Low Complexity Multicast Beamforming in Massive MIMO Multi-Cell Networks

by

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Abstract

In this thesis, we consider downlink multicast beamforming in massive multi-input-multi-output (MIMO) multi-cell networks. Both non-cooperative and cooperative beamforming scenarios are considered. For the non-cooperative scenario, aiming at maximizing the minimum signal-to-interference-plus-noise (SINR) among users, we propose two multicast beamforming structures: weighted maximum ratio transmission (MRT) and weighted zero-forcing (ZF). Based on the weighted MRT beamforming structure, we propose a multicast beamforming design that transforms the beamforming problem into an optimization problem of weights and solve it via semi-definite relaxation (SDR) and successive convex approximation (SCA). To further reduce the computational complexity and the communication overhead, a distributed weighted MRT beamforming design based on signal-to-leakage ratio (SLR) is developed. Furthermore, we propose a distributed ZF beamforming design to maximize the minimum SINR among users based on the weighted ZF structure. The asymptotically optimal solution for the weighted ZF method with infinite number of antennas is derived. We also extend our work to the cooperative beamforming scenario and develop the weighted MRT approach for cooperative beamforming.

Compared with the traditional method which directly solves beamforming problem via SDR approach, our proposed methods have a low computational complexity for massive MIMO systems. Particularly, the computational complexity of weighted MRT methods does not grow with the number of antennas. Therefore our proposed methods are suitable for multi-cell networks equipped with large scale of antennas. Simulation results show that our proposed multicast beamforming solutions yield comparable or better performance than existing approaches but have significantly lower complexity for practical systems with a large but finite number of antennas.

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Chapter 1

Introduction

1.1 Overview

Owing to the fast development of wireless applications as well as the increase in the number of users and multimedia contents, there has been a tremendous growth in data traffic over the past decade, particularly in mobile networks. Video data for mobile devices has been playing a significant role in the growth. According to the *Csico Visual Networking Index* forecast for 2016-2021 [1], wireless data will increase at a compound annual growth rate of 46 percent, and video data is anticipated to account for 82 percent of all consumer internet traffic by 2021, growing from 73 percent in 2016. The emerging technologies such as virtual reality, autonomous vehicle and cloud service, will also have higher requirements for wireless communication. In order to cater the surging demand, numbers of wireless communication techniques have been studied and developed. Recently, multicasting has been emerging as the efficient transmission to delivered common data to multiple users [2]. It is considered as a promising technique for next generation wireless communication.

Over the past decade, the multiple-input-multiple-output (MIMO) communication technique has been an important area of research due to its potential for high throughput, increased diversity, and interference suppression. A MIMO system is a network with multiple transmit antennas and receiver antennas, e.g., a transmitter with multiple antennas and multiple receivers with one antenna. In a MIMO system, there are multiple signal paths between a transmitter and a receiver. When channel information is known, MIMO beamforming can be used to improve the signal gain at the receiver. Recently, massive MIMO has been envisioned as a key technology for next generation communication. As a scaled up version of the MIMO system, the massive MIMO system comprises a large number of antennas, e.g. a base station (BS) equipped with over a hundred antennas. The massive MIMO technique can reap all the benefits of conventional MIMO technique but in a much larger scale. Besides, there are other benefits including the use of inexpensive low-power components, low latency and improved robustness [3].

Multicast beamforming in a massive MIMO system is a promising solution for high data rate transmission of popular contents [4]. By beamforming, signal energy can be focused into a small region of space to achieve a huge improvement in throughput and transmission energy efficiency. The multicast beamforming problem is generally a NP-hard problem [5], and the conventional approach for finding near-optimal solutions is semidefinite relaxation (SDR). However, the computational complexity of SDR approach increases significantly as the number of antennas grows, which prevents its application in a massive MIMO system. Moreover, the large number of antennas also induces higher cost for BS coordination and cooperation to exchange channel information. Low complexity approaches for massive MIMO beamforming are desirable.

In this thesis, we investigate the downlink multicast beamforming in massive MIMO systems. To address the aforementioned challenges, we propose a weighted max ratio transmission (MRT) approach and a weighted zero-forcing (ZF) approach for multicast beamforming in massive MIMO multi-cell networks. Low complexity solutions based on weighted MRT and weighted ZF are derived. Furthermore, we extend the weighted MRT approach to cooperative multicast beamforming in massive MIMO systems, where BSs can form a cluster to send common contents cooperatively to a group of users.

1.2 Multicast Beamforming Techniques

Nowadays mobile networks have been shifted from users-centric to content-centric, which is manifested by the huge traffic of live streaming and popular video sharing. To handle this new challenging feature, multicasting has become a promising approach to cater the increasing demand [6]. In contrast to the traditional users-centric unicast technique, which transmits the separate signal to each individual user, multicasting can deliver the common contents to a group of users synchronously. By exploiting the broadcast nature of the network, multicasting is able to lower transmission burden and to improve energy efficiency.

Early in 2005, the Multimedia Broadcast/Multicast Service (MBMS) standard for terminal, radio network and user service was defined by Third Generation Partnership Project (3GPP). Lately, evolved MBMS (eMBMS) was developed from MBMS for Long-Term Evolution (LTE). The higher bit rates and the more flexible network operation in LTE enable eMBMS to bring improved performance to cellular networks.

For example, eMBMS can reduce network backhaul by delivering premium video content to multi users, and by pushing content services via user equipment caching [7]. As video contents continue to grow as a dominant part of mobile usage, multicasting becomes an increasingly attracting solution for future networks.

In order to guarantee the quality of service (QoS) at each user in multicast networks, multicast beamforming is applied at the transmitter. Similar to unicast beamforming, transmission power minimization and maximization of minimum signal-to-interference-plus-noise ratio (SINR) are two main beamforming problems for multicasting. In the power minimization problem, the optimal beamforming vector, which minimizes the transmission power while subject to specific SINR constraints at users, is desired. In the maximization of minimum SINR problem, the optimal beamforming vector needs to be obtained to maximize the minimum SINR at users under transmission power constraints. Transmit power minimization problem and maximization of minimum SINR problem are two parallel optimization problems, and both are NP-hard. Numerical algorithms and signal processing techniques have been sought to obtain approximately optimal solutions.

1.3 Massive MIMO System

With the rapid development of internet applications and the increase in number of wireless devices, the wireless network has become more crowd than ever, especially for dense urban area. However, available spectrums for wireless communication remain limited in regardless of the increasing demand. Due to the growing number of devices in a limited spectrum, interference becomes a crucial limiting role in mobile networks.

There are two conventional techniques used by cellular networks to mitigate intra-cell interference. One is transmitting signal to each user in different time slots (time-division multiplexing, TDM), the other one is transmitting signal to each user over different frequency bands (frequency-division multiplexing, FDM). Recently, massive MIMO has been emerging as a promising key technology for the 5th generation wireless system [3,8]. By equipping a large number of antennas at the transmitter, massive MIMO can achieve spatial multiplexing, which enables transmitting different signal to multiple users over same time and frequency resources. By applying beamforming at transmitter side, each transmit antenna can shape its emitting signal according to the channel condition. As the signals are received by receiver antennas, signals from different path add up constructively at its target users, while it add up destructively at the unintended users. Therefore, intra-cell interference can be suppressed without consuming extra time or frequency resources.

Massive MIMO systems come with many benefits and potentials. By spatial multiplexing, massive MIMO can significantly improve the SINR at users, and the network capacity can increase over 10 times [9]. Moreover, energy efficiency is dramatically improved. At the limit of infinite number of antennas, the power consumption at transmitter can be reduced to arbitrary small. In terms of cost, massive MIMO is economic for using a number of small and low cost antennas instead of a few large and high quality antennas. The transmitter in a massive MIMO system can comprise over a hundred independent hot-swappable antennas, making it extremely robust and low cost to maintain.

1.4 Motivation and Objectives

As two of the promising techniques for future communication, multicast beamforming and massive MIMO have been increasingly popular in recent research. However, few works have consider multicast beamforming in massive MIMO systems, which is the combination of the two techniques. With the increasing demand and the forthcoming 5G communication era, it is important to develop massive MIMO multicast beamforming techniques for performance maximization.

Multicast beamforming for MIMO systems faces many challenges. The beamforming problems, to minimize transmit power subject to meeting prescribed SINR at users, or to maximize the minimum SINR at users under transmit power constraints, are both known to be NP-hard.

The conventional approach for multicast beamforming is applying SDR to relax the beamforming problem, and then solve the relaxed problem via semidefinite programming (SDP). Eventually a good sub-optimal beamforming vector is recovered from the solution given by SDP. For massive MIMO systems which comprise over a hundred antennas, the convention SDR approach incurs extremely high computational complexity as the problem size becomes large, making it impractical for massive MIMO systems. Low complexity approaches for massive MIMO multicast beamforming are required.

Multi-cell interference mitigation is also one of challenges for the multicast beamforming in cellular networks. Beamforming with BS coordination or BS cooperation in a MIMO system can shape the beam width to reduce interference. A recent study [4] on multi-cell massive MIMO multicast shows that, the inter-cell interference vanishes as the number of BS antennas goes to infinity, and the asymptotically optimal beamforming solution is obtained in closed-form as a linear combination of the channel vectors. However, both existing studies [10] and our study show that the multicast inter-cell interference vanishes at a rather slow rate as the number of BS antennas increases, and the asymptotically optimal beamforming solution is rather suboptimal and may not be a good choice for practical systems with a large but finite number of antennas.

Considering all above factors, we investigate the multicast beamforming in massive MIMO multi-cell networks. Our goal is to design a low-complexity multicast beamforming solutions to maximize the minimum SINR of all users subject to transmitting power constraints. BS coordination and cooperation will be considered to mitigate interference and to improve signal gain.

1.5 Contributions

In this thesis, we focus on the downlink multicast beamforming in massive MIMO multi-cell networks. We aim to develop low complexity multicast beamforming solutions for maximizing the minimum SINR among users under transmitting power budgets. Both non-cooperative and cooperative multicast beamforming scenarios are considered.

1.5.1 Non-Cooperative Scenario

For non-cooperative multicast beamforming scenarios, We propose two multicast beamforming structures: weighted MRT and weighted ZF. Based on the two structures, we develop centralized and distributed beamforming design for coordinated multicasting.

Centralized Approach

Using the weighted MRT structure, we develop a centralized beamforming approach that transforms the multicast beamforming optimization problem into a optimization problem of weights. The optimal weights can be obtained via the SDR approach, or be obtained via successive convex approximation (SCA). The problem size of weighted MRT method is independent of the number of antennas, thus it is suitable for massive MIMO multi-cell networks.

Distributed Approach

- **SLR-Based Weighted MRT**

  We develop a distributed beamforming design using the signal-to-leakage ratio (SLR) as the design metric. The SLR-based design converts the centralized multicast beamforming problem into individual beamforming problems, where each BS maximizes the minimum SLR at its serving users. Again using weighted MRT structure, the individual beamforming problem is then transformed into a optimization problem of weights and solved by the SDR approach. This distributed solution requires no communication or information sharing among BSs. The complexity of the SLR-based weighted MRT method does not grow with the number of BS antennas and is suitable for massive MIMO systems.

- **Weighted ZF**

  The ZF approach can eliminate inter-cell interference while no BS communication is needed. Using weighted ZF structure, we develop a distributed beamforming method that transforms the multicast beamforming optimization problem into a optimization problem of weights. The weight optimization problem can be solved by the SDR approach. Furthermore, we develop the asymptotically optimal solution for the weighted ZF method at the limit of infinite antennas.

1.5.2 Cooperative Scenario

We proposed a weighted MRT beamforming structure for cooperative multicast beamforming scenarios, which allow a cluster of BSs cooperatively sending common signals to a group of users. Similarly, the beamforming problem is transformed into a optimization problem of weights, which is then solved by the SDR approach. Compared with solving beamforming problem directly with SDR approach, our proposed weighted MRT method has a significantly smaller problem size in massive MIMO systems, and its complexity does not grow with the number of antennas.

Simulations show that our proposed solutions deliver comparable performance to the traditional direct SDR approach but with significantly lower complexity for massive MIMO systems. Additionally, our proposed solutions substantially outperform the non-coordinated solutions such as the traditional single-cell direct SDR approach and the existing asymptotically optimal beamforming solution.

1.5.3 Publications

- J. Yu and M. Dong, "Low-Complexity Weighted MRT Multicast Beamforming in Massive MIMO Cellular Networks," *in Proc. 2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Calgary, Apr 2018.
- J. Yu and M. Dong, "Distributed Low-Complexity Multi-cell Coordinated Multicast Beamforming with Large-Scale Antennas," *in Proc. 2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Kalamata, Jun 2018.

1.6 Thesis Outline

The rest of this thesis is organized as follows. In Chapter 2, a literature review on related wireless communication techniques is present. In Chapter 3, the weighted MRT and weighted ZF beamforming structures are proposed for non-cooperative massive MIMO multicasting. Centralized and distributed beamforming designs are developed based on our proposed structures. In Chapter 4, the cooperative weighted MRT beamforming approach is proposed. The conclusion of the thesis is provided in Chapter 5.

1.7 Notation

In this thesis, transpose, Hermitian and trace of $\mathbf{A}$ are denoted by $\mathbf{A}^T$, $\mathbf{A}^H$ and $\mathrm{tr}[\mathbf{A}]$. Notation $\mathrm{vec}(\mathbf{A})$ denotes vectorizing a matrix $A = [\mathbf{a}_1, \ldots, \mathbf{a}_N]$ into a vector $[\mathbf{a}_1^T, \ldots, \mathbf{a}_N^T]^T$. Notion $\mathrm{bldg}[\mathbf{A}_1, \ldots, \mathbf{A}_N]$ indicates constructing a block diagonal matrix by $\mathbf{A}_1, \ldots, \mathbf{A}_N$. A $N \times N$ identity matrix is denoted by $\mathbf{I}_N$. A semi-definite matrix $\mathbf{A}$ is denoted as $\mathbf{A} \succeq 0$. Notion $\mathbf{x} \sim \mathcal{CN}(a, \mathbf{Y})$ means $\mathbf{x}$ is drawn from the complex Gaussian distribution with mean $\mathbf{a}$ and covariance matrix $\mathbf{Y}$. $\mathrm{E}[\mathbf{x}]$ stands for the expected value of variable $\mathbf{x}$.

Chapter 2

Literature Review

2.1 Beamforming Techniques

The classical beamforming originated in the early spatial filters design, which forms pencil beams in order to receive a signal radiating from a specific location and to attenuate signals from other locations [11]. Beamforming is applicable to both transmission and reception.

In downlink transmission, beamforming is a technique used in multi-antenna systems to improve the signal-to-noise ratio (SNR) at receiver and to suppress co-channel interference. By exploiting spatial characteristics of propagation channels, the spectrum efficiency and power effieincy in a MIMO system can be improve by beamforming [12–15].

Nowadays, beamforming is a versatile technique for transmitting and relaying signals in the multiple-antenna systems [16–28]. Based on the transmitting strategies, beamforming can be classified into unicast beamforming and multicast beamforming.

2.2 Unicast Beamforming

Downlink unicast beamforming is a traditional beamforming technique in cellular networks, where the transmitter equipped with multiple antennas uses different beam vectors to send independent data to each user [12]. Such beamforming is considered as space-division multiple access (SDMA).

For the scenario where the antenna array is only applied at the transmitter side, downlink beamforming approaches are proposed in [29–31] for minimizing the transmission power while satisfying the QoS at each receiver. Convex optimization is introduced in [32] to solve the beamforming problems of [30] and [31]. Beamforming optimization approaches are developed in [33, 34] for maximizing the minimum SINR among receivers under transmit power constraints.

The more general scenario is studied in [35–38], where both the transmitter and the receivers are equipped with multiple antennas. Beamforming problems with different objectives and constraints have been studied. In [37], beamforming vectors are jointly optimized to minimize transmission power under SINR constraints at the receivers. Other optimization objectives, such as maximizing spectral efficiency under SINR constraints are studied in [35]. Under the constraints of fixed transmission power, authors in [36] develop the beamforming scheme to maximize spectral efficiency. Zero-forcing methods for downlink beamforming in multi-user MIMO channels are proposed in [38] to optimize the maximum transmission rate and to minimize transmission power problem.

Unicast beamforming is also considered in relay networks. A rank-two Alamouti-based distributed relay beamforming scheme is proposed in [25] to minimize per relay power. Authors in [27] propose a low complexity cooperative beamforming scheme for multi-cluster relay interference networks. In [23, 26], beamforming problems for multi-antenna relay processing are investigated.

Channel state information (CSI) is required to perform downlink beamforming. However, in the practical wireless systems, CSI must be estimated and estimation error exists. To counter the erroneous channel knowledge, robust beamforming designs based on convex optimization are developed in [39] and [40].

2.3 Multicast Beamforming

As content-centric applications such as video streaming become prevailing, multicast transmission is considered as a promising technique to delivery such contents.

A seminal work for multicast beamforming is performed by [5], which investigates the transmission power minimization problem and the max-min SINR problem in a single group single cell environment. The researchers prove that both optimization problems are NP-hard, and the SDR approach is used to solve the problems [41]. Other approaches, such as stochastic beamforming (SBF) and Alamouti-based beamforming are proposed in [42] for the multicast beamforming optimization. A low complexity approach based on channel orthogonalization and local refinement is proposed in [43] for the system with a large number of users. Multicast beamforming in the system with large scale of antennas is studied in [44]. To find a low complexity beamforming solution, authors in [44] develop a successive convex approximation (SCA) strategy to arrive a convergent iterative second-order cone programming (SOCP) solution.

Multicast beamforming for multi-group single cell environment is investigated in [45–49]. In [45] both the transmission power minimization problem and the max-min SINR problem are considered. For the case where dirty paper precoding is employed at the transmitter, a beamforming scheme with an optimal power allocation strategy is proposed in [47]. Multicast beamforming for the max-min SINR problem under per-antenna constraints is studied in [46], and the SDR approach is used to give an approximate solution. For the scenario where the transmitter is equipped with large scale of antennas, authors in [49] propose a low complexity algorithm by leveraging the alternating direction method of multipliers together with the convex-concave procedure (CCP).

In a multi-cell network, inter-cell interference is taken into consideration for multicast beamforming. Coordinated multicast beamforming, where beamforming vectors among multiple cells are jointly designed to reduce inter-cell interference, is considered in [50, 51] for maximizing the minimum SINR among users. Cooperative multicast beamforming, where BSs form a cluster to transmit common data to a group of users, is recently considered jointly with caching to minimize the network cost in [6]. In these works, conventional finite number of BS antennas is assumed, and the SDR approach is adopted to find the good suboptimal beamforming vectors.

Multicast beamforming designs for dual-hop relay networks are also investigated recently. A Lagrangian dual approach is proposed in [24] to find an approximate solution. In [52], an iterative algorithm is developed to minimize the maximal mean-squared error of the signal waveform estimation among all receivers.

2.4 Beamforming Design Objectives

Many different design objectives have been proposed for multicast beamforming by researchers. However, the most commonly studied multicast beamforming optimization problems are the power minimization problem and the maximization of minimum SINR problem.

2.4.1 Power Minimization Problem

In the power minimization problem, beamforming vectors are optimized to minimize the transmission power while satisfying the QoS level at receivers. The SINR is the most commonly considered metric for QoS in this problem.

For traditional unicast beamforming, The power minimization problem is studied in [31, 33, 38, 53, 54] with different strategies and approaches. Semi-definite programming (SDP) is proposed to solve the power minimization problem in [32, 55]. In [56] the power minimization problem with per-antenna power constraints is investigated.

The power minimization problem in multicast beamforming is originally studied in [5], where it is proven to be NP-hard and approximated by SDR and Gaussian randomization techniques. In [57], a distributed SCA method is proposed to solve the power minimization problem in relay networks. Multicast beamforming to cochannel user groups is investigated in [58], and SDR and Gaussian randomization is applied to yield a quasi-optimal solution to the power minimization problem.

2.4.2 Maximization of Minimum SINR Problem

In the maximization of minimum SINR problem, beamforming vectors are optimized jointly to maximize the minimum SINR among users in the network under transmission power constraints. For simplicity, we refer to the maximization of minimum SINR problem as the max-min fair problem.

Many approaches are proposed for the max-min fair problem in traditional unicast beamforming. A iterative approach to solve the max-min fair problem is studied in [59, 60]. In [61, 62], ℓ_1-norm optimization is proposed to find suboptimal solutions. A power allocation algorithm for multiuser orthogonal frequency division multiplexing (OFDM) to minimize total transmission power is proposed in [63]. SDP is introduced to solve the max-min problem in [32, 55], where the researchers shows SDP relaxation achieves the global optimum.

For multicast beamforming, the max-min fair problem is proven to be NP-hard in [5]. The optimal solution is approximated by the SDR and Gaussian randomization techniques. Beamforming for the max-min fair problem in the multigroup scenario is studied in [46,64], where per-antenna constraints are applied. Coordinated beamforming with a single group per cell is investigated in [51], where a distributed beamforming approach for the max-min fair problem is derived. In [65], a distributed beamforming approach is proposed by applying the alternating direction method of multipliers (ADMM). The researchers in [65] further propose a simple distributed beamforming design for the max-min fair problem with per-antenna constraints.

2.5 Coordinated Multipoint Transmission

The history of BS cooperation dates back to previous decades, where the concept of macroscopic diversity was proposed [66]. To date, coordinated multipoint (CoMP) transmission, where multiple BSs cooperate by exchanging signaling and/or user data with the core or backhaul networks, has proven to be a very beneficial solution for interference management [67]. To be specific, the BS cooperation for downlink transmission can be divided into two categories: *coordinated beamforming* and *joint transmission/ cooperative beamforming*.

Coordinated beamforming in the cellular network allows BSs to transmit signal coordinately to reduce inter-cell interference, thus the SINR at users can be improved effectively. In coordinated beamforming, there is no need for sharing of the transmission data or signal-level synchronization among BSs. Therefore, it just requires a relatively small amount of backhual communication, and can be considerably beneficial for a MIMO system with plenty users [68]. In [69], a beamforming scheme is proposed to mitigate the multi-cell interference by exploiting signal leakage information. A distributed coordinated beamforming design to minimize the maximum BS antenna power without user data exchange between BSs is discussed in [70]. Authors in [71] propose a distributed method to minimize sum transmission power under given QoS requirements. In [72], a decentralized method is developed to maximize the minimum rate for users and to cancel inter-cell interference. Authors in [73] give fast iterative algorithms to maximize the minimum user rate. The optimal assignment of each mobile to a BS station has been studied in [74].

Cooperative beamforming allows BSs to form a cluster to transmit a common data to users simultaneously in order to improve signal strength and to reduce inter-cell interference. By cooperative beamforming, a BS is able to turn its inter-cell interference into signaling for a users in other cell, and a user can receive its desired data from more than one BS. To enable MIMO cooperation, BSs are connected by high-capacity delay-free links to a central processor (CP). Both CSI and data signals for intended users are shared among BSs, and their beamforming vector are jointly designed. Cooperative downlink transmission is consider in [75] for a capacity-limited backhaul and partial channel knowledge at BSs and users. Authors in [76] study the overall capacity requirements for a backhaul network for supporting different CoMP schemes. The relation between the desired SINR at users and the required backhaul capacity is investigated in [77]. Authors in [78] evaluate the data rates the user can achieve in a CoMP system with constrained backhaul. In [79], the CoMP performance on a topologically constrainted backhaul, where links exist only between neighbor BSs, is studied.

2.6 Massive MIMO System

In the MIMO systems, spatial diversity and spatial multiplexing are two techniques for improving performance [80]. These two techniques can be combined with beamforming to obtain the spatial diversity gain, the spatial multiplexing gain and the array gain simultaneously in a MIMO system. Although there are transmission schemes composed with both spatial diversity and spatial multiplexing, authors in [81] show that there is a fundamental tradeoff between how much of each technique can get.

Multi-user MIMO beamforming with a large number of antennas at the BS is advocated in [82, 83] for the time-division duplex (TDD) scenario. Recently, massive MIMO systems have been increasingly popular in research [84–87].

Pilot symbol is used in massive MIMO systems to acquire channel CSI. However, due to the frequency reuse, pilot contamination will occur in cellular networks, resulting in the imperfect acquisition of channel CSI. A TDD protocol with pilots is proposed in [83] for massive MIMO systems. A compressive sensing-based channel estimation approach is proposed in [88]. In [83], the acquisition of CSI and the limitation imposed by pilot contamination are studied for a noncooperative multi-cell massive MIMO system. It is shown that in the limit of infinite antennas, the effects of uncorrelated noise vanishes, and only the inter-cell interference caused by pilot contamination remains. The impact of pilot contamination when the number of antennas at BS and the number of users grow to infinity while maintaining a fixed ratio is studied in [89, 90].

Energy and spectral efficiency of massive MIMO systems are investigated in [91, 92]. In [91] it is shown that in the uplink transmission ZF generally outperforms MRC by its ability to eliminate intra-cell interference. However, the performance difference becomes unobvious as pilot contamination grows strong. The Antenna selection for improving energy efficiency is investigated in [93,94], where circuit power is considered.

A general guideline for massive MIMO designs is drawn in [95], for configurations of different numbers of users, different numbers of antennas, and different coherence interval lengths. Performance of different combination of transmission approaches is studied, where different transmission techniques (unicast or multicast), pilot assignment strategies (dedicated or shared pilot assignment) and beamforming techniques (MRT or ZF) are combined.

For the single cell scenario, a low complexity singe group multicast beamforming approach is proposed in [44] by using SCA and SOCP. An ADMM-based fast algorithm for multi-group multicast beamforming is developed in [96]. In [97], a two-layer low complexity beamforming algorithm for massive MIMO systems is proposed, and the duality between the power minimization problem and the max-min SINR problem is presented.

For the multi-cell scenario, coordinated multicast beamforming is studied in [4,98], where it is shown that, the inter-cell interference vanishes as the number of BS antennas goes to infinity, and the asymptotically optimal beamforming solution is obtained in closed-form as a linear combination of the channel vectors. However, both existing studies [10] and our study show that the multicast inter-cell interference vanishes at a rather slow rate as the number of BS antennas increases, and the asymptotically optimal beamformer is rather suboptimal and may not be a good choice for practical systems with a large but finite number of antennas. In [99–101] the beamforming designs take practical antenna array structures into account.

There is a great potential for multicast beamforming in massive MIMO systems. However, low complexity beamforming schemes are in need of further research, especially for practical systems with a large but finite number of antennas. Therefore, our goal in this thesis is to develop low complexity approaches for multicast beamforming in massive MIMO systems.

Chapter 3

Low Complexity Non-Cooperative Multicast Beamforming for Massive-MIMO Multi-Cell Networks

In this chapter, based on the multi-cell downlink multicasting scenario, we propose two beamforming designs to maximize the minimum SINR among users in the network, which have the considerably low complexity compared to the conventional approach. Our proposed designs also allow base station (BS) coordination to mitigate the inter-cell interference, therefore outperform non-coordinated beamforming approaches significantly.

3.1 System Model

We consider the downlink multicasting in a cellular network consisting of N cells and a group of K users per cell. The BS in each cell is equipped with M antennas, where $M \gg 1$ for a massive MIMO system. Each user is equipped with a single antenna. We assume that all BSs and users are perfectly synchronized in time and use the same spectrum for transmission.

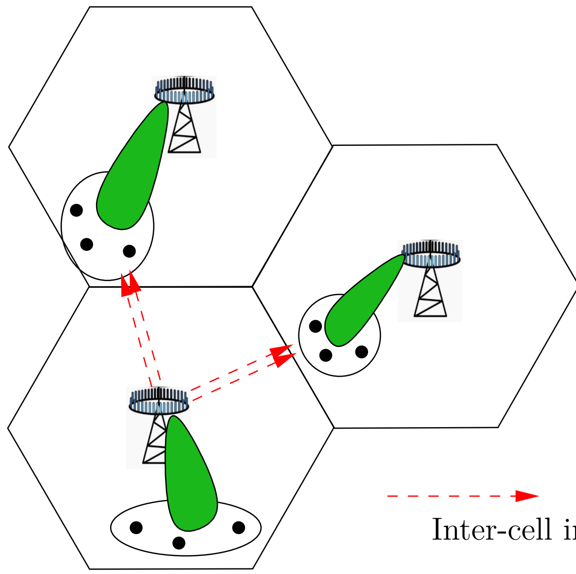

Figure 3.1: A multi-cellular downlink non-cooperative multicast beamforming scenario.

Each BS sends the common data to the group of K users in its own cell, as shown in Fig. 3.1. Define $\mathcal{N} \triangleq \{1, \cdots, N\}$,and $\mathcal{K} \triangleq \{1, \cdots, K\}$. Let $\mathbf{h}_{njk}$ denote the $M \times 1$ channel vector from BS n to user k in cell j, for $n, j \in \mathcal{N}$ and $k \in \mathcal{K}$. Cell j is defined by the cell which BS j serves. Let s_n denote the multicast information symbol from BS n with $\mathbb{E}[|s_n|^2] = 1$. Let $\mathbf{w}_n$ denote the $M \times 1$ multicast beamforming vector at BS n. The received signal at user k in cell n is given by

$$y_{nk} = \mathbf{h}_{nnk}^H \mathbf{w}_n s_n + \sum_{i \neq n}^{N} \mathbf{h}_{ink}^H \mathbf{w}_i s_i + n_{nk}, \quad k \in \mathcal{K}, \ n \in \mathcal{N} \tag{3.1}$$

where n_{nk} is the receiver additive white Gaussian noise at user k in cell n with zero mean and variance σ^2. The first term in (3.1) is the desired signal for user k and the second term is the interference from the BSs of the neighboring cells. The transmit power at BS n is limited by its maximum power P_n, and we have $\|\mathbf{w}_n\|^2 \leq P_n$, for $n \in \mathcal{N}$.

From (3.1), the received SINR at user k in cell n is given by

$$\text{SINR}_{nk} = \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{i \neq n}^{N} |\mathbf{h}_{ink}^H \mathbf{w}_i|^2 + \sigma^2}. \tag{3.2}$$

For multicast beamfroming, the performance at each cell is characterized by the minimum SINR among all users in the cell. Our objective is to design the beamforming vectors $\{\mathbf{w}_n\}$ of all BSs to maximize the minimum SINR of the network, under the transmit power constraints. The optimization problem is formulated by

$$\begin{aligned} \mathcal{P}_{\text{NC}}: \quad \max_{\{\mathbf{w}_n\}} \; \min_{k \in \mathcal{K}, n \in \mathcal{N}} \; & \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{i \neq n}^{N} |\mathbf{h}_{ink}^H \mathbf{w}_i|^2 + \sigma^2} \\ \text{s.t.} \; & \|\mathbf{w}_n\|^2 \leq \; P_n, \; n \in \mathcal{N}. \end{aligned} \tag{3.3}$$

3.2 Weighted MRT Approach

The optimization problem $\mathcal{P}_{\text{NC}}$ is a non-convex and NP-hard problem, and the optimal solution typically cannot be obtained. To find a good sub-optimal solution, a typical approach is to apply the SDR approach to find a sub-optimal $\{\mathbf{w}_n\}$. However, the complexity of the SDR approach grows with the size of the problem which is determined by M. For massive MIMO systems, as $M \gg 1$, the SDR approach incurs very high computational complexity, thus directly obtaining $\{\mathbf{w}_n\}$ through SDR is not suitable for large-scale antenna systems. Below, we first propose a low complexity multicast beamforming design via a special multicast beamforming structure to find a sub-optimal solution $\{\mathbf{w}_n\}$ whose complexity does grow with the number of antennas. Since the proposed design is a centralized method, we then develop a distributed low complexity method to find $\{\mathbf{w}_n\}$ to further reduce the complexity and communication overhead.

3.2.1 Weighted MRT Multicast Design

Instead of directly solving for $\{\mathbf{w}_n\}$ in problem $\mathcal{P}_{\text{NC}}$, we propose the structure of $\mathbf{w}_n$ at each BS n as a weighted sum of the channel vectors between BS n and its each serving users, given by

$$\mathbf{w}_n^{\text{WMRT}} \triangleq \sum_{k=1}^{K} a_{nk}\mathbf{h}_{nnk}, \;\; n \in \mathcal{N}. \tag{3.4}$$

where a_{nk} is the complex weight for the channel between BS n and its user k. We name this as the weighted MRT structure.

Define $\mathbf{H}_{nn} \triangleq [\mathbf{h}_{nn1}, \cdots, \mathbf{h}_{nnK}]$ as the $M \times K$ channel matrix between BS n and its serving user group. Define $\mathbf{a}_n \triangleq [a_{n1}, \cdots, a_{nK}]^T$ as the $K \times 1$ weight vector associated with the beamforming vector $\mathbf{w}_n^{\text{WMRT}}$ at BS n. Based on $\mathbf{w}_n^{\text{WMRT}}$ in (3.4), the SINR expression in (3.2) can now be rewritten as

$$\text{SINR}_{nk} = \frac{|\mathbf{h}_{nnk}^H \mathbf{H}_{nn}\mathbf{a}_n|^2}{\sum_{i\neq n}^{N} |\mathbf{h}_{ink}^H \mathbf{H}_{ii}\mathbf{a}_i|^2 + \sigma^2} = \frac{\mathbf{a}_n^H \mathbf{A}_{nnk}\mathbf{a}_n}{\sum_{i\neq n}^{N} (\mathbf{a}_i^H \mathbf{A}_{ink}\mathbf{a}_i) + \sigma^2} \tag{3.5}$$

where $\mathbf{A}_{ink} \triangleq \mathbf{H}_{ii}^H \mathbf{h}_{ink}\mathbf{h}_{ink}^H \mathbf{H}_{ii}, i, n \in \mathcal{N}$. The transmitting power of BS n can be written as

$$\|\mathbf{w}_n^{\text{WMRT}}\|^2 = \|\mathbf{H}_{nn}\mathbf{a}_n\|^2 = \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \tag{3.6}$$

where $\mathbf{B}_n \triangleq \mathbf{H}_{nn}^H \mathbf{H}_{nn}$.

The optimization problem $\mathcal{P}_{\text{NC}}$ is now transformed into a optimization problem of weight vectors $\{\mathbf{a}_n\}_{n\in\mathcal{N}}$ for the same objective. The max-min SINR optimization problem $\mathcal{P}_{\text{NC}}$ can now be rewritten as

$$\mathcal{P}_{\text{WMRT1}}: \max_{\{\mathbf{a}_n\}} \min_{k\in\mathcal{K}, n\in\mathcal{N}} \frac{\mathbf{a}_n^H \mathbf{A}_{nnk}\mathbf{a}_n}{\sum_{i\neq n}^{N} (\mathbf{a}_i^H \mathbf{A}_{ink}\mathbf{a}_i) + \sigma^2}$$

$$\text{s.t. } \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \leq \ P_{\text{tot}}, \quad n \in \mathcal{N}.$$

Note that the problem size of $\mathcal{P}_{\text{WMRT1}}$ is NK based on the optimization variables in $\{\mathbf{a}_n\}$, as opposed to NM in $\mathcal{P}_{\text{NC}}$. The number of constraints in both $\mathcal{P}_{\text{WMRT1}}$ and $\mathcal{P}_{\text{NC}}$ is same. The problem size of $\mathcal{P}_{\text{WMRT1}}$ is independent of the number of BS antennas M, making this approach especially attractive for massive MIMO systems. Problem $\mathcal{P}_{\text{WMRT1}}$ can be further transformed into the following

$$\begin{aligned} \mathcal{P}_{\text{WMRT2}} : \ & \min_{\{\mathbf{a}_n\}} t \\ & \text{s.t. } \frac{\mathbf{a}_n^H \mathbf{A}_{nnk} \mathbf{a}_n}{\sum_{i \neq n}^{N} (\mathbf{a}_i^H \mathbf{A}_{ink} \mathbf{a}_i) + \sigma^2} \geq \frac{1}{t}, \ k \in \mathcal{K}, n \in \mathcal{N} \\ & \quad \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \leq \ P_n, \ n \in \mathcal{N}, \\ & \quad t > 0. \end{aligned}$$

Although $\mathcal{P}_{\text{WMRT2}}$ is a problem with much a smaller size, it is still a non-convex and NP-hard problem. In the following subsections, we consider two approaches to find a good sub-optimal solution $\{\mathbf{a}_n\}$.

Solving for $\{\mathbf{a}_n\}$ with SDR Approach

The SDR approach can be applied to solve $\mathcal{P}_{\text{WMRT2}}$. Define $\mathbf{X}_n \triangleq \mathbf{a}_n \mathbf{a}_n^H$, $n \in \mathcal{N}$, and drop the rank constrain $\text{Rank}(\mathbf{X}_n) = 1$, we relax $\mathcal{P}_{\text{WMRT2}}$ into the following problem

$$\begin{aligned} \mathcal{P}_{\text{WMRT3}} : \ & \min_{\{\mathbf{X}_n\}, t} \ t \\ & \text{s.t. } \text{tr}\Big[t \mathbf{A}_{nnk} \mathbf{X}_n - \sum_{i \neq n}^{N} \mathbf{A}_{ink} \mathbf{X}_i\Big] \geq \sigma^2, \ k \in \mathcal{K}, n \in \mathcal{N} \\ & \quad \text{tr}\Big[\mathbf{B}_n \mathbf{X}_n\Big] \leq \ P_{\text{tot}}, \ n \in \mathcal{N}, \\ & \quad \mathbf{X}_n \succeq 0, \end{aligned}$$

$$t > 0.$$

Although $\mathcal{P}_{\text{WMRT3}}$ is not jointly convex w.r.t. $\mathbf{X}_n$ and t, when t is fixed, it is convex w.r.t. $\mathbf{X}_n$. Thus, we are able to find $\{\mathbf{X}_n\}$ by applying the bi-section search over t, along with solving a feasibility test problem, given by

$$\begin{aligned} &\text{Find } \{\mathbf{X}_n\} \\ &\text{s.t. } \operatorname{tr}[t\mathbf{A}_{nnk}\mathbf{X}_n - \sum_{i \neq n}^{N} \mathbf{A}_{ink}\mathbf{X}_i] \geq \sigma^2, \ k \in \mathcal{K}, n \in \mathcal{N} \\ &\quad\ \operatorname{tr}[\mathbf{B}_n\mathbf{X}_n] \leq \ P_{\text{tot}}, \ n \in \mathcal{N}, \\ &\quad\ \mathbf{X}_n \succeq 0, \\ &\quad\ t > 0. \end{aligned}$$

The above problem is a semi-definite programming (SDP) problem which can be solved efficiently with standard SDP solvers by applying interior point methods [102]. After the good sub-optimal solution $\{\mathbf{X}_n^*\}$ is obtained. We need to recover the optimal weight vectors $\{\mathbf{a}_n^*\}$ from the solution. If $\mathbf{X}_n^*$ is rank one, the weight vector $\mathbf{a}_n^*$ for BS n can be directly recovered from $\mathbf{X}_n^* = \mathbf{a}_n\mathbf{a}_n^H$. Otherwise, the Gaussian randomization method [42] can be applied to find a good sub-optimal solution. Details of the Gaussian randomization procedure for recovering $\{\mathbf{a}_n^*\}$ is shown in Algorithm 1.

Solving for $\{\mathbf{a}_n\}$ with Successive Convex Approximation (SCA).

The SDR approach with Gaussian randomization shown above can provide a good approximate solution when the problem size is small. However,the performance of the SDR approach deteriorates as the problem size becomes large. Therefore, for a system with a larger number of users per group K, the SDR approach may not have

Algorithm 1 Recovering $\{\mathbf{a}_n^*\}$ from $\{\mathbf{X}_n^*\}$ with Gaussian Randomization Procedure

Set L.
for $n = 1 \ldots N$ **do**
  if rank$(\mathbf{X}_n^*) == 1$. **then**
    Directly obtain $\mathbf{a}_n^*$ by $\mathbf{X}_n^* = \mathbf{a}_n^* \mathbf{a}_n^{*H}$.
    Let $\mathbf{a}_n^{(l)} = \mathbf{a}_n^*,\ l = 1 \ldots L$.
  else
    Generate the i.i.d random weight vector $\hat{\mathbf{a}}_n^{(l)} \sim \mathcal{CN}(0, \mathbf{X}_n^*),\ l = 1 \ldots L$.
    Scale $\hat{\mathbf{a}}_n^{(l)}$ to satisfy the power constraint by

$$\mathbf{a}_n^{(l)} = \sqrt{\frac{P_n}{\hat{\mathbf{a}}_n^{(l)H} \mathbf{B}_n \hat{\mathbf{a}}_n^{(l)}}} \hat{\mathbf{a}}_n^{(l)},\ l = 1 \ldots L.$$

  end if
end for
Calculate the minimum SINR with $\{\mathbf{a}_1^{(l)}, \ldots, \mathbf{a}_N^{(l)}\}$ by

$$\text{SINR}_{\min}^{(l)} = \min_{k \in \mathcal{K}, n \in \mathcal{N}} \frac{\mathbf{a}_n^H \mathbf{A}_{nnk} \mathbf{a}_n}{\sum_{i \neq n}^N (\mathbf{a}_i^H \mathbf{A}_{ink} \mathbf{a}_i) + \sigma^2}, \quad l = 1, \ldots, L.$$

Let $l^* = \arg\ \max_{l=1 \ldots L}\ \text{SINR}_{\min}^{(l)}$; Set $\mathbf{a}_n^* = \mathbf{a}_n^{(l^*)},\ n = 1, \ldots, N$.

the desirable performance. In this section, we consider using a SCA approach to solve $\mathcal{P}_{\text{WMRT2}}$. The optimization problem $\mathcal{P}_{\text{WMRT2}}$ can be written equivalently as

$$\begin{aligned} \mathcal{P}_{\text{WMRT4}} : \min_{\{\mathbf{a}_n\}, t} \quad & t \\ \text{s.t.} \quad & \sum_{j \neq n}^N \mathbf{a}_j^H \mathbf{A}_{ink} \mathbf{a}_j - t \mathbf{a}_n^H \mathbf{A}_{nnk} \mathbf{a}_n \leq -\sigma^2, \quad k \in \mathcal{K} \\ & \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \leq\ P_n, \\ & t > 0. \end{aligned} \tag{3.7}$$

Since $\mathbf{A}_{nnk}$ is a positive semi-definite matrix, for any arbitrary vector $\mathbf{v}_n \in \mathbb{C}^{K \times 1}$, we have the following property $(\mathbf{a}_n - \mathbf{v}_n)^H \mathbf{A}_{nnk} (\mathbf{a}_n - \mathbf{v}_n) \geq 0$. By re-arranging the terms we have

$$-\mathbf{a}_n^H \mathbf{A}_{nnk} \mathbf{a}_n \leq -2\Re\mathfrak{e}(\mathbf{v}_n^H \mathbf{A}_{nnk} \mathbf{a}_n) + \mathbf{v}_n^H \mathbf{A}_{nnk} \mathbf{v}_n, \tag{3.8}$$

where the equality holds when $\mathbf{a}_n = \mathbf{v}_n$. Based on (3.7) and (3.8), we consider the following inequalities

$$\sum_{j\neq n}^{N} \mathbf{a}_j^H \mathbf{A}_{jnk}\mathbf{a}_j - 2t\Re\mathfrak{e}(\mathbf{v}_n^H \mathbf{A}_{nnk}\mathbf{a}_n) \leq -t\mathbf{v}_n^H \mathbf{A}_{nnk}\mathbf{v}_n - \sigma^2,\ k \in \mathcal{K} \tag{3.9}$$

Note that if $\{\mathbf{a}_n\}$ satisfies (3.9), then it satisfies (3.7) as well. Thus given any $\mathbf{v}_n$, using (3.9) instead of (3.7), we approximate $\mathcal{P}_{\text{WMRT4}}$ by the following problem

$$\begin{aligned} \mathcal{P}_{\text{WMRT5}} : \min_{\{\mathbf{a}_n\},t} \quad & t \\ \text{s.t.} \quad & \sum_{j\neq n}^{N} \mathbf{a}_j^H \mathbf{A}_{jnk}\mathbf{a}_j - 2t\Re\mathfrak{e}(\mathbf{v}_n^H \mathbf{A}_{nnk}\mathbf{a}_n) \leq -t\mathbf{v}_n^H \mathbf{A}_{nnk}\mathbf{v}_n - \sigma^2, \quad k \in \mathcal{K} && (3.10) \\ & \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \leq \ P_n, && (3.11) \\ & t > 0. \end{aligned}$$

According to the inequality (3.8), we see that any feasible solution for $\mathcal{P}_{\text{WMRT5}}$ is also feasible for $\mathcal{P}_{\text{WMRT4}}$, as well as for $\mathcal{P}_{\text{WMRT2}}$. Note when $\mathbf{v}_n = \mathbf{a}_n$, (3.8) holds with equality, (3.10) is equivalent to (3.7), and $\mathcal{P}_{\text{WMRT5}}$ and $\mathcal{P}_{\text{WMRT4}}$ are equivalent.

Note that, when t is fixed, $\mathcal{P}_{\text{WMRT5}}$ is convex in $\{\mathbf{a}_n\}$. To solve $\mathcal{P}_{\text{WMRT5}}$, do bi-section search on t, and solve the feasibility problem. Let $\{\mathbf{a}_n^*\}$ denotes the optimal solution for $\mathcal{P}_{\text{WMRT5}}$. By setting $\mathbf{v}_n = \mathbf{a}_n^*$ in $\mathcal{P}_{\text{WMRT5}}$, a new problem with updated $\mathbf{v}_n$ is formed, marked by $\mathcal{P}_{\text{WMRT5}}^{+}$. Again, with bi-section search and feasibility check, a new optimal solution $\{\mathbf{a}_n^*\}$ for the new problem $\mathcal{P}_{\text{WMRT5}}^{+}$ can be obtained. By iteratively updating $\mathbf{v}_n = \mathbf{a}_n^*$ and solving the updated $\mathcal{P}_{\text{WMRT5}}$, we can eventually obtain a good sub-optimal solution for $\mathcal{P}_{\text{WMRT4}}$ and $\mathcal{P}_{\text{WMRT2}}$. Based on the above discussion, we summarize in Algorithm 2 the SCA method to solve $\mathcal{P}_{\text{WMRT2}}$.

In the following, we show the iteration in Algorithm 2 converges. Note that in the ith iteration of Algorithm 2, by applying (3.8) into (3.10) in $\mathcal{P}_{\text{WMRT5}}$, we will have

Algorithm 2 SCA Method for Weighted Multicast Beamforming $\mathcal{P}_{\text{WMRT2}}$

1: **Initialization** : Set $i = 0$ and ϵ. Choose arbitrary $\{\mathbf{v}_n^{(0)}\}$ such that $\mathbf{v}_n^{(0)H}\mathbf{B}_n\mathbf{v}_n^{(0)} \leq P_n$, for $n \in \mathcal{N}$.
2: Solve the following problem via the bi-section search over t and feasibility check; obtain optimal solution $\{\mathbf{a}_n^{*(i)}\}$.

$$\begin{aligned}\mathcal{P}_{\text{WMRT5}} : \min_{\{\mathbf{a}_n\},t} \quad & t \\ \text{s.t.} \quad & \sum_{j\neq n}^{N} \mathbf{a}_j^H \mathbf{A}_{jnk}\mathbf{a}_j - 2t\mathfrak{Re}(\mathbf{v}_n^{(i)H}\mathbf{A}_{nnk}\mathbf{a}_n) \leq -t\mathbf{v}_n^{(i)H}\mathbf{A}_{nnk}\mathbf{v}_n^{(i)} - \sigma^2, \ k \in \mathcal{K}, n \in \mathcal{N}, \\ & \mathbf{a}_n^H\mathbf{C}_n\mathbf{a}_n \leq \ P_n, \\ & t > 0.\end{aligned}$$

3: Set $\mathbf{v}_n^{(i+1)} = \mathbf{a}_n^{*(i)}$.
4: $i \leftarrow i + 1$.
5: If $\max_{n\in\mathcal{N}} \frac{\|\mathbf{a}_n^{*(i)} - \mathbf{v}_n^{(i)}\|}{\|\mathbf{a}_n^{*(i)}\|} \leq \epsilon$, **stop**; otherwise, **repeat** Steps $2 - 5$.

the following inequalities

$$\sum_{j\neq n}^{N} \mathbf{a}_j^{*(i)H}\mathbf{A}_{jnk}\mathbf{a}_j^{*(i)} - t^{*(i)}\mathbf{a}_n^{*(i)H}\mathbf{A}_{nnk}\mathbf{a}_n^{*(i)} \leq -\sigma^2, \quad k \in \mathcal{K}, n \in \mathcal{N}, \tag{3.12}$$

where $\mathbf{a}_n^{*(i)}$ and $t^{*(i)}$ are the optimal solutions for $\mathcal{P}_{\text{WMRT5}}$ in the ith iteration. In the $(i+1)$th iteration, since $\mathbf{v}_n^{(i+1)} = \mathbf{a}_n^{*(i)}$, the constraint (3.10) in $\mathcal{P}_{\text{WMRT5}}$ can be written as

$$\sum_{j\neq n}^{N} \mathbf{a}_j^H\mathbf{A}_{jnk}\mathbf{a}_j - 2t\mathfrak{Re}(\mathbf{a}_n^{*(i)H}\mathbf{A}_{nnk}\mathbf{a}_n) \leq -t\mathbf{a}_n^{*(i)H}\mathbf{A}_{nnk}\mathbf{a}_n^{*(i)} - \sigma^2, \quad k \in \mathcal{K}, n \in \mathcal{N}. \tag{3.13}$$

By choosing $\mathbf{a}_n^{(i+1)} = \mathbf{a}_n^{*(i)}, t^{(i+1)} = t^{*(i)}$ in (3.13) , we have (3.12), i.e. (3.10) in the $(i+1)$th iteration is reduced to (3.12), which is satisfied from the ith iteration. Thus, $(\{\mathbf{a}_n^{*(i)}\}, t^{*(i)})$ is a set of feasible solution for the $(i+1)$th iteration for $\mathcal{P}_{\text{WMRT5}}$, and $\mathcal{P}_{\text{WMRT2}}$. Consequently, we always have $t^{*(i+1)} \leq t^{*(i)}$. Therefore, by doing the successive iteration, the optimal objective t of $\mathcal{P}_{\text{WMRT5}}$ is non-increasing and eventually the iteration is guaranteed to converge.

3.2.2 SLR-Based Distributed Coordinated Multicast Design

There are two main challenges in solving $\mathcal{P}_{\rm NC}$. First, because the problem is NP-hard, and the complexity of numerical algorithms is often sensitive to the problem size, and becomes high for $M \gg 1$, traditional algorithms are not suitable for large-scale antenna systems. Secondly, the coordinated beamforming design in $\mathcal{P}_{\rm NC}$ is a centralized solution which requires the knowledge of channels between all BSs and users to jointly optimize $\{\mathbf{w}_n\}$. This can create significant communication overhead, backhaul traffic burden and additional delay. The first challenge is well addressed by our proposed approach in Section 3.2, where the problem size is significantly reduced by the weighted MRT structure. However, weighted MRT approach is still a centralized solution, which demands inter-BS communication. Distributed low-complexity algorithms at each BS n to determine $\mathbf{w}_n$ are desirable, especially for M being large. In order to find a low complexity distributed beamforming solution, we consider the signal-to-leakage ratio (SLR) under the beamforming vector at each BS, and optimize the multicast beamforming vector to maximize the minimum SLR among users.

Distributed SLR-Based Multicast Beamforming

With beamforming vector $\mathbf{w}_n$ at BS n, the interference caused to the unintended user(s) is defined as leakage. We consider three types of leakage:

- Type 1 (T1): Consider the interference to individual user from BS n to user i in cell j. The SLR for user k in cell n to user i in cell j is given by

$$\mathrm{SLR}^{(\mathrm{T1})}_{nk,ji} = \frac{|\mathbf{h}^H_{nnk}\mathbf{w}_n|^2}{|\mathbf{h}^H_{nji}\mathbf{w}_n|^2 + \sigma^2}, \tag{3.14}$$

where $|\mathbf{h}_{nji}^H \mathbf{w}_n|^2$ is the interference caused to user i in cell j.

- Type 2 (T2): Consider the interference to cell j from BS n. The SLR for user k in cell n to cell j is given by

$$\text{SLR}_{nkj}^{(\text{T2})} = \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{i \in \mathcal{K}} |\mathbf{h}_{nji}^H \mathbf{w}_n|^2 + \sigma^2}, \tag{3.15}$$

where $\sum_{i \in \mathcal{K}} |\mathbf{h}_{nji}^H \mathbf{w}_n|^2$ is the total interference caused to users in cell j.

- Type 3 (T3): Consider the interference to all other cells from BS n. The SLR for user k in cell n to all other cells is given by

$$\text{SLR}_{nk}^{(\text{T3})} = \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{j \in \mathcal{N}, j \neq n} \sum_{i \in \mathcal{K}} |\mathbf{h}_{nji}^H \mathbf{w}_n|^2 + \sigma^2}, \tag{3.16}$$

where $\sum_{j \in \mathcal{N}, j \neq n} \sum_{i \in \mathcal{K}} |\mathbf{h}_{nji}^H \mathbf{w}_n|^2$ is the total interference caused to all users in other cells.

Instead of considering the minimum SINR in the network, we design beamforming vectors $\{\mathbf{w}_n\}$ to maximize the minimum SLR in the network, under the transmit power constraints. Using $\text{SLR}_{nk}^{(\text{T3})}$ of type T3 as an example, the SLR-based optimization problem is formulated by

$$\begin{aligned} \max_{\{\mathbf{w}_n\}} \min_{k,n} \quad & \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{j \in \mathcal{N}, j \neq n} \sum_{i \in \mathcal{K}} |\mathbf{h}_{nji}^H \mathbf{w}_n|^2 + \sigma^2} \\ \text{s.t.} \quad & \|\mathbf{w}_n\|^2 \leq \; P_n, n \in \mathcal{N}. \end{aligned}$$

Since $\text{SLR}_{nk}^{(\text{T3})}$, for $k \in \mathcal{K}$, is only a function of $\mathbf{w}_n$, the coordinated multicast beamforming optimization can be separated into the sub-problems, one for each $\mathbf{w}_n$ to be solved at each BS n independently. Similar conclusion holds for SLR T1 and T2. Thus, beamforming vector $\mathbf{w}_n$ optimization at BS n is formulated as follows

1) Maximizing the minimum SLR to each user:

$$\mathcal{P}_1^{\mathrm{T1}}: \max_{\mathbf{w}_n} \min_{k,i,j,j\neq n} \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{|\mathbf{h}_{nji}^H \mathbf{w}_n|^2 + \sigma^2}$$
$$\text{s.t. } \|\mathbf{w}_n\|^2 \leq P_n. \tag{3.17}$$

2) Maximizing the minimum SLR to each cell:

$$\mathcal{P}_1^{\mathrm{T2}}: \max_{\mathbf{w}_n} \min_{k,j,j\neq n} \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{i\in\mathcal{K}} |\mathbf{h}_{nji}^H \mathbf{w}_n|^2 + \sigma^2}$$
$$\text{s.t. } \|\mathbf{w}_n\|^2 \leq P_n. \tag{3.18}$$

3) Maximizing the minimum SLR to all other cells:

$$\mathcal{P}_1^{\mathrm{T3}}: \max_{\mathbf{w}_n} \min_{k} \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{j\in\mathcal{N},j\neq n} \sum_{i\in\mathcal{K}} |\mathbf{h}_{nji}^H \mathbf{w}_n|^2 + \sigma^2}$$
$$\text{s.t. } \|\mathbf{w}_n\|^2 \leq P_n. \tag{3.19}$$

Note that to solve for the $\mathbf{w}_n$ at each BS n in each problem above, only the channel vectors between BS n and the users are needed. Thus, using SLR metric, each BS can solve for its multicast beamforming vector distributively ,while the BS coordination to mitigate inter-cell interference is maintained.

Distributed SLR-Based Multicast Beamforming with Weighted MRT Structure

Again, the complexity of the SDR approach grows with the size of the problem. For a large-scale antenna system with $M \gg 1$, directly using the SDR approach for $\mathcal{P}_1^{\mathrm{T1}} - \mathcal{P}_1^{\mathrm{T3}}$ still incurs high computational complexity, therefore it is not suitable for large M. Instead, we apply our proposed weighted MRT structure in Section 3.2.1 for $\mathbf{w}_n$, e.g.

$$\mathbf{w}_n^{\text{WMRT}} \triangleq \sum_{k=1}^{K} a_{nk}\mathbf{h}_{nnk}, \ n \in \mathcal{N}. \tag{3.20}$$

which will convert the beamforming vector optimization into a optimization problem of weights, thus the computational complexity does not grow with M.

Recall that $\mathbf{a}_n = [a_{n1}, \cdots, a_{nK}]^T$ is the $K \times 1$ weight vector associated with beamforming vector $\mathbf{w}_n$ at BS n. Denote $\mathbf{H}_{nj} \triangleq [\mathbf{h}_{nj1}, \cdots, \mathbf{h}_{njK}]$ as the $M \times K$ channel matrix between BS n and the user group in cell j. The SLR expression in (3.14) for T1 can now be rewritten as

$$\begin{aligned} \text{SLR}_{nk,ji}^{(\text{T1})} &= \frac{|\mathbf{h}_{nnk}^H \mathbf{H}_{nn}\mathbf{a}_n|^2}{|\mathbf{h}_{nji}^H \mathbf{H}_{nn}\mathbf{a}_n|^2 + \sigma^2} \\ &= \frac{\mathbf{a}_n^H(\mathbf{H}_{nn}^H \mathbf{h}_{nnk}\mathbf{h}_{nnk}^H \mathbf{H}_{nn})\mathbf{a}_n}{\mathbf{a}_n^H(\mathbf{H}_{nn}^H \mathbf{h}_{nji}\mathbf{h}_{nji}^H \mathbf{H}_{nn})\mathbf{a}_n + \sigma^2}. \end{aligned} \tag{3.21}$$

The corresponding SLR expressions for T2 and T3 in (3.15) and (3.16) can be similarly obtained, respectively. The transmission power of BS n is accordingly rewritten as

$$\|\mathbf{w}_n\|^2 = \|\mathbf{H}_{nn}\mathbf{a}_n\|^2 \leq P_n. \tag{3.22}$$

Based on the above, the optimization problems $\mathcal{P}_1^{\text{T1}} - \mathcal{P}_1^{\text{T3}}$ can be reformulated and are equivalent to the following weight optimization problems for $\mathbf{a}_n$ at each BS n:

$$\begin{aligned} \mathcal{P}_2^{\text{T1}} : \ & \min_{\mathbf{a}_n} \quad t \\ \text{s.t.} \quad & \frac{\mathbf{a}_n^H(\mathbf{H}_{nn}^H \mathbf{h}_{nnk}\mathbf{h}_{nnk}^H \mathbf{H}_{nn})\mathbf{a}_n}{\mathbf{a}_n^H(\mathbf{H}_{nn}^H \mathbf{h}_{nji}\mathbf{h}_{nji}^H \mathbf{H}_{nn})\mathbf{a}_n + \sigma^2} \geq \frac{1}{t}, \quad k, i \in \mathcal{K}, \ j \in \mathcal{N}, j \neq n \\ & \mathbf{a}_n^H \mathbf{H}_{nn}^H \mathbf{H}_{nn}\mathbf{a}_n \leq \ P_n, \ t > 0. \end{aligned}$$

$$\mathcal{P}_2^{\text{T2}} : \ \min_{\mathbf{a}_n} \quad t$$

$$\text{s.t.} \quad \frac{\mathbf{a}_n^H(\mathbf{H}_{nn}^H\mathbf{h}_{nnk}\mathbf{h}_{nnk}^H\mathbf{H}_{nn})\mathbf{a}_n}{\mathbf{a}_n^H\mathbf{H}_{nn}^H\mathbf{H}_{nj}\mathbf{H}_{nj}^H\mathbf{H}_{nn}\mathbf{a}_n + \sigma^2} \geq \frac{1}{t}, \quad k \in \mathcal{K},\ j \in \mathcal{N}, j \neq n$$

$$\mathbf{a}_n^H\mathbf{H}_{nn}^H\mathbf{H}_{nn}\mathbf{a}_n \leq\ P_n,\ t > 0.$$

$$\mathcal{P}_2^{\mathrm{T3}} : \min_{\mathbf{a}_n} \quad t$$

$$\text{s.t.} \ \frac{\mathbf{a}_n^H(\mathbf{H}_{nn}^H\mathbf{h}_{nnk}\mathbf{h}_{nnk}^H\mathbf{H}_{nn})\mathbf{a}_n}{\mathbf{a}_n^H\left(\sum_{j\in\mathcal{N},j\neq n}\mathbf{H}_{nn}^H\mathbf{H}_{nj}\mathbf{H}_{nj}^H\mathbf{H}_{nn}\right)\mathbf{a}_n + \sigma^2} \geq \frac{1}{t}, \quad k \in \mathcal{K}$$

$$\mathbf{a}_n^H\mathbf{H}_{nn}^H\mathbf{H}_{nn}\mathbf{a}_n \leq\ P_n,\ t > 0.$$

Note that there are K optimization variables in the above problems instead of M in $\mathcal{P}_1^{\mathrm{T1}} - \mathcal{P}_1^{\mathrm{T3}}$, which provides significant complexity reduction in the large-scale antenna systems.

To solve the above problems, we apply the SDR approach. In the following, we focus on $\mathcal{P}_2^{\mathrm{T3}}$; the other two problems can be solved similarly. Define

$$\mathbf{X}_n \triangleq \mathbf{a}_n\mathbf{a}_n^H, \quad n \in \mathcal{N} \tag{3.23}$$

$$\mathbf{C}_{n^-} \triangleq \sum_{j\in\mathcal{N},j\neq n}\mathbf{H}_{nn}^H\mathbf{H}_{nj}\mathbf{H}_{nj}^H\mathbf{H}_{nn}, \tag{3.24}$$

and again, define

$$\mathbf{A}_{njk} \triangleq \mathbf{H}_{nn}^H\mathbf{h}_{njk}\mathbf{h}_{njk}^H\mathbf{H}_{nn} \tag{3.25}$$

$$\mathbf{B}_n \triangleq \mathbf{H}_{nn}^H\mathbf{H}_{nn}. \tag{3.26}$$

By removing the rank-1 constraint on $\mathbf{X}_n$, we transform and relax $\mathcal{P}_2^{\mathrm{T3}}$ into the following problem

$$\mathcal{P}_3^{\mathrm{T3}} : \quad \min_{\mathbf{X}_n,t} \quad t$$

$$\text{s.t.} \quad \mathrm{tr}[t\mathbf{A}_{nnk}\mathbf{X}_n - \mathbf{C}_{n^-}\mathbf{X}_n] \geq \sigma^2,\ k \in \mathcal{K}$$

$$\mathrm{tr}[\mathbf{B}_n\mathbf{X}_n] \leq \ P_n$$

$$\mathbf{X}_n \succeq 0, \quad t > 0.$$

For a fixed t, the above problem becomes a feasibility test problem given by

$$\begin{aligned} &\text{Find } \mathbf{X}_n \\ &\text{s.t. } \mathrm{tr}[t\mathbf{A}_{nnk}\mathbf{X}_n - \mathbf{C}_{n^-}\mathbf{X}_n] \geq \sigma^2, \ k \in \mathcal{K} \\ &\mathrm{tr}[\mathbf{B}_n\mathbf{X}_n] \leq \ P_n, \end{aligned} \tag{3.27}$$

which is a semi-definite programming (SDP) problem and can be solved efficiently with standard SDP solvers by applying interior point methods. Thus, $\mathcal{P}_3^{\mathrm{T3}}$ can be solved by applying the bi-section search over t, along with solving the above feasibility test problem for each t.

Let $\mathbf{X}_n^*$ denote the optimal solution for $\mathcal{P}_3^{\mathrm{T3}}$. The optimal weight vector solution $\mathbf{a}_n^*$ can be obtained from $\mathbf{X}_n^*$. The weight vector $\mathbf{a}_n^*$ can be directly recovered from $\mathbf{X}_n^* = \mathbf{a}_n\mathbf{a}_n^H$, if $\mathbf{X}_n^*$ has rank one. Otherwise, a Gaussian randomization procedure [42] similar with Algorithm 1 can be applied to find a good suboptimal solution. The beamforming vector $\mathbf{w}_n$ is then obtained at each BS n by (3.4), i.e.

$$\mathbf{w}_n^{\mathrm{WMRT}} \triangleq \sum_{k=1}^{K} a_{nk}\mathbf{h}_{nnk}, \ n \in \mathcal{N}.$$

In addition, the SCA approach in Section 3.2.1 can be similarly applied to solve $\mathcal{P}_2^{\mathrm{T1}} - \mathcal{P}_2^{\mathrm{T3}}$ as well.

3.3 Weighted ZF Multicast Beamforming Approach

In massive MIMO multi-cell networks, despite the narrower beam formed by the large antenna array, multi-cell interference is still a main issue causing performance degradation. The Orthogonalization of transmit beamformer to the interference channel is realized only for M being very large ($M > 1000$). Our study reveals that multi-cell interference decline at a rather slow rate over the increase in antenna number M, and the interference is nonnegotiable. A ZF approach, which eliminates the inter-cell interference, may be effective for achieving a good performance in large scale antenna systems. In this section, we propose a low-complexity weighted ZF beamforming method for BS to distributively maximize the minimum SINR among users in a cell.

3.3.1 Weighted ZF Multicast Design

We propose a beamforming structure $\mathbf{w}_n^{\mathrm{WZF}}$ at BS n based on the weighted sum of orthogonal vectors $\{\mathbf{z}_{nk}\}$ to interference space, given by

$$\mathbf{w}_n^{\mathrm{WZF}} \triangleq \sum_{k=1}^{K} b_{nk}\mathbf{z}_{nk}, \; n \in \mathcal{N}. \tag{3.28}$$

where $\{b_{nk}\}$ are the complex weights. Since $\mathbf{z}_{nk}$ is the orthogonal vector to interference channels, the following equation holds

$$\mathbf{z}_{nk}^H \mathbf{G}_{n-} = 0, \quad n \in \mathcal{N}, k \in \mathcal{K}, \tag{3.29}$$

where

$$\mathbf{G}_{n-} \triangleq [\mathbf{H}_{n1}, \ldots, \mathbf{H}_{n(n-1)}, \mathbf{H}_{n(n+1)}, \ldots, \mathbf{H}_{nN}] \tag{3.30}$$

is a $M \times (N-1)K$ interference channel matrix, which includes all interference channel vectors from BS n to other-cell users. We construct $\mathbf{z}_{nk}$ by projecting the channel vector $\mathbf{h}_{nnk}$ into the nullspace of $\mathbf{G}_{n-}$ for BS n, given by following [95]

$$\mathbf{z}_{nk} \triangleq (\mathbf{I}_M - \mathbf{G}_{n-}(\mathbf{G}_{n-}^H\mathbf{G}_{n-})^{-1}\mathbf{G}_{n-}^H)\mathbf{h}_{nnk} \qquad \text{for } k = 1 \in K. \tag{3.31}$$

Vectorize the weights $\{b_{nk}\}$ in (3.28) into a $K \times 1$ vector by $\mathbf{b}_n \triangleq [b_{n1}, \cdots, b_{nK}]^T$. Then the SINR expression in (3.2) can be rewritten as

$$\text{SINR}_{nk} = \frac{|\mathbf{h}_{nnk}^H \mathbf{Z}_n \mathbf{b}_n|^2}{\sigma^2} = \frac{\mathbf{b}_n^H \mathbf{D}_{nk} \mathbf{b}_n}{\sigma^2}, \tag{3.32}$$

where

$$\mathbf{Z}_n \triangleq (\mathbf{I}_M - \mathbf{G}_{n-}(\mathbf{G}_{n-}^H \mathbf{G}_{n-})^{-1} \mathbf{G}_{n-}^H) \mathbf{H}_{nn}, \tag{3.33}$$

$$\mathbf{D}_{nk} \triangleq \mathbf{Z}_n^H \mathbf{h}_{nnk} \mathbf{h}_{nnk}^H \mathbf{Z}_n. \tag{3.34}$$

Note that the interference part is eliminated since the beamformer is zero-forced by (3.31). Define $\mathbf{E} \triangleq \mathbf{Z}_n^H \mathbf{Z}_n$, the transmission power at BS n is given by

$$\|\mathbf{w}_n\|^2 = \|\mathbf{Z}_n \mathbf{b}_n\|^2 = \mathbf{b}_n^H \mathbf{E}_n \mathbf{b}_n. \tag{3.35}$$

From (3.28) and (3.35), the optimization problem $\mathcal{P}_{\text{NC}}$ is transformed into a optimization problem of weights and can be decoupled into the following distributed problems at each BS n

$$\mathcal{P}_{\text{WZF1}} : \max_{\mathbf{a}_n} \min_{k \in \mathcal{K}} \frac{\mathbf{b}_n^H \mathbf{D}_{nk} \mathbf{b}_n}{\sigma^2}$$
$$\text{s.t.} \quad \mathbf{b}_n^H \mathbf{E}_n \mathbf{b}_n \leq P_n, \quad n \in \mathcal{N}.$$

Again, note that the number of optimization variables in $\mathcal{P}_{\text{WZF1}}$ is the number of users in group, instead of the number of antennas M. Furthermore, the problem size of $\mathcal{P}_{\text{WZF1}}$ is also smaller than problem size of $\mathcal{P}_{\text{NC}}$ because of its distributed structure, especially for number of BSs N is large. Having a considerably smaller problem size, our proposed weighted ZF method could achieve a significant complexity saving for massive MIMO systems, where $M \gg 1$ and $NM > K$. $\mathcal{P}_{\text{WZF1}}$ also consists of fewer constraints than $\mathcal{P}_{\text{NC}}$, which is a benefit from its distributed structure.

To solve $\mathcal{P}_{\text{WZF1}}$, a SDR approach similar to Section 3.2.1 can be applied. We define $\mathbf{X}_n \triangleq \mathbf{b}_n \mathbf{b}_n^H$. The optimization problem $\mathcal{P}_{\text{WZF1}}$ can rewritten as following

$$\begin{aligned} \mathcal{P}_{\text{WZF2}} : \min_{\mathbf{X}_n, t} \ & t \\ \text{s.t. } & \text{tr}\Big[t\mathbf{D}_{nnk}\mathbf{X}_n\Big] \geq \sigma^2, \ k \in \mathcal{K} \\ & \text{tr}\Big[\mathbf{E}_n\mathbf{X}_n\Big] \leq \ P_n, \ \mathbf{X}_n \succeq 0, \\ & \text{Rank}(\mathbf{X}_n) = 1, \quad t > 0. \end{aligned} \tag{3.36}$$

By applying the SDR approach, we drop rank one constraint of $\mathcal{P}_{\text{WZF2}}$, and then turn the problem into a SDP problem by fixing t, given by

$$\begin{aligned} & \text{Find } \mathbf{X}_n \\ & \text{s.t. } \text{tr}[t\mathbf{D}_{nnk}\mathbf{X}_n] \geq \sigma^2, \ k \in \mathcal{K} \\ & \text{tr}[\mathbf{E}_n\mathbf{X}_n] \leq \ P_n \ . \end{aligned} \tag{3.37}$$

The SDP problem can be efficiently solved by SDP solvers. By applying the bisection search over t, a optimal solution $\mathbf{X}_n^*$ for $\mathcal{P}_{\text{WZF2}}$ can be obtained. The weight vector $\mathbf{b}_n$ can be recovered from $\mathbf{X}_n^*$ in a gaussian randomization procedure similar with Algorithm 1.

3.3.2 Asymptotic ZF Approach

To further reduce the computational complexity, in this section, we develop an asymptotical approach for the weighted ZF method, for the number of antennas $M \to \infty$. As M grows, the channels between BS and users become asymptotically orthogonal to each other. Therefore, by exploiting the orthogonality between channels in massive MIMO systems, we propose the asymptotically optimal solution for weighted ZF beamforming.

The asymptotically optimal beamforming vector is a linear combination of the channels between the BS and its serving users [4], given by

$$\mathbf{w}_n^{\mathrm{ABF}} = \sum_{k=1}^{K} b_{nk}\mathbf{h}_{nnk} \ , \tag{3.38}$$

where b_{nk} is the weight for each channel. Assume $\mathbf{h}_{njk} = \sqrt{\beta_{njk}}\mathbf{g}_{njk}$, where $\mathbf{g}_{njk}^H \sim \mathcal{CN}(0, \mathbf{I}_M)$, and β_{njk} is large scale channel attenuation from BS n to user k in cell j.

Consider the ZF beamformer $\mathbf{z}_{nk}$ in (3.31). As $M \to \infty$, the ZF beamformer $\mathbf{z}_{nk}$ converges to the channel itself,

$$\lim_{M\to\infty} \mathbf{z}_{nk} = \lim_{M\to\infty} (\mathbf{I}_M - \mathbf{G}_{-n}(\mathbf{G}_{-n}^H\mathbf{G}_{-n})^{-1}\mathbf{G}_{-n}^H)\mathbf{h}_{nnk} \tag{3.39}$$

$$= \lim_{M\to\infty} \mathbf{h}_{nnk} - \mathbf{G}_{-n}(\mathbf{G}_{-n}^H\mathbf{G}_{-n})^{-1}\mathbf{G}_{-n}^H\mathbf{h}_{nnk} \tag{3.40}$$

$$= \mathbf{h}_{nnk} \ , \tag{3.41}$$

where to arrive (3.41), we use

$$\lim_{M\to\infty} \mathbf{G}_{n-}\mathbf{h}_{nnk} = 0. \tag{3.42}$$

Therefore, the linear combination of $\mathbf{z}_{nk}$ is also a asymptotically optimal beamforming structure for massive MIMO systems, given by

$$\mathbf{w}_n^{\mathrm{AZF}} = \sum_{k=1}^{K} b_{nk}\mathbf{z}_{nk} \ . \tag{3.43}$$

Note that $\mathbf{w}_n^{\mathrm{AZF}} = \mathbf{w}_n^{\mathrm{ABF}}$ in the limit of infinite number of antennas M.

By applying (3.43) into (3.32) and let $M \to \infty$, we can derive the asymptotic SINR for user k in cell n as following

$$\begin{aligned}
\lim_{M\to\infty} \text{SINR}_{nk} &= \lim_{M\to\infty} \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sigma^2} \\
&= \lim_{M\to\infty} \frac{|\mathbf{h}_{nnk}^H (\sum_{i=1}^K b_{ni}\mathbf{z}_{ni})|^2}{\sigma^2} \\
&= \lim_{M\to\infty} \frac{|\mathbf{h}_{nnk}^H (\sum_{i=1}^K b_{ni}\mathbf{h}_{nni})|^2}{\sigma^2} \\
&= \lim_{M\to\infty} \frac{|b_{nk}\mathbf{h}_{nnk}^H \mathbf{h}_{nnk}|^2}{\sigma^2} \\
&= \lim_{M\to\infty} \frac{|b_{nk}\mathbf{h}_{nnk}^H \mathbf{h}_{nnk}|^2}{\sigma^2} \\
&= \frac{b_{nk}^2 \beta_{nnk}^2 M^2}{\sigma^2}
\end{aligned} \tag{3.44}$$

Similarly, the transmission power at BS n has the following structure

$$\begin{aligned}
\lim_{M\to\infty} \|\mathbf{w}_n\|^2 &= \lim_{M\to\infty} \|\sum_{i=1}^K b_{ni}\mathbf{z}_{ni}\|^2 \\
&= \lim_{M\to\infty} \|\sum_{i=1}^K b_{ni}\mathbf{h}_{nni}\|^2 \\
&= \lim_{M\to\infty} (\sum_{i=1}^K b_{ni}\mathbf{h}_{nni}^H)(\sum_{i=1}^K b_{nj}\mathbf{h}_{nni}) \\
&= \sum_{i=1}^K b_{ni}^2 \beta_{nni} M \ .
\end{aligned} \tag{3.45}$$

From observation we can see that there is a relation between the power constraint at BS n and the sum of SINRs of its serving users. Define

$$\lambda_{nk} \triangleq \frac{b_{nk}^2 \beta_{nnk} M}{P_n}, n \in \mathcal{N}, k \in \mathcal{K} \ . \tag{3.46}$$

The SINR at user k in cell n can be rewritten as

$$\lim_{M\to\infty} \text{SINR}_{nk} = \frac{\lambda_{nk}\beta_{nnk} M P_n}{\sigma^2} \ , \tag{3.47}$$

And the transmission power can be rewritten as

$$\lim_{M\to\infty} \|\mathbf{w}_n\|^2 = \sum_{i=1}^{K} \lambda_{ni} P_n \ . \tag{3.48}$$

By applying (3.47) and (3.48), asymptotically the optimization problem $\mathcal{P}_{\text{NC}}$ can be transformed into a distributed optimization problem over $\{\lambda_{ni}\}$ at BS n, given by

$$\begin{aligned} \mathcal{P}_{\text{AZF1}}: \ \max_{\{\lambda_{ni}\}} \ \min_{k\in\mathcal{K}} \ & \frac{\lambda_{nk}\beta_{nnk} M P_n}{\sigma^2} \\ \text{s.t.} \ & \sum_{i=1}^{K} \lambda_{ni} \leq \ 1, \quad n \in \mathcal{N}. \end{aligned}$$

Note that the inequation constraint in $\mathcal{P}_{\text{AZF1}}$ is attained at equality for the optimal solution $\{\lambda^*_{ni}\}$. Thus, we can replace the inequality sign with the equal sign. Therefore, $\mathcal{P}_{\text{AZF1}}$ becomes an optimization problem given by

$$\begin{aligned} \mathcal{P}_{\text{AZF2}}: \ \max_{\{\lambda_{nk}\}} \ \min_{k\in\mathcal{K}} \ & \frac{\lambda_{nk}\beta_{nnk} M P_n}{\sigma^2} \\ \text{s.t.} \ & \sum_{i=1}^{K} \lambda_{ni} = \ 1, \quad n \in \mathcal{N}. \end{aligned}$$

It can be shown that the optimal solution $\{\lambda^*_{nk}\}$ for $\mathcal{P}_{\text{AZF2}}$ is obtained when all users' SINRs are equal, ie,

$$\frac{\lambda_{n1}\beta_{nn1} M P_n}{\sigma^2} = \ldots = \frac{\lambda_{nK}\beta_{nnK} M P_n}{\sigma^2} \ . \tag{3.49}$$

Thus, the optimal solution $\{\lambda^*_{nk}\}$ is given by

$$\lambda^*_{nk} = \frac{1}{\beta_{nnk} \sum_{i=1}^{K} \frac{1}{\beta_{nni}}} \ , \quad k \in \mathcal{K} \ . \tag{3.50}$$

Substituting (3.50) into (3.46), we can obtain the optimal weights $\{b_{nk}\}$, given by

$$b^*_{nk} = \sqrt{\frac{P_n}{M\beta^2_{nnk} \sum_{i=1}^{K} \frac{1}{\beta_{nn1}}}}$$

$$= \frac{1}{\beta_{nnk}}\sqrt{\frac{P_n u_n}{M}} \ , \quad k \in \mathcal{K} \ , \tag{3.51}$$

where

$$u_n \triangleq \frac{1}{\sum_{i=1}^{K} \frac{1}{\beta_{nni}}} . \tag{3.52}$$

Bringing the above solution into (3.38), we obtain the asymptotically optimal ZF beamforming vector, given by

$$\begin{aligned} \mathbf{w}_n^{\mathrm{AZF}*} &= \sum_{k=1}^{K} b_{nk}^{*} \mathbf{z}_{nk} \\ &= \sum_{k=1}^{K} \frac{1}{\beta_{nnk}} \sqrt{\frac{P_n u_n}{M}} \mathbf{z}_{nnk} \qquad (3.53) \\ &= \sqrt{\frac{P_n u_n}{M}} \sum_{i=1}^{K} \frac{1}{\beta_{nnk}} \mathbf{z}_{nnk} \ . \qquad (3.54) \end{aligned}$$

The asymptotically minimum SINR at BS n can be obtained in a similar way, given by

$$\min \mathrm{SINR}_n = \frac{\lambda_{nk}\beta_{nnk} M P_n}{\sigma^2} = \frac{M P_n}{u_n \sigma^2} \ , \quad \forall\ k \in \mathcal{K} \ . \tag{3.55}$$

For a MIMO system with finite but large number of antennas M, e.g. $M \geq 100$, we can approximately obtain $\mathbf{w}_n^{\mathrm{AZF}}$ using (3.54), where the large scale channel attenuation β_{njk} is statistically obtained at BS n. Since $\mathbf{w}_n^{\mathrm{AZF}}$ is a closed-form solution, the computational complexity is dramatically simplified compared with solving the optimization problem $\mathcal{P}_{\mathrm{WZF1}}$ to obtain the optimal weights.

3.4 Complexity Analysis and Comparison

3.4.1 Centralized Multicast Beamforming via Weighted MRT

The centralized weighted MRT method largely reduces the problem size of multicast beamforming problem for a massive MIMO systems, where $M \gg 1$ and $M > K$. The

Computational complexity of weighted MRT depends on K and N, and it does not grow with M. Specifically, to solve $\mathcal{P}_{\text{WMRT2}}$ with the SDR approach, the complexity to solve each SDP in $\mathcal{P}_{\text{WMRT3}}$ is $\mathcal{O}(K^5N^3)$, as opposed to $\mathcal{O}(M^4KN^3)$ for directly using the SDR method to solve $\mathcal{P}_{\text{NC}}$. Compared with directly solving the beamforming problem with the SDR approach, whose complexity grows with M, the computational saving of the weighted MRT method is significant. Instead of using the SDR approach, weighted MRT can also be solved by applying the SCA iteration to find the optimal solution. The complexity and performance of weighted MRT via SCA will be shown in the simulation.

3.4.2 Distributed SLR-based Coordinated Multicast Beamforming via Weighted MRT

The SLR-based distributed method allows a BS to independently solve for its beamforming vector without sharing information with other BSs. The problem size of the SLR-based beamforming problem only depends on K and is independent of M. Compared with the centralized weighted MRT method, the distributed structure of SLR-based method can significantly reduce communication overhead without sacrificing BS coordination to mitigate the multi-cell interference. Computational complexity is further reduced by the SLR-based method, which has a smaller number of variables than centralized weighted MRT method. The number of optimization variables in $\mathcal{P}_1^{\text{T3}}$ is K, compared to NK in $\mathcal{P}_{\text{WMRT1}}$ and NM in $\mathcal{P}_{\text{NC}}$.

The complexity and performance of the SDR approach depend on the number of constraints. Comparing the three problems $\mathcal{P}_1^{\text{T1}} - \mathcal{P}_1^{\text{T3}}$, we note that $\mathcal{P}_1^{\text{T1}}$ has the largest number of constraints $(N-1)K^2+1$, while $\mathcal{P}_1^{\text{T3}}$ has the lowest number of constraints $K+1$. Thus the $\mathcal{P}_1^{\text{T3}}$ has the lowest complexity. Thus, from the aspect of computational complexity, SLR in T3, *i.e.,* measuring the total leakage to all the neighboring cells in the SLR, is the preferred metric, and accordingly, $\mathcal{P}_1^{\text{T3}}$ is the preferred SLR-based distributed multicast beamforming method.

It is worth mentioning that the number of constraints in $\mathcal{P}_{\text{WMRT1}}$ and $\mathcal{P}_{\text{NC}}$ is $NK+1$, which is nearly N times of that in $\mathcal{P}_1^{\text{T3}}$. Specifically, the computational complexity to solve each SDP for $\mathcal{P}_2^{\text{T3}}$ is $\mathcal{O}(K^5)$, as opposed to $\mathcal{P}_{\text{WMRT2}}$ in $\mathcal{O}(K^5N^3)$ and $\mathcal{O}(M^4KN^3)$ in $\mathcal{P}_{\text{NC}}$. Therefore, the computational complexity in $\mathcal{P}_1^{\text{T3}}$ is even further reduced when compared with $\mathcal{P}_{\text{WMRT1}}$, which is a result of fewer constraints.

3.4.3 Distributed Coordinated Multicast Beamforming via Weighted ZF

As a distributed beamforming design, the weighted ZF method also allows a BS to solve for its beamforming vector independently, thus it shares the same benefits from distributed structure like the SLR-based method. The main computational complexity of weighted ZF method comes from two parts. Part (1) is from $\mathcal{P}_{\text{WZF1}}$ by solving the SDP problem after the relaxation. Complexity of solving $\mathcal{P}_{\text{WZF1}}$ is similar with solving $\mathcal{P}_1^{\text{T3}}$, given that they have the same number of variables and the same number of constraints. Part (2) is from (3.31), where the inversion of a $(N-1)K \times (N-1)K$ matrix is needed. Matrix inversion has low complexity in the software simulation but may incur high computational complexity in the hardware implementation. The realization of matrix inversion could be rather complicated and less accurate in the aspect of practical applications Thus, in the weighted ZF method, matrix inversion is also considered as one source of complexity, especially for practical applications.

The asymptotic ZF method provides a asymptotically optimal closed-form solution, thus its main complexity is only from part (2) the matrix inversion. Since Matlab can solve matrix inversion efficiently, the complexity for part (2) is very low. Thus the asymptotic ZF method has the lowest computational complexity among our proposed methods in the simulation.

3.5 Simulation and Results

For the simulation study, we consider a multi-cell environment with 3 BSs ($N = 3$) conducting multicast beamforming. Each cell has a unit cell radius. We randomly drop K users in each cell. The default transmission power is set such that $P_n/\sigma^2 = 10$ dB, $\forall n$. the channel vectors between each BS and each user $\mathbf{h}_{nik}$ are i.i.d. generated as complex Gaussian with zero mean and covariance $\beta_{nik}\mathbf{I}$, where variance β_{nik} is determined by the pathloss model $\beta_{nik} = K_o d_{nik}^{-\kappa}$, with d_{nik} being the distances between BS n and user k in cell i, and κ being the path loss exponent set to $\kappa = 3.5$. Constant K_o is set such that with a single antenna under unit transmission power at BS, the received SNR at the edge of each cell is -5 dB. Performance is averaged over random channel realizations and user drops.

For comparison, we consider the following existing beamforming methods:

i) *Centralized coordinated method*:

C1) Using SDR to jointly solve for $\{\mathbf{w}_n\}$ in $\mathcal{P}_{\text{NC}}$ directly, namely direct SDR.

ii) *Distributed/decentralized methods*:

D1) Using SDR to solve the multicast beamforming problem directly for single cell (*i.e.,* non-coordinated) [5], namely distributed direct SDR.

D2) Asymptotic multicast beamforming (BF) solution as $M \rightarrow \infty$ (non-coordinated) [51], namely asymptotic BF.

Note that these two distributed methods are non-coordinated methods. (e.g single-cell solution)

3.5.1 Perfect CSI Setup

In this section, we assume all the channel state information (CSI) are perfectly known at each BS.

Weighted MRT Method

We first discuss the performance of weighted MRT via the SDR approach. In the simulation, we consider SDR as the default approach for weighted MRT. Simulation of Weighted MRT via the SCA method will be discussed later for comparison.

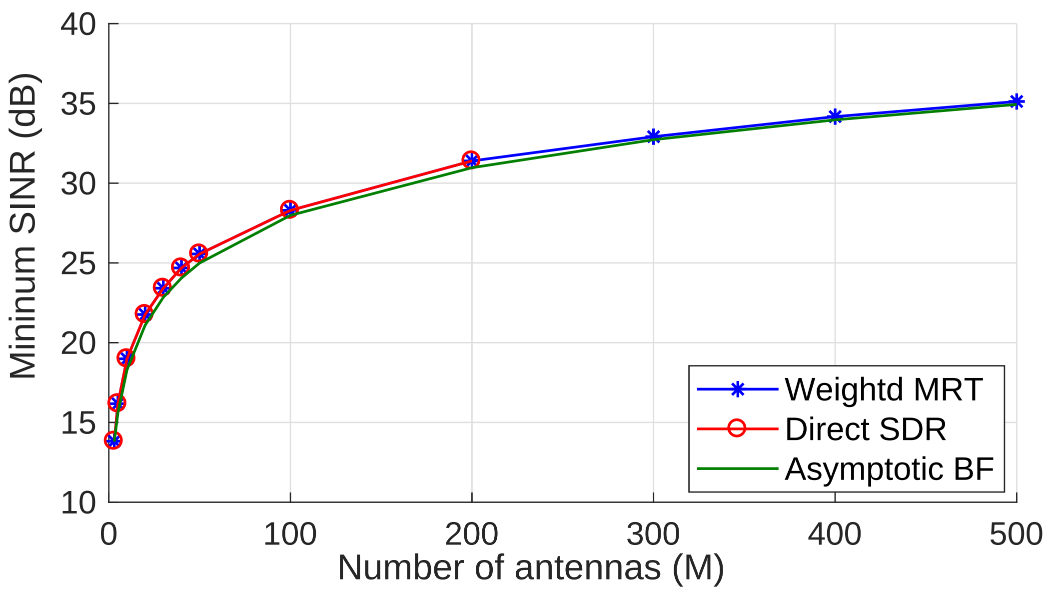

Figure 3.2: Minimum SINR vs. M for the single cell case.

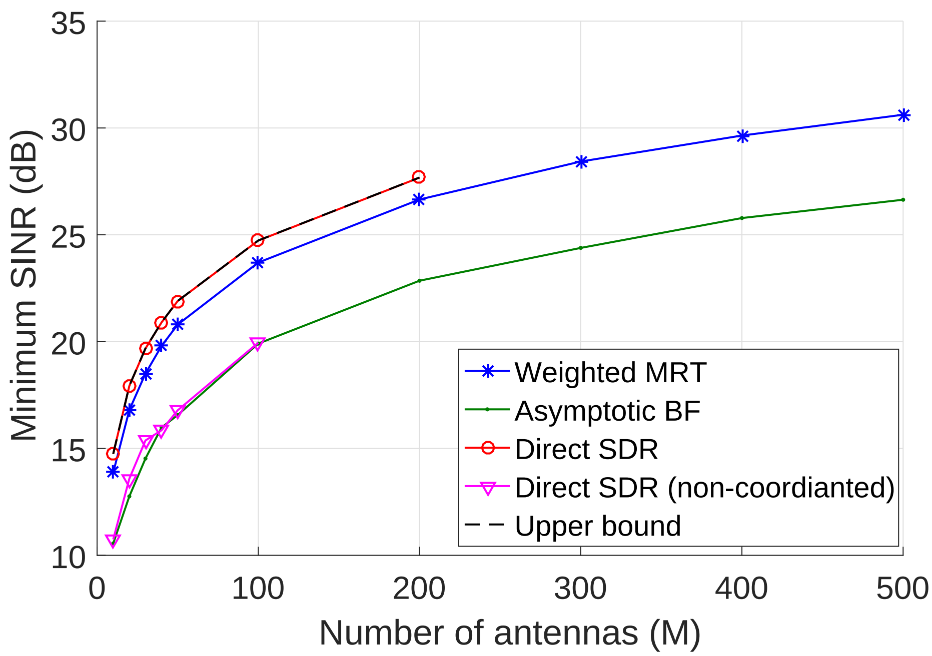

Figure 3.3: Minimum SINR vs. M for the non-cooperative scenario ($N = 3$, $K = 3$).

We first consider a single-cell setup without inter-cell interference with $K = 3$ users. In Fig. 3.2, we plot the minimum SINR performance versus M for the weighted MRT method, the direct SDR method and the asymptotic BF method. We see that all three methods provide very close performance to each other. However, their computational complexity is substantially different, which can be seen later. Fig. 3.2 indicates that the three methods provide the same-level performance when there is no interference. However, their performance under interference varies, which will be shown next.

Fig. 3.3 shows the performance of different methods in a multi-cell scenario with $N = 3$ and $K = 3$. The SDR-based upper bound is plotted as a benchmark. We see that our proposed weighted MRT method results in a small loss (~ 1 dB) compared with the centralized direct SDR method (C1) for $M \leq 200$, while using the latter

M	Weighted MRT (s)	SLR T3 (s)	Weighted ZF (s)	Direct SDR (s)
10	6.02	3.61	3.51	9.68
20	5.92	3.34	3.55	24.08
40	5.84	3.53	3.32	153.2
50	5.87	3.56	3.34	311.5
100	5.87	3.54	3.32	2487
200	5.87	3.57	3.34	18339
500	5.93	3.61	3.32	N/A

Table 3.1: Comparison of average computation time for non-cooperative scenario ($N = 3$).

for $M > 200$ becomes computationally prohibitive. The asymptotic BF (D2) method and distributed direct SDR (D1) method have very similar performance, and our proposed method significantly outperforms the two methods by about 4 dB. The reason is that the inter-cell interference reduces at a very slow rate as M increases, and the asymptotic solution (assuming inter-cell interference vanishes) is considerably sub-optimal for practical large value of M. The average computation time for the weighted MRT and the direct SDR method are shown Table. 3.1. The complexity of weighted MRT is low and is almost unchanged as M increases, while the complexity of direct SDR increases significantly with M and becomes impractical for finite but large M.

In contract to the single cell scenario, the performances of different methods diverge in the multi-cell scenario, where inter-cell interference exists. Asymptotic BF (D2) and distributed direct SDR (D1) become less effective in the multi-cell scenario, because of their lack of interference control. Our proposed weighed MRT method remains comparable performance as the direct SDR (C1) method.

Now we discuss the performance and complexity difference between solving problem $\mathcal{P}_{\text{WMRT2}}$ with the SDR approach and the SCA approach. Fig. 3.4 shows the performance of the weighted MRT method via the two approaches as the number of users per cell K increases. Performance of weighted MRT via the SCA approach has the similar performance with the SDR approach when K is small, such as $K = 3$. However, weighed MRT via the SDR approach starts to deteriorate as the number of constraints grows with K, showing around 1dB loss to its upper bound and 0.7dB loss to the SCA approach for $K = 15$. Although the SCA approach appears beneficial for the system with larger K, the simulation shows the computation time for the SCA approach is significantly higher than SDR approach, as can be seen in Fig 3.5.

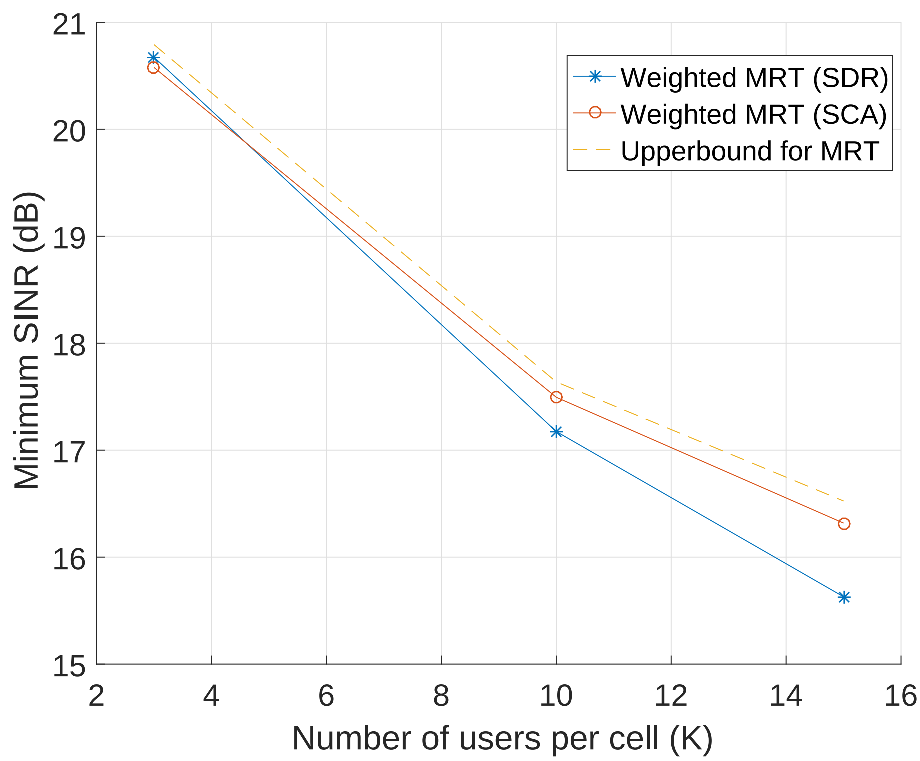

Figure 3.4: Minimum SINR vs. K for weighted MRT via different approaches ($N = 3$).

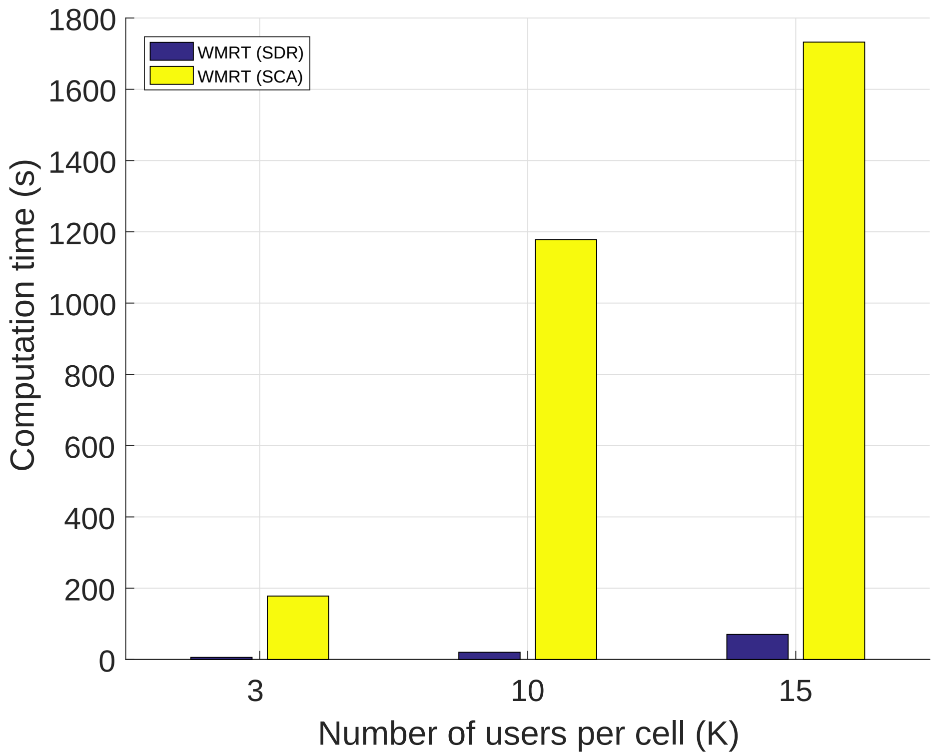

Figure 3.5: Comparison of average computation time for the SDR and SCA approaches ($N = 3$).

Distributed SLR-Based Methods

Fig. 3.6 shows the minimum SINR vs. M by the three SLR-based methods for $K = 3$, along with previously proposed methods as comparison. As we can see, among the three different SLR metrics, T3 gives the best performance. Therefore, from the aspect of computational complexity and performance, T3 is our preferred metric and the default method for SLR-based beamforming in the rest of simulation. As shown in Fig. 3.6, comparing the performance of the SLR T3 method with centralized beamforming methods, for $M \leq 200$, the SLR T3 method results in around 1 dB loss compared to the centralized weighted MRT , and an additional 1 dB loss compared to the centralized SDR approach (C1) (and upper bound). The loss in the former

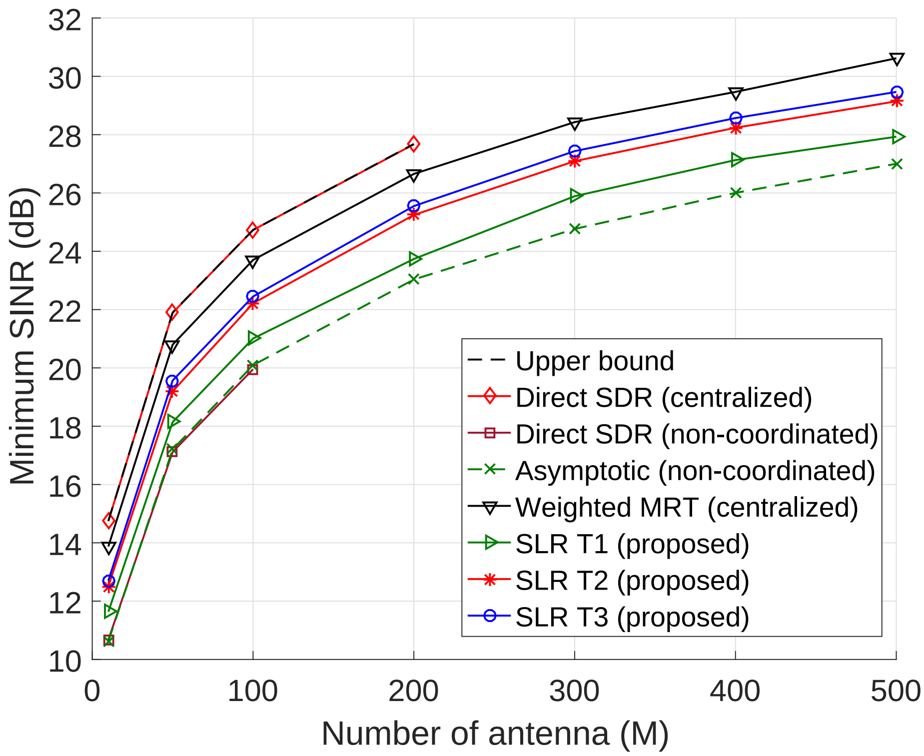

Figure 3.6: Minimum SINR vs. the number of antennas M ($N = 3$, $K = 3$).

is due to the SLR-based distributed method instead of joint optimization, and the latter is due to the weighted MRT beamforming structure. As we can see, the loss is relatively small. The SLR-based method does not require any channel information sharing and joint processing among BSs, thus it significantly reduces the network burden. Furthermore, compared with the two distributed methods (D1 and D2), our proposed SLR T3 method significantly outperforms them by more than 2.5 dB.

Fig. 3.7 shows the average computation time for the three SLR methods T1-T3. The complexity of the SLR-based methods remain approximately unchanged as the number of antennas M increases, which is due to the weighted MRT beamforming structure. This is in sharp contrast with the conventional direct SDR methods (C1 and D1) where the complexity grows significantly with M, and for $M > 200$, they

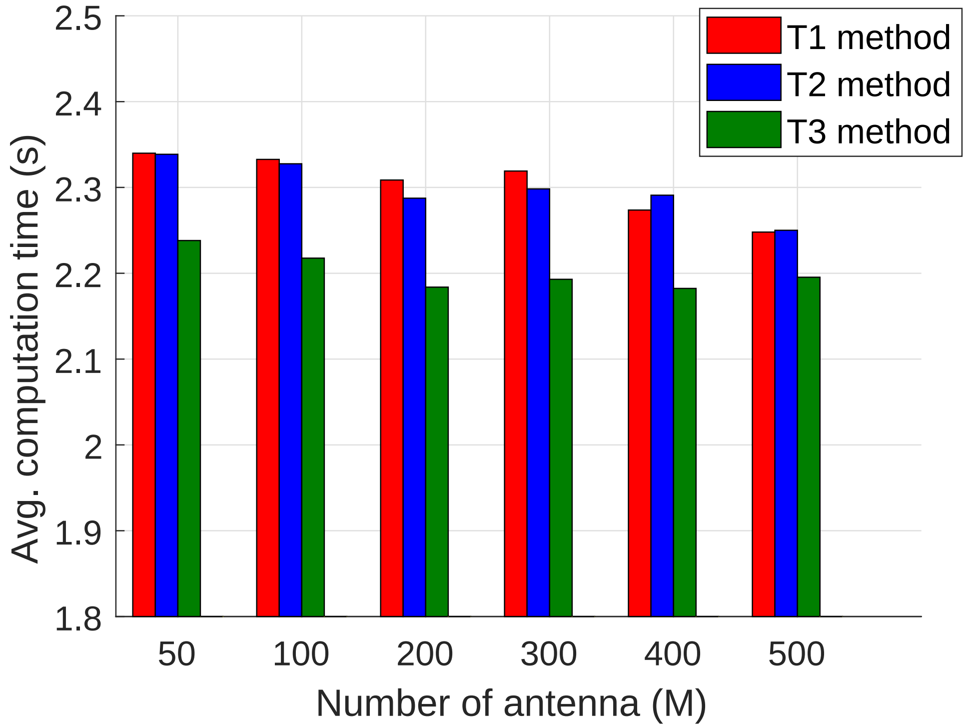

Figure 3.7: Comparison of average computation time for SLR metrics T1-T3 ($N = 3$, $K = 3$).

become computationally prohibitive to obtain a solution. Among T1-T3, we see that the average computation time of using T3 is the lowest among the three due to substantially fewer constraints.

Fig. 3.8. shows the performance of different methods as the number of users per cell K increases. We see that the performance gap between the distributed SLR T3 method and the centralized weighted MRT method remains roughly unchanged for different K. The SDR-based upper bound is given for $M = 50$, while obtaining the upper bound for larger M value is computationally prohibitive. Fig. 3.9 shows the performance for different transmission power P_n/σ^2 for $M = 100, 200, 500$. Increasing the transmission power causes higher inter-cell interference, and potentially more performance loss for the distributed method. We see from Fig. 3.9 that, for P_n/σ^2

from 0 dB to 15 dB, the minimum SINR increases, and the additional loss by the SLR-based method is mild, which indicates that the SLR-based method can sufficiently suppress the interference to prevent more performance deterioration.

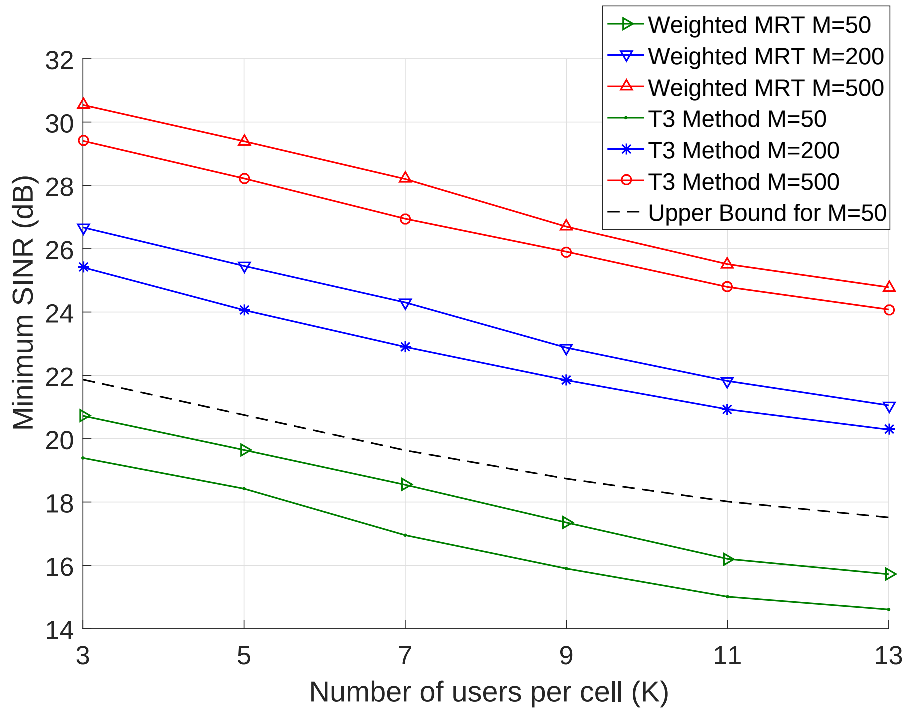

Figure 3.8: Minimum SINR performance at different K ($N = 3$).

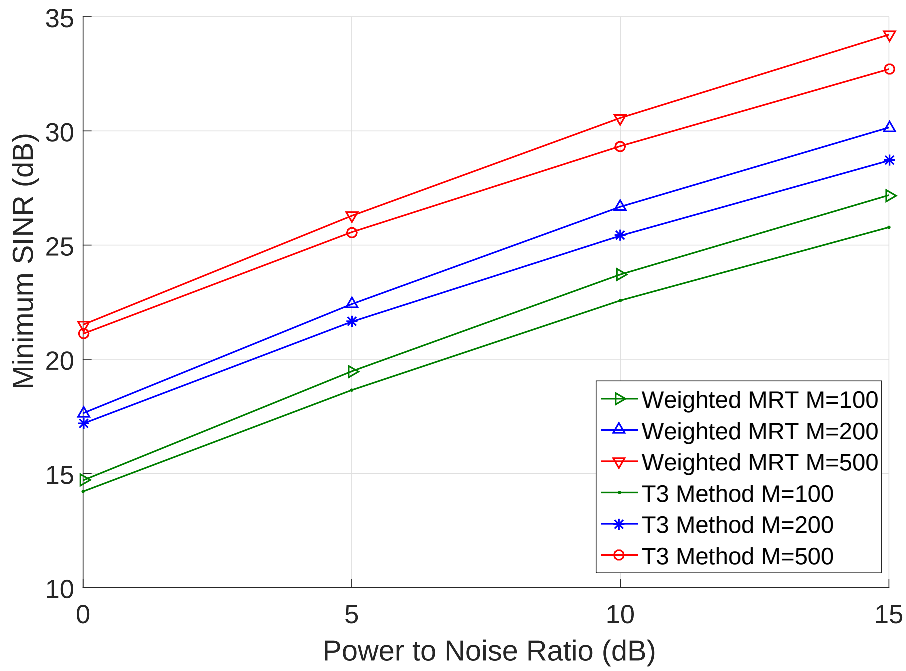

Figure 3.9: Minimum SINR performance at different P_n/σ^2 ($N = 3, K = 3$).

Weighted ZF Method

In Fig. 3.10, we plot the minimum SINR vs. M by the weighted ZF method, along with centralized SDR, weighted MRT and asymptotic BF as comparison. As we can see, although weighed ZF performs worse than weighed MRT for $M \leq$20, when the number of antenna M increases, the performance of the weighted ZF improves and is very close to the centralized direct SDR method (C1) and the upper bound. For $M > 20$, the weighted ZF method outperforms the weighted MRT method. There is only $\sim$ 0.2 dB loss compared to the centralized SDR method and the upper bound at $M = 100$. The improvement for the weighted ZF method over M is due to the higher degree of freedom. When the number of user per cell K is fixed, for larger M the orthogonal space to interference channels has a higher dimension, with the higher degree of freedom to optimize the beamforming vector. In other words, zero forcing allows the BS to further exploit the channel diversity in order to find a good sub-optimal beamforming vector.

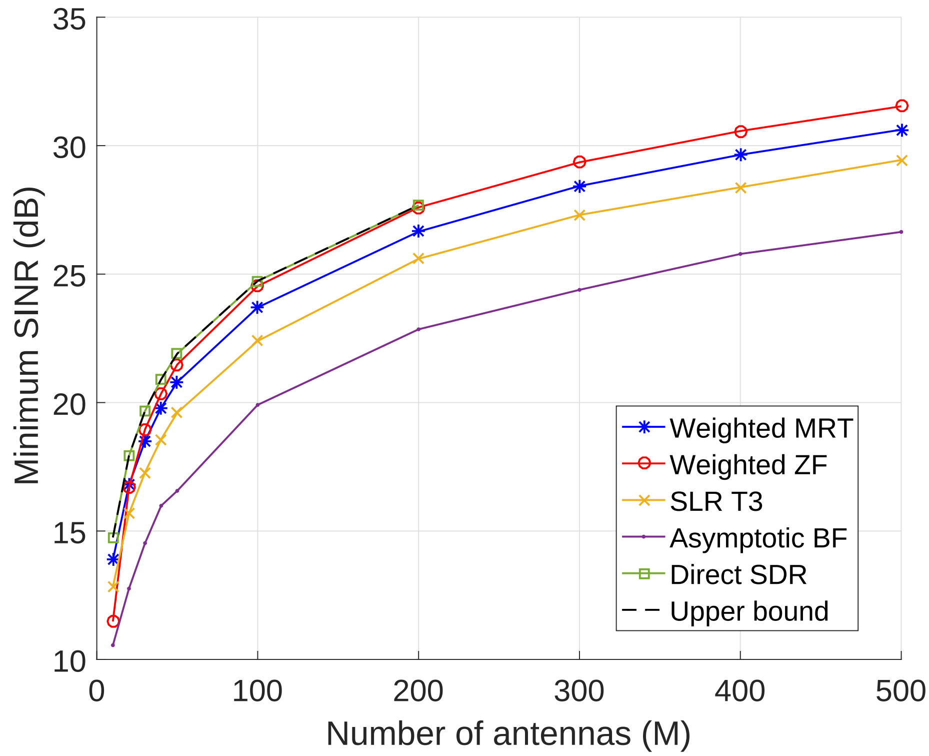

Figure 3.10: Minimum SINR vs. the number of antennas M ($N = 3$, $K = 3$).

Computation time for the weighted ZF method over different numbers of antennas M is shown in Fig. 3.11 and Table 3.1. As we can see, weighted ZF has the lower complexity than weighted MRT, and has similar computation time with the SLR T3 method. The saving is due to its distributed structure, which results in fewer constraints and a smaller problem size than $\mathcal{P}_{\text{NC}}$ with weighted MRT and the centralized direct SDR method. As can be seen, the computation time of the weighted ZF method keeps almost unchange as M increases in the simulation. Note that although the computational complexity for weighted ZF is low in the simulation, it may be undesirable in practical hardware implementation because of large matrix inversion.

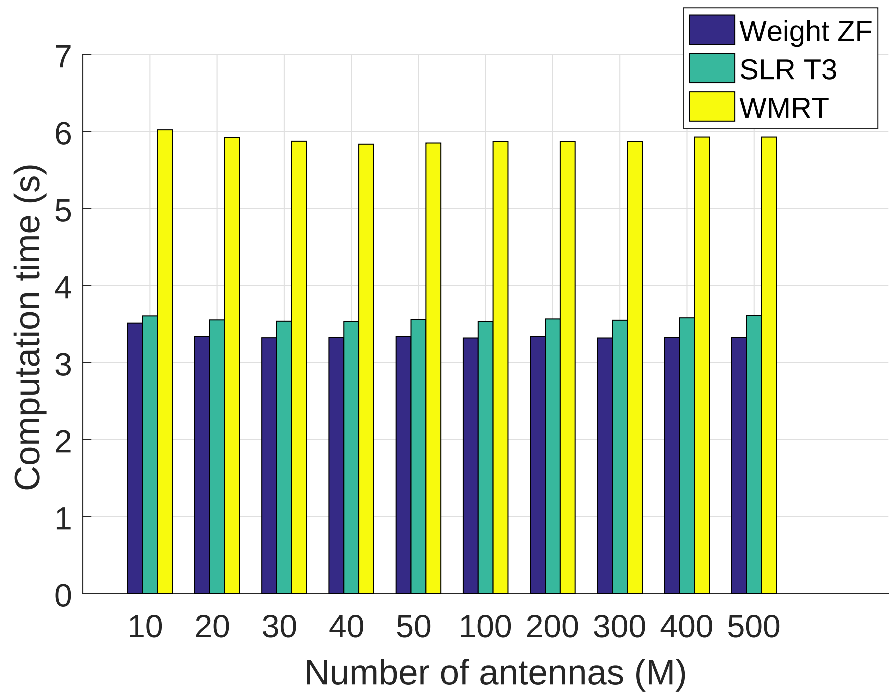

Figure 3.11: Comparison of average computation time for different methods ($N = 3$, $K = 3$).

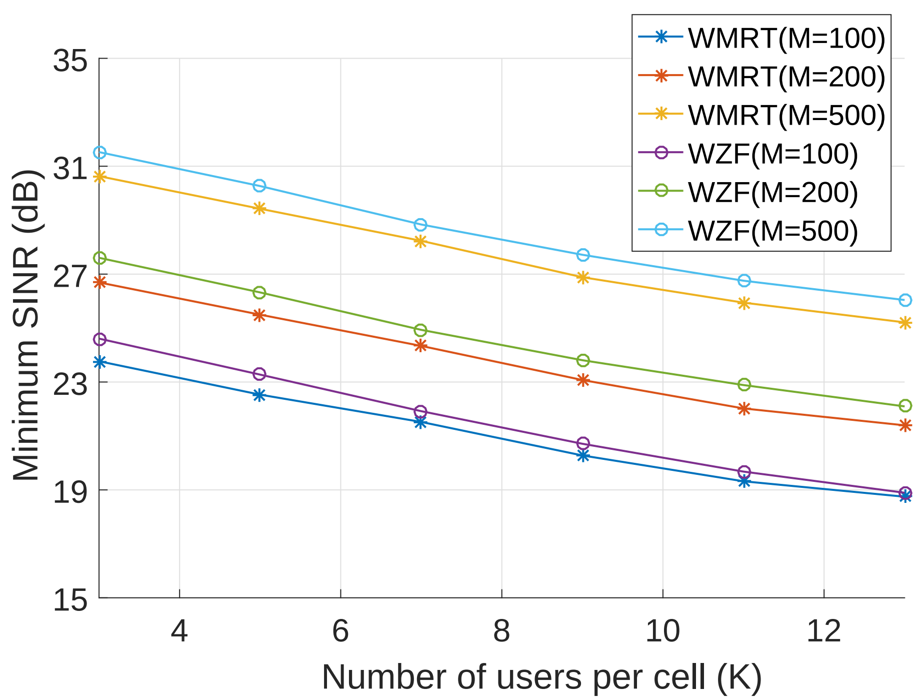

Figure 3.12: Minimum SINR performance at different K ($N = 3$).

Fig.3.12 shows performance of weighted ZF and weighed MRT as the number of

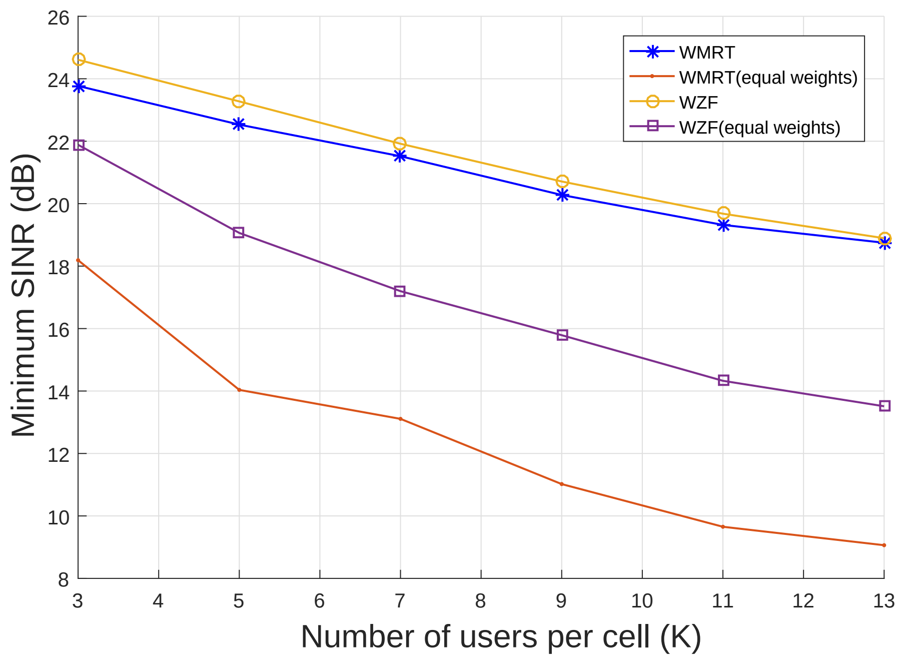

Figure 3.13: Minimum SINR performance at different K ($N = 3$, $M = 100$).

user per cell K increases. For a system with $M = 100$ antennas and $K = 3$ users per cell, there is a 0.9 dB advantage for weighted ZF against weighted MRT. However, the advantage for the weighted ZF method diminishes as K increases. As the number of users per cell K is large, the weighted ZF method requires a larger M to maintain its performance advantage. This is because when K increases, the dimension of the orthogonal space to interference channels decreases, and a lower dimension in the orthogonal space leads to a limitation in the beamforming optimization. Performance of weighted ZF and weighted MRT with equal weights is plotted in Fig. 3.13 for comparison. Note that when the weights are equal, weighted ZF and weighted MRT are reduced to the conventional MRT method and the conventional ZF method for multicast. Equal weight methods have more than 3 dB loss when compared with weighted ZF, and the gap continues to widen as K increases. Thus our proposed weighted methods significantly outperform the conventional MRT method and the conventional ZF method for multicast.

Fig. 3.14 shows the performance of weighted ZF and weighted MRT as the transmission power to noise P_n/σ^2 ratio increases. As we can see, when P_n/σ^2 is small, weighted ZF and weighted MRT have the similar performance. As P_n/σ^2 increases, the performance gap between weighted ZF and weighted MRT is widened. There is an advantage of more than 1 dB for weighted ZF against weighted MRT at $P_n/\sigma^2 = 15$dB. In Fig. 3.15, the performance for weighted ZF and weighted MRT with equal weights is plotted. There is a 2.5dB loss for equal weighted ZF, compared to our proposed weighted ZF method. We can see that equal weight methods have considerably big performance loss to our proposed methods.

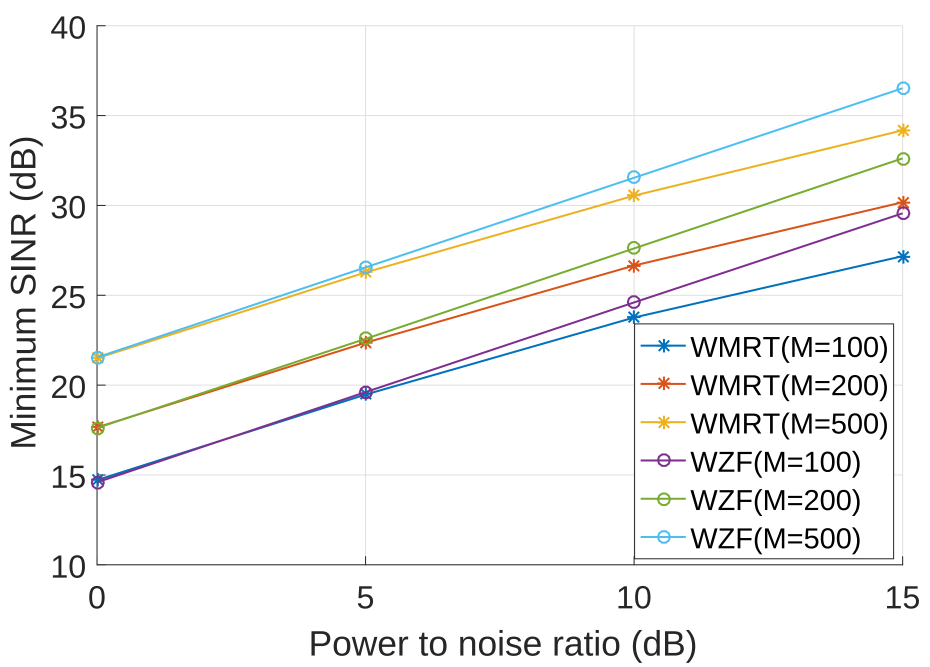

Figure 3.14: Minimum SINR performance at different P_n/σ^2 ($K = 3$).

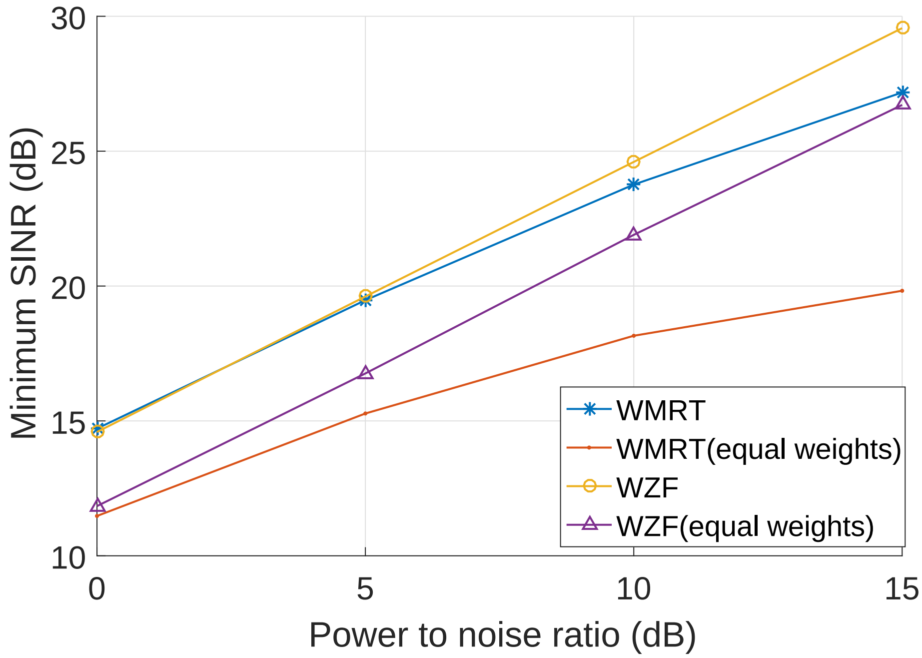

Figure 3.15: Minimum SINR performance at different P_n/σ^2 ($N = 3$, $M = 100$).

Asymptotic ZF method

Fig. 3.16 shows the performance of asymptotic ZF method as M increases, along with the previously proposed methods for comparison. As we can see, the performance of asymptotic ZF lies between weighted ZF and weighted MRT, with 0.4 dB loss to weighted ZF and 0.5 dB advantage against weighted MRT. Since the asymptotic ZF method provides a closed-form solution for the beamforming vector, it offers the simple computation complexity like asymptotic BF, but it has a 4.5dB gain in minimum SINR against the latter.

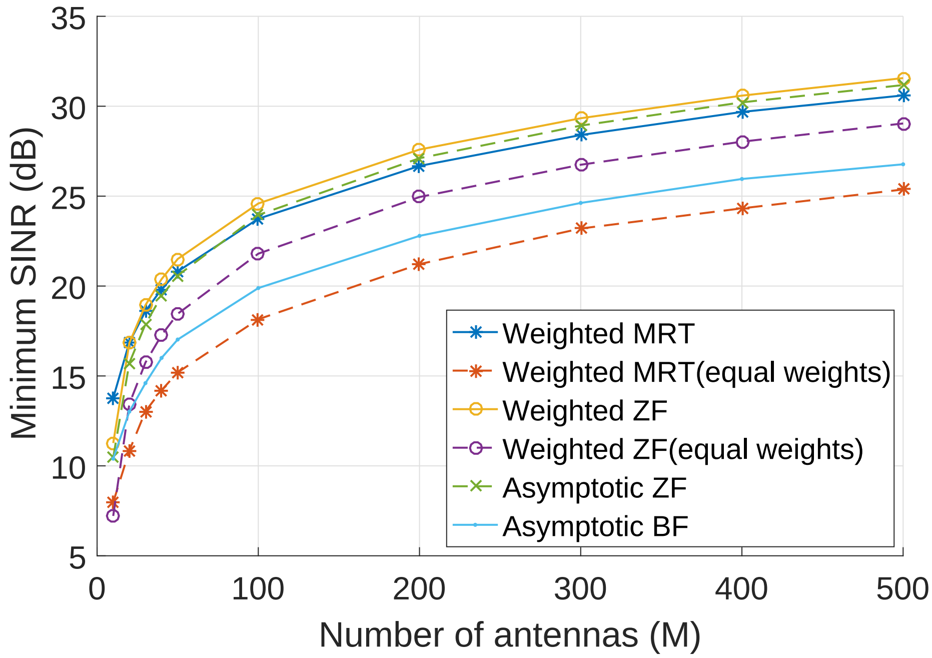

Figure 3.16: Minimum SINR vs. the number of antennas M ($N = 3$, $K = 3$).

3.5.2 Imperfect CSI Setup

In practical network systems, a BS can not obtain the CSI perfectly. Imperfect CSI might impact the performance of different beamforming methods. We assume the channel estimation error to be Gaussian independent to the channel. In Section 3.3.2, the channel between BS n and user k is generated by the path loss modeled as $\mathbf{h}_{nik} \sim \mathcal{CN}(\mathbf{0}, \beta_{nk}\mathbf{I})$, where β_{nk} is a function of distance d_{nk}. Thus, it is equivalent to write the channel model as a function of d_{nk}, given by $\mathbf{h}_{nik} \sim \mathcal{CN}(\mathbf{0}, \beta(d_{nk})\mathbf{I})$. We assume the cell radius to be r. The channel estimation can be modeled as following

$$\hat{\mathbf{h}}_{nik} = \mathbf{h}_{nik} + \check{\mathbf{h}}_{nik} \ , \tag{3.56}$$

where $\check{\mathbf{h}}_{nik} \sim \mathcal{CN}(\mathbf{0}, \mu\beta(r)\mathbf{I})$ is the channel estimation error, and μ indicates the level of estimation error. Based on this model, the channel estimation will be more inaccurate for distant users while the opposed for nearby users, which is consistent with the pilot-based channel estimation performance.

Partial Interference

We first consider the scenario where BSs have perfect CSI for its serving users but imperfect channel estimation for users in other cell, and we name this scenario as partial interference.

Fig. 3.17 and Fig. 3.18 show the minimum SINR of our proposed methods, with estimation error factor $\mu = 0.2,\ 0.4$, respectively. Performance of the centralized direct SDR method (C1) and the asymptotic BF (D2) is also plotted for comparison. Note that the asymptotic BF method is not affected by partial interference CSI, since it only requires channel information of its serving users. As we can see, when the estimation error μ increases, the minimum SINR of all methods reduces. The weighted MRT method appears more sensitive than the other methods, and its performance reduces to the same level with the SLR T3 method for $\mu = 0.4$. The weighted ZF method keeps the close performance to the centralized direct SDR method (C1) for different μ. The performance of weighted MRT and weighted ZF over μ is shown in Fig. 3.19. As we can see, the decrease rate of their performance over μ becomes slower as μ increases. Fig. 3.20 and Fig. 3.21 show the performance of the asymptotic ZF method and the equal weight methods for different μ. Asymptotic ZF only has a small loss to weighted ZF, but has a over 2 dB advantage against asymptotic BF for $\mu = 0.4$. The equal weight methods perform significantly sub-optimal compared with our proposed methods. For $\mu = 0.4$, asymptotic BF has similar performance with equal weight ZF.

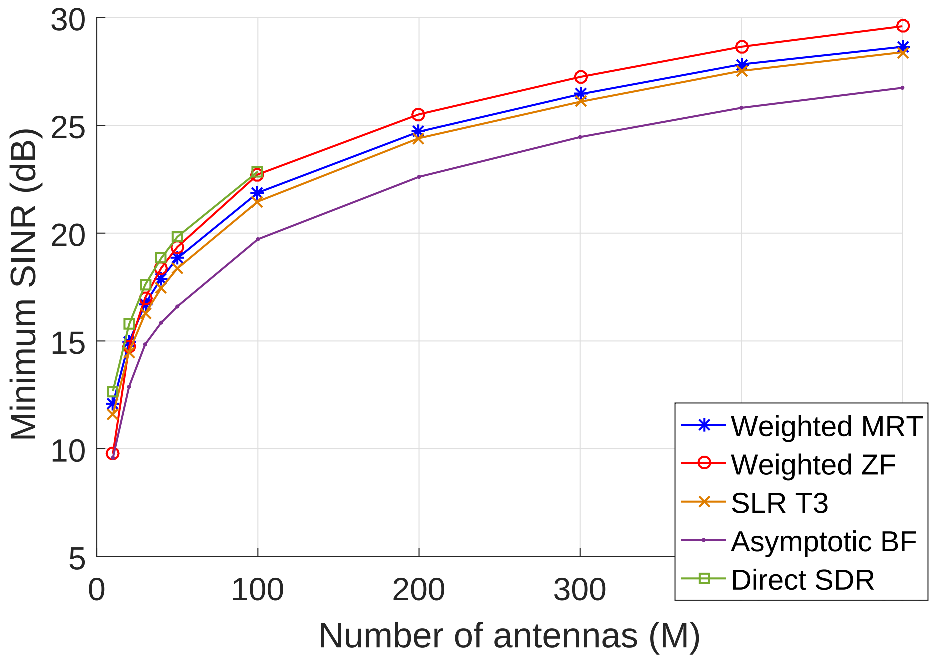

Figure 3.17: Minimum SINR vs. M for partial interference ($N = 3$, $K = 3$, $\mu = 0.2$).

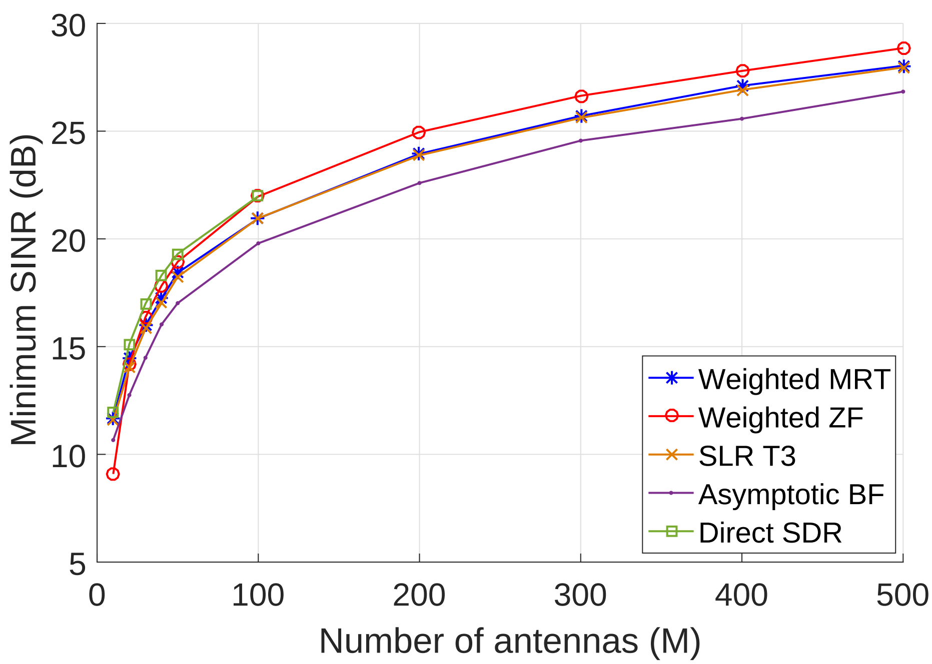

Figure 3.18: Minimum SINR vs. M for partial interference ($N = 3$, $K = 3$, $\mu = 0.4$).

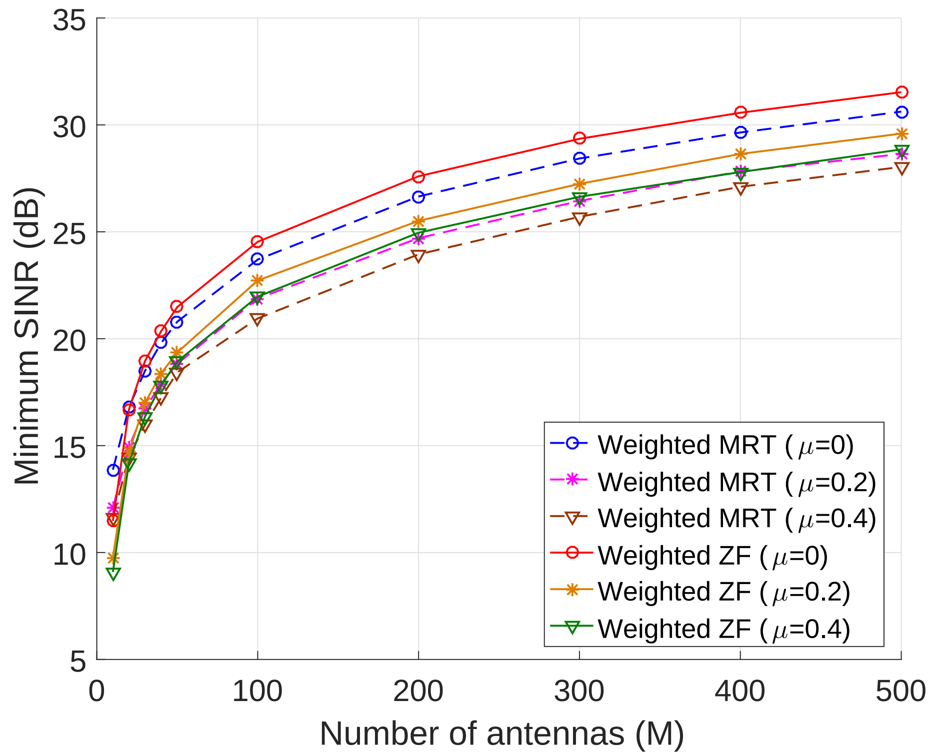

Figure 3.19: Minimum SINR vs. M with different μ for partial interference ($N = 3$, $K = 3$).

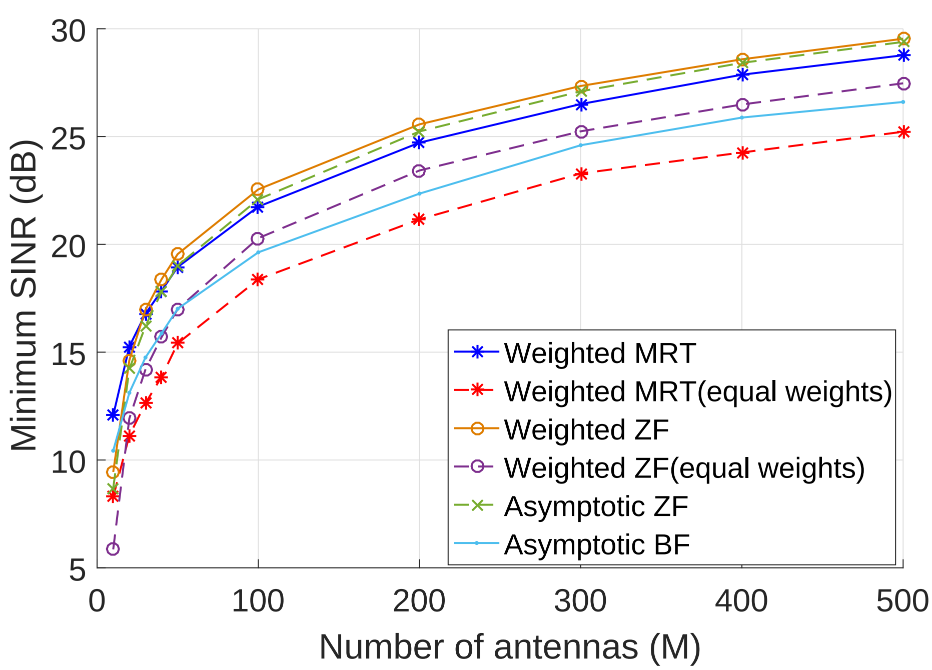

Figure 3.20: Minimum SINR vs. M for partial interference ($N = 3$, $K = 3$, $\mu = 0.2$).

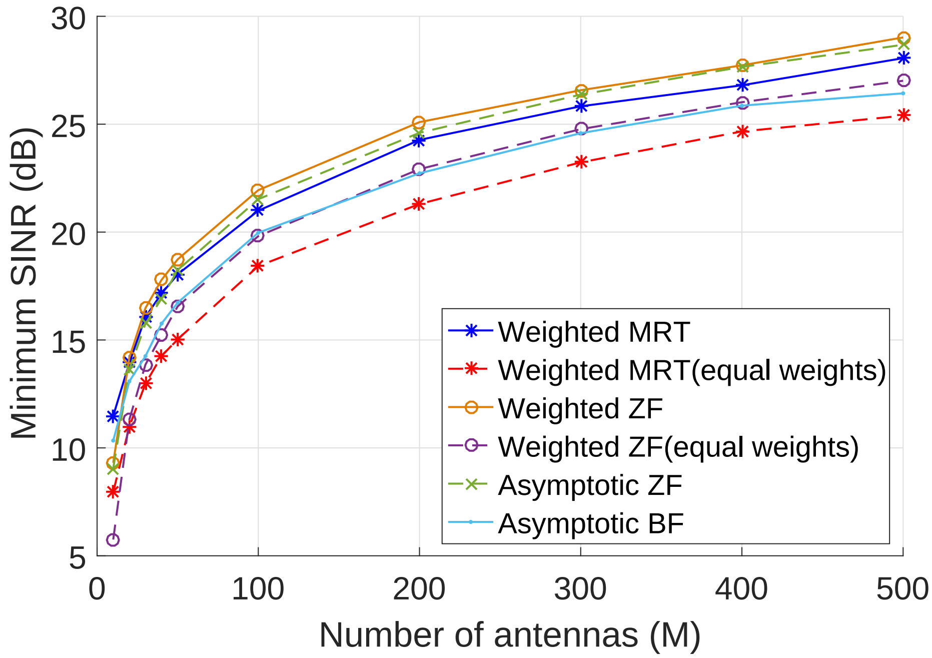

Figure 3.21: Minimum SINR vs. M for partial interference ($N = 3$, $K = 3$, $\mu = 0.4$).

Fully Imperfect CSI

For the fully imperfect CSI scenario, all channel information obtained by a BS is disturbed and represented by the estimation channel model $\hat{\mathbf{h}}_{nik}$. In this case, channel estimation error exists for the channels between BS and all users. However, due to the estimation model the estimation is more accurate for its own users than the users in other cell.

Fig. 3.22 and Fig. 3.23 show the minimum SINR vs. the number of antennas M for $\mu = 0.2, 0.4$, respectively. We can see that the performance gaps between different methods shrink as μ increases, but the relative performance of different methods keeps unchanged. Similar to the partial interference scenario, the SLR T3 method appears to be more robust against the channel estimation error than weighted MRT. The weighted MRT method and the SLR T3 method have similar performance for $\mu = 0.4$

in the fully imperfect CSI case.

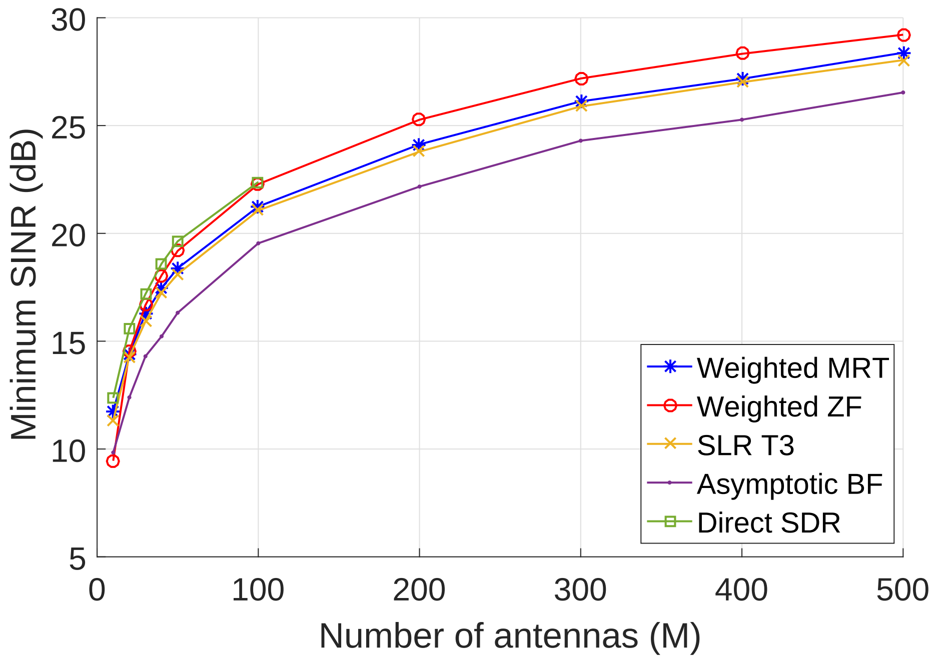

Figure 3.22: Minimum SINR vs. M for fully imperfect CSI ($N = 3$, $K = 3$, $\mu = 0.2$).

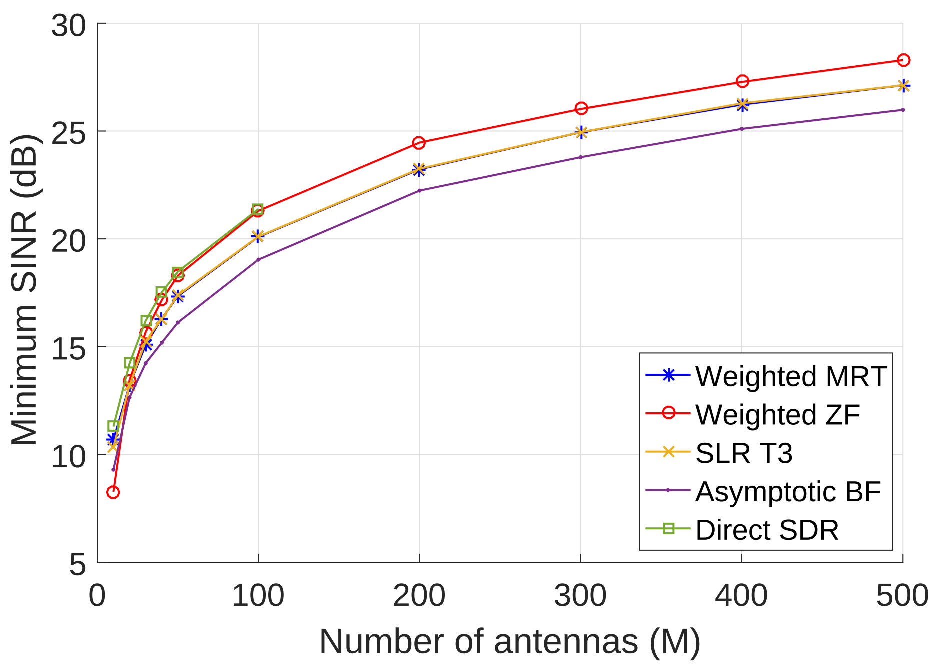

Figure 3.23: Minimum SINR vs. M for fully imperfect CSI ($N = 3$, $K = 3$, $\mu = 0.4$).

3.6 Summary

In this chapter, we considered the non-cooperative multicast beamforming designs in a massive MIMO multi-cell networks. Aiming to maximize the minimum SINR among users, we proposed the weighted MRT beamforming approach which can be optimized through a weight optimization problem via the SDR approach, whose problem size is independent of the number of BS antennas. To further reduce the computational complexity and BS communication , we have considered the SLR metric and have proposed a coordinated multicast beamforming design to maximize the minimum SLR among users. This allows coordinated beamforming problem to be solved distributively and independently at each BS for its own beamforming vectors. The weighed ZF structure is proposed as another distributed beamforming design to maximize the minimum SINR among users. The weighted ZF method can eliminate the inter-cell interference, which is effective in a interference heavy network. The asymptotically optimal solution for weighted ZF is given in closed-form. Simulation shows the performance of our proposed methods are comparable to the centralized direct SDR method but with much lower complexity. Our proposed solutions also significantly outperform the conventional non-coordinated methods.

Chapter 4

Low Complexity Cooperative Multicast Beamforming for Massive-MIMO Multi-Cell Networks

In cooperative massive MIMO multicast networks, BSs are clustered together to jointly transmit the signal to a group of users. The cooperations among BSs enable them to achieve higher data rates and diversity than individual transmission [103–105]. In this section, we propose the weighted MRT approach for cooperative multicast beamforming in massive MIMO multi-cell networks.

4.1 System Model

In the cooperative multicasting scenario, multiple BSs form a cluster to cooperatively multicast data to users, as show in Fig. 4.1. We consider the general case where users are divided into groups, with J users per group. Each user group is served by a cluster of BSs. This setup includes the case where users requesting the same content in different cells are considered in a same group and served by a specific cluster of BSs.

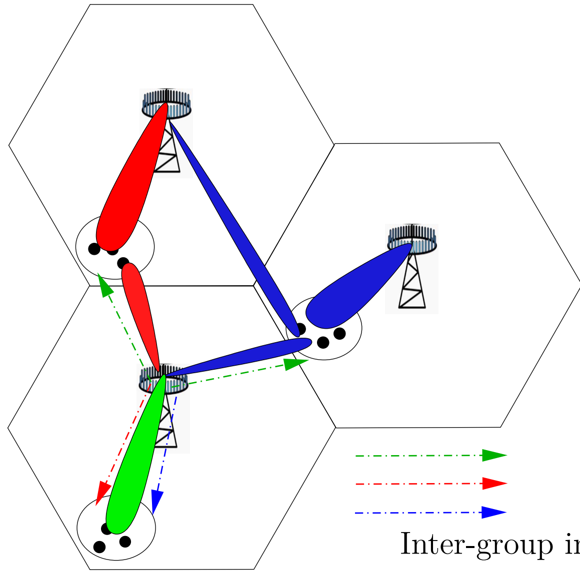

Figure 4.1: A multi-cellular downlink cooperative multicast beamforming scenario.

Assume there are N BSs, forming C clusters, where $C \leq N$. Each BS cluster serves one user group, and each user group is only served by one particular BS cluster. Due to this one-one correspondence, we use the same index for the user group and its serving BS cluster. Denote the sets of BS cluster indexes and the user indexes per group as $\mathcal{C} = \{1, \cdots, C\}$ and $\mathcal{J} = \{1, \cdots, J\}$. Let $\mathcal{Q}_c$ denote the set of BS indices for BS cluster c, where $\mathcal{Q}_c \subseteq \mathcal{N}$, for $c \in \mathcal{C}$ ($\mathcal{N} = \{1 \ldots N\}$ as defined in Chapter 3). Note that a BS could be in multiple BS clusters to serve multiple user groups simultaneously. Thus, the sets $\mathcal{Q}_1 \ldots \mathcal{Q}_c$ may overlap with each other. Let $\mathcal{B}_n$ denote the set of cluster indexes that BS n belongs to, i.e., $\mathcal{B}_n = \{c : n \in \mathcal{Q}_c,\ \forall c \in C\}$. Let $\tilde{\mathbf{w}}_{nc}$ denote the beamforming vector at BS n for BS cluster c. Let $\tilde{\mathbf{h}}_{ncj}$ denote the channel vector from BS n to the user j in group c. The received signal at user j in group c from BS cluster c, for $c \in \mathcal{C}, j \in \mathcal{J}$, is given by

$$y_{cj} = \sum_{n \in \mathcal{Q}_c} \tilde{\mathbf{w}}_{nc}^H \tilde{\mathbf{h}}_{ncj} s_c + \sum_{i \neq c}^{C} \sum_{n \in \mathcal{Q}_i} \tilde{\mathbf{w}}_{ni}^H \tilde{\mathbf{h}}_{ncj} s_i + n_{cj} \tag{4.1}$$

where s_c is the common multicast symbol from BS cluster c, and n_{cj} is the received Gaussian additive noise at user j in group c. The transmission power constraint at BS n is given by

$$\sum_{c\in\mathcal{B}_n} \|\tilde{\mathbf{w}}_{nc}\|^2 \le P_n. \tag{4.2}$$

Note that there can be multiple beamforming vectors at BS n for different BS clusters. Therefore, the transmission power at BS n is the sum of the transmit power by different beamforming vectors at BS n.

Based on (4.1), the SINR at user j in group c under the cooperative multicasting is given by

$$\mathrm{SINR}_{cj} = \frac{|\sum_{n\in\mathcal{Q}_c} \tilde{\mathbf{w}}_{nc}^H \tilde{\mathbf{h}}_{ncj}|^2}{\sum_{i\neq c}^{C} |\sum_{n\in\mathcal{Q}_i} \tilde{\mathbf{w}}_{ni}^H \tilde{\mathbf{h}}_{ncj}|^2 + \sigma^2}, \quad j\in\mathcal{J},\ c\in\mathcal{C}. \tag{4.3}$$

Our goal is to maximize the minimum SINR among all users in the network. The optimization problem is formulated by

$$\begin{aligned} \mathcal{P}_{\mathrm{CP}}:\quad \max_{\{\tilde{\mathbf{w}}_{nc}\}}\ \min_{j\in\mathcal{J},c\in\mathcal{C}}\ & \frac{|\sum_{n\in\mathcal{Q}_c} \tilde{\mathbf{w}}_{nc}^H \tilde{\mathbf{h}}_{ncj}|^2}{\sum_{i\neq c}^{C} |\sum_{n\in\mathcal{Q}_i} \tilde{\mathbf{w}}_{ni}^H \tilde{\mathbf{h}}_{ncj}|^2 + \sigma^2} \\ \text{s.t.}\ & \sum_{c\in\mathcal{B}_n} \|\tilde{\mathbf{w}}_{nc}\|^2 \le P_{\mathrm{tot}}, \quad n\in\mathcal{N}. \end{aligned} \tag{4.4}$$

4.2 Cooperative Weighted MRT Multicast Design

The optimization problems $\mathcal{P}_{\mathrm{CP}}$ is a non-convex and NP-hard problem, and the optimal solutions typically cannot be obtained. To find a good sub-optimal solution again, a typical approach is to apply the SDR approach to find a sub-optimal solution $\{\tilde{\mathbf{w}}_{nc}\}$. However, similar to the non-cooperative case in Chapter 3, the complexity of the SDR approach grows with the number of antennas M. For massive MIMO systems, as $M \gg 1$, the SDR approach incurs very a high computational complexity, thus directly obtaining $\{\tilde{\mathbf{w}}\}_{nc}$ through SDR is not practical for massive MIMO systems. In the following, we propose a low complexity cooperative multicast beamforming design for massive MIMO systems via the special mutlicast beamforming structure similar to the one proposed in Chapter 3, to find a good sub-optimal solution, whose complexity does not grow with the number of antennas.

Similar to the non-cooperative scenario, we construct the beamforming vector $\tilde{\mathbf{w}}_{nc}$ as a weighted sum of the channel vectors between BS n and its serving users in group c, given by

$$\tilde{\mathbf{w}}_{nc} \triangleq \sum_{j=1}^{J} \tilde{a}_{ncj}\tilde{\mathbf{h}}_{ncj}, \;\; n \in \mathcal{Q}_c, c \in \mathcal{C} \tag{4.5}$$

where $\tilde{\alpha}_{ncj}$ is the complex weight for the channel vector between BS n and user j in group c.

Define $\tilde{\mathbf{H}}_{nc} \triangleq [\tilde{\mathbf{h}}_{nc1}, \cdots, \tilde{\mathbf{h}}_{ncJ}]$ as the channel matrix between BS n and user group c. Define $\tilde{\mathbf{a}}_{nc} \triangleq [\tilde{a}_{nc1}, \cdots, \tilde{a}_{ncJ}]^T$ as the weight vector for the beamforming vector of BS n to group c. The SINR of user j in group c in (4.3) can now be rewritten as

$$\mathrm{SINR}_{cj} = \frac{|\sum_{n\in\mathcal{Q}_c} \tilde{\mathbf{h}}_{ncj}^H \tilde{\mathbf{H}}_{nc}\tilde{\mathbf{a}}_{nc}|^2}{\sum_{i\neq c}^{C} |\sum_{n\in\mathcal{Q}_i} \tilde{\mathbf{h}}_{nij}^H \tilde{\mathbf{H}}_{ni}\tilde{\mathbf{a}}_{ni}|^2 + \sigma^2}. \tag{4.6}$$

To facilitate the notations, let $Q_c = |\mathcal{Q}_c|$, i.e. the size of BS cluster c, and we denote the BS indices in the BS cluster set as $\mathcal{Q}_c = \{n_1, \cdots, n_{Q_c}\}$, where n_k is the BS index for kth BS in BS cluster c, for $k = 1, \cdots, Q_c$. We further define $\tilde{\mathbf{a}}_c \triangleq \mathrm{vec}([\tilde{\mathbf{a}}_{n_1c}, \cdots, \tilde{\mathbf{a}}_{n_{Q_c}c}])$ as the weight vector associated with the beamforming vectors for BS cluster c. Define $\mathbf{g}_{icj} \triangleq \mathrm{vec}([\tilde{\mathbf{H}}_{n_1i}^H \tilde{\mathbf{h}}_{n_1cj}, \cdots, \tilde{\mathbf{H}}_{n_{Q_c}i}^H \tilde{\mathbf{h}}_{n_{Q_c}cj}])$. Note that vector $\mathbf{g}_{ccj}$ contains the correlation of the channel vector from each BS cluster c to its user j and the channel vectors from that BS to all users in group c. Using $\tilde{\mathbf{a}}_c$ and

$\mathbf{g}_{icj}$, the SINR_{cj} expression in (4.6) can be further rewritten as

$$\text{SINR}_{cj} = \frac{\tilde{\mathbf{a}}_c^H \tilde{\mathbf{A}}_{ccj} \tilde{\mathbf{a}}_c}{\sum_{i \neq c}^{C} \tilde{\mathbf{a}}_i^H \tilde{\mathbf{A}}_{icj} \tilde{\mathbf{a}}_i + \sigma^2} \tag{4.7}$$

where $\tilde{\mathbf{A}}_{icj} \triangleq \mathbf{g}_{icj} \mathbf{g}_{icj}^H$. Similarly, the transmission power at BS n can be rewritten as

$$\sum_{c \in \mathcal{B}_n} \|\tilde{\mathbf{w}}_{nc}\|^2 = \sum_{c \in \mathcal{B}_n} \tilde{\mathbf{a}}_{nc}^H \tilde{\mathbf{H}}_{nc}^H \tilde{\mathbf{H}}_{nc} \tilde{\mathbf{a}}_{nc} = \sum_{c \in \mathcal{B}_n} \tilde{\mathbf{a}}_c^H \mathbf{F}_{nc} \tilde{\mathbf{a}}_c \tag{4.8}$$

where $\mathbf{F}_{nc} \triangleq \text{bldg}(\mathbf{0}, \cdots, \mathbf{0}, \tilde{\mathbf{H}}_{nc}^H \tilde{\mathbf{H}}_{nc}, \mathbf{0}, \cdots \mathbf{0})$ is a block diagonal matrix consisting of Q_c diagonal blocks of size $J \times J$ each; Matrix $\tilde{\mathbf{H}}_{nc}^H \tilde{\mathbf{H}}_{nc}$ is located at the kth diagonal block, where k is determined by the inverse mapping from BS index n to the kth element in $\mathcal{Q}_c = \{n_1, \cdots, n_{Q_c}\}$, where $n_k = n$. The rest diagonal blocks are $J \times J$ zero matrices.

Using (4.7) and (4.8), the optimization problem $\mathcal{P}_{\text{CP}}$ for the cooperative multicasting scenario is now transformed to

$$\begin{aligned}
\mathcal{P}_{\text{CP2}} : \min_{\{\tilde{\mathbf{a}}_c\}} \quad & t \\
\text{s.t.} \quad & \frac{\tilde{\mathbf{a}}_c^H \tilde{\mathbf{A}}_{ccj} \tilde{\mathbf{a}}_c}{\sum_{i \neq c}^{C} (\tilde{\mathbf{a}}_i^H \tilde{\mathbf{A}}_{icj} \tilde{\mathbf{a}}_i) + \sigma^2} \geq \frac{1}{t}, \; j \in \mathcal{J}, c \in \mathcal{C} \\
& \sum_{c \in \mathcal{B}_n} \tilde{\mathbf{a}}_c^H \mathbf{F}_{nc} \tilde{\mathbf{a}}_c \leq P_n, \; n \in \mathcal{N}, \\
& t > 0.
\end{aligned}$$

Compared with the original problem $\mathcal{P}_{\text{CP}}$, the transformed problem $\mathcal{P}_{\text{CP2}}$ is of size $J \sum_{c=1}^{C} Q_c$ based on the optimization variables $\{\mathbf{b}_c\}$, which only depends on N and J and is independent of the number of BS antennas M. This makes the cooperative weighted MRT approach particularly suitable for massive MIMO systems.

Now $\mathcal{P}_{\text{CP2}}$ has a very similar structure as $\mathcal{P}_{\text{NCP3}}$ in the non-cooperative case in Chapter 3. Likewise, we define $\mathbf{Y}_c \triangleq \tilde{\mathbf{a}}_c \tilde{\mathbf{a}}_c^H$, $c \in \mathcal{C}$, and use the SDR approach to find a solution for $\mathcal{P}_{\text{CP2}}$ as

$$\begin{aligned}\mathcal{P}_{\text{CP3}}: \min_{\{\mathbf{Y}_c\},t} \; & t \\ \text{s.t. } & \text{tr}\Big[t\tilde{\mathbf{A}}_{ccj}\mathbf{Y}_c - \sum_{i\neq c}^{C}\tilde{\mathbf{A}}_{icj}\mathbf{Y}_i\Big] \geq \sigma^2, \; j \in \mathcal{J}, c \in \mathcal{C} \\ & \text{tr}\Big[\sum_{c\in\mathcal{B}_n}\mathbf{F}_{nc}\mathbf{Y}_c\Big] \leq \; P_n, \; n \in \mathcal{N}, \\ & \mathbf{Y}_n \succeq 0, \; n \in \mathcal{N} \\ & t > 0.\end{aligned}$$

Note that $\mathcal{P}_{\text{CP3}}$ is not jointly convex w.r.t $\mathbf{Y}_c$ and t. However, when t is fixed, $\mathcal{P}_{\text{CP3}}$ is convex w.r.t $\mathbf{Y}_c$. A good sub-optimal solution for $\mathbf{Y}_c$ can be obtained by applying bi-section search over t along with a feasibility test problem, given by

$$\begin{aligned}\text{Find } & \{\mathbf{Y}_c\} \\ \text{s.t. } & \text{tr}[t\tilde{\mathbf{A}}_{ccj}\mathbf{Y}_c - \sum_{i\neq c}^{C}\tilde{\mathbf{A}}_{icj}\mathbf{Y}_i] \geq \sigma^2, \; j \in \mathcal{J}, c \in \mathcal{C} \\ & \text{tr}[\sum_{c\in\mathcal{B}_n}\mathbf{F}_{nc}\mathbf{Y}_c] \leq \; P_n, \; n \in \mathcal{N}, \\ & \mathbf{Y}_c \succeq 0, \; n \in \mathcal{N} \\ & t > 0.\end{aligned}$$

The above problem is a SDP problem and can be efficiently solved by standard SDP solvers. Along with interior point methods [102], a good sub-optimal solution $\{\mathbf{Y}_n^*\}$ can be obtained by SDP solvers. The weight vector $\{\tilde{\mathbf{a}}_c^*\}$ can be extracted from $\{\mathbf{Y}_c^*\}$. If $\mathbf{Y}_c^*$ is rank one, the weight vector $\tilde{\mathbf{a}}_c^*$ can be directly recovered from $\mathbf{Y}_c^* = \tilde{\mathbf{a}}_c^*\tilde{\mathbf{a}}_c^{*H}$. Otherwise, a sub-optimal solution can be obtained by the Gaussian

Algorithm 3 Recovering $\{\tilde{\mathbf{a}}_c^*\}$ from $\{\mathbf{Y}_c^*\}$ with Gaussian Randomization Procedure

Set L.
for $c = 1 \dots C$ **do**
  if $\text{rank}(\mathbf{Y}_c^*) == 1.$ **then**
    Directly obtain $\tilde{\mathbf{a}}_c^*$ by $\mathbf{Y}_n^* = \tilde{\mathbf{a}}_c^* \tilde{\mathbf{a}}_c^{*H}$.
    Let $\tilde{\mathbf{a}}_c^{(l)} = \tilde{\mathbf{a}}_c^*,\ l = 1 \dots L$.
  else
    Calculate the power constraint for beamformer in cluster c at BS n

$$P_{nc} = \text{tr}\Big[\mathbf{F}_{nc}\mathbf{Y}_c\Big] \quad n \in Q_c.$$

    Generate i.i.d random weight vector $\hat{\mathbf{a}}_c^{(l)} \sim \mathcal{CN}(0, \mathbf{Y}_c^*),\ l = 1 \dots L$.
    Scale $\hat{\mathbf{a}}_c^{(l)}$ to satisfy the power constraint given by follow

$$\hat{\mathbf{a}}_c^{(l)H}\mathbf{F}_{nc}\hat{\mathbf{a}}_c^{(l)} = P_{nc}, \quad n \in Q_c,\ l = 1 \dots L.$$

    Let $\tilde{\mathbf{a}}_c^{(l)} = \hat{\mathbf{a}}_c^{(l)}$.
  end if
end for
Calculate the minimum SINR with $\{\tilde{\mathbf{a}}_1^{(l)}, \dots, \tilde{\mathbf{a}}_C^{(l)}\}$ by

$$\text{SINR}_{\min}^{(l)} = \min_{j \in \mathcal{J}, c \in \mathcal{C}} \frac{\tilde{\mathbf{a}}_c^H \tilde{\mathbf{A}}_{ccj} \tilde{\mathbf{a}}_c}{\sum_{i \neq c}^{C} (\tilde{\mathbf{a}}_i^H \tilde{\mathbf{A}}_{ick} \tilde{\mathbf{a}}_i) + \sigma^2}, \text{ for } l = 1, \dots, L.$$

Let $l^* = \arg\ \max_{l=1 \dots L}\ \text{SINR}_{\min}^{(l)}$; Set $\tilde{\mathbf{a}}_c^* = \tilde{\mathbf{a}}_c^{(l^*)},\ c = 1, \dots, C$.

randomization procedure [42]. Details of the Gaussian randomization procedure for recovering $\{\mathbf{b}_c^*\}$ is provided in Algorithm 3.

4.2.1 Complexity Discussion

The problem size of $\mathcal{P}_{\text{CP2}}$ only depends on J and N, but independent of M. Therefore, the complexity of the proposed weighted MRT method does not grows with M. To be specific, the complexity to solve each SDP in $\mathcal{P}_{\text{CP3}}$ is $\mathcal{O}(J^5(\sum_{c=1}^{C}|Q_c|)^2 C)$, while it is $\mathcal{O}(M^4 J(\sum_{c=1}^{C}|Q_c|)^2 C)$ in directly solving $\mathcal{P}_{\text{CP}}$ with the SDR approach. It is considerably smaller than that of $\mathcal{P}_{\text{CP}}$ for a network with large scale of antennas, where $M \gg 1$ and $M > J$. Compared with directly solving $\mathcal{P}_{\text{CP}}$ with the SDR

M	Non-cooperative		Cooperative	
	Weighted MRT (s)	Direct SDR (s)	Weighted MRT (s)	Direct SDR (s)
10	6.02	9.68	5.12	42.1
20	5.92	24.08	5.06	338.7
40	5.84	153.2	7.85	3510
50	5.87	311.5	5.23	5094
100	5.87	2487	5.26	N/A
200	5.87	18339	5.60	N/A
500	5.93	N/A	5.72	N/A

Table 4.1: Comparison of Average Computation Time ($N = 3$).

approach, whose complexity grows with M, the computation saving of the weighted MRT method is significant, especially for the cooperative scenario where multiple beamforming vectors need to be jointly optimized.

4.3 Simulation Results

For simulation, we set the transmission power at BS n to $P_n/\sigma^2 = 10$ dB, $\forall n$. The channel vectors between each BS and each user $\mathbf{h}_{ncj}$ are i.i.d. generated as complex Gaussian with zero mean and covariance $\beta_{ncj}\mathbf{I}$, where variance β_{ncj} is determined by the path loss model $\beta_{ncj} = K_o d_{ncj}^{-\kappa}$, with d_{ncj} being the distances between BS n and user j in group c, and κ being the path loss exponent set to $\kappa = 3.5$. Constant K_o is set such that with a single antenna under unit transmission power at the BS, the received SNR at the edge of each cell is -5 dB. Performance is averaged over random channel realizations and user drops.

We consider $N = 3$ and set $C = 3$ clusters, where each cluster includes all 3

BSs, and each cluster is serving a group of users with $J = 3$ users per group. For comparison, we consider the direct SDR method to obtain $\{\tilde{\mathbf{w}}_{nc}\}$ in $\mathcal{P}_{\text{CP}}$, which is directly solving $\mathcal{P}_{\text{CP}}$ with the SDR approach. The minimum SINR among users vs. the number of antennas M by different beamforming methods is shown in Fig. 4.2. As can be seen, the performance gap between direct SDR and weighted MRT is about 1.5 dB at $M = 50$. However, as will be shown later the weighted MRT method has the considerably much lower complexity. Performance of the direct SDR method with $M > 50$ is computational prohibitive due to its high complexity. Similar to the non-cooperative scenario, we can derive the cooperative asymptotically optimal beamforming solution as $M \to \infty$. Its performance is shown in Fig. 4.2, which is significantly worse than the weighted MRT, as well as the asymptotical BF solution in the non-cooperative scenario in Chapter 3. This is due to the increased interference when a BS participates multiple clusters, which decreases very slowly with M and cannot be captured in the asymptotic solution for finite but large M.

Comparing the performance of the weighted MRT method in the non-cooperative scenario shown in Fig. 3.3 and the cooperative scenarios shown in Fig. 4.2, we observe that there is about 1dB gain due to cooperation among 3 BSs.

The average computation time for the weighted MRT method and the direct SDR method are shown in Table. 4.1. As we can see, the computation time for the weighted MRT method in the cooperative scenario remains relatively small and nearly unchanged over M. However, the computation time for the direct SDR method increases dramatically over M. Comparing the noncooperative and cooperative scenarios, we can see that the computation time for weighted MRT remains the same level for both cases. However, the computation time for direct SDR in the cooperative scenario is much higher than that in the non-cooperative scenario due to the larger problem size introduced by the BS cooperation.

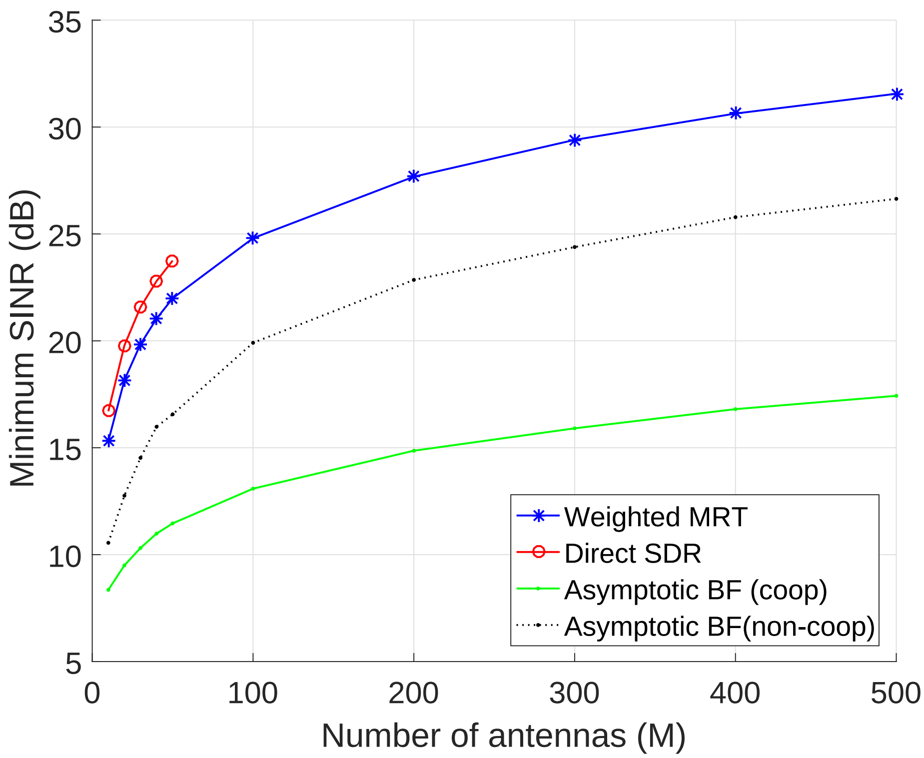

Figure 4.2: Minimum SINR vs. M for the cooperative scenario ($N = 3$, $J = 3$).

The CDF of the minimum SINR among different channel realizations for different methods is shown in Fig. 4.3. As we can see, the weighted MRT method and the direct SDR method get a steep CDF curve. The asymptotic BF method has a wider CDF distribution of the minimum SINR, thus it might result in a higher probability of poor SINR at users. The more concentrated CDF distribution of the minimum SINR with the weighted MRT method shows that our proposed method provides the higher and also more consistent performance and thereby improves the overall network performance.

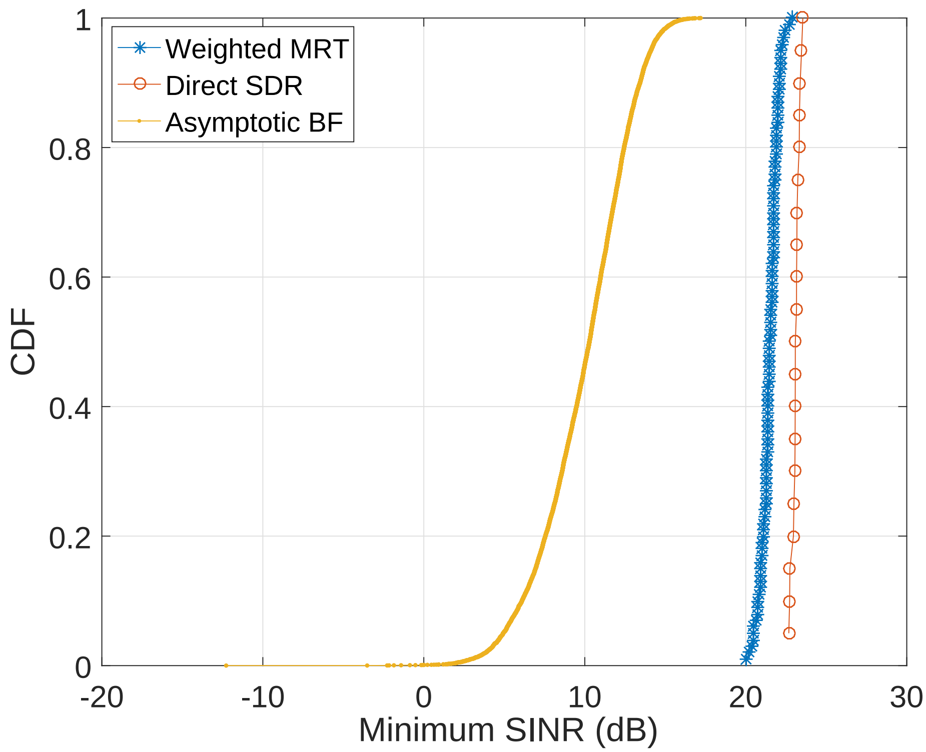

Figure 4.3: Comparison of CDF of minimum SINR for the cooperative scenario ($N = 3,\ J = 3$).

Fig.4.4 shows the minimum SINR as the power to noise ratio P_n/σ^2 grows. As we can see, the addition loss of our proposed method to the direct SDR method is mild as P_n/σ^2 increases, which indicates that the weighted MRT method can keep good performance though interference is strong.

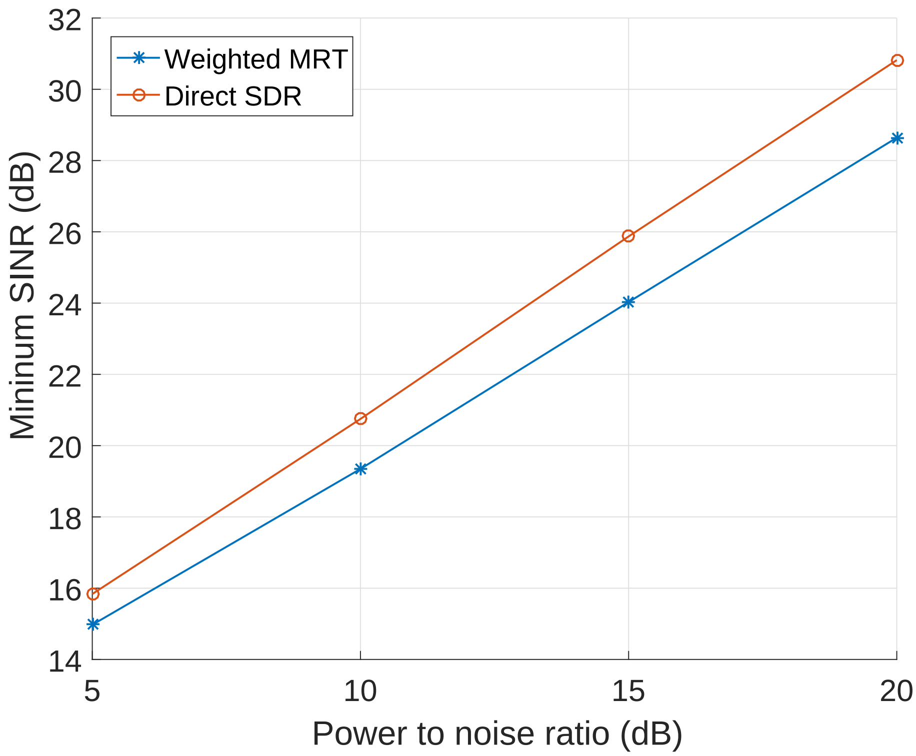

Figure 4.4: Minimum SINR vs. power to noise ratio ($N = 3$, $J = 3$).

The performance of weighted MRT vs. number of users per cluster J is shown in Fig.4.5. As can be seen, the minimum SINR of the weighted MRT method decreases as the J increases, but the performance gap for weighted MRT with different M remain constant over J. Fig.4.6 shows the computation time of weighted MRT as J increases. The computation time grows as J increases because of the larger problem size in $\mathcal{P}_{\text{CP2}}$, but it is still considerably low when compared to the direct SDR method.

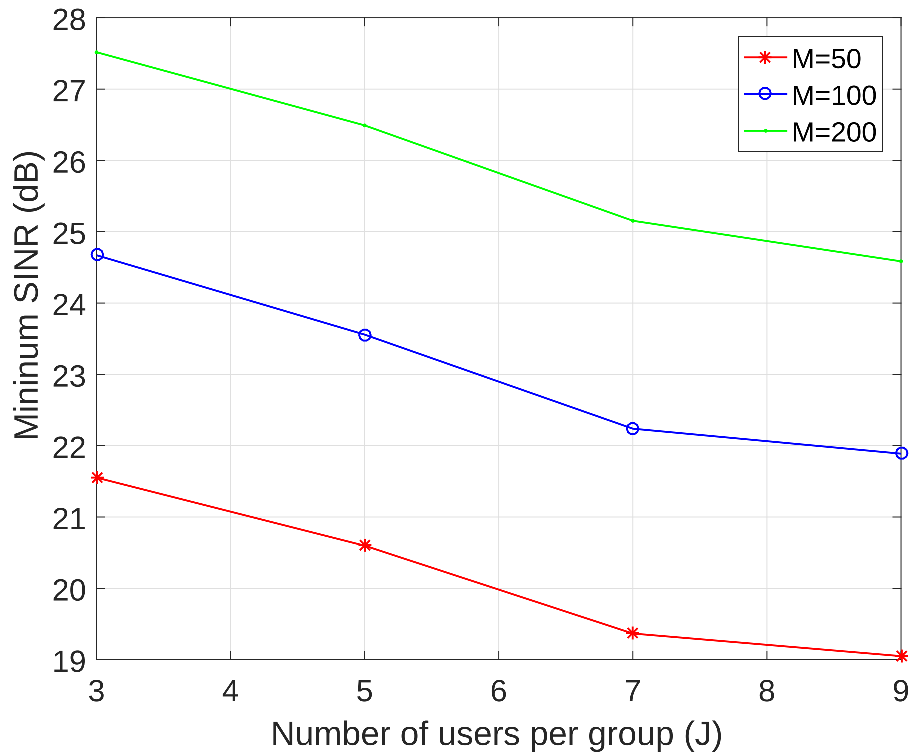

Figure 4.5: Minimum SINR vs. number of per cell J in 3 BSs cooperative scenario.

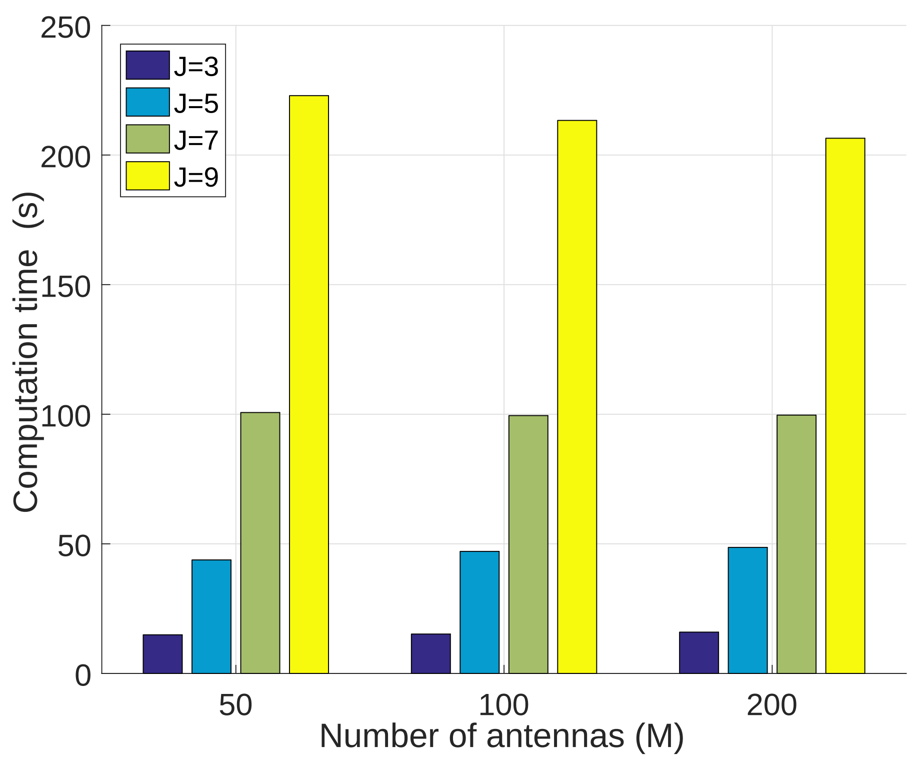

Figure 4.6: Comparison of average computation time for weighted MRT with different J in 3 BSs cooperative scenario.

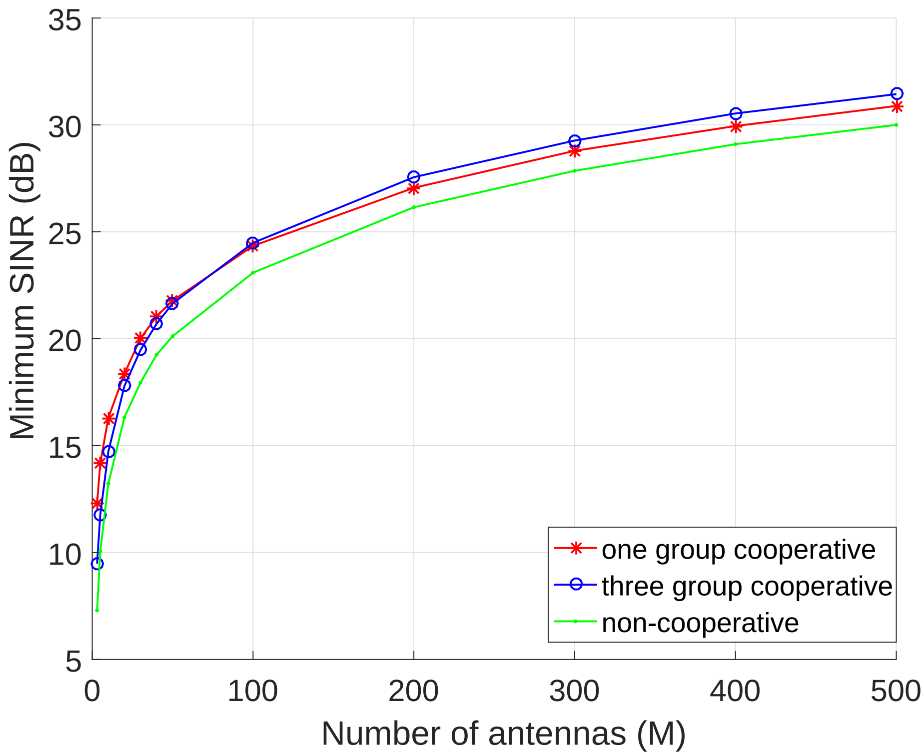

Figure 4.7: minimum SINR of different grouping strategy ($N = 3$, $J = 3$).

Fig.4.7 shows the performance of weighted MRT with different user grouping strategies. Besides the *three groups cooperative* cluster/grouping setup we described at the beginning of Section 4.3, we also consider two other grouping strategies for comparison. The *one group cooperative* is a strategy where all users in the three cells form a group in the network. In this case, all users are requiring the same content, and the three BSs form a cluster to jointly multicast to all users simultaneously. Note that there is no interference in this grouping strategy since the whole network is considered as a cluster. The non-cooperative weighted MRT method is also considered, where the users are served by the nearest BS. As we can see, the one group strategy has a small performance advantage against the three groups strategy for M less than 100.

However, for $M > 100$, the three group strategy outperforms the one group strategy since the inter-group interference reduces as M grows.

4.4 Summary

In this chapter, we proposed the weighted MRT beamforming design for cooperative multicast in massive MIMO multi-cell networks. The weighted MRT structure transforms the beamforming problem into a low complexity optimization problem of weights, whose problem size is independent with the number of antennas M. Therefore, compared to the conventional SDR method, the weighted MRT method largely reduces the computation complexity of multicast beamforming in massive MIMO systems. Simulation results show that the performance of the weighted MRT method is comparable to direct SDR method. However, weighted MRT has the significantly lower computational complexity, making it attracting for multi-cell networks with large scale of antennas at the BS.

Chapter 5

Conclusion

In this thesis, we considered the non-cooperative and cooperative multicast beamforming in massive MIMO multi-cell networks. Aiming to maximize the minimum SINR among users under transmission power constraints, we proposed low complexity beamforming approaches for finding good sub-optimal beamforming solutions.

We first considered the non-cooperative multi-cell scenario. Two beamforming structures, namely weighted MRT and weighted ZF, were proposed. Applying the weighted MRT structure, we transformed the beamforming problem into a optimization problem of weights, which can be solved by the SDR approach or the SCA approach. To further reduce the computation complexity and BS communication, we have considered the SLR metric and have proposed a distributed multicast beamforming method with weighted MRT to maximize the minimum SLR among users. This allows the coordinated beamforming to be solved distributively and independently at each BS. Our proposed weighted MRT methods have very low computational complexity that does not depend on the number of antennas. Furthermore, based on the weighted ZF structure, we have proposed a low complexity distributed beamforming method to maximize the minimum SINR among users while eliminating inter-cell interference. Additionally, the asymptotically optimal solution for the weighted ZF method was derived in closed-form. The ZF methods have low complexity and require no BS communication.

We then extended our work to the cooperative massive MIMO beamforming scenario. We proposed the weighted MRT structure and derived a low complexity beamforming method for the cooperative scenario to maximize the minimum SINR among users. Similarly, the cooperative beamforming problem was transformed into a optimization problem of weights, then it was solved by the SDR approach. Simulation shows that our proposed methods result in good performance comparable to the traditional method directly using the SDR approach , but the complexity of our methods is significantly lower. Additionally, our proposed methods outperform the equal weighted MRT method, the equal weighted ZF method and the non-coordinated methods.

5.1 Feature Work

There are more problems to be investigated for the multicast beamforming in massive MIMO systems. The following future work can be considered. Firstly, the weighted ZF beamforming approach will be extend to the cooperative scenario. It is interesting to derive the asymptotically optimal solution for the cooperative weighted ZF approach as the number of antennas goes to infinity. Secondly, for solving the weight optimization problems in the proposed methods, the algorithms with lower complexity than SDR approach can be applied, e.g., the ADMM algorithm. Furthermore, the proposed methods can also be extended to the multi-group-multi-cell scenarios.

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