
350 Victoria St., Toronto, ON, Canada, M5B 2K3.*

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<sup>2</sup> Corresponding author. Email: [salma.karray@uoit.ca](mailto:salma.karray@uoit.ca), Tel: +905-721-8668.

## **Abstract**

This paper investigates reward expiry for loyalty programs. It provides insights into the profitability of setting reward expiry for competing firms and identifies conditions under which such a policy would be beneficial. We develop and solve a game-theoretic model that reflects consumer behavior in choosing products and redeeming rewards. Applying a new iterative algorithm, we get the Nash equilibrium outputs for three scenarios (games): (1) neither firm sets an expiry date, (2) both firms set an expiry date, and (3) only one firm sets an expiry date. Comparison of the firms' profits across scenarios shows that the firms' prices and profits are affected by the loyalty program of the competing firm and by consumers' valuation of rewards and of time to rewards. In particular, a firm offering rewards that do not expire should increase its price if the competing firm changes its reward policy from no expiry to expiry, even when the expiry period is quite long. Finally, when customers highly value rewards and time, reward expiry is a dominant strategy for both firms. This means that firms would benefit from setting expiry on their loyalty rewards only if their customers highly value both rewards and time. Alternatively, both firms' rewards should not expire if consumers have low valuations of both rewards and time.

**Keywords:** Loyalty programs, Iterative algorithm, Multinomial logit models, Game theory, Reward expiry, Operations Management.

## **1. Introduction**

Recent industry reports show that firms spending for loyalty programs (LPs) has exceeded \$1 billion (Kumar, 2008), with the number of memberships nearing 3 billion in the US in 2012 (Berry, 2013). The popularity of LPs is due to their effectiveness in building and reinforcing customer

loyalty by offering benefits to consumers in exchange for their repeat patronage to the company (Rust et al., 2000; Sharp & Sharp, 1997; Buckinx & Van den Poel, 2005; Li et al., 2006; Van den Poel & Lariviere, 2004).

Given the considerable investments that firms are allocating to LPs, it is important for managers to understand the implications of such programs for the firms' revenues and profits. This is especially important since these programs can offer different benefits to consumers and present different features that can affect customers' purchasing and redemption decisions. In particular, one can observe varying reward expiry policies by firms across different industries. For example, in the airline industry, JetBlue has recently changed its loyalty program to match the policy of its main competitor, Delta, by cancelling the expiry of its travelers' reward points (Yahoo news, 2013). However, other companies such as United, Alaska Airlines and Air France maintained the expiry of their rewards. We observe similar variations in reward expiry policies for LPs offered by gas companies. In the US, points earned through the BP Driver reward card program expire after one year, Shell Fuel rewards expire after one month, while Esso points do not expire. It is unclear how competing firms decide of incorporating reward expiry in their loyalty programs, whether such policies benefit competing firm and if so under what conditions.

The purpose of this study is to analyze the implications of reward expiry for competing firms. Specifically, we aim to identify whether firms should set expiry on their rewards or not given the competitor's chosen policy. We focus on reward policies where a fixed number of purchasing is necessary before a consumer can claim his/her rewards. This is the common loyalty program of the kind 'buy n, get one free' used in the restaurant and hoteling industries (e.g., McDonald's, Starbucks and Second Cup loyalty cards). In such LPs, some companies impose reward expiry while others do not. For example, while McDonald's McCafe rewards do not

expire<sup>3</sup>, Starbucks' Stars reward and 7-Eleven's loyalty rewards do. Starbucks Stars rewards are earned and accumulated at every purchase. They can be redeemed as a free product once the customer has accumulated enough rewards to cover the cost of a free product. The policy posted on the Starbucks Stars reward program's website specifies that "If you do not earn at least three hundred (300) new Stars in each consecutive 12- month period, your Stars will expire and your Star balance will automatically reset to zero (0)."<sup>4</sup> 7-eleven loyalty rewards are earned at every drink purchase (using a virtual loyalty card via an App). The customer has to accumulate rewards to get a free drink after accumulating rewards from six purchases. According to 7-Eleven's loyalty program's terms of use, "unused reward drink will expire 366 days after it appears in the App."<sup>5</sup>

The purpose of this study is to analyze the implications of reward expiry for competing firms. Specifically, we aim to identify whether firms should set expiry on their rewards or not given the competitor's chosen policy. To do so, we develop a game-theoretic model and obtain the optimal reward expiry duration as well as price for competing firms offering similar products at uniform pricing (e.g., coffee shops and gas stations). We investigate the effects of setting reward expiry on the competing firms' profits as well as on consumers' redemptions. Finally, we identify the Nash equilibrium solution and discuss conditions under which both firms, none, or only one should set an expiry on its loyalty rewards. Such analysis will provide managers with insight into whether reward expiry can be profitable, and if so, under what conditions. It will also provide guidelines to competing firms on how to react to competitor's reward expiry policies.

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<sup>3</sup> [http://www.mcdonalds.ca/ca/en/help/mccafe\\_loyalty\\_program.html](http://www.mcdonalds.ca/ca/en/help/mccafe_loyalty_program.html)

<sup>4</sup> <https://www.starbucks.ca/card/rewards/rewards-program-ts-and-cs>

<sup>5</sup> <https://www.7-eleven.com/Home/Terms>

## **2. Literature Review**

We review the literature related to three main areas. First, we report the main findings related to loyalty programs. Second, we discuss research to date about the implications of reward expiry. Third and last, we expose the analytical literature about loyalty programs that used game theoretic modeling.

### **2.1. Loyalty programs**

Loyalty programs are offered by companies to engage customers. This is done by offering rewards to those customers who repeat purchase the company's product. Multiple studies in the marketing literature have shown the beneficial impact of these programs on consumers' behavior (Leenheer et al., 2007; Liu, 2007; Dorotic et al., 2012). LPs are found to favorably increase consumers' purchases (Lal & Bell, 2003; Lewis, 2004; Liu, 2007; Taylor & Neslin, 2005) and the company's market share (Leenheer et al., 2007).

The existing research on LP rewards mainly consists in empirical findings and is focused on the attractiveness of different reward types and their impact on consumer responses (Kim et al., 2001; Kivetz & Simonson, 2002; Noble et al., 2014; Wagner et al., 2009). For example, the literature reports different reward types and redemption policies employed in LPs. Rewards can be monetary, e.g., discounts, coupons, cash, or non-monetary, e.g., upgrades, access to special events (Jones et al., 2006; Mimouni-Chaabane & Volle, 2010). Furthermore, rewards redemption can be immediate or delayed. An example of the latter is the 'buy n times, get one free' reward policy commonly used in the restaurant industry for frequently purchased products such as coffee (e.g., McDonald's, Starbucks and Second Cup loyalty cards).

Despite this large literature on the effects of loyalty programs, reward redemption effects themselves have received relatively little attention in the existing literature (Bijmolt et al., 2012;

Dorotic et al., 2014; Smith & Sparks, 2009a). Some notable exceptions are the recent works by Stourm et al. (2015) and Dorotic et al. (2014). Using retail LP data, Stourm et al. (2015) finds evidence that customers use different “mental accounts” to evaluate cash and loyalty points. This result is in line with the mental accounting theory (Thaler, 1985), according to which customers’ increased utility from a gain, or disutility (pain) from a loss (payment), can vary depending on which currency (cash or reward) is being exchanged for the payment (Soman, 2003; Drèze & Nunes, 2009).

The mental accounting theory can play an important role in customers’ redemption decisions since at every purchasing occasion, customers have to weigh their gain (either from accumulating points or from receiving a cash discount) versus their loss (either from redeemed rewards or from the missed opportunity of price savings). Dorotic et al. (2014) empirically studied the redemption behavior of over three thousand members of an LP and showed evidence of time pressure effects, which affects consumers’ evaluations of rewards soon to expire. These effects also influence consumers’ redemption vs. stockpiling decisions. More discussion will follow in the next section.

## **2.2. Reward expiry**

The marketing literature provides contradictory findings related to the effects of reward expiry. Some research finds evidence that reward expiry may decrease customers’ satisfaction and motivation and create frustration (Stauss et al., 2005), which can lead to lower purchases (Dorotic et al., 2014). The fear of such negative effects may have encouraged some LPs to stretch their reward expiry periods or to abolish reward expiry altogether (Bijmolt et al., 2012).

A few other studies find positive effects of reward expiry on consumer buying behavior and explain that reward expiry can create a time pressure mechanism that results in increasing

consumer purchases (e.g., Kopalle & Neslin, 2003; Drèze & Nunes, 2009; Dorotic et al., 2014). The goal-gradient theory explains this behavior; the closer a customer gets to the expiry period, the more he/she will feel the pressure to accumulate points, and the more likely that he/she will purchase the product of the firm offering the loyalty program with the soon to expire rewards (Kivetz et al., 2006; Kopalle et al., 2012). This time pressure mechanism can have positive effects on consumer purchases as it keeps consumers engaged with the LP and can lead to increased purchases in the period preceding redemption. Customers' reward accumulation and redemption behavior can then be explained by their anticipation of obtaining future rewards and/or by switching costs, which together constitute the pressure to collect a sufficient amount of points for a reward (Hartmann & Viard, 2008; Kopalle et al., 2012; Lewis, 2004).

Due to these complex effects, the implications of reward expiration in LPs remain largely unexplored. In their empirical study, Dorotic et al. (2014) note that "*whether firms should encourage reward redemption and consider long-term expiration policies ranks among the least understood aspects of LPs*". This paper answers Dorotic et al.'s call to explore this issue.

### **2.3. Game theoretic models about LPs**

Unlike the large empirical literature about LPs discussed above, there are only a few analytical studies about LPs. Among these, a small number has used game theoretic models to investigate the implications of loyalty programs for competing firms (e.g., Caminal & Matutes, 1990; Klemperer, 1987b, 1995; Kim et al., 2001; Singh et al., 2008). In our knowledge, no analytical study, for one or multiple firms, has looked at the effects of reward expiry on firms' profitability and consumer redemptions. This makes our research the first to investigate the implications of reward expiry for competing firms using a game theoretic approach.

The early work related to game-theoretic analysis of LPs is in the economics field (e.g., Klemperer, 1987b; Caminal & Matutes, 1990; Klemperer, 1995). **In this literature, the consumer demand is derived from a Hotelling model, in which consumers' preferences are uniformly distributed on a unit line.** The modeled LPs are over two purchasing periods such as consumers who purchase in the first period can get a price discount as a reward in the second period. The main findings suggest that LPs that offer rewards as price discount on the next purchase introduce consumer-switching costs. These change consumer sensitivity to price, which then affect the firms' pricing policies, consumer welfare and the level of competition between the firms.

Following these economics studies, a few researchers have used a similar modeling approach to investigate the managerial implications of LPs for competing firms. In particular, Kim et al. (2001) study optimal reward type (price discount vs. free product) and LP profitability for undifferentiated competing firms. Using a similar modelling approach, they extend the previously developed Hotelling models by considering that the market is formed by two groups (segments) of consumers: heavy and light users. Heavy users buy a product in each period, whereas light users only buy in the first period, then exit the market and are replaced by a new group of consumers who buy in the next period. Their results show that the optimal reward type and the profitability of the LP for the competing firms mainly depends on the size of the consumer segments and their price sensitivity. Singh et al. (2008) use a similar model to Kim et al.'s to study whether firms should introduce an LP offering a price discount as reward. Although they do not solve for the optimal reward amount, they include an asymmetric game where one firm offers the LP and the other does not. They show that firms might be better off with no loyalty program even if the competitor is offering one.



## 2.4. Contributions

The contribution of this paper is threefold. First, we **analytically** explore for the first time the profitability of reward expiry in LPs. Second, contrary to previous studies that predominantly consider rewards as price discounts on the next purchase, we consider rewards of the form ‘buy n, get one free’. **Third, unlike previous game-theoretic studies about LPs that relied on a Hotelling framework, we analytically obtain consumers’ demand using a discrete choice model (multinomial logit (MNL)). This approach allows us to capture customers’ stockpiling and redemption behaviors, which affect consumer demand in a market where rewards can be accumulated over time.** In particular, we take into account the mental accounting theory to represent different consumer valuations of rewards and cash. We also model the time pressure effect that has been proved to affect consumer purchasing and redemption behavior.

The objective of this paper is to assess whether reward expiry for LPs of the kind ‘buy n times, get one free’ is profitable for competing firms. Using our rich model that reflects consumer behavior related to the use of LPs, we hope that our results will provide managers with insight into whether reward expiry can be profitable and, if so, under what conditions.

To analyze the impact of reward expiry on the competing firms’ profits and identify whether expiry is an equilibrium strategy for each of the firms, we solve three Nash games. In two of these games, both firms choose the same reward expiry policy (both either setting or not setting reward expiry). In the third game, the firms’ policies are asymmetric (one chooses reward expiry and the other does not). Comparison of equilibrium outputs across games provides insights into the effects of setting reward expiry on the firms’ revenues and profits and on consumers’ redemptions and the market conditions conducive for such policies.

The rest of this article is arranged as follows. Section 3 presents the model, Section 4 explains the method used to solve the three games, Section 5 and 6 deal with numerical results and discussions of the findings. Section 7 summarises and concludes.

### **3. Model formulation**

#### **3.1. Model set-up and assumptions**

We consider that the market is served by two competing firms named  $a$  and  $b$ . Both firms are assumed to be rational decision makers, to have complete information about the market, and to make their decisions simultaneously. They both offer loyalty programs according to which consumers can redeem their rewards accumulated over a certain number of purchases and receive a free product as a reward.

We assume that the competing firms offer undifferentiated products with a constant price during the purchasing periods to focus on mature industries characterized by dominant firms with similar market shares (e.g., coffee shops). Firms in such environment use marketing strategies aimed at increasing customers' loyalty and are therefore concerned about issues related to the effective design of their loyalty programs. Finally, similar to previous studies (e.g., Singh et al., 2008; Gandomi & Zolfaghari, 2013), and to exclude the impact of market expansion, the market size is normalized to one unit and kept constant. Table 1 includes a summary of the notations used for the model.

Table 1: List of notations

Notation	Definition
$i$	Firm index, $i \in \{a, b\}$
$j$	Customer index, $j$ is an integer higher than 1
$N_i$	Reward period: Required number of purchases to receive Firm $i$ 's reward
$p_i$	Firm $i$ 's price
$T_i$	Expiry length of Firm $i$ 's reward
$n_i$	Number of purchases from Firm $i$ after the last redemption till the time of being eligible to receive the reward
$t_i$	Number of periods left for the customer to redeem his/her Firm $i$ 's reward
$R_i$	Firm $i$ 's revenue
$C_i$	Firm $i$ 's reward cost
$I_i$	Firm $i$ 's profit
$\alpha_v$	Customers' reward valuation coefficient, $\alpha_v \in (0,1)$
$\alpha_d$	Customers' time valuation coefficient, $\alpha_d \in (0,1)$
$U_z^j$	Customer $j$ 's utility of choosing alternative $z$ , $z \in \{A0, A1, B0, B1\}$
$D_z^j$	Deterministic part of customer $j$ 's utility of choosing alternative $z$
$\varepsilon_z^j$	Random part of customer $j$ 's utility of choosing alternative $z$
$T_{max}$	Maximum expiry length
$q_z^j$	Probability of choosing alternative $z$ by customer $j$
$v_i^j$	Value of Firm $i$ 's product for customer $j$
$M_k(n_a, n_b, t_a, t_b)$	Set of customers whose parameters are $(n_a, n_b, t_a, t_b)$ in period $k$
$M(n_a, n_b, t_a, t_b)$	Set of customers whose parameters are $(n_a, n_b, t_a, t_b)$ in a stationary demand condition
$N_{(k)}(n_a, n_b, t_a, t_b)$	Number of customers in set $M_{(k)}(n_a, n_b, t_a, t_b)$
$Q_{z(k)}(n_a, n_b, t_a, t_b)$	Probability of choosing alternative $z$ by the customers in set $M_{(k)}(n_a, n_b, t_a, t_b)$
$V(x, t_1, t_2)$	Value function of $x$ rewards that can be redeemed after $t_1$ and before $t_2$ periods
$W(x, t)$	Lost value when customer gets one period closer to the expiry of $x$ rewards that will be expired after $t$ periods

The firms' decision variables are represented by their prices  $p_i$  for Firm  $\in \{a, b\}$ , the loyalty reward period offered by each firm ( $N_i, i \in \{a, b\}$ ), and the reward expiry period ( $T_i, i \in \{a, b\}$ ). Both firms offer loyalty rewards to their consumers in form of a free product after a certain number of purchases ( $N_i$  for Firm  $i$ ).<sup>6</sup> This represents a common LP used by coffee shop chains such as Starbucks and Second cup, and retailers such as Seven eleven.

To model the reward expiry period, we consider that the unit of time is the period of purchasing. Each firm can set expiry on the rewards earned by its customers by restricting the number of purchasing periods in which that reward should be redeemed. Let the positive integer  $T_i$  represent the expiry periods offered by Firm  $i$ . More details about the firms' loyalty programs will be provided later on in this section.

To study whether firms should set expiry on their rewards in a competitive market, we need to solve a Nash game played by Firm  $a$  and Firm  $b$  where each player can either set or not set a reward expiry date. Let Expiry (E) and No Expiry (NE) be the possible strategies for each firm and ( $I_i$ ) be the profit of Firm  $i$ . The payoff matrix of this game can be derived as shown in Table 2.

Table 2: The payoff matrix of the game

		Firm $b$	
		No Expiry (NE)	Expiry (E)
Firm $a$	No Expiry (NE)	$(I_a^{NE,NE}, I_b^{NE,NE})$	$(I_a^{NE,E}, I_b^{NE,E})$
	Expiry (E)	$(I_a^{E,NE}, I_b^{E,NE})$	$(I_a^{E,E}, I_b^{E,E})$

<sup>6</sup> Note that lower values of  $N_i$  denote earlier reward given by Firm  $i$ .

The scenario where both firms do not set reward expiry (NE, NE) is denoted by Scenario 1 (S1), in which case  $T_a = T_b = \infty$ . The scenario where both firms set expiry (E, E) is Scenario 3 (S3), such as  $T_a, T_b \neq \infty$ . The case where Firm *a* does not set expiry while Firm *b* does (NE, E) is Scenario 2 (S2), which is characterized by  $T_a = \infty$  and  $T_b \neq \infty$ . Finally, the scenario where Firm *a* sets expiry while Firm *b* does not (E, NE) is equivalent to scenario S2 since firms are symmetric in this case except in their reward expiry decision. So the results for this case for Firm *a* (*b*) are equivalent to those for Firm *b* (*a*) in S2.

To solve this game, we then study the three different scenarios (games). In the first scenario (S1), each firm has two decision variables (price and the time of reward). In scenario S2, Firm *b* sets reward expiry while Firm *a* does not. In this case, Firm *a* has two decision variables (price and the time of reward) and Firm *b* makes three decisions (price, reward period, and expiry period). Finally, in scenario S3, each firm has three decision variables (price, reward period, and expiry period). Table 3 summarizes the three scenarios and the corresponding decision variables.

We assume that, in the market served by the two firms, each consumer buys one unit of the product in each period of time from either Firm *a* or Firm *b*. Consequently, customers' valuation for the product is sufficiently high to exceed both firms' products' prices. Note that this assumption does not imply that the market is formed by two consumer segments of heavy and light users as in the previous literature that modeled rewards as a discount on the next purchase (Singh et al., 2008; Kim et al., 2001). This is because we consider a different kind of rewards of the kind 'buy *n*, get one free', which are commonly used in markets of frequently purchased products (e.g., coffee). Moreover, in each period, customers select the firm from which they purchase and decide either to redeem or not to redeem their cumulated rewards (if any). In this formulation, we allow consumers the possibility to freely switch between firms in each period of time. Therefore, we do

not impose any exogenous restrictions on consumers' loyalty or purchasing behavior but rather let these behaviors to be endogenously derived by consumers' utility functions.

Table 3: Scenario definitions and firms' decision variables

Scenario	Definition	Firm $a$ 's decision variables	Firm $b$ 's decision variables
S1	Neither firm sets expiry	$N_a$ and $p_a$	$N_b$ and $p_b$
S2	Only one firm sets expiry	$N_a$ and $p_a$	$N_b, p_b$ and $T_b$
S3	Both firms set expiry	$N_a, p_a$ and $T_a$	$N_b, p_b$ and $T_b$

### 3.2. Consumer choice

At each period of purchasing, the customer chooses one of the following four alternatives: (1) purchase from Firm  $a$  and not redeem (denoted by A0), (2) purchase from Firm  $a$  and redeem (denoted by A1), (3) purchase from Firm  $b$  and not redeem (denoted by B0), and, (4) purchase from Firm  $b$  and redeem (denoted by B1). Obviously, a customer who does not have Firm  $a$ 's or Firm  $b$ 's rewards can only choose between alternatives A0 and B0.

Consumer choice is modeled according to the multinomial logit (MNL) model, which indicates that customers choose the option that results in the maximum utility. The MNL model is related to the revenue management models developed by Talluri & van Ryzin (2004) and widely used in the literature (e.g., de Palma et al., 1985; Anderson et al., 1985; Verma & Thompson, 1999; Liu & van Ryzin, 2008; van Ryzin & Vulcano, 2008; Hongmin & Woonghee Tim, 2011; Meissner & Strauss, 2012; Meissner et al., 2012; Topaloglu, 2013; Rusmevichientong et al., 2014). **We have chosen the MNL model because it allows us to capture customers' stockpiling and redemption behaviors, which are important considerations for 'buy n, get one free' type of loyalty programs and the reward expiry problem studies in this paper.**

The MNL model serves as an analytical tool to describe the demand of consumers for the firms' competing products. We start by formulating the consumer utilities obtained for the competing firms' products and rewards. These utilities are then used to obtain the probabilities of consumer choices for purchases and reward redemption from each firm, which later will serve to obtain the expected number of sales for each firm, the costs of rewards paid to consumers, and the firms' profit functions.

Let  $U_z^j$  be the utility that consumer  $j$  obtains from alternative  $z$  such as, for any  $z \in \{A0, A1, B0, B1\}$ ,  $U_z^j = D_z^j + \varepsilon_z^j$ , where  $D_z^j$  is the deterministic utility part and  $\varepsilon_z^j$  is the random (unknown) component. Each customer chooses the alternative that provides him/her with the maximum utility. For instance, when choosing between alternatives A0 and B0, customer  $j$  would choose A0 if and only if  $U_{A0}^j \geq U_{B0}^j$ . Therefore, the probability of choosing alternative A0 by customer  $j$  is the probability of having  $U_{A0}^j \geq U_{B0}^j$ , which is equivalent  $\varepsilon_{B0}^j - \varepsilon_{A0}^j \leq D_{A0}^j - D_{B0}^j$ . According to the logit model, each  $\varepsilon_z^j$  is an independently and identically distributed Type I extreme value (also called Gumbel) with a specific location parameter and scale parameter 1 (see Ben-Akiva & Lerman 1985). As a result,  $\varepsilon_{B0}^j - \varepsilon_{A0}^j$  follows a logistic distribution with mean zero and scale 1. Therefore, based on the CDF of the logistic distribution, the probability of  $\varepsilon_{B0}^j - \varepsilon_{A0}^j \leq D_{A0}^j - D_{B0}^j$  is equal to  $\exp(D_{A0}^j) / (\exp(D_{A0}^j) + \exp(D_{B0}^j))$ . The same logic applies for the situation where the customer chooses among multiple alternatives (see Train, 2009). Consequently, denoting  $q_z^j$  as the probability of choosing alternative  $z \in \{A0, A1, B0, B1\}$  by customer  $j$ , we get:

$$q_z^j = \exp(D_z^j) / (\exp(D_{A0}^j) + \exp(D_{A1}^j) + \exp(D_{B0}^j) + \exp(D_{B1}^j)). \quad (1)$$

Based on the above equation, we can derive the probability of choosing alternative  $z$  by customer  $j$  using the deterministic component of customer's utility in each alternative, which is

the surplus of the value gained by choosing that alternative subtracted by the value paid for it. Next, to model and formulate the customers' gains and losses, we formulate the consumer value functions.

### 3.3. Consumer value functions

Based on the psychological theory of mental accounting (Thaler, 1985), we consider that customers have a higher valuation for cash than for reward. To model this mentality, consumers value  $x$  amount of reward as  $\alpha_v x$ , where the parameter  $\alpha_v$  denotes customers' sensitivity to reward relative to cash and varies in the range of (0,1).

To model the effect of time pressure on customers' utility, we assume that customers' valuation of reward is affected by the time left to redeem the reward, (e.g., Besanko & Winston, 1990), with closer periods to redemption resulting in higher utility for the customer, which creates the pressure to accumulate points. We apply a common discounting formula (Crosson & Needles, 2008), and consider that one unit of reward earned at time  $t$  is valued at of  $1/(1 + \alpha_d)^t$  at the present time, where  $\alpha_d$  is equivalent to the interest rate for one period or discount rate, and is therefore between 0 and 1. Note that  $\alpha_d$  represents consumer sensitivity to time, with higher values of  $\alpha_d$  leading to lower values of rewards. Therefore, everything else being the same, the lower is  $\alpha_d$ , the more valuable become the rewards earned by consumers, and the higher is the time pressure effect.

Based on the above explanations and considering both the mental accounting theory and consumers' discounting of rewards, one can derive the value function,  $\hat{V}(x, t)$ , representing consumers' evaluation of  $x$  units of reward after  $t$  periods as follows:

$$\hat{V}(x, t) = \alpha_v x / (1 + \alpha_d)^t, \quad (2)$$



with  $\alpha_v$  and  $\alpha_d$  representing customers' sensitivity to reward value and to time pressure, respectively.

Next, we model the effect of reward expiry. From equation (2), the value of  $x$  units of rewards that can be redeemed at the current period and never expire is equal to  $\alpha_v x$ . However, in order to take into account reward expiry, the value function needs to depend on two different periods: the period after which the reward can be redeemed,  $t_1$ , and the period after which the reward will be expired,  $t_2$ . Therefore, we rewrite the value function in equation (2) as  $V(x, t_1, t_2)$ , which is given by the discounted value of rewards earned at  $t_1$  minus the discounted value of the rewards expired (lost) at  $t_2$  and is as follows:

$$V(x, t_1, t_2) = \alpha_v x ((1 + \alpha_d)^{-t_1} - (1 + \alpha_d)^{-t_2}). \quad (3)$$

As mentioned before, we consider customers' utility in choosing alternative  $z$  as the surplus of the value gained by choosing this alternative subtracted by the value paid for it. Besides modeling a customer's gain and loss of cash and of rewards associated with purchasing and redeeming rewards, our model also takes into account consumer disutility arising from getting closer to the reward expiry period. This is to represent the pressure a customer feels when his/her reward gets closer to expiry. To do so, we define the function  $W(x, t)$ , to represent the customer loss of  $x$  value of rewards that will be expired after  $t$  periods, when the customer gets one unit of time closer to the reward expiry period. Referring to equation (3), one can conclude:

$$W(x, t) = V(x, 0, t) - V(x, 0, t - 1) = \alpha_v x ((1 + \alpha_d)^{-t+1} - (1 + \alpha_d)^{-t}). \quad (4)$$

### 3.4. Deterministic components of customer's utilities

Let  $n_i^j$  be the number of purchases by customer  $j$  from Firm  $i$  from the time of the last redemption until the time requested for receiving the reward ( $n_i^j \leq N_i$ ). This means that customer  $j$  is  $(N_i - n_i^j)$  periods away from qualifying to receive Firm  $i$ 's rewards respectively. Consequently, the

value of an additional purchase increases when the customer gets closer to the reward. In this regard, our model follows the goal gradient theory (Kivetz et al., 2006) indicating that the closer a customer is to a reward, the more likely a new purchase is.

After reaching the required number of purchases to receive the reward ( $n_i^j = N_i$ ), each purchase gets the customer one period closer to expiry, which causes pressure to redeem. To model this fact, we define the time parameter of  $t_i^j$ , which denotes customer  $j$ 's distance to the Firm  $i$ 's reward expiry period if he/she has the reward. This parameter is a positive integer with values that are lower than or equal to the firms' expiry length ( $T_i$ ) for customers who have accumulated rewards, and zero for those who do not have reward. As a result, for a customer  $j$  who accumulated enough rewards to redeem, he/she possesses  $p_i$  in reward value after  $n_i^j$  purchases which can be redeemed at the current period and will be expired after  $t_i^j$  periods. Based on these definitions, a customer's situation can be specified by the four parameters ( $n_a^j, n_b^j, t_a^j, t_b^j$ ).

Based on the above definitions, and after formulating the value functions for the gain and loss of cash and rewards,  $D_z^j$  —the deterministic part of customer  $j$ 's utility in choosing alternative  $z \in \{A0, A1, B0, B1\}$  — represents customer  $j$ 's surplus and is equal to the value gained by choosing alternative  $z$  subtracted by the value paid for it. Therefore,  $D_z^j$  is formulated as follows in equations (5-8).

$$D_{A0}^j = \begin{cases} v_a^j - p_a - W(p_a, t_a^j) - W(p_b, t_b^j) & \text{if } n_a^j = N_a \text{ \& } n_b^j = N_b \\ v_a^j - p_a + V(p_a, N_a - n_a^j, N_a - n_a^j + T_a) - W(p_b, t_b^j) & \text{if } n_a^j < N_a \text{ \& } n_b^j = N_b \\ v_a^j - p_a - W(p_a, t_a^j) & \text{if } n_a^j = N_a \text{ \& } n_b^j < N_b \\ v_a^j - p_a + V(p_a, N_a - n_a^j, N_a - n_a^j + T_a) & \text{if } n_a^j < N_a \text{ \& } n_b^j < N_b \end{cases} \quad (5)$$

$$D_{A1}^j = \begin{cases} v_a^j - V(p_a, 0, t_a^j) - W(p_b, t_b^j) & \text{if } n_a^j = N_a \text{ \& } n_b^j = N_b \\ v_a^j - V(p_a, 0, t_a^j) & \text{if } n_a^j = N_a \text{ \& } n_b^j < N_b \\ -\infty & \text{if } n_a^j < N_a \end{cases} \quad (6)$$

$$D_{B0}^j = \begin{cases} v_b^j - p_b - W(p_b, t_b^j) - W(p_a, t_a^j) & \text{if } n_b^j = N_b \text{ \& } n_a^j = N_a \\ v_b^j - p_b + V(p_b, N_b - n_b^j, N_b - n_b^j + T_b) - W(p_a, t_a^j) & \text{if } n_b^j < N_b \text{ \& } n_a^j = N_a \\ v_b^j - p_b - W(p_b, t_b^j) & \text{if } n_b^j = N_b \text{ \& } n_a^j < N_a \\ v_b^j - p_b + V(p_b, N_b - n_b^j, N_b - n_b^j + T_b) & \text{if } n_b^j < N_b \text{ \& } n_a^j < N_a \end{cases} \quad (7)$$

$$D_{B1}^j = \begin{cases} v_b^j - V(p_b, 0, t_b^j) - W(p_a, t_a^j) & \text{if } n_b^j = N_b \text{ \& } n_a^j = N_a \\ v_b^j - V(p_b, 0, t_b^j) & \text{if } n_b^j = N_b \text{ \& } n_a^j < N_a \\ -\infty & \text{if } n_b^j < N_b \end{cases} \quad (8)$$

where  $v_i^j$  is customer  $j$ 's valuation for Firm  $i$ 's product.

In equations (5) and (7), the customer  $j$ 's surplus in purchasing from Firm  $a$  ( $b$ ) and not redeeming includes the value of the product ( $v_{a(b)}^j$ ), the loss of cash as price paid for the product ( $-p_{a(b)}$ ), the value of getting closer to the reward ( $V(p_{a(b)}, (N_{a(b)} - n_{a(b)}^j), (N_{a(b)} - n_{a(b)}^j + T_{a(b)}))$ ) if the customer has not completed the required number of purchases to redeem the reward ( $n_{a(b)}^j < N_{a(b)}$ ), the disutility arising from the pressure of getting closer to the expiry of the firm's reward ( $-W(p_{a(b)}, t_{a(b)}^j)$ ) if the customer has completed the required number of purchases to get reward ( $n_{a(b)}^j = N_{a(b)}$ ), and from the pressure of getting closer to the expiry of the other firm's reward ( $-W(p_{b(a)}, t_{b(a)}^j)$ ) if the customer is qualified to receive that reward ( $n_{b(a)}^j = N_{b(a)}$ ).

In equations (6) and (8), the customer  $j$ 's surplus in redeeming the reward of Firm  $a$  ( $b$ ) includes the value of the product ( $v_{a(b)}^j$ ), the loss of the accumulated reward ( $-V(p_{a(b)}, 0, t_{a(b)}^j)$ ), the disutility arising from the pressure of getting closer to the other firm's reward expiry ( $-W(p_{b(a)}, t_{b(a)}^j)$ ) if the customer is qualified to receive that reward ( $n_{b(a)}^j = N_{b(a)}$ ). When the customer has no reward from Firm  $a$  ( $b$ ) ( $n_{a(b)}^j < N_{a(b)}$ ), redemption (alternative A1 (B1)) is not an option. To show this, a negative infinity value has been assigned to  $D_{A1}^j$  and  $D_{B1}^j$  under this

condition, which leads to dropping the term ( $\exp(D_{A1(B1)}^j)$ ) from Equation (1) when the customer has no reward to redeem.

Based on equations (1-8), the probability ( $q_z^j$ ) of customer  $j$  choosing alternative  $z \in \{A0, A1, B0, B1\}$  can be calculated. We assume that  $v_a^j = v_b^j$  since both firms sell the same product/service. It should also be mentioned that equations (5-8) are derived for scenario S3, where both firms set a reward expiry. In the scenarios where one or both firms do not set expiry (scenarios S1 and S2),  $T_a$  and  $T_b$  are set to infinity, and the terms ( $N_{a(b)} - n_{a(b)}^j + T_{a(b)}$ ) also turn to infinity.

### 3.5. Profit functions

Each firm's profit is equal to the revenues from selling its product minus the costs of redeemed rewards. Next, we use the probabilities of customer choice for each alternative obtained in the previous section to get the quantity of products sold (in units) for each firm and the number of redeemed rewards.

As mentioned before, customer  $j$  is specified by the following four parameters:  $n_a^j, n_b^j, t_a^j, t_b^j$ . So, in period  $k$ , all customers who have the same  $(n_a^j, n_b^j, t_a^j, t_b^j)$  have an equal probability of choosing alternative  $z$  and can therefore be grouped. Denoting this group as  $M_k(n_a, n_b, t_a, t_b)$  and the probability of this group choosing  $z$  as  $Q_{z,k}(n_a, n_b, t_a, t_b)$ , we get:

$$\begin{cases} M_k(n_a, n_b, t_a, t_b) = \{\forall \text{ customer } j \text{ in period } k | n_a^j = n_a, n_b^j = n_b, t_a^j = t_a, t_b^j = t_b\}, \\ Q_{z,k}(n_a, n_b, t_a, t_b) = q_z^j, \quad \forall z \in \{A0, A1, B0, B1\}, n_a^j = n_a, n_b^j = n_b, t_a^j = t_a, t_b^j = t_b. \end{cases}$$

Based on the above explanations, we derive the flow chart of customers between two subsequent periods (see Figure A1 in the Appendix). We then use it to identify the expected set of customers in the next period resulting from a given customer set  $M_k(n_a, n_b, t_a, t_b)$  at a specific period  $k$ , and the probability of transition to each possible set in period  $k + 1$ . Denote by  $N_k(n_a, n_b, t_a, t_b)$  the number of customers in group  $M_k(n_a, n_b, t_a, t_b)$  in period  $k$ , one can then

derive all possible equations representing the transition between  $N_k(n_a, n_b, t_a, t_b)$  and  $N_{k+1}(n_a', n_b', t_a', t_b')$ . For instance one can say:

$$N_{k+1}(2,3,0,0) = N_k(1,3,0,0) * Q_{A0,k}(1,3,0,0) + N_k(2,2,0,0) * Q_{B0,k}(2,2,0,0).$$

In our model, we assume that the firms' variables do not change between different periods. Consequently, a stationary demand condition can be assumed in which, given a specific set of market conditions  $(n_a, n_b, t_a, t_b)$ , the number of customers is constant in two subsequent purchasing periods, such as  $N_k(n_a, n_b, t_a, t_b) = N_{k+1}(n_a, n_b, t_a, t_b) = N(n_a, n_b, t_a, t_b)$ . This assumption allows us to derive a number of independent equations corresponding to all possible values of  $N(n_a, n_b, t_a, t_b)$  and to all possible combinations of  $(n_a, n_b, t_a, t_b)$ . Solving these equations simultaneously, we obtain different values of  $N(n_a, n_b, t_a, t_b)$ , which can then be used to calculate the firms' revenues (denoted by  $R_a$  and  $R_b$ ), costs (denoted by  $C_a$  and  $C_b$ ), and profits (denoted by  $I_a$  and  $I_b$ ) in the stationary demand condition as shown in equations (9-11), respectively.

$$\begin{cases} R_a = p_a \sum_{n_a} \sum_{n_b} \sum_{t_a} \sum_{t_b} N(n_a, n_b, t_a, t_b) (Q_{A0}(n_a, n_b, t_a, t_b) + Q_{A1}(n_a, n_b, t_a, t_b)) \\ R_b = p_b \sum_{n_a} \sum_{n_b} \sum_{t_a} \sum_{t_b} N(n_a, n_b, t_a, t_b) (Q_{B0}(n_a, n_b, t_a, t_b) + Q_{B1}(n_a, n_b, t_a, t_b)) \end{cases} \quad (9)$$

$$\begin{cases} C_a = p_a \sum_{n_a} \sum_{n_b} \sum_{t_a} \sum_{t_b} N(n_a, n_b, t_a, t_b) Q_{A1}(n_a, n_b, t_a, t_b) \\ C_b = p_b \sum_{n_a} \sum_{n_b} \sum_{t_a} \sum_{t_b} N(n_a, n_b, t_a, t_b) Q_{B1}(n_a, n_b, t_a, t_b) \end{cases} \quad (10)$$

$$\begin{cases} I_a = R_a - C_a = p_a \sum_{n_a} \sum_{n_b} \sum_{t_a} \sum_{t_b} N(n_a, n_b, t_a, t_b) Q_{A0}(n_a, n_b, t_a, t_b) \\ I_b = R_b - C_b = p_b \sum_{n_a} \sum_{n_b} \sum_{t_a} \sum_{t_b} N(n_a, n_b, t_a, t_b) Q_{B0}(n_a, n_b, t_a, t_b) \end{cases} \quad (11)$$

The profit functions in Equation (11) show that the firms' decision variables (price, reward period and expiry length) affect the expected probability of consumers' choice for each alternative ( $Q_z$ ) and the number of purchases for each product ( $N$ ), which in turn affect each firm's revenues and costs (rewards paid to consumers), therefore profits.

Finally, It should be noted that although in equations (9-11) the parameters  $n_{a(b)}$  and  $t_{a(b)}$  change in the ranges of  $[0, N_{a(b)}]$  and  $[0, T_{a(b)}]$  respectively, some special combinations of  $(n_a, n_b, t_a, t_b)$  are not logically possible. For example  $(N_a, N_b, T_a, T_b)$  is not possible because customers only purchase from one firm at each period and therefore cannot become eligible to receive the reward of both firms at the same time. Using the same logic, one can argue that  $T_b - t_b \neq T_a - t_a$  when  $n_a = N_a$  and  $n_b = N_b$ .

#### 4. Solving the model

To study the profitability of setting reward expiry or not, we solve a Nash game played by Firm *a* and Firm *b* where each player can either set or not set reward expiry (Table 2). To find such an equilibrium, we obtain, then compare the firms' optimal profits in scenarios S1, S2 and S3. In each scenario, we solve a subgame in which the optimal values of the firms' decision variables are determined by maximizing simultaneously the firms' profits, each one in terms of its own decision variables. For instance, in scenario S3 (where both firms set reward expiry), the firms' profits are functions of the decision variables of Firm *a* ( $N_a$ ,  $p_a$  and  $T_a$ ) and of Firm *b* ( $N_b$ ,  $p_b$  and  $T_b$ ). Therefore, for given values of parameters ( $\alpha_v$  and  $\alpha_d$ ), the solution in S3 is obtained by solving the two following optimization problems simultaneously:

$$(I) \max_{N_a, T_a, p_a} I_a$$

$$\text{s.t. } N_a \text{ and } T_a \text{ are integers, } p_a > 0.$$

$$(II) \max_{N_b, T_b, p_b} I_b$$

$$\text{s.t. } N_b \text{ and } T_b \text{ are integers, } p_b > 0.$$

In order to solve the firms' problems simultaneously in each scenario, we employ an iterative algorithm explained in Table 4. This algorithm is based on the definition of a Nash equilibrium,

which is the condition under which neither firm can increase its profit by unilaterally deviating from that condition to any other possible one (Nash, 1951).

An analytical solution to find the best response (optimal decision variables), if available, is difficult to derive because the profit functions are highly nonlinear. Therefore, we resort to numerical analyses by considering reasonable limits for the decision variables. We set the required number of purchases to receive Firm  $i$ 's reward, represented by the positive integer  $N_i$ , to be equal or less than 10. When  $N_i = 10$ , Firm  $i$  gives reward after 10 purchases, which is commonly used in many loyalty programs such as the ones offered by McDonald, Starbucks and Second Cup.

Next, looking at Firm  $i$ 's expiry lengths,  $T_i$ , we consider an upper limit equal to  $T_{max}$ . At equal or higher values than  $T_{max}$ , the expiry length is considered to be very large and is perceived by customers as if there was no reward expiry, meaning that customers do not differentiate between a point that expires after  $T_{max}$  and a point that never expires. According to the customers' value functions in Equation (2), as the expiry length of a reward point increases, its value increases, and the value reaches its maximum amount when the expiry length is infinity. Therefore, to set  $T_{max}$ , we find the expiry length at which the value of a point reaches a very high percentage of its maximum value. Since the value function is decreasing in the customer's sensitivity to reward distance ( $\alpha_d$ ),  $T_{max}$  is chosen as the expiry length under which one point has 99 percentage of its maximum value for the most non-sensitive customers to reward distance (those whose  $\alpha_d$  is minimum). Based on this definition, one can conclude:

$$T_{max} \geq \text{ceil}(\ln(100)/\ln(1 + \min(\alpha_d))). \quad (12)$$

Finally, we do not consider a maximum limit for the prices, and for each combination of ( $N_{a(b)}$ ,  $T_{a(b)}$ ), optimal prices in the range of  $(0, \infty)$  are found that maximize each firm's profit. Employing Matlab's `fmincon` function, which uses an interior-point algorithm based on Byrd et al (2000), we

are able to reach the optimal price with the accuracy of 32 decimal digits. As a result, we obtain and report the optimal decisions ( $N_{a(b)}$ ,  $T_{a(b)}$  and  $p_{a(b)}$ ).

Using the algorithm (e.g., Table 4 for S3), we find the closest condition to the equilibrium with an error of (0.001). Although we do not obtain the exact equilibrium, we are able to check for the convergence of the algorithm. If it does not converge, it can be concluded that either there are more than one equilibrium solution or there is no equilibrium.

Table 4: Equilibrium finder algorithm (S3)

task	
1	Find optimal response $[\widetilde{N}_a, \widetilde{T}_a, \widetilde{p}_a]$ for initial strategy of $[N_b = 1, T_b = 1, p_b = 0]$
2	Find optimal response $[\widetilde{N}_b, \widetilde{T}_b, \widetilde{p}_b]$ for the strategy of $[\widetilde{N}_a, \widetilde{T}_a, \widetilde{p}_a]$
3	$x = \text{abs}([\widetilde{N}_a, \widetilde{T}_a, \widetilde{p}_a] - [\widetilde{N}_b, \widetilde{T}_b, \widetilde{p}_b])$
4	<i>While <math>x \geq \text{converge threshold}</math>, do</i>
5	Find optimal response $[\widetilde{N}_a, \widetilde{T}_a, \widetilde{p}_a]$ for strategy of $[\widetilde{N}_b, \widetilde{T}_b, \widetilde{p}_b]$
6	Find optimal response $[\widetilde{N}_b, \widetilde{T}_b, \widetilde{p}_b]$ for the strategy of $[\widetilde{N}_a, \widetilde{T}_a, \widetilde{p}_a]$
7	$x = \text{abs}([\widetilde{N}_a, \widetilde{T}_a, \widetilde{p}_a] - [\widetilde{N}_b, \widetilde{T}_b, \widetilde{p}_b])$
8	<i>end while</i>

In our framework, the parameters  $\alpha_v$  and  $\alpha_d$  are bounded in the (0,1) interval. To make this range more realistic, we consider values in the range of [0.5, 0.9] for  $\alpha_v$  and [0.1, 0.5] for  $\alpha_d$  with a step size of 0.1. This results in 25 different combinations of these parameters. In order to give a better sense of these parameters and justify the mentioned limits, notice that  $\alpha_v = 0.5$  means that the customer evaluates one dollar in reward as 50 cents, and  $\alpha_d = 0.5$  means that a reward loses half of its value after one unit of time. Given these ranges, inequality (12) results in  $T_{max}$  equal to 50.



At the last step, in order to find the equilibrium solution for the general game (Expiry or No Expiry), we compare the optimal profits for Firm  $a$  and Firm  $b$  in scenarios S1, S2, and S3.

## 5. Results

We use equations (1-11), and apply the solution method explained in the previous section to get each firm's optimal decision variables and profits in each scenario (S1, S2, and S3). These outcomes are obtained for different combinations of the parameters representing consumer sensitivity to rewards ( $\alpha_v$ ) and to time ( $\alpha_d$ ). We then solve the general game in which the firms decide about choosing reward expiry or not for all 25 combinations of parameters  $\alpha_v$  and  $\alpha_d$ . We do so by comparing these outcomes for each firm, and under different scenarios (subgames). The following sections present results for each scenario separately, and then in comparison with each other.

### 5.1. Scenario 1 (S1): neither firm sets reward expiry

In this scenario, both firms do not set an expiry date for the rewards they offer their customers. They decide the required number of purchases before a customer can be rewarded a free product,  $N_a$  and  $N_b$ , and choose their prices,  $p_a$  and  $p_b$ , which maximize their profits when the market reaches a stationary demand condition. In the beginning of the selling horizon, both firms set their decision variables simultaneously without knowing each other's decisions. Using the algorithm explained in Table 4, we seek a Nash equilibrium such as the optimal decision of each firm is the best response to the other firm's strategy for each of the 25 considered value combinations of parameters  $\alpha_v$  and  $\alpha_d$ . Finally, we compute the firms' optimal profits,  $I_a$  and  $I_b$ .

As expected, the firms' optimal strategies are equal under Scenario S1 since firms are symmetric in this set-up. Therefore, we drop the subscripts in this scenario to denote the optimal strategies of each firm. The optimal decisions and profits are shown in Table 5.

Table 5: Optimal decision variables and profits in Scenario S1 for combinations of  $\alpha_v$  &  $\alpha_d$ 

$\alpha_v$	$\alpha_d$														
	0.1			0.2			0.3			0.4			0.5		
	N	p	I	N	p	I	N	p	I	N	p	I	N	p	I
0.5	5	3.14	1.317	4	2.865	1.159	3	2.765	1.057	3	2.624	1.005	3	2.522	0.966
0.6	5	3.399	1.428	4	3.043	1.234	3	2.905	1.116	3	2.737	1.053	3	2.616	1.007
0.7	5	3.483	1.468	4	3.125	1.273	3	2.959	1.144	3	2.796	1.083	3	2.672	1.036
0.8	6	3.368	1.46	4	3.063	1.255	4	2.864	1.175	3	2.776	1.084	3	2.672	1.045
0.9	7	3.184	1.41	5	2.928	1.247	4	2.781	1.149	3	2.683	1.06	3	2.615	1.033

When reward expiry is not offered by both firms, the results in Table 5 reveal that the equilibrium pricing and reward strategies as well as the firms' profits are strongly related to consumers' valuations of rewards compared to cash ( $\alpha_v$ ), and to consumers' discount rate ( $\alpha_d$ ). These effects are summarized in the proposition below.

**Proposition 1:** In scenario S1, everything else being the same, the equilibrium solution reacts as follows to changes in  $\alpha_d$  and  $\alpha_v$ :

- 1.a. each firm's price, profit and reward period ( $N$ ) decrease with higher levels of  $\alpha_d$ .
- 1.b. each firm's price and profit increase as  $\alpha_v$  takes higher values for  $\alpha_v \leq 0.7$ , then decreases with higher values of  $\alpha_v$  that exceed 0.7.

Proposition 1.a shows the effects of the discount rate at equilibrium. In markets characterized by "impatient" consumers who strongly prefer immediate rewards (high  $\alpha_d$ ), both firms should charge lower prices and offer the reward sooner to consumers at equilibrium. Given that both firms choose the same strategies at equilibrium, the market is divided between them. Therefore, a decrease in price leads to lower revenues. This in turn results in shrinking the firms' profits at equilibrium for higher levels of  $\alpha_d$ . Hence, firms that offer LP rewards with no expiry find it more profitable to offer such programs in markets where consumers have the lowest

discount rates. Finally, the reward period at equilibrium increases as  $\alpha_d$  increases, but its value becomes stagnant ( $N = 3$ ) for high values of the discount rate. The explanation for this finding is that the consumers' high discount rate leads to lower valuations of the rewards. Therefore, the reward period of the firm needs to be short to avoid further devaluing the value of the rewards offered to consumers.

Looking at the effects of reward valuation compared to cash ( $\alpha_v$ ), **the result in proposition 1.b shows that**, everything else being the same, firms should charge higher prices when consumers have moderate valuations of rewards compared to cash, while prices should be lower for markets characterized either by very low or very high consumer valuations of rewards. In fact, the firms' optimal price increases as  $\alpha_v$  takes higher values for  $\alpha_v \leq 0.7$ , then decreases with higher values of  $\alpha_v$  that exceed 0.7. Finally, variations in  $\alpha_v$  affect the firms' optimal profits in a similar way than prices. Therefore, both firms will earn the highest profits under market conditions where they can charge the highest prices to consumers, which is for moderate values of  $\alpha_v$ . This result shows the importance of considering different mental accounts to represent consumers' valuations of cash and reward. In fact, without such an approach (i.e., if  $\alpha_v$  was assumed equal to one), the firms would charge a lower price than what consumers are willing to pay for the product, therefore missing out on profit opportunity.

Finally, the results for scenario S1 also suggest that the influence of consumers' valuations of rewards compared to cash and discount rate on the firms' optimal strategies and profits are interdependent. In particular, when the consumer discount rate is low, firms should extend their reward period ( $N$ ) with higher consumer rewards valuations ( $\alpha_v$ ). However, the optimal reward period becomes insensitive to  $\alpha_v$  when the discount rate is high, in which cases the firms should

offer a free product as reward in the third period regardless of consumers' valuation of rewards compared to cash.

### 5.2. Scenario 2 (S2): only one firm sets reward expiry

In Scenario S2, both firms offer rewards after a required number of purchases, however, only Firm *b* sets an expiry date for the reward. Firm *b* then decides of its price ( $p_b$ ), the required number of purchases to receive the reward ( $N_b$ ), and the expiry length ( $T_b$ ), while Firm *a* decides of its price ( $p_a$ ) and the required number of purchases to receive the reward ( $N_a$ ). Using the methods explained in Table 4, we seek a Nash equilibrium for the game played by Firm *a* and *b*, where the optimal decision of each firm is the best response to the other one's strategy. Similar to the previous scenario, the first step is identifying the firms' optimal decision variables in Scenario S2 for each of the 25 different combinations of parameters  $\alpha_v$  and  $\alpha_d$ . Then, the firms' optimal profits ( $I_a$  and  $I_b$ ) are computed. Given that the firms are not symmetric in S2, their optimal choices are different. The results are shown in Table 6 for Firm *a* and in Table 7 for Firm *b*.

Table 6: Firm *a*'s optimal decision variables and profits in Scenario S2 for  $\alpha_v$  &  $\alpha_d$

$\alpha_v$	$\alpha_d$														
	0.1			0.2			0.3			0.4			0.5		
	$N_a$	$p_a$	$I_a$	$N_a$	$p_a$	$I_a$	$N_a$	$p_a$	$I_a$	$N_a$	$p_a$	$I_a$	$N_a$	$p_a$	$I_a$
0.5	5	3.141	1.317	4	2.866	1.16	3	2.766	1.058	3	2.625	1.005	3	2.523	0.967
0.6	5	3.401	1.429	4	3.044	1.235	3	2.907	1.116	3	2.739	1.053	3	2.617	1.008
0.7	6	3.444	1.482	4	3.127	1.274	3	2.974	1.150	3	2.811	1.088	3	2.687	1.041
0.8	6	3.386	1.472	4	3.132	1.292	4	2.938	1.200	3	2.849	1.116	3	2.733	1.069
0.9	7	3.284	1.479	5	3.083	1.330	4	2.941	1.222	4	2.778	1.145	3	2.725	1.083

Table 7: Firm  $b$ 's optimal decision variables and profits in Scenario S2 for  $\alpha_v$  &  $\alpha_d$ 

$\alpha_v$	$\alpha_d$																			
	0.1				0.2				0.3				0.4				0.5			
	$N_b$	$T_b$	$p_b$	$I_b$	$N_b$	$T_b$	$p_b$	$I_b$	$N_b$	$T_b$	$p_b$	$I_b$	$N_b$	$T_b$	$p_b$	$I_b$	$N_b$	$T_b$	$p_b$	$I_b$
0.5	5	50	3.129	1.312	4	50	2.867	1.160	3	50	2.767	1.058	3	50	2.625	1.005	3	50	2.523	0.967
0.6	5	50	3.391	1.424	4	50	3.044	1.235	3	50	2.907	1.116	3	50	2.739	1.053	3	50	2.617	1.008
0.7	5	50	3.512	1.485	4	50	3.127	1.273	3	19	2.979	1.150	3	15	2.815	1.089	3	15	2.692	1.042
0.8	6	50	3.397	1.468	4	19	3.177	1.288	3	11	3.124	1.210	3	10	2.902	1.121	3	10	2.779	1.075
0.9	7	33	3.373	1.464	4	13	3.32	1.335	3	9	3.229	1.240	3	8	3.031	1.178	3	8	2.851	1.100

When reward expiry is offered by only one firm (Firm  $b$ ), the results in Tables 6 and 7 reveal the following main results.

**Proposition 2:** In scenario S2, everything else being the same, the equilibrium solution reacts as follows to changes in  $\alpha_d$  and  $\alpha_v$ :

2.a. Firm  $a$ 's price, profit and reward period ( $N_a$ ) decrease with higher levels of  $\alpha_d$ . The optimal expiry period for Firm  $b$  ( $N_b$ ) either decreases or stays the same as  $\alpha_d$  increases.

2.b. Firm  $a$ 's price and profit increase (decrease) as  $\alpha_v$  takes higher values for low (high) values of  $\alpha_v$ . Firm  $b$ 's reward and expiry periods decrease while its price and profit increase with higher values of  $\alpha_v$ .

The result in proposition 2 shows first that, as in S1, the optimal pricing and reward strategies and therefore profits of each firm are strongly related to consumers' valuations of rewards compared to cash ( $\alpha_v$ ), and to their discount rate ( $\alpha_d$ ). Second, the decision of Firm  $b$  to impose expiry of its rewards affects each firm's strategies and profits differently.

Looking at the effects of the discount rate ( $\alpha_d$ ) on the equilibrium solution for each firm, the result in proposition 2.a shows that everything else being the same, each firm's price, profit and reward period decreases with higher levels of  $\alpha_d$ . As in the previous scenario (S1), each firm

finds it more profitable to offer its LP in markets where consumers have the lowest discount rates. Furthermore, the optimal expiry period for Firm  $b$  ( $N_b$ ) either decreases or stays the same as  $\alpha_d$  increases, suggesting that in this case, Firm  $b$  should offer lower expiry and reward periods to impatient consumers to fully benefit of the time pressure mechanism.

The effects of the reward valuation parameter ( $\alpha_v$ ) on the equilibrium solution in S2 shows different impacts for the firm that offers expiry and for the one that does not. **Proposition 2.b shows that** for Firm  $a$ , its equilibrium price and reward period react similarly to  $\alpha_v$  than in S1, although the threshold on  $\alpha_v$  now depends on the value of  $\alpha_d$ . Contrary to S1, Firm  $a$  does not earn its highest profit for those values of parameters at which its price is the highest. Indeed, Firm  $a$ 's profit increases with larger values of  $\alpha_v$  for almost all values of  $\alpha_d$ . This is because Firm  $a$ 's profit is not driven solely by its price anymore since Firm  $b$ 's decision to impose expiry on its reward has led to an asymmetric game where the market shares are not equal. Therefore, a high price strategy can drive consumers away from Firm  $a$  and increase the share of its competitor, which makes a lower price the optimal solution in this scenario for Firm  $a$ .

For Firm  $b$ , for almost all values of  $\alpha_d$ , both its reward and expiry periods decrease with higher consumer valuation of reward, while its equilibrium price and profit increase. With expiry, Firm  $b$ 's profit and price are both highest at the highest level of  $\alpha_v$ , which shows again a different pattern from the one observed in scenario S1. However, similar to S1, Firm  $b$  will earn its highest profit level at those market conditions when it can also charge the highest price at equilibrium. Despite the difference in market shares between firms in this scenario, Firm  $b$ 's reward expiry can motivate consumers' purchase behavior enough to justify paying the high price charged for its product.

Therefore, everything else being the same, the firm that does not set reward expiry should charge higher prices when consumers have moderate valuations of rewards compared to cash, while firms offering rewards that expire should charge their highest prices in markets characterized by very high consumer valuations of rewards. However, both firms would earn their highest profits in markets where  $\alpha_v$  is very high. Further, these results also suggest that the influence of consumers' discount rate and their valuations of rewards compared to cash on the firms' optimal strategies and profits are interdependent and together influence each firm's optimal results differently. For Firm *a*, the optimal reward period ( $N_a$ ) becomes insensitive to  $\alpha_v$  only when the time pressure effect is very high ( $\alpha_d = 0.5$ ). However, for Firm *b*, the optimal reward period ( $N_b$ ) does not change with  $\alpha_v$  for any  $\alpha_d > 0.1$ .

Finally, comparisons of the equilibrium strategies and profits of each firm shows the following results. The firm that offers reward expiry charges a higher price<sup>7</sup>, and offers the same or a lower reward period than the one that does not. The rationale for this result is provided by the time pressure mechanism that influences consumers' purchasing and redemption behavior, which is more significant for Firm *b*'s customers than for those purchasing Firm *a*'s product. Comparisons of profits at equilibrium confirm this intuition since Firm *b* earns higher profits than Firm *a* in cases where the time pressure parameter is high enough ( $\alpha_d \geq 0.3$ ). Alternatively, when consumers' sensitivity to time pressure is too low to incite enough purchases, reward expiry makes Firm *b* earn a lower profit than Firm *a*, especially, since the Firm *b*'s price is also higher under these conditions than Firm *a*'s price, which drives these customers further away from Firm *b*' product.

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<sup>7</sup> Exceptions are noted for  $\alpha_v = 0.1, 0.2$  and  $\alpha_d = 0.1$ .

### 5.3. Scenario 3 (S3): both firms set reward expiry

In Scenario S3, both firms set an expiry date for the reward they offer. Therefore, each firm sets three decision variables: price, required number of purchasing to get the reward, and expiry length. Since Scenario S3 is a symmetric condition, as expected, the optimal decision variables are equal for both firms.

Table 8: Each firm's optimal decision variables and profits in Scenario S3 for  $\alpha_v$  &  $\alpha_d$

$\alpha_v$	$\alpha_d$																			
	0.1				0.2				0.3				0.4				0.5			
	N	T	p	I	N	T	p	I	N	T	p	I	N	T	p	I	N	T	p	I
0.5	5	50	3.129	1.312	4	50	2.867	1.160	3	50	2.767	1.058	3	50	2.625	1.005	3	50	2.523	0.967
0.6	5	50	3.392	1.425	4	50	3.044	1.235	3	50	2.907	1.116	3	50	2.739	1.053	3	50	2.617	1.008
0.7	5	50	3.506	1.477	4	50	3.127	1.274	3	19	2.993	1.156	3	15	2.827	1.093	3	12	2.705	1.046
0.8	6	50	3.416	1.480	4	19	3.252	1.327	3	11	3.213	1.240	3	9	3.007	1.162	3	8	2.844	1.101
0.9	7	31	3.53	1.558	4	13	3.535	1.440	3	9	3.463	1.335	3	7	3.201	1.235	3	7	2.985	1.157

As in the previous scenarios, when reward expiry is offered by both firms, the results in Table 8 reveal that the equilibrium pricing and reward strategies as well as the firms' profits are strongly related to consumers' valuations of rewards compared to cash ( $\alpha_v$ ), and to consumers' discount rate ( $\alpha_d$ ). These effects are summarized in proposition 3 below.

**Proposition 3:** In scenario S3, everything else being the same, the equilibrium solution reacts as follows to changes in  $\alpha_d$  and  $\alpha_v$ :

3.a. Each firm's price, profit and reward period and reward expiry period decrease with higher levels of  $\alpha_d$ .

3.b. As  $\alpha_v$  takes higher values, each firm's reward and expiry periods decrease, while price and profit increase.



The finding in proposition 3.a shows that when reward expiry is offered by both firms, the firms' prices and profits decrease with higher values of the consumer discount rate ( $\alpha_d$ ). As in the previous scenarios (S2), each firm finds it more profitable to offer its LP in markets where consumers are least sensitive to time. Furthermore, the firms' reward and expiry periods at equilibrium also decrease or stay the same as ( $\alpha_d$ ) increases. This means that both firms should offer a lower expiry and reward period to impatient consumers to fully benefit of the time pressure mechanism in this scenario.

Looking at the effects of reward valuation compared to cash ( $\alpha_v$ ) on equilibrium strategies, proposition 3.b shows that both firms' reward and expiry periods decrease with higher consumer valuation of reward, while their price and profit increase. In this case, both firms can charge the highest price and thereby earn the largest profit in markets where consumers highly values rewards. This shows again a different pattern from the one observed in the previous scenarios when there is no reward expiry (i.e., in S1 and Firm *a* in S2). However, similar to S1, both firms will earn their highest profit level at those market conditions when they can also charge the highest price at equilibrium.

#### 5.4. General game

In the previous sections, three sub-games were discussed and solved (scenarios S1, S2, and S3). Comparing optimal prices in S1 and those of Firm *a* in Scenario S2, one can conclude that a firm that applies no reward expiry should offer a higher price if its competitor sets expiry in comparison to the condition where its competitor also offers a no expiry policy. Similar comparisons between optimal prices in Scenario S3 and those of Firm *b* in Scenario S2 show that a firm that applies reward expiry should offer a higher price if its competitor also sets expiry compared with the condition where its competitor applies no expiry policy.

Next, knowing the firms' profits in each of the scenarios, we can solve the general game in which the two firms decide about setting or not setting reward expiry. Based on Tables 4 to 7, we obtain the payoff matrix of this game for each combination of  $\alpha_v$  and  $\alpha_d$ .

Reward expiry by both firms (S3 or (E, E)) is the Nash equilibrium if the following inequalities are true:  $I_a^{E,E} > I_a^{NE,E}$  and  $I_b^{E,E} > I_b^{E,NE}$ , meaning that each firm does not have an incentive (i.e., does not earn higher profit) to not set reward expiry; and, setting expiry by both firms (S3) is Pareto improving compared to the case where they both do not set reward expiry (S1).

Studying the payoff matrices obtained for each of the 25 combinations of parameters  $\alpha_v$  and  $\alpha_d$ , one can identify the equilibrium solution for the general game as described in the following proposition.

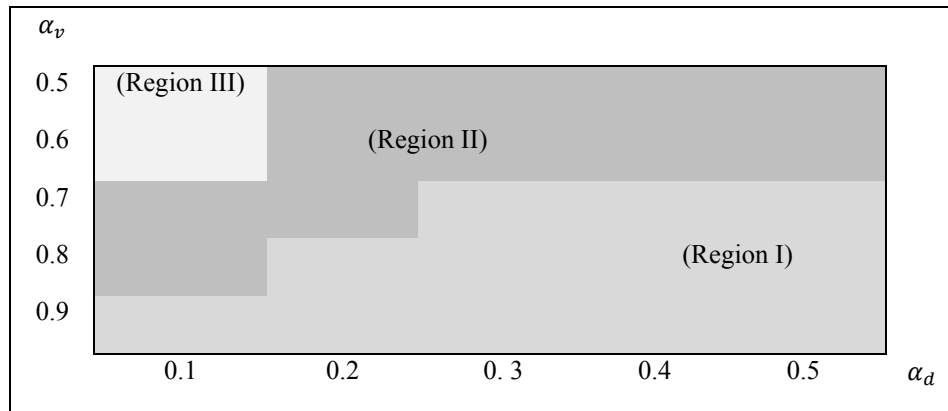
**Proposition 4:** The equilibrium solution for the general game (whether to set reward expiry or not) depends on the values of  $\alpha_v$  and  $\alpha_d$  as described in Table 9.

Table 9 shows the equilibrium solution for the game for different values of  $\alpha_v$  and  $\alpha_d$  (the specific results are included in Table A.1 in the Appendix). In Region I, setting expiry is a dominant equilibrium, i.e., each firm's profit is highest when both firms impose expiry on their rewards. This area is characterized by high customers' sensitivity levels to both time and reward.

Further, in both Regions II and III, the optimal expiry length in S3 and in S2 for the firm that sets expiry are equal to the upper bound value ( $T_{max} = 50$ ). Therefore, we cannot be sure about the existence of the equilibrium solution in this case. However, given an optimal  $T = 50$ , one can say that in Region II, setting expiry by both firms is also a dominant strategy<sup>8</sup>. Finally, in Region III, where customers are not sensitive to reward and time, not setting expiry is the dominant Nash equilibrium.

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<sup>8</sup> An exception is when  $\alpha_d = 0.1$  and  $\alpha_v = 0.7$ . In this case, there is no equilibrium solution.

Table 9: Different regions of the game between Firm  $a$  and Firm  $b$ 

## 6. Discussion

Our findings suggest that companies should actively assess the level of consumers' valuations of rewards compared to cash and consumers discount rates when designing their loyalty programs. A key managerial implication from this study is that competing firms should adjust their optimal prices, reward and expiry periods given consumers' preferences for rewards and sensitivity to time pressure, e.g., by offering distinct LP terms in markets differing along these dimensions.

The importance of these two criteria in identifying the firms' optimal strategies and best response to the competition can first be seen in the insights obtained from the symmetric game with no expiry (S1). The results we obtain in this case are different from previous studies in the literature that did not model reward expiry, nor the mental accounting and goal gradient theories, and focused on rewards in form of discounts on the next purchase (Singh et al., 2008; Kim et al., 2001). In these works, the implicit assumption is that  $\alpha_d = 0$  and  $\alpha_v = 1$ . Although our modeling approach is very different from these studies, a notable difference in our results to theirs is how variations in consumers' preferences can explain differences in loyalty programs offered by competing firms across different markets. For example, managers of competing firms should offer

LPs with extended reward periods and simultaneously charge high prices if consumer discount rate is low and consumers highly value rewards, while simultaneously charge high prices only when consumers have moderate valuations of rewards compared to cash.

The impact of consumer valuation of rewards compared to cash and their discount rate is also prevalent when one or both firms imposes expiry on its rewards, albeit the sensitivity of the equilibrium solution to these consumer preferences changes in such scenarios. In particular, the firms' strategies and profits react similarly to changes in the consumer discount rate; i.e., prices and profits increase with higher values of  $\alpha_d$  when reward expiry is implemented by one or both firm. However, the consumer valuation of rewards compared to cash influences differently equilibrium strategies and outputs with reward expiry, as prices and profits should be higher for higher levels of  $\alpha_v$  for the firm(s) using the expiry policy.

In addition to the effect of expiry on the sensitivity of the equilibrium solutions to changes in consumer preferences, the results we obtain in scenario S2 suggest that reward expiry (even when the expiry period is long) has an impact on the consumers' purchase and redemption behavior, and consequently on the firms' strategies, and profits. First, our results challenge the common assumption in practice that firms can gain a competitive advantage by offering a lenient LP policy (with no expiry). In particular, our findings indicate that the competitor's decision to set reward expiry can be a disadvantage for the firm with the more lenient policy when consumer discount rates are high. This is because reward expiry in these conditions increases the time pressure of accumulating rewards and can lead to higher purchases. The firm with the reward expiry policy should also charges a higher price compared to the competitor in this case to take full advantage of the time pressure mechanism. Ultimately, this benefits the firm with reward expiry. Alternatively, reward expiry can also lead to a lower profit than the competitor's if

consumer discount rates are low, in which case consumers are less sensitive to the time pressure effect and are more attracted to the lenient LP. These results explain the diverging results in the literature about the controversial effects of reward expiry (Kopalle & Neslin, 2003; Drèze & Nunes, 2009; Noble et al., 2014; Stauss et al., 2005; Dorotic et al., 2014; Bijmolt et al., 2012). For managers in charge of implementing and designing LPs, this means that reward expiry should be considered as a strategic decision that should be made according to consumers' preferences and the competitor's actions.

Given these asymmetric effects of reward expiry on the firms' strategies and profits, it is important to consider the equilibrium solution for each firm in the game where each can set or not set expiry. Our results indicate that setting reward expiry is a dominant equilibrium when customers value rewards highly enough and their discount rates are not too low. Otherwise, lifting the expiry restriction is more profitable for both firms. These results provide further explanations about the effects of reward expiry by showing that it can, not only provide a competitive advantage to the firm setting expiry unilaterally, but can also improve the profits of both competing firms.

## **7. Conclusion**

This paper studies the effectiveness of designing loyalty programs where expiry is set on rewards offered to customers. The motivation for this research originates from the observation of different reward expiry policies across firms in different industries, as well as the lack of clear guidance in the literature about the impact of reward expiry on firms' profitability and customers' redemption of rewards (e.g., Breugelmans et al., 2015). In fact, empirical studies report that reward expiry can have positive and negative effects on consumer purchases. While it may frustrate customers and lower their satisfaction and motivation to buy the product, it can also make customers feel pressured to redeem their points before expiry thereby increasing consumer repeat purchases.

These effects are therefore largely guided by consumers' valuations of rewards and of time pressure.

We investigate the optimality of setting reward expiry for competing firms offering a loyalty program of the type 'buy n times, get one free'. We develop a game-theoretic model that reflects consumer redemption/stockpiling and purchasing behaviors. In particular, our model takes into account the empirical observation that consumers value rewards differently from cash and feel a time pressure effect when confronted by time limitations to buy or accumulate/redeem rewards (Stourm, 2015). Consumers' utility from purchasing and redeeming rewards is used to calculate the probability of consumer purchase and reward redemption from each firm, which then is used to get each firm's profit. Our comprehensive model has the following unique properties. First, consumer choice is derived using utility functions that reflect both the mental accounting theory and the time pressure mechanism. Second, the model takes into account random effects in consumer utility and choice. Third, while most of the literature to date models reward as a discount on the next purchase, we analyze LPs of the kind 'buy n, get one free', in which rewards can be accumulated over a larger number of periods.

We apply a numerical algorithm to obtain Nash equilibrium solutions for prices, reward and expiry periods (if any) and profits in three scenarios (subgames): (1) neither firm sets reward expiry, (2) both firms set reward expiry; and (3) only one firm sets reward expiry. Comparisons of equilibrium profits across scenarios provide the equilibrium solution for the main game in which each firm decides whether or not to set reward expiry, taking into account the reaction of the competing firm.

Our main findings indicate that each firm's price and profit are affected by the expiry policy of the competing firm's LP, and by consumers' valuation of rewards and discount rate. In

particular, a key managerial implication from this study is that competing firms should adjust their optimal prices, reward and expiry periods given consumer evaluations of rewards and sensitivity to time pressure. **In particular, the price, profit and reward period of the firm that does not offer expiry decrease with higher levels of customer sensitivity to distance. As  $\alpha_v$  takes higher values, reward and expiry period of the firms that offer expiry decrease, while their price and profit increase.**

Further, our findings challenge the common assumption in practice that firms can gain a competitive advantage by offering a lenient LP policy (with no expiry) while the competitor sets reward expiry. This is because expiry can increase the pressure for consumers to accumulate rewards, therefore leading to higher purchases, even at higher prices. Reward expiry can also lead to lower profit than the competitor's if consumer discount rates are low, in which case consumers are less sensitive to the time pressure effect and are more attracted to the lenient LP. These results explain the diverging results in the literature about the positive and negative effects of reward expiry. For managers in charge of implementing and designing LPs, this means that reward expiry should be considered as a strategic decision that depends on consumers' preferences and on the competitor's actions. Finally, our findings indicate that setting reward expiry is a dominant equilibrium when customers value rewards highly enough and their discount rates are not too low. Otherwise, lifting the expiry restriction is more profitable for both firms. These results further explain the effects of reward expiry by showing that it can, not only provide a competitive advantage to the firm setting expiry unilaterally, but also improve the profits of both competing firms.

This research provides for the first time an analytical framework to assess the benefits from loyalty reward expiry for competing firms using a consumer-oriented approach. This analysis

could be extended in several ways. For instance, our model is useful to firms operating in mature industries and facing similar competitors such as the case for food or entertainment industries. Firms in such industries usually use a uniform pricing strategy and use marketing strategies aimed at increasing customers' loyalty. Future research can adapt our model to study other set-ups such as asymmetric products where customers might have significantly different evaluations for each firm's product. This can change our results since some empirical studies have showed that LPs mainly benefit large-share brands and those firms with a previously established competitive advantage (Sharp & Sharp, 1997; Meyer-Waarden & Benavent, 2010; Leenheer et al., 2007). An extension of this work can also consider a dynamic model to represent markets where prices are frequently adjusted.

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141.html.

## Appendix

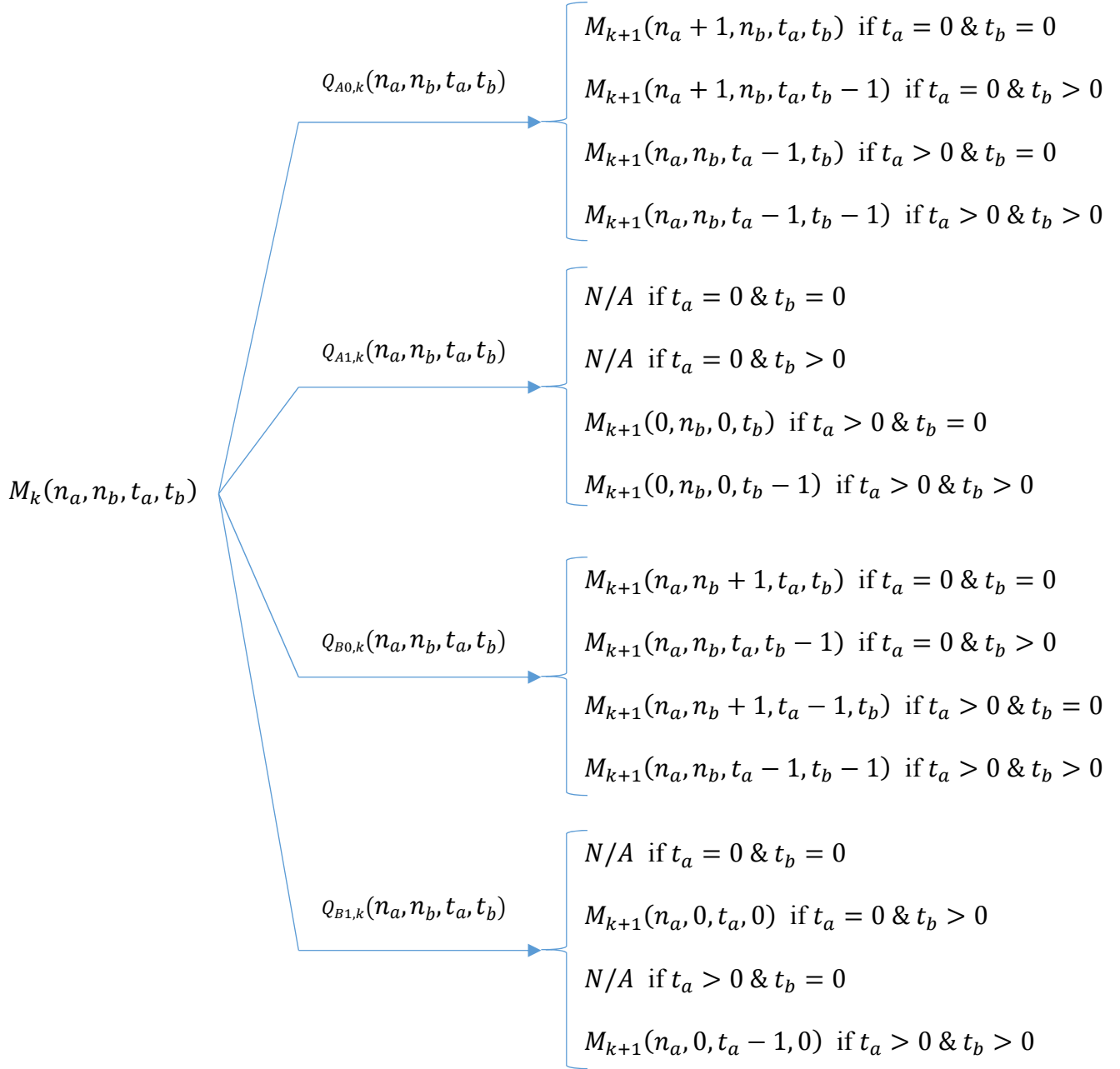
Figure A1: Flow chart between purchasing periods  $k$  and  $k + 1$

Table A.1: Payoff matrix of the game between Firm  $a$  and Firm  $b$ 

$\alpha_d \backslash \alpha_v$	0.1		0.2		0.3		0.4		0.5	
0.5	[3,3]	[4,1]	[1,1]	[3,2]	[1,1]	[3,2]	[1,1]	[3,2]	[1,1]	[3,2]
	[1,4]	[2,2]	[2,3]	[4,4]	[2,3]	[4,4]	[2,3]	[4,4]	[2,3]	[4,4]
0.6	[3,3]	[4,1]	[1,1]	[4,2]	[1,1]	[3,2]	[1,1]	[3,2]	[1,1]	[3,2]
	[1,4]	[2,2]	[2,4]	[3,3]	[2,3]	[4,4]	[2,3]	[4,4]	[2,3]	[4,4]
0.7	[1,1]	[3,4]	[1,1]	[3,2]	[1,1]	[3,2]	[1,1]	[2,3]	[1,1]	[2,3]
	[4,3]	[2,2]	[2,3]	[4,4]	[2,3]	[4,4]	[3,2]	[4,4]	[3,2]	[4,4]
0.8	[1,1]	[3,2]	[1,1]	[3,2]	[1,1]	[2,3]	[1,1]	[2,3]	[1,1]	[2,3]
	[2,3]	[4,4]	[2,3]	[4,4]	[3,2]	[4,4]	[3,2]	[4,4]	[3,2]	[4,4]
0.9	[1,1]	[3,2]	[1,1]	[2,3]	[1,1]	[2,3]	[1,1]	[2,3]	[1,1]	[2,3]
	[2,3]	[4,4]	[3,2]	[4,4]	[3,2]	[4,4]	[3,2]	[4,4]	[3,2]	[4,4]

In Table A.1, each firm's profit obtained in each of the four possible combinations of strategies are ranked from lowest (rank of 1) to highest (rank of 4) for each combination of  $\alpha_v$  and  $\alpha_d$ . The result is denoted by  $(k_a, k_b)$  for  $k_a, k_b = 1, \dots, 4$ , where  $k_a$  is the rank of preference of the strategy for Firm  $a$  and  $k_b$  is the rank of preference of the strategy for Firm  $b$ .